Few-body universality and "super" Efimov effect

Yusuke Nishida (Tokyo Tech)

7th International Conference
on the Exact Renormalization Group

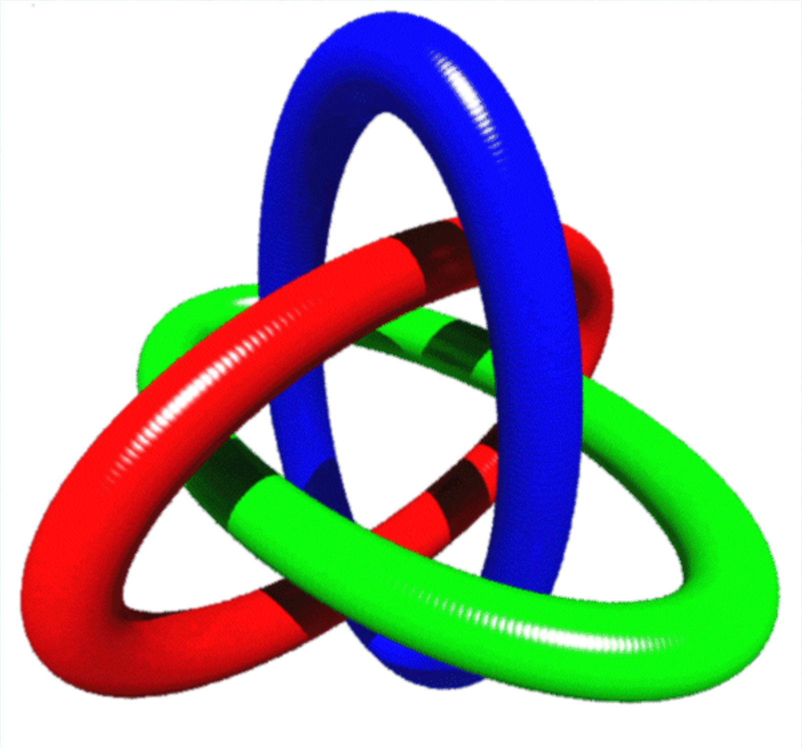
September 22-26 (2014)

Plan of this talk

1. Universality in physics

2. What is the Efimov effect ?

   - universality
   - discrete scale invariance
   - RG limit cycle



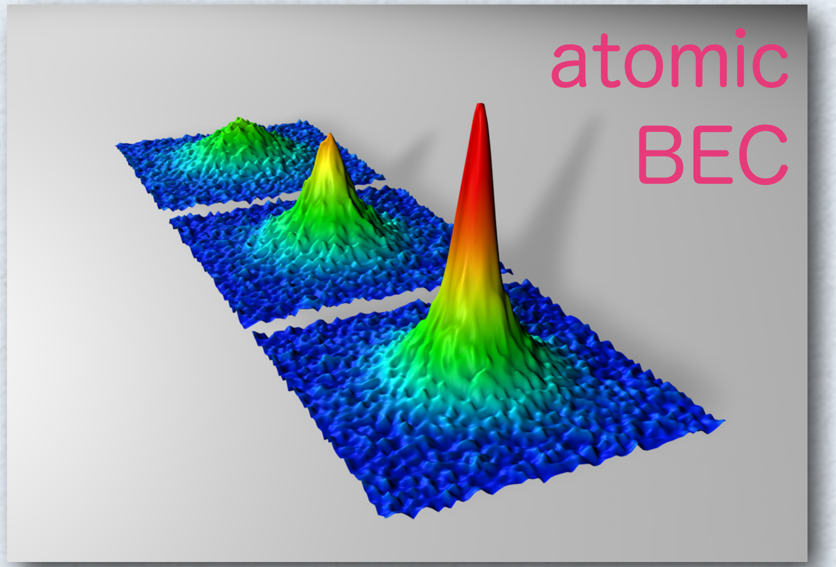
3. New discovery “Super Efimov effect”

