Critical exponents in quantum Einstein gravity[a]

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[a] S.N., B. Fazekas, L. Juhasz, K. Sailer, Phys. Rev. D**88** 116010 (2013)

Outline

- Quantum Einstein gravity
- Functional renormalization group
    - Wetterich equation
    - regulators
- Asymptotic safety in QEG
    - UV criticality
    - crossover criticality
    - IR criticality

Quantum Einstein gravity

- In gravitational interaction the spacetime metrics becomes a dynamical object. The Einstein equation is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}$$

  - G Newton constant: $G = 6.67 \times 10^{-11} \, \frac{\mathrm{m}^3}{\mathrm{kgs}^2}$
  - Λ cosmological constant: $\Lambda \approx 10^{-35} \mathrm{s}^{-2}$
- quantum Einstein gravity (QEG) = gravitational interaction + quantum field theory
- We use the elements of quantum field theory
  - the metrics $g_{\mu\nu}$ plays the role of the field variable in the path integral
  - the action is a diffeomorphism invariant action
  - couplings, the cosmological constant λ, Newton constant g

Quantum Einstein gravity

- The model cannot be extended beyond the ultraviolet (UV) limit, since around the Gaussian fixed point (GFP)
$$\lambda = \Lambda k^{-2} \text{ relevant,}$$
$$g = G k^{d-2} \text{ irrelevant.}$$
- The loop correction do not modify the tree level scaling behaviors at the GFP.
- The Newton coupling g is irrelevant it diverges as $g \sim k^2$ the theory seems perturbatively non-renormalizable.
- The classical description of gravity is an effective theory which looses its validity around the Planck scale, therefore it is not complete.

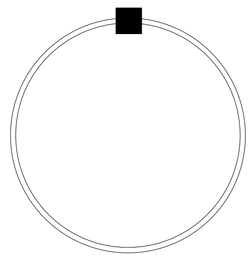
Asymptotic safety + functional RG

- If there is a UV attractive non-gaussian fixed point (NGFP) in QEG, then the model can be extended to arbitrarily high energies, without divergences.
- The scaling properties around the UV NGFP could overwrite the tree level scalings that were obtained around the GFP.
- We use functional RG instead of perturbative RG since it can provide nonperturbative flow equations and universal (less sensitive) results.

Renormalization

- The functional renormalization group method is a fundamental element of quantum field theory.
- We know the high energy (UV) action, which describes the small distance interaction between the elementary excitations. We look for the low energy IR (or large distance) behavior.
- The RG method gives a functional integro-differential equation for the effective action, which is called the *Wetterich equation*

$$\dot{\Gamma}_k = \frac{1}{2}\mathrm{Tr}\frac{\dot{\mathcal{R}}_k}{\mathcal{R}_k + \Gamma_k''} = \frac{1}{2}$$



,

where $' = \partial/\partial\varphi$, $^{\cdot} = \partial/\partial t$, and the symbol Tr denotes the momentum integral and the summation over the internal indices.

Regulators

The IR regulator has the form $\mathcal{R}_k[\phi] = \frac{1}{2}\phi \cdot \mathcal{R}_k \cdot \phi$. It is a momentum dependent mass like term, which serves as an IR cutoff and has the following properties

- $\lim_{p^2/k^2 \to 0} \mathcal{R}_k > 0$: it serves as an IR regulator
- $\lim_{k^2/p^2 \to 0} \mathcal{R}_k \to 0$: in the limit $k \to 0$ we obtain back the form of Z
- $\lim_{k^2 \to \infty} \mathcal{R}_k \to \infty$: for the microscopic action $S = \lim_{k\to\Lambda} \Gamma_k$ (it serves as a UV regulator, too)

The compactly supported smooth (css) regulator has the form (I. Nandori, JHEP **04** (2013) 150)

$$r_{css} = \frac{R}{p^2} = \frac{s_1}{\exp[s_1 y^b/(1 - s_2 y^b)] - 1}\theta(1 - s_2 y^b),$$

with $y = p^2/k^2$. Its limiting cases provide us the following commonly used regulator functions

$$\begin{aligned}
\lim_{s_1 \to 0, s_2 = 1} r_{css} &= \left(\frac{1}{y^b} - 1\right)\theta(1 - y) \;\; \text{Litim} \\
\lim_{s_1 \to 0, s_2 \to 0} r_{css} &= \frac{1}{y^b} \;\; \text{power law} \\
\lim_{s_1 = 1, s_2 \to 0} r_{css} &= \frac{1}{\exp[y^b] - 1} \;\; \text{exponential}
\end{aligned}$$

If $b = 1$ then $\lim_{y\to 0} yr = 1$ and $\lim_{y\to\infty} yr = 0$.

