The Holographic view on RG Flows

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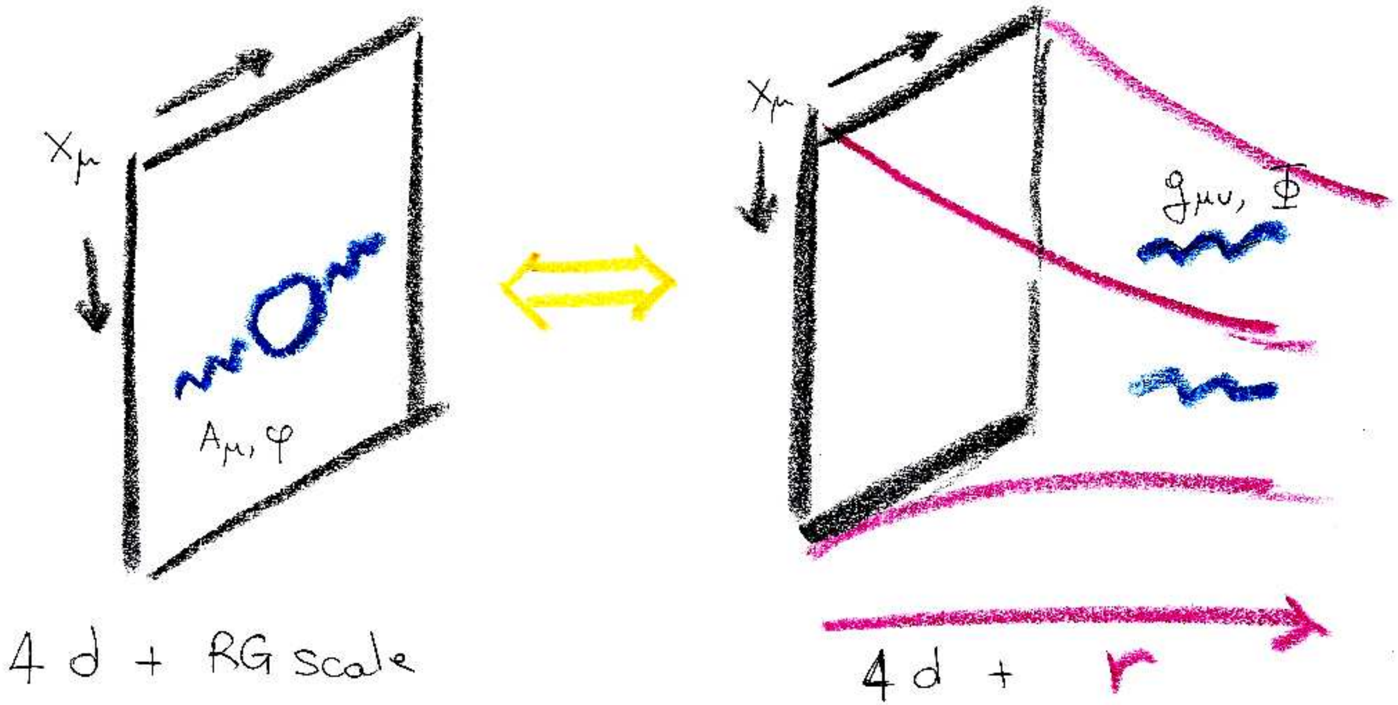
APC, U. Paris VII

ERG 2014, Lefkada, Sep 26th 2014

AdS/CFT

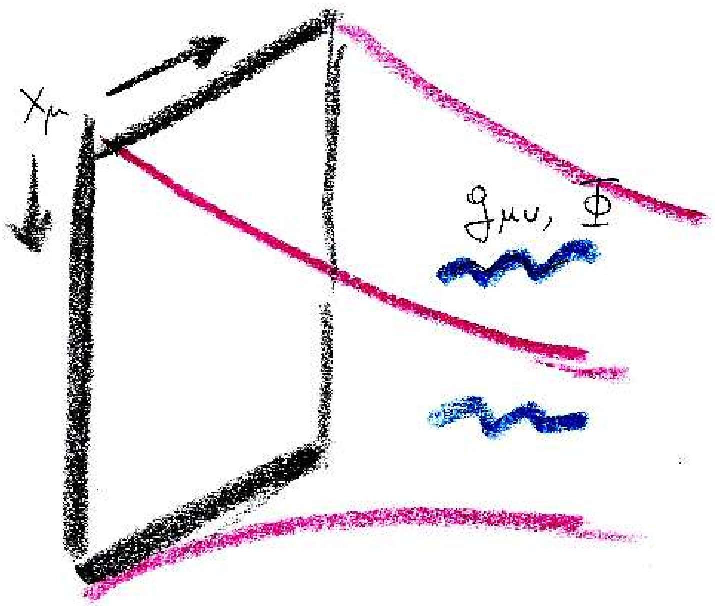
The AdS/CFT duality: conjecture that certain quantum field theories are equivalent to theories of gravity in higher dimensions Maldacena ’98.



“boundary theory” “bulk theory”

Equivalent means that the two theories contain the same degrees of freedom, but arranged in differnt ways.

AdS/CFT



$$ds^2 = \frac{\ell^2}{r^2}\left(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

- A conformal field theory in d dimension has a dual geometric description in terms of *Anti de Sitter space* AdS_{d+1}
- x^μ are mapped to the CFT space-time coordinates; r is mapped to the CFT scale.
- Scale invariance is realized as an isometry:

$$r \to \lambda r \qquad x^\mu \to \lambda x^\mu$$

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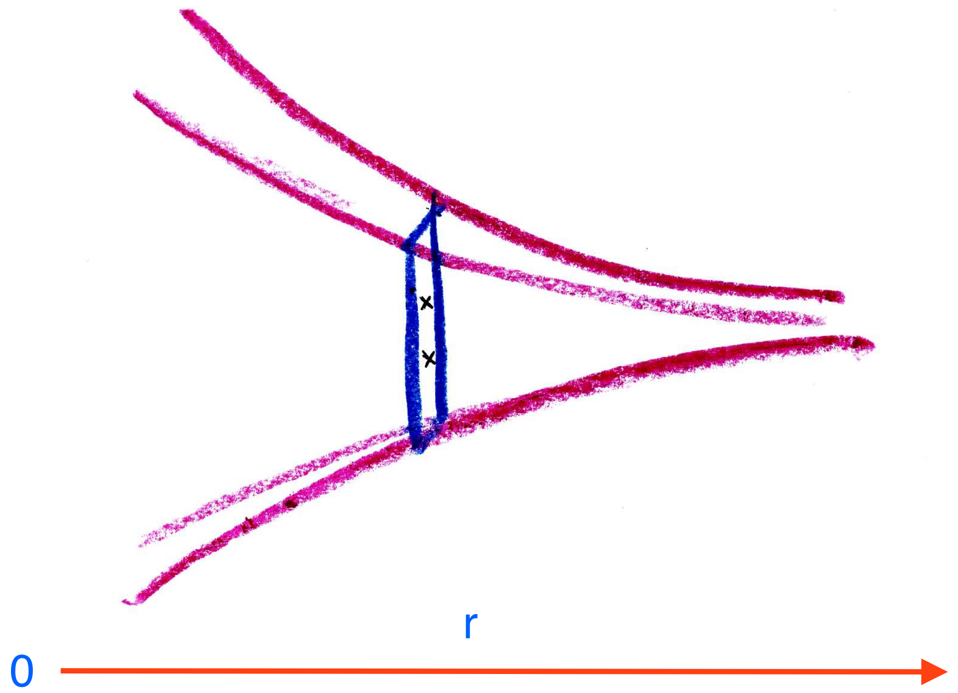

Moving towards $r = 0$ (AdS boundary) equivalent to reducing the distance in space-time.

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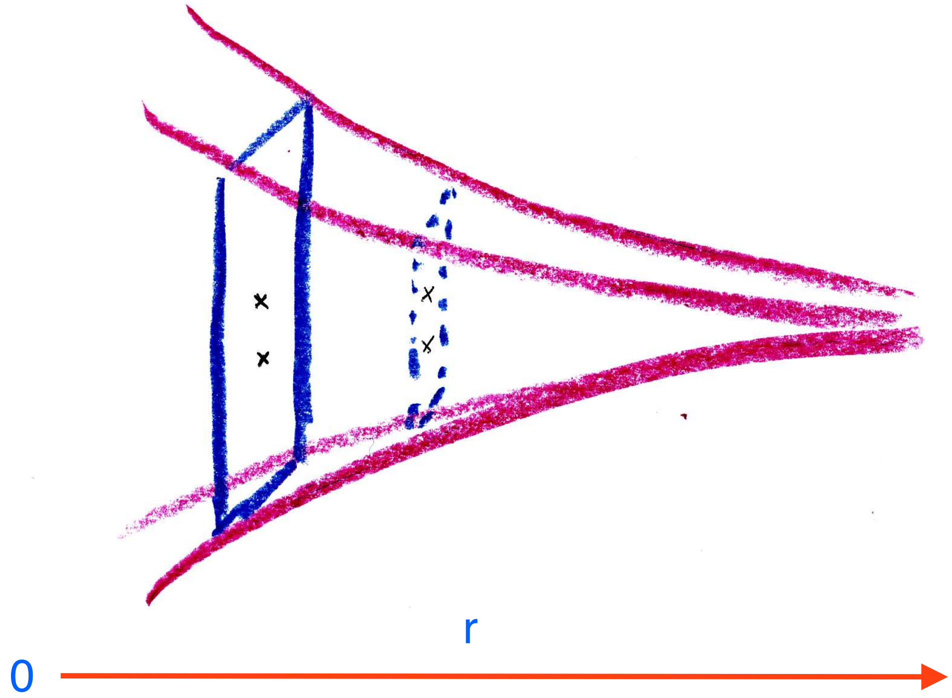

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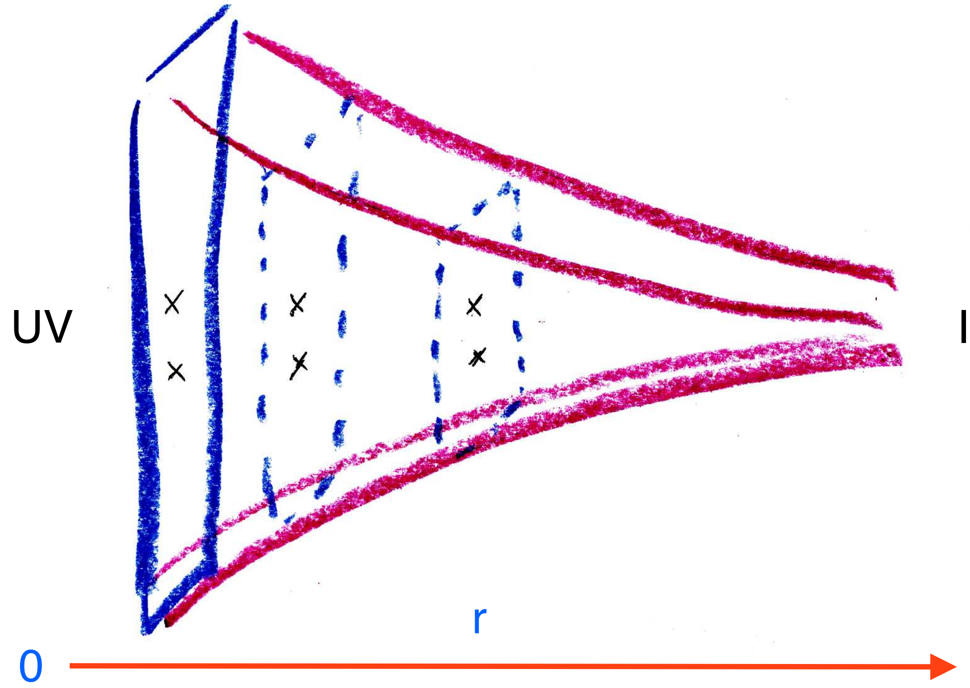