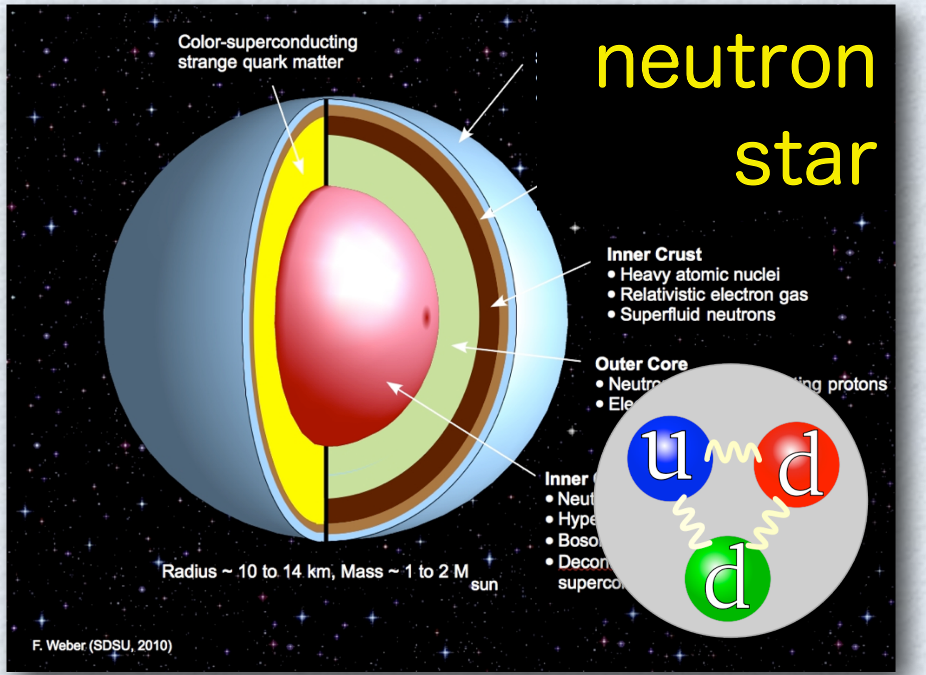
Introduction

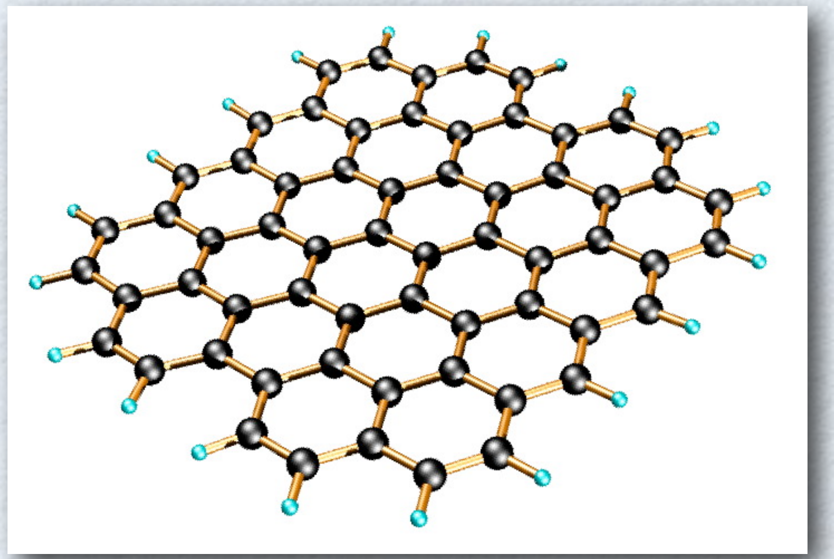
1. **Universality in physics**
2. What is the Efimov effect ?
3. Super Efimov effect

(ultimate) Goal of research

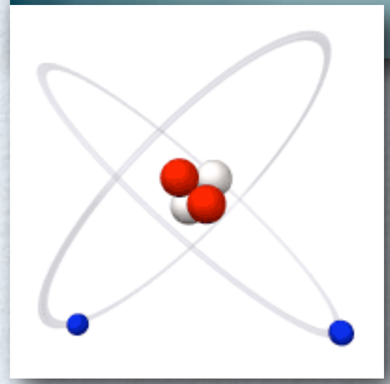
Understand physics of few and many particles governed by quantum mechanics