QEG effective action

The QEG effective action with Einstein–Hilbert truncation is

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^d x \sqrt{\det g_{\mu\nu}} (2\Lambda_k - R) \tag{1}$$

We use the forms of the QEG evolution equations (M. Reuter, F. Saueressig, PRD **65** (2002) 065016)

$$\begin{aligned}
\dot{\lambda} &= 2(2-\eta)\lambda + \frac{1}{2}(4\pi)^{1-d/2} g[2d(d+1)\Phi^1_{d/2}(-2\lambda) - 8d\Phi^1_{d/2}(0)] \\
&\quad -d(d+1)\eta\tilde{\Phi}^1_{d/2}(-2\lambda)], \\
\dot{g} &= (d-2+\eta)g, \quad \eta = \frac{gB_1(\lambda)}{1-gB_2(\lambda)} \text{ anomalous dimension}
\end{aligned}$$

The functions $B_1(\lambda)$ and $B_2(\lambda)$ are

$$\begin{aligned}
B_1(\lambda) &= \frac{1}{3}(4\pi)^{1-d/2}[d(d+1)\Phi^1_{d/2-1}(-2\lambda) \\
&\quad -6d(d-1)\Phi^2_{d/2}(-2\lambda) - 4d\Phi^1_{d/2-1}(0)24\Phi^2_{d/2}(0)], \\
B_2(\lambda) &= -\frac{1}{6}(4\pi)^{1-d/2}[d(d+1)\tilde{\Phi}^1_{d/2-1}(-2\lambda) - 6d(d-1)\tilde{\Phi}^2_{d/2}(-2\lambda)],
\end{aligned}$$

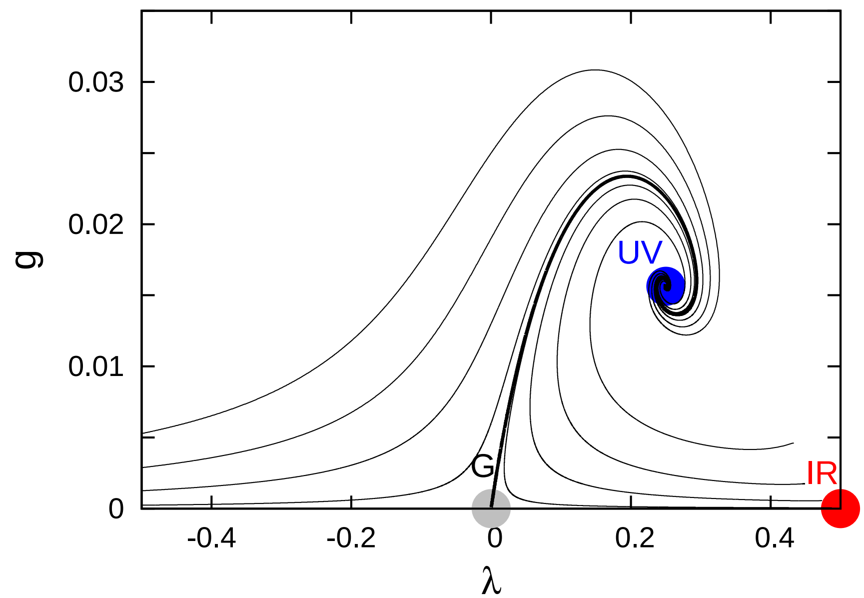
with the threshold functions $\Phi^p_n(\omega)$ and $\tilde{\Phi}^p_n(\omega)$.

QEG phase space

UV: (UV) attractive focal point: $\lambda^*_{UV} = 1/4$, $g^*_{UV} = 1/64$, eig.values: $\theta_{1,2} = (-5 \pm i\sqrt{167})/3$,

G: hyperbolic point: $\lambda^*_G = 0$, $g^*_{UV} = 0$ eig.values: $s_{G1} = -2$ and $s_{G2} = d - 2$

IR: (IR) attractive fixed point: $\lambda^*_{IR} = 1/2$, $g^*_{IR} = 0$ eig.values: $s_{IR1} = 0$ and $s_{IR2} = 1/2$