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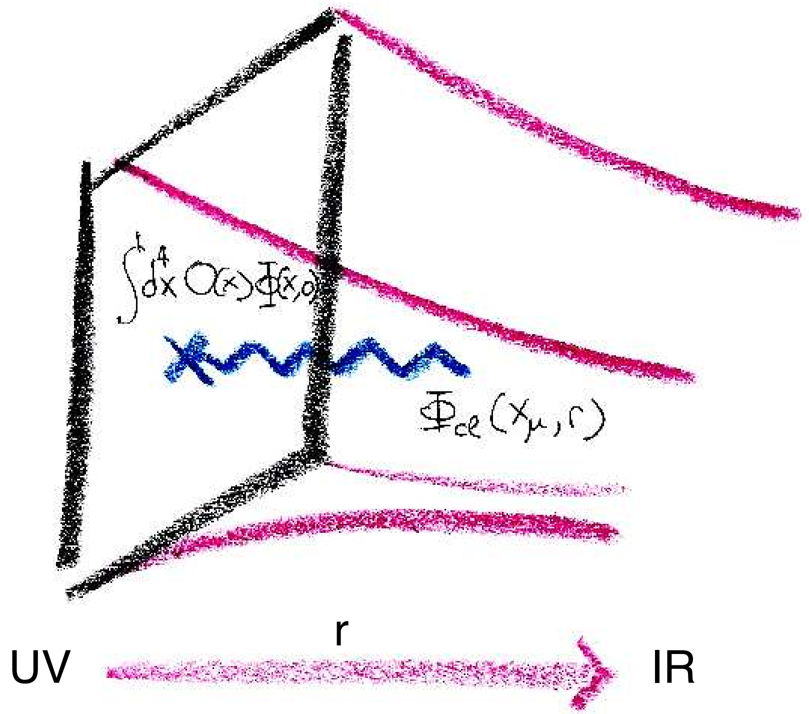
Field/Operator correspondence

- An operator $O(x)$ corresponds to a dynamical bulk field $\Phi(x, r)$
- $\Phi(x, 0)$ represents a source for O in the CFT.



The QFT sources become dynamical fields in higher dimensional curved spacetime

Example: massive scalar in AdS

$$S_{grav}[\Phi] = \frac{1}{2}\int d^d x dr \ \left[g^{ab}\partial_a\Phi\partial_b\Phi - m^2\Phi^2\right]$$

$$\partial_r^2\Phi + \frac{3}{r}\partial_r\Phi + \partial_\mu\partial^\mu\Phi = 0$$

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$\Phi(x,r)$ is a scalar under the dilatation isometry
$\Rightarrow \alpha(x)$ has scaling dimension $d - \Delta$
$\Rightarrow O(x)$ has dimension Δ

Stress Tensor

In any CFT there is a symetric stress tensor $T_{\mu\nu}$. The source is the boundary theory metric $\gamma_{\mu\nu}(x)$. This can be naturally identified as part of the bulk metric:

$$ds^2 = \frac{\ell^2}{r^2}\left[dr^2 + \gamma_{\mu\nu}(x,r)dx^\mu dx^\nu\right] \qquad \gamma_{\mu\nu}(x) \equiv \gamma_{\mu\nu}(x,0)$$

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$\Rightarrow$ The bulk theory must have dynamical gravity

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Typically the CFT admits a large-N limit in which $M_p \sim N^2$.
At large-N the map is between a quantum CFT and classical AdS gravity

Generating functional

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- Solve classical bulk field equations for $\Phi(x,r)$ with fixed boundary conditions in the UV ($r \to 0$):

$$\Phi_\alpha(x,r) \to \alpha(x) r^{d-\Delta}, \qquad r \to 0$$

- Evaluate the bulk action on the solution.

$$S_{grav}\left[\Phi_\alpha(x,r)\right] = \text{funcional of } \alpha(x)$$

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Couplings vs. Fields

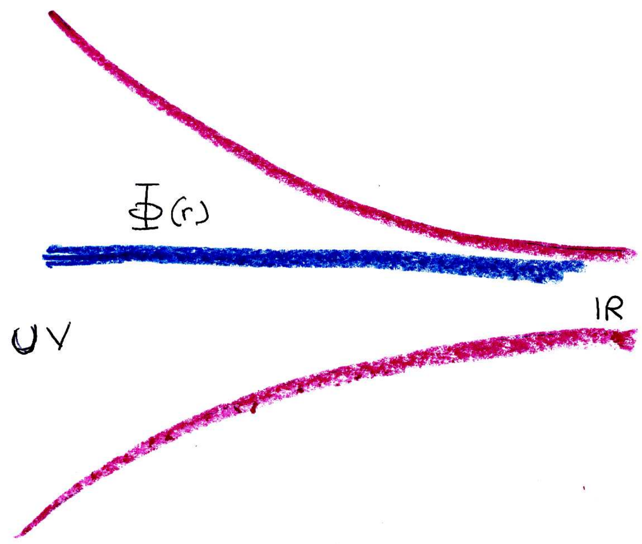
$$\Phi(x,r) = \alpha r^{(d-\Delta)} + \ldots \qquad \Leftrightarrow \qquad S_{CFT} = S_0 + \int d^4x \, \alpha O(x)$$

Couplings vs. Fields

$$\Phi(x,r) = 0 \qquad \Leftrightarrow \qquad S_{CFT} = S_0$$

$\alpha = 0$ corresponds to an underformed CFT.

$\Rightarrow$ Bulk scalar is constant, spacetime is AdS



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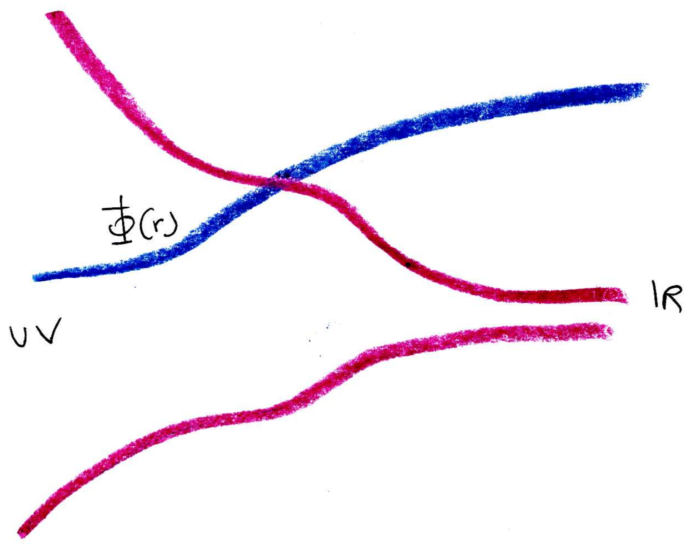
$$\Phi(x,r) = \alpha r^{(d-\Delta)} + \dots \quad \Leftrightarrow \quad S_{CFT} = S_0 + \int d^4x\, \alpha O(x)$$

$\alpha \neq 0$ corresponds to a relevant coupling for the CFT.

$\Rightarrow$ a profile $\Phi(r)$ and deformation of AdS geometry in the interior.