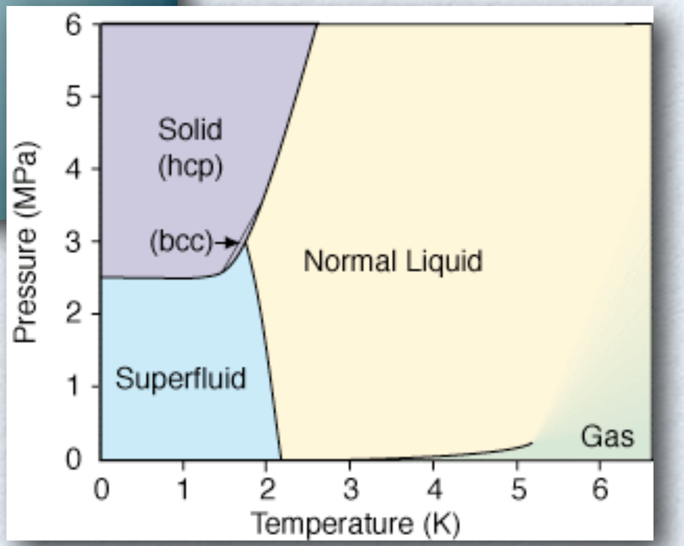




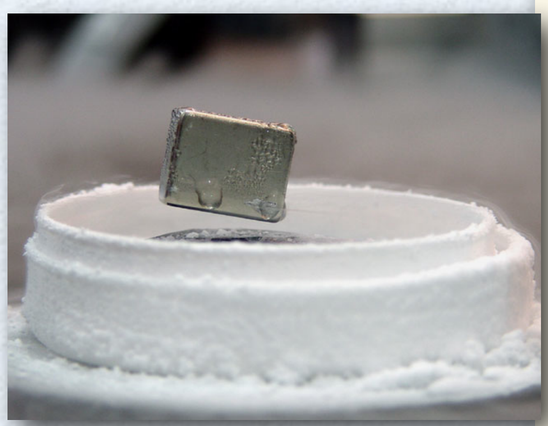


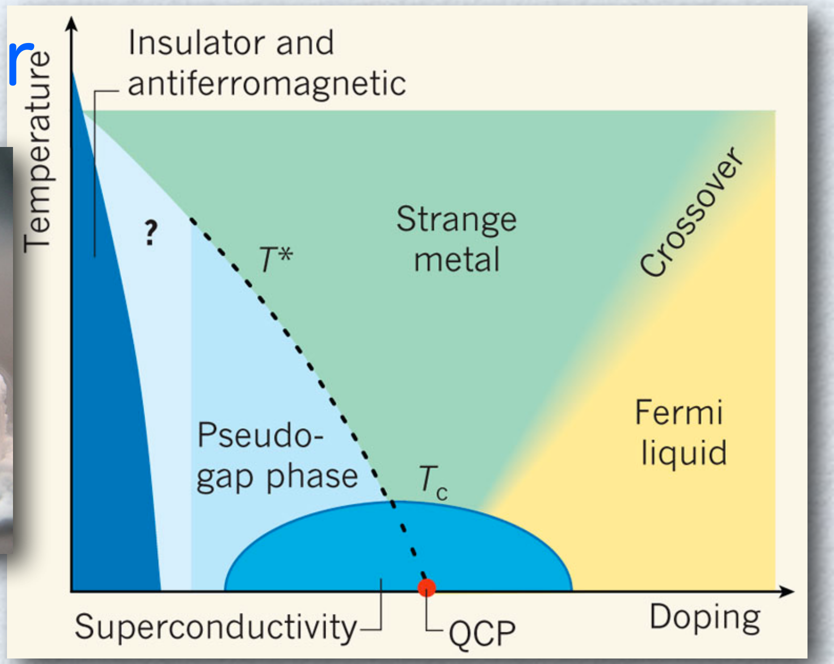
graphene