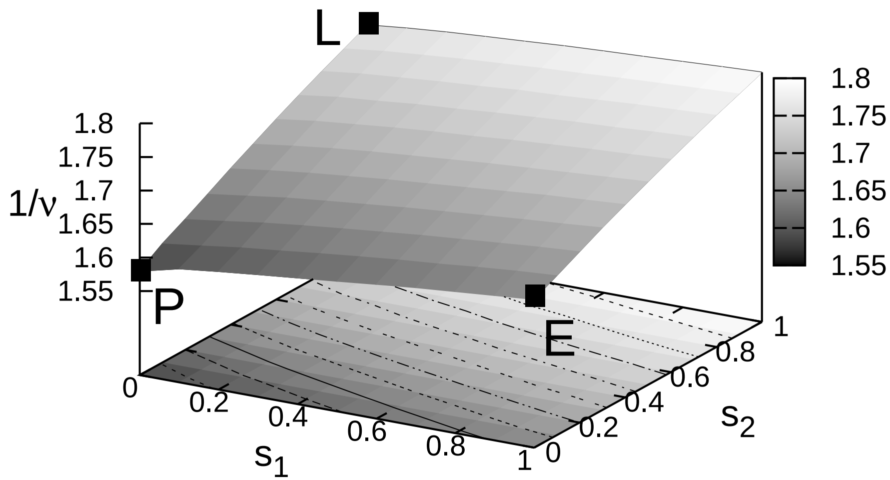


UV criticality

In the case of UV NGFP the eigenvalues of the corresponding stability matrix can be written as

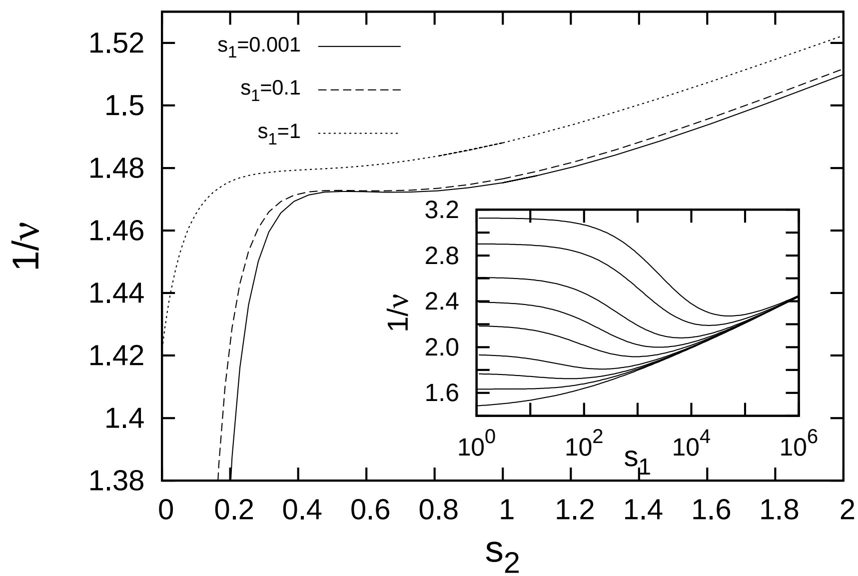
$$\theta_{1,2} = \theta' \pm i\theta'', \quad \text{and} \quad \nu = 1/\theta'$$

- $s_1 = s_2 = 0 \to$ **P**ower law regulator
- $s_1 = 0,\ s_2 = 1 \to$ **L**itim's regulator
- $s_1 = 1,\ s_2 = 0 \to$ **E**xponential regulator



UV criticality

Taking into account wider regions in s_1 and s_2



The inflection point, where we have the minimival sensitivity to the regulator parameters corresponds to a rescaled Litim's regulator of the form

$$r_{opt} = \left(\frac{1}{y} - \frac{1}{2}\right)\theta(1 - y/2)$$

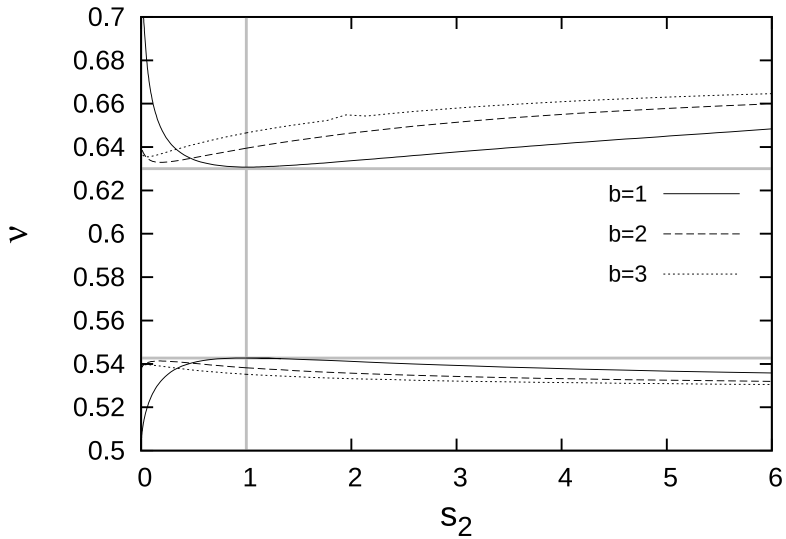
Remarks:

- ν can be any real number:
  - $\nu < 0$ makes the NGFP UV repulsive
  - $\nu = 0$ gives limit cycle around the NGFP
- the product $\lambda^* g^*$ shows less sensitivity:
  - if $s_1 \to \infty$ then λ^* and g^* become s_2 independent and scale according to $\lambda^* \sim s_1^{-0.89}$ and $g^* \sim s_1^{0.89} \to \lim_{s_1 \to \infty} \lambda^* g^* \approx 0.133$
  - at the extremum we have $\lambda^* g^* = 0.136$

Crossover criticality

3-dimensional ϕ^4 model with the potential

$$\tilde{V} = \sum_{i=1}^{N} \frac{\tilde{g}_i}{(2i)!}\phi^{2i}$$



Evolution equations:

$$\begin{aligned} \dot{\tilde{g}}_1 &= -2\tilde{g}_1 + \tilde{g}_2\bar{\Phi}^2_{3/2}(\tilde{g}_1) \\ \dot{\tilde{g}}_2 &= -\tilde{g}_2 + 6\tilde{g}_2^2\bar{\Phi}^3_{3/2}(\tilde{g}_1)\ldots \end{aligned}$$

Remarks:

- 2 couplings:
  - maxima of exponent ν in s_2
  - largest value is found at at $b = s_2 = 1$ and $s_1 \to 0$
- 4 couplings:
  - minima of exponent ν in s_2
  - smallest value is found at at $b = s_2 = 1$ and $s_1 \to 0$

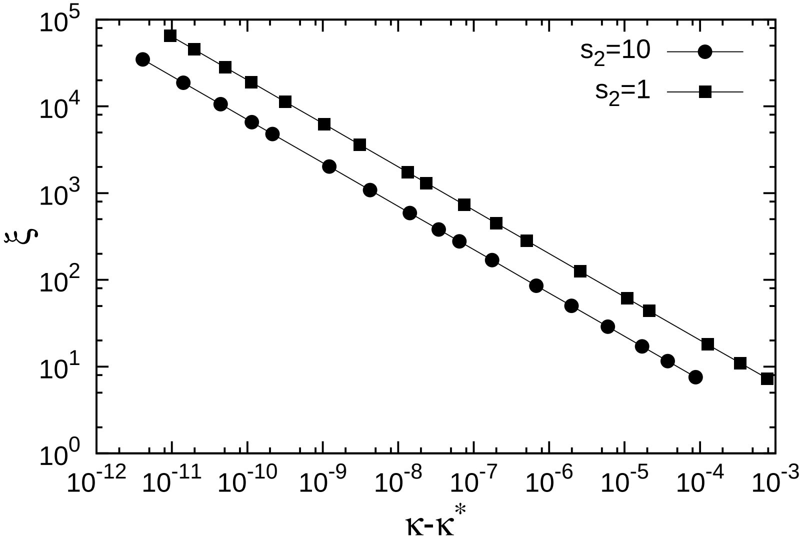
Both correspond to the Litim's regulator

IR criticality

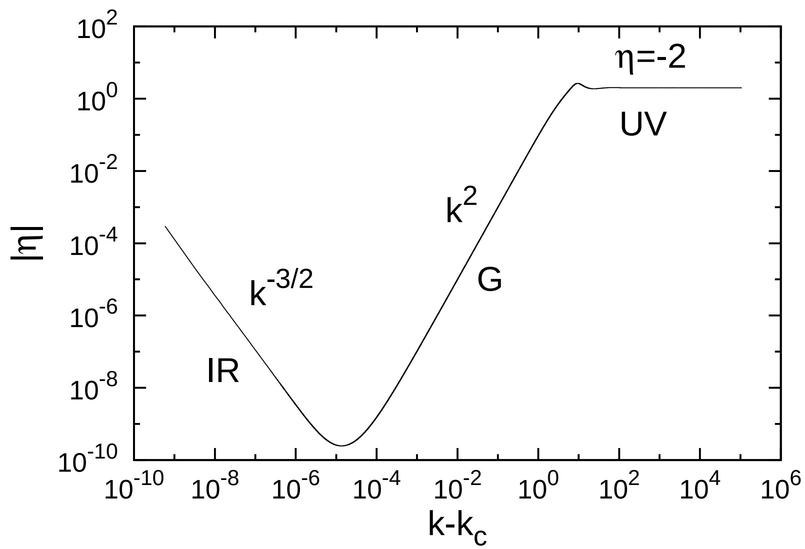
The correlation length scales as

$$\xi \sim (\kappa - \kappa^*)^{-\nu},$$

with the exponent $\nu = 1/2$, $\kappa = g\lambda$, and $\kappa^* = g^*\lambda^*$ equals the value which is taken at the fixed point and $\xi = 1/k_c$ is the reciprocal of the scale k_c where the evolution stops.



exponent	**UV**	**G**	**IR**
ν	0.679	0.5	0.5
η	-2	k^2	$(k-k_c)^{-3/2}$



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Thank You for Your attention