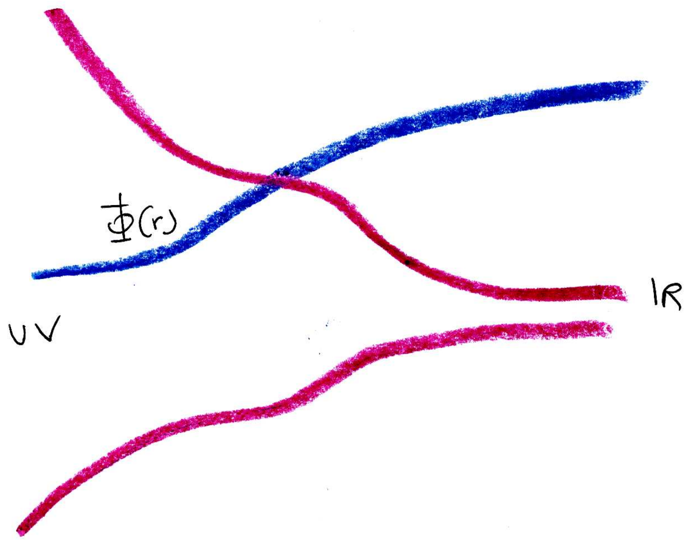
Running away from AdS

- α represents the bare UV coupling
- Around fixed points $\Phi(r)$ represents the running coupling:

$$\mu = 1/r, \qquad \Phi(\mu) = \alpha\mu^{\Delta-d} + O\left(\mu^{-\Delta}\right) \qquad \beta(\Phi) = (\Delta - d)\Phi$$

$$\Phi \simeq \alpha\mu^{(\Delta_{UV}-d)}$$

$$\mu \to \infty$$



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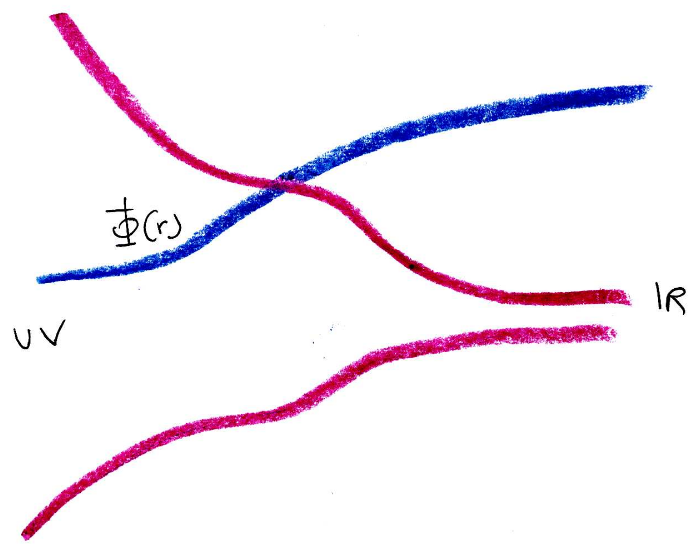

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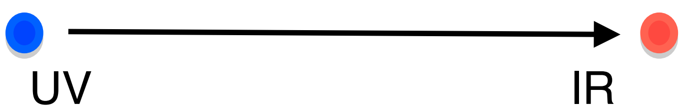
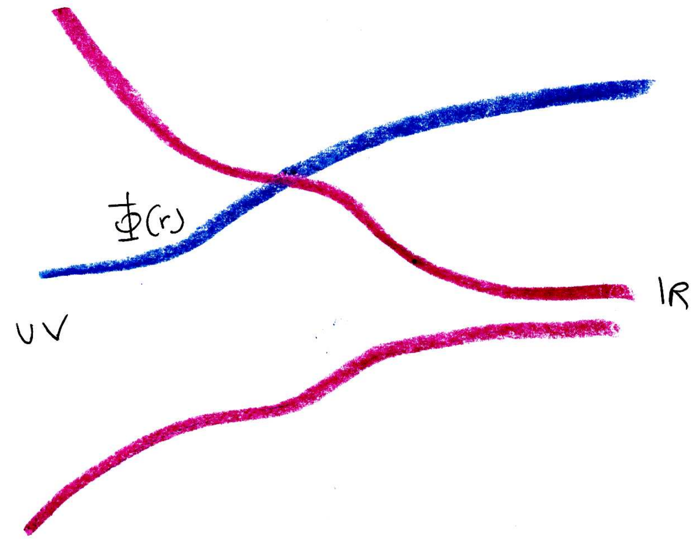
$$\mu \to 0$$

- Naturally identify $\Phi(r)$ with running coupling all along the flow
- Field theory couplings become dynmical fields. Couplings are naturally *space-time dependent*.

Hologrphic Renormalization Group

The holographic renormalization group is the way to translate between the field theory description of the running of couplings and the geometric radial evolution encoded in the bulk field equations.





$\frac{d}{d\log\mu}\Phi = \beta(\Phi) \Leftrightarrow$ Classical bulk evolution equation for $\Phi(r)$

A geometrization of the RG-flow.

Local RG Equations

Bulk Einstein's equations give rise to geometric flow equations for the local sources on a fixed-r slice (scalars $\Phi_I(x,r)$ and the induced metric $\gamma_{\mu\nu}(x,r)$)

$$\begin{cases} G_{ab} = 8\pi\kappa T_{ab} \\ \nabla^2\Phi + m^2\Phi = 0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} \dot{\gamma}_{\mu\nu}(x,r) = B_{\mu\nu} \\ \dot{\Phi}_I(x,r) = B_I(x,r) \end{cases}$$

The β-functions $B_{\mu\nu}(x), B_I(x)$, are written in terms of local slice-covariant boundary quantities constructed with $\Phi(x)$ and the induced metric $\gamma_{\mu\nu}$.

Not just a metaphor

This may seem like a nice analogy or a made-up set of rules.

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In all cases where both sides are known explicitly (as different limits of the same string setup) the spectrum of operators, anonalous dimensions, correlators, anomalies, non-perturbative sectors... match exactly

This includes (but is not limited to) $\mathcal{N} = 4$ Super Yang-Mills and its various relevant deformations.