superconductor

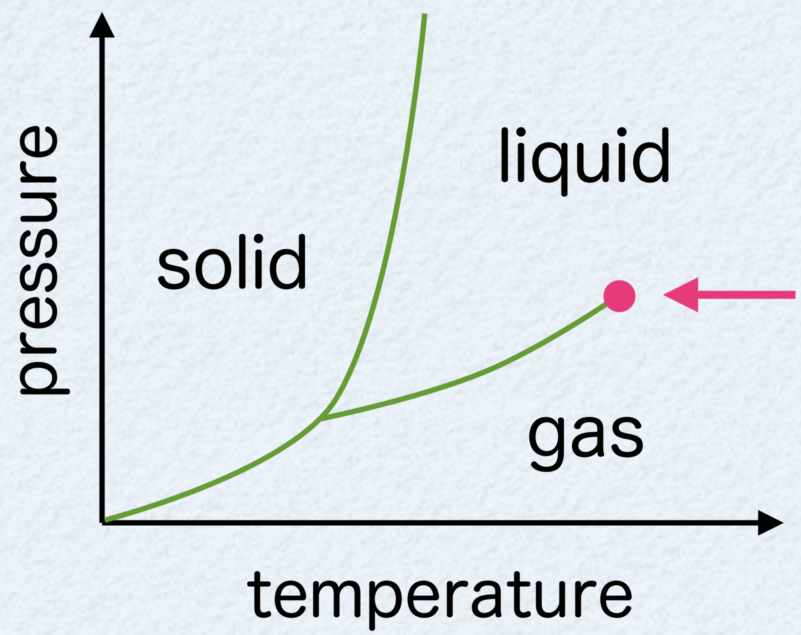


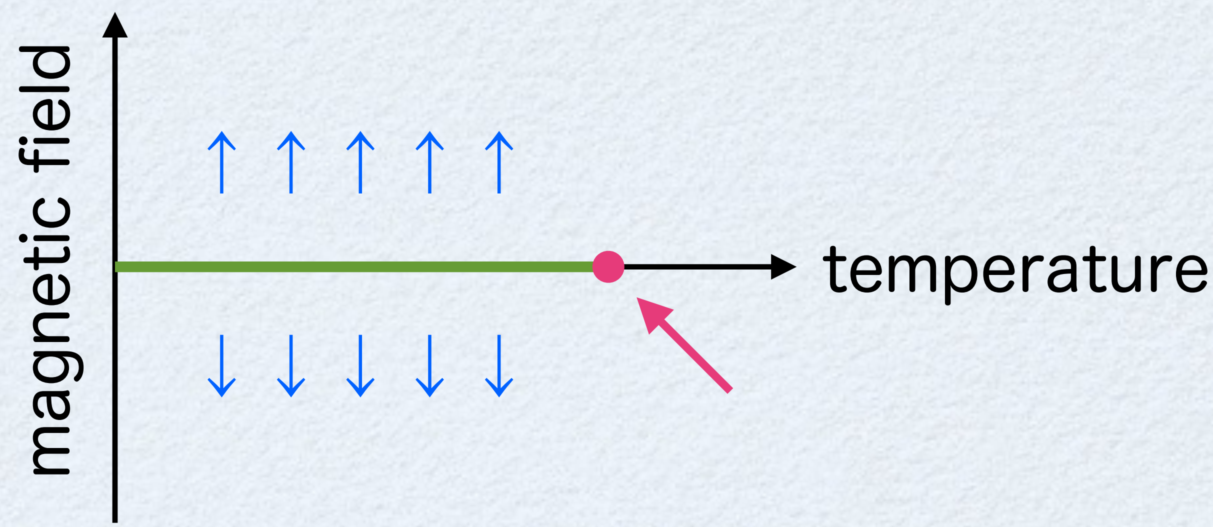


When physics is universal ?