Setup

Simple setup: $d+1$-dimensional Einstein Gravity plus one scalar field:

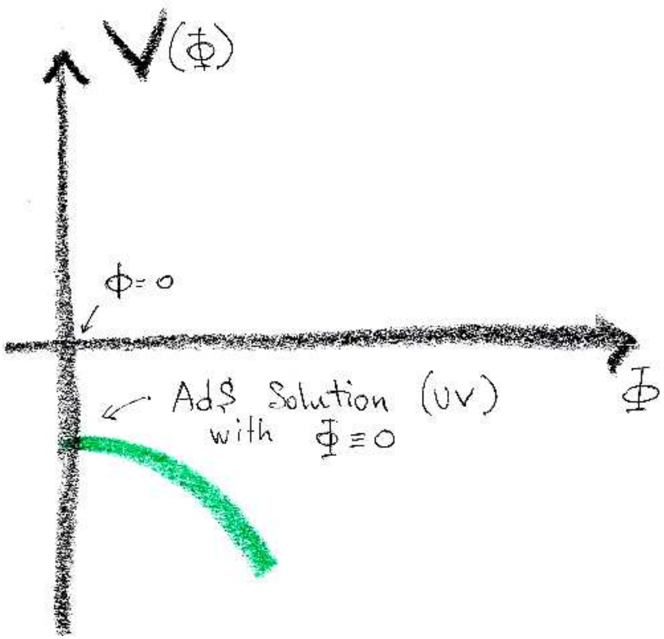
$$S_{grav} = M_p^{d-1} \int d^d x \int du \sqrt{-g} \left[R - \frac{1}{2}(\partial\Phi)^2 - V(\Phi) \right]$$

- Holographic coordinate u ranging from $-\infty$ to u_{IR}
- Only one scalar $\leftrightarrow$ focus on a single operator O in the field theory.$\Phi(u) =$ running coupling associated to O
- The potential $V(\phi)$ encodes the dimension of the operator and the way the coupling runs.

AdS solutions

If $V(\Phi)$ has an extremum at Φ_* ($V'(\Phi*) = 0$ with $V(\Phi*) = V_* < 0$)

$$ds^2 = du^2 + e^{-u/\ell} dx_d^2, \quad \Phi(u, x_\mu) = \Phi_*, \quad m^2\ell^2 = \Delta(\Delta - d)$$

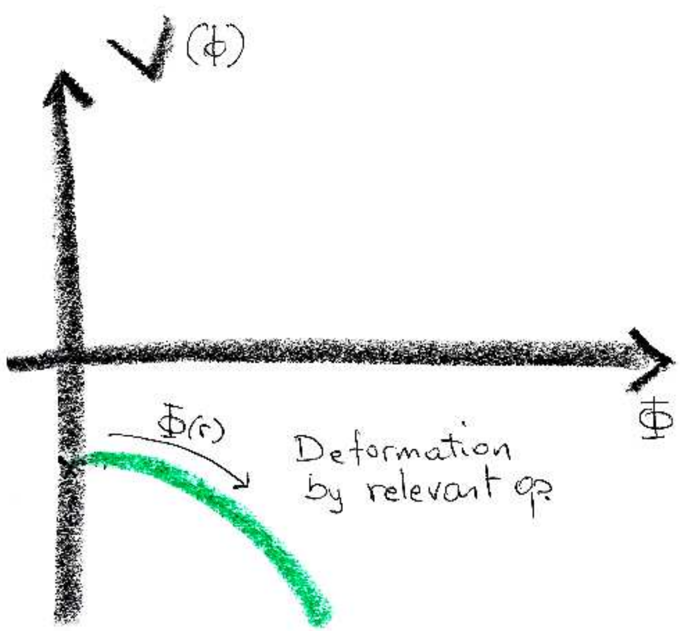


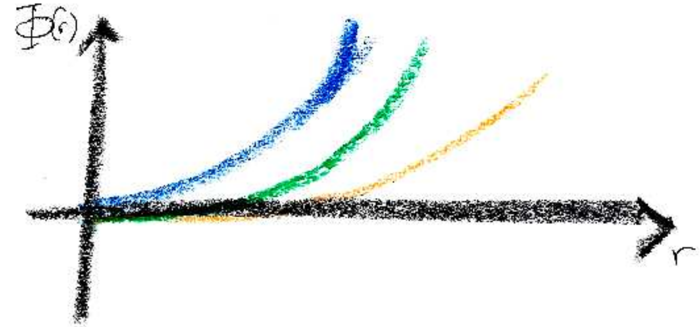
Theory at a conformal fixed point. If $m^2 < 0$, we have a relevant operator, and we expect IR deformations to exist.

Deformations of AdS

Generic Poincaré-invariant solution:

$$ds^2 = du^2 + e^{A(u)}\eta_{\mu\nu}dx^\mu dx^\nu, \quad \Phi = \Phi(u)$$





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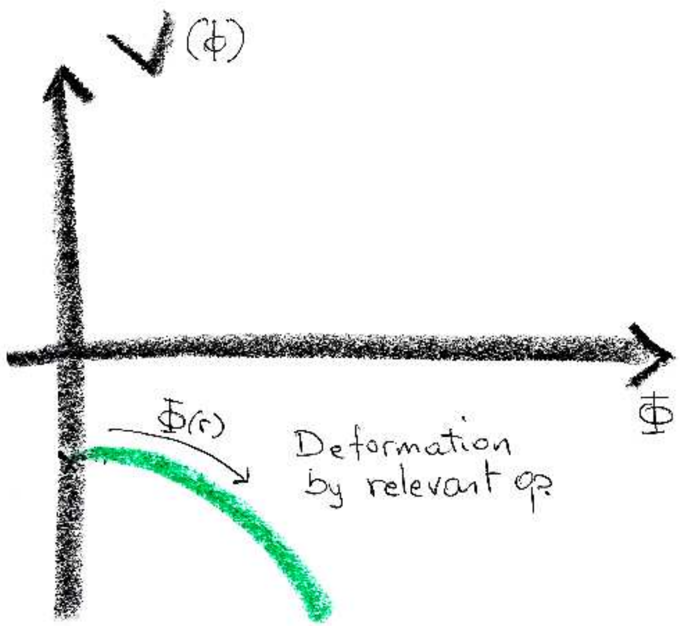

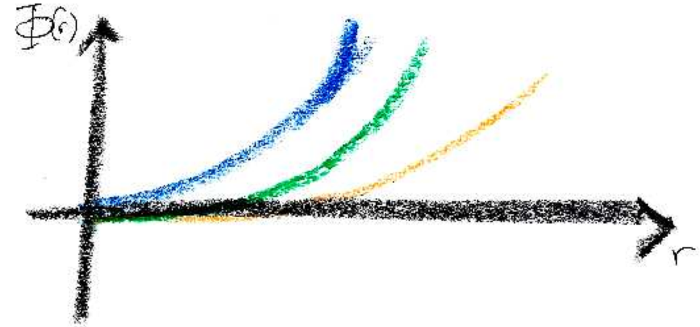

The UV (IR) is represented by the region where $e^{A(u)} \to +\infty$ ($\to 0$). Intuitively, we can think of e^A as the energy scale.

RG-flow solutions

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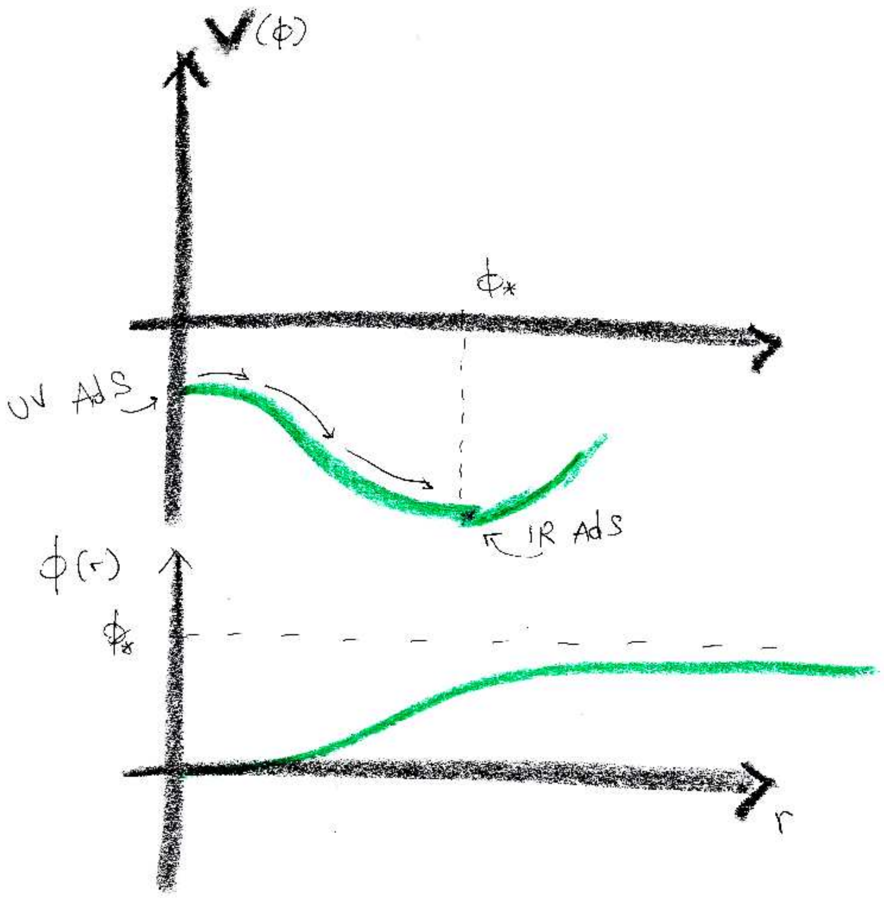

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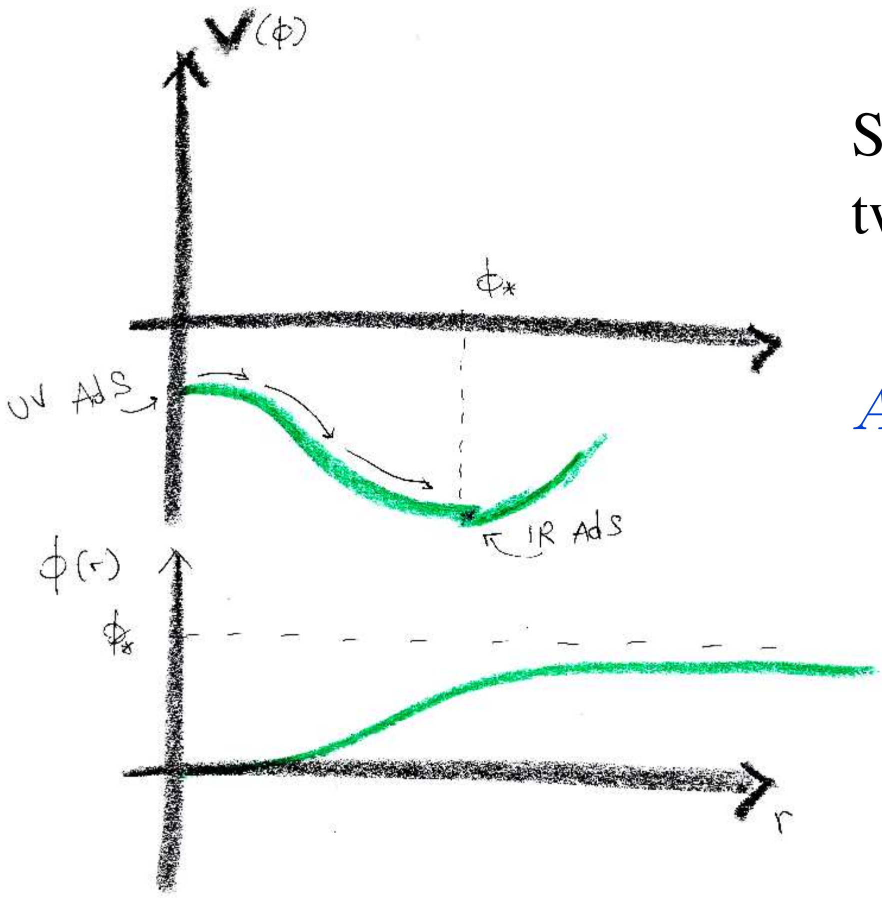


Solutions that interpolate between the two fixed points:

$$A(u) \sim -u \begin{cases} u \to -\infty & UV \\ u \to +\infty & IR \end{cases}$$

$$\phi(u) \to \begin{cases} 0 & UV \\ \phi_* & IR \end{cases}$$

Superpotential

For a homogeneous ansatz, Einstein's equations can be put in first order form in terms of an auxiliary function $W(\Phi)$:

$$\dot{\Phi} = W'(\Phi), \qquad \dot{A} = -\frac{1}{2(d-1)} W(\Phi)$$

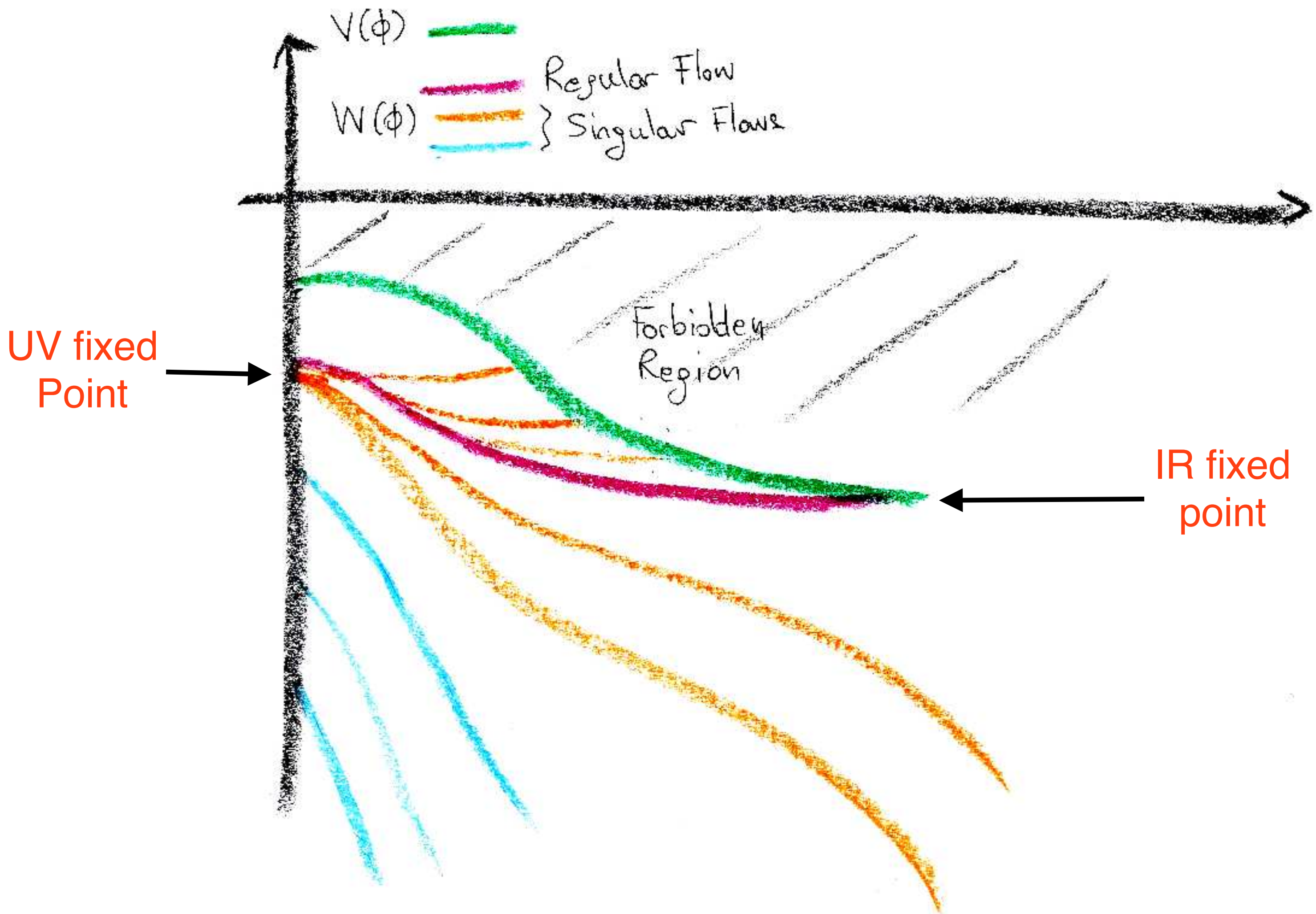
$$-\frac{d}{4(d-1)} W^2 + \frac{1}{2} \left(W'\right)^2 = V$$

Once $W(\Phi)$ is found the other equations can be integrated trivially: using Φ as a coordinate:

$$A(\Phi) = A_0(\Phi_0) - \frac{1}{2(d-1)} \int_{\Phi_0}^{\Phi} d\phi \frac{W(\phi)}{W'(\phi)},$$

Different solutions with the same $W(\Phi)$ all look the same up to an additive integration consant in A.

Superpotential solutions



The UV AdSis an *attractor* for the superpotential equation. ⇔
The UV fixed point is stable under relevant deformations.

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(Renormalized) generating functional: $S^{(ren)}_{grav}[\alpha]$

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Map $\alpha \leftrightarrow (A(u), \Phi(u))$ allows to rewrite it as a function of the field on any interior slide:

$$S_{grav}^{(ren)}[A, \Phi] = \int d^d x\, e^{dA} \exp\left[-\frac{d}{2(d-1)} \int^{\Phi} \frac{W}{W'}\right]$$

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$S_{grav}^{(ren)}[A, \Phi]$ constant along the radial flow:

$$\frac{d}{dA} S_{grav}^{(ren)}[A, \Phi(A)] = \left[\frac{\partial}{\partial A} + \frac{d\Phi}{dA}\frac{\partial}{\partial \Phi}\right] S_{grav}^{(ren)}[A, \Phi(A)] = 0$$

Beyond zeroth order

We want to generalize this approach to the case of spacetime-dependent couplings, by keeping d-dimensional bulk covariance. work with E. Kiritsis and Wenliang Li

The data will be the d-dimensional metric $\gamma_{\mu\nu}(x, u)$ and scalar field $\Phi(x, u)$ evaluated on a space-time slice in the bulk.

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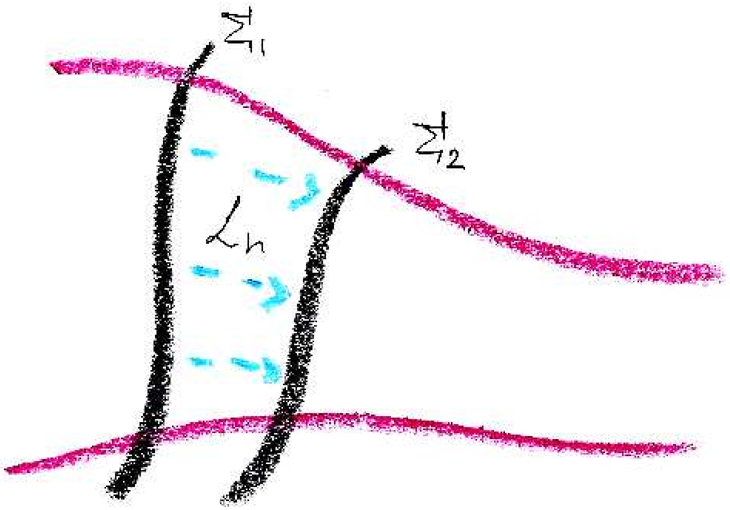
We take a solution with a general space-time metric $\gamma_{\mu\nu}(x,u)$, in ADM form:

$$ds^2 = N^2 du^2 + \gamma_{\mu\nu}(x)(dx^\mu + N^\mu du)(dx^\nu + N^\nu du), \qquad \Phi = \Phi(u,x)$$

Flow equations



The flow equations tell how to go from one hypersurface of the ADM slicing to another one nearby, as a function only on the invariants on the *slice*.