A1. Continuous phase transitions ⇔ $\xi / r_0 \to \infty$

E.g. Water  vs. Magnet 



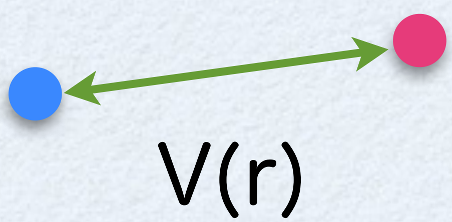


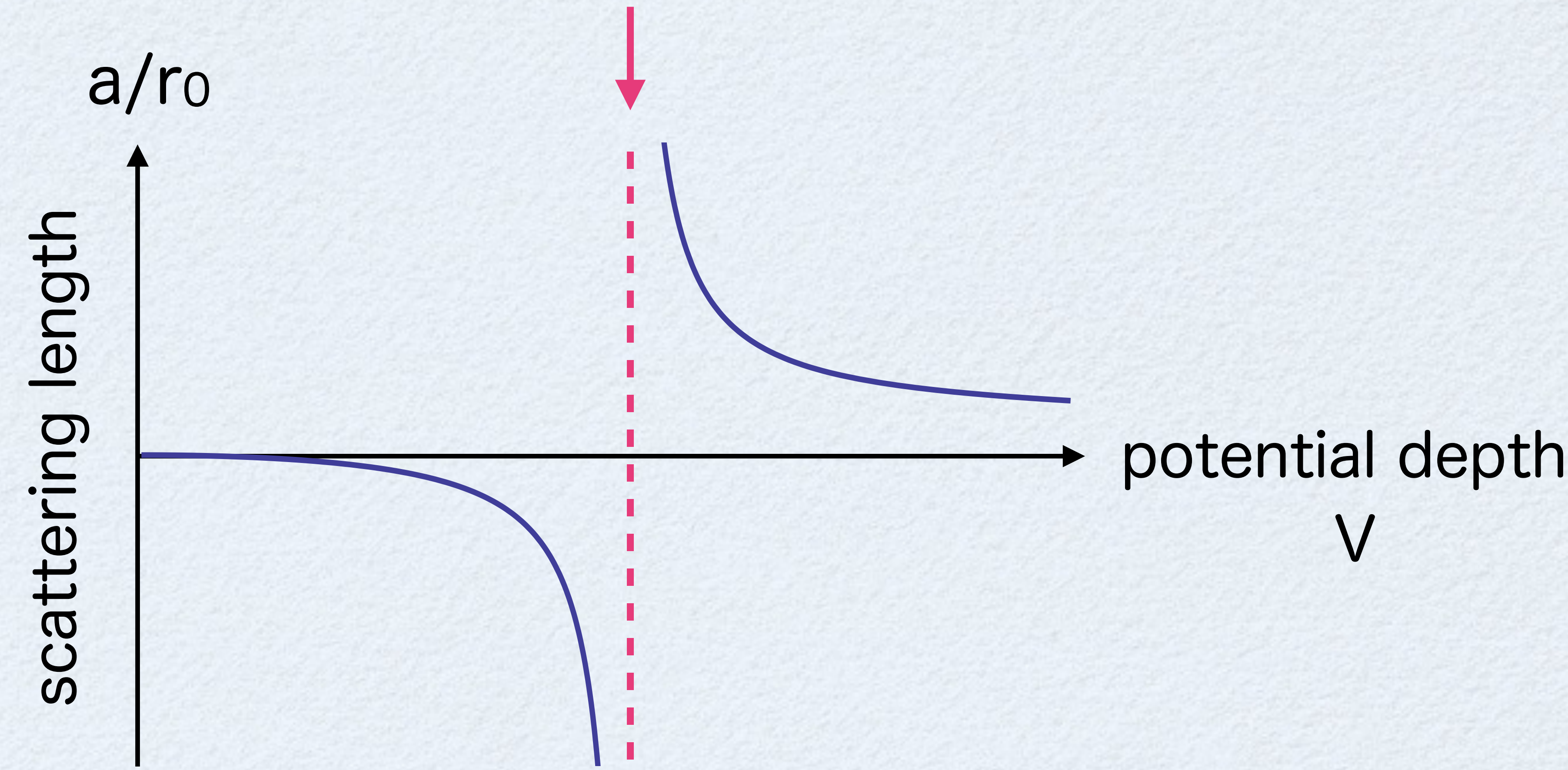
Water and magnet have the same exponent $\beta \approx 0.325$