They can be derived using Einstein's constraint equations order by order in a *derivative expansion* on the slice.

Second order flow equations

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$$\begin{aligned}\pounds_n \gamma_{\mu\nu} &= -\frac{1}{d-1}\gamma_{\mu\nu}\left(W + fR + \frac{W}{2W'}f'(\gamma^{\rho\sigma}\partial_\rho\Phi\partial_\sigma\Phi)\right) \\ &\quad +2fR_{\mu\nu} + \left(\frac{W}{W'}f' - 2f''\right)\partial_\mu\Phi\partial_\nu\Phi - 2f'\nabla_\mu\partial_\nu\Phi \\ \pounds_n \Phi &= W' - f'R + \frac{1}{2}\left(\frac{W}{W'}f'\right)'(\gamma^{\rho\eta}\partial_\rho\Phi\partial_\eta\Phi) + \frac{W}{W'}f'(\gamma^{\rho\eta}\nabla_\rho\partial_\eta\Phi)\end{aligned}$$

$W(\Phi)$ and $f(\Phi)$ are solutions of:

$$\frac{d}{4(d-1)}W^2 - \frac{1}{2}W'^2 = -V, \qquad W'f' - \frac{d-2}{2(d-1)}Wf = 1$$

Beta-functions

$$\Delta_\mu \gamma_{\mu\nu} = 2\gamma_{\mu\nu} + \beta_{\mu\nu}, \qquad \Delta_\mu \Phi = \beta_\Phi$$

$$\beta_\Phi = -2(d-1)\frac{W'}{W} - \frac{2(d-1)}{W}\left(f' + \frac{W'}{W}f\right)R + \ldots$$

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- To zeroth order we recover the results of the homogeneous calculation
- The metric gets an anomalous change beyond a Weyl rescaling due to the curvature terms. This resembles the case of Ricci flows.

Two-Derivavive Generating Functional

- The renormalized partition function takes has the local covariant form:

$$S^{(ren)} \equiv \log \mathcal{Z}_{QFT}[\gamma, \Phi] = \int d^d x \sqrt{\gamma} \left[Z_0(\Phi) + Z_1(\Phi) R + Z_2(\Phi)(\partial\Phi)^2 \right] + \ldots$$

- $Z_i(\Phi)$ are complicated but known functions of Φ, written in terms of W and f. Up to the three scheme-dependent multiplicative quantities D_i, their functional form is universal.
- $\log \mathcal{Z}$ obeys the local RG-invariance equation

$$\left(2\gamma^{\mu\nu} \frac{\delta}{\delta \gamma^{\mu\nu}} - \beta_{\mu\nu} \frac{\delta}{\delta \gamma_{\mu\nu}} - \beta_\Phi \frac{\delta}{\delta \phi} \right) \log \mathcal{Z} = \ Anomaly$$

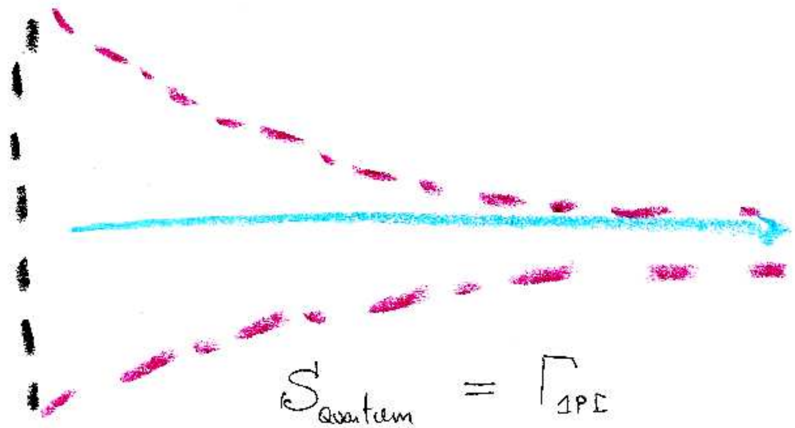
with the holographic β-functions appearing.

Conclusion and Open Questions

- AdS/CFT: a dynamical theory for QFT *sources*
- Local RG flow equations arise from Einstein equations. How exactly does the IR regularity condition select a solution?
- Ongoing work trying to understand how geometry emerges from field theory side (work by S.S. Lee)
- Relation with with the Wilsonian framework ? (work by Polchinski and Heemskerk)
- Relation between HRG and ERG ?
- The flow equations are a rewriting of Einstein's equations, and are cast in a form that resembles the conformal conditions in σ-models. What are the fixed points of these generalised flows? what is their physical meaning ?
- The formalism for the derivative expansion is limited to solutions built around a Poincaré invariant vacuum state. Can we gereralize to less symmetric cases (e.g. black holes) ?

Wilsonian picture

So far we have computed the quantum effective action by integrating the solution from a UV cutoff to the IR.



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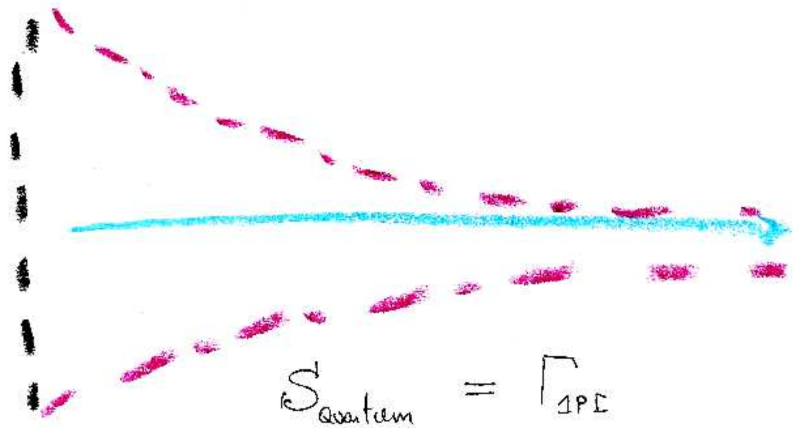

What about the Wilsonian action?