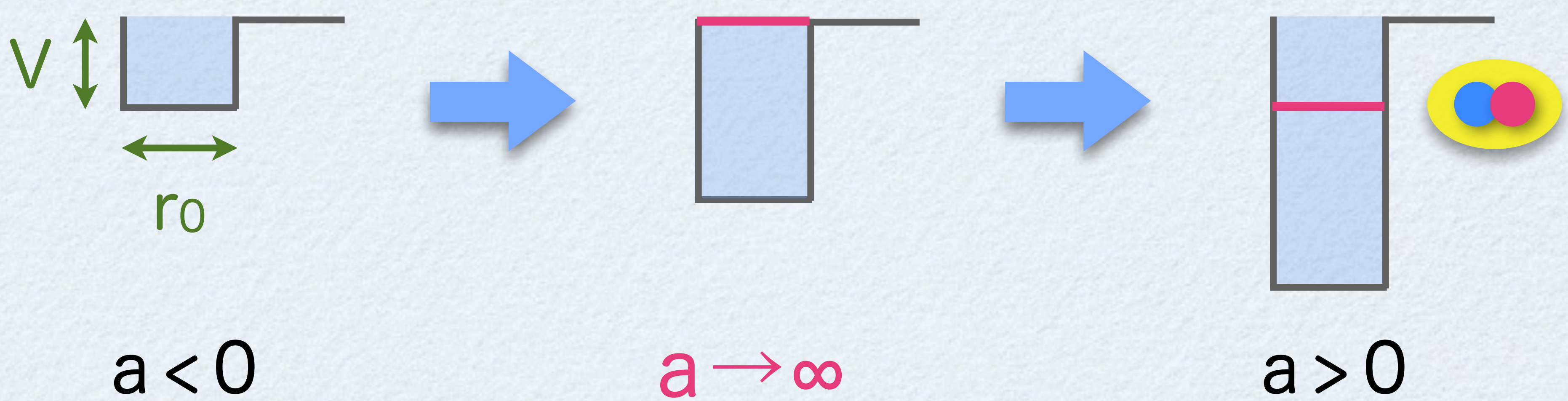
$$\rho_{\mathrm{liq}} - \rho_{\mathrm{gas}} \sim (T_{\mathrm{c}} - T)^{\beta} \qquad M_{\uparrow} - M_{\downarrow} \sim (T_{\mathrm{c}} - T)^{\beta}$$

When physics is universal ?

A2. Scattering resonances ⇔ $a/r_0 \to \infty$

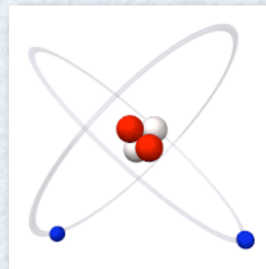
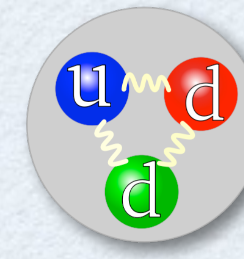






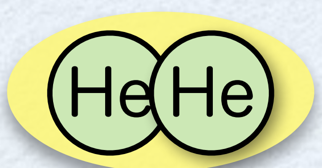
When physics is universal ?

A2. Scattering resonances ⇔ $a/r_0 \to \infty$

E.g. ^{4}He atoms  vs. proton/neutron 

van der Waals force:

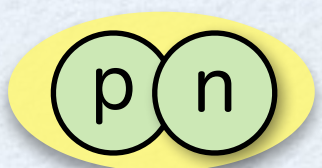
$a \approx 1\times10^{-8}$ m $\approx 20\ r_0$



$E_{binding} \approx 1.3\times10^{-3}$ K

nuclear force:

$a \approx 5\times10^{-15}$ m $\approx 4\ r_0$



$E_{binding} \approx 2.6\times10^{10}$ K

Atoms and nucleons have the same form of binding energy

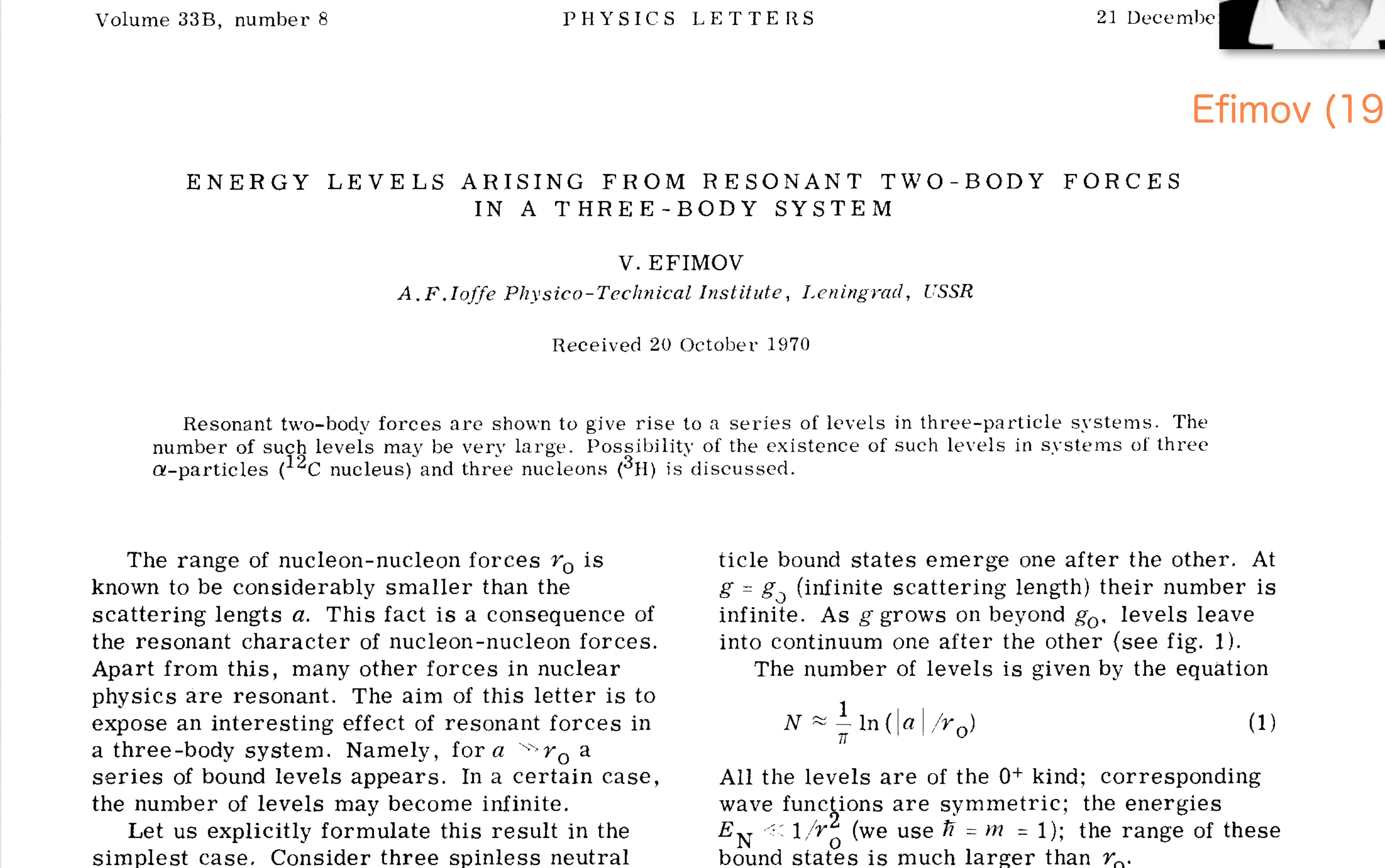
$$E_{\rm binding} \to -\frac{\hbar^2}{m\,a^2} \qquad (a/r_0 \to \infty)$$

→ Physics only depends on the scattering length “a”

Efimov effect

1. Universality in physics
2. **What is the Efimov effect ?**
3. Super Efimov effect

Efimov effect



Volume 33B, number 8 PHYSICS LETTERS 21 Decembe

ENERGY LEVELS ARISING FROM RESONANT TWO-BODY FORCES IN A THREE-BODY SYSTEM

V. EFIMOV
A.F.Ioffe Physico-Technical Institute, Leningrad, USSR

Received 20 October 1970

Resonant two-body forces are shown to give rise to a series of levels in three-particle systems. The number of such levels may be very large. Possibility of the existence of such levels in systems of three α-particles (^{12}C nucleus) and three nucleons (^{3}H) is discussed.

The range of nucleon-nucleon forces r_0 is known to be considerably smaller than the scattering lengts a. This fact is a consequence of the resonant character of nucleon-nucleon forces. Apart from this, many other forces in nuclear physics are resonant. The aim of this letter is to expose an interesting effect of resonant forces in a three-body system. Namely, for $a \gg r_0$ a series of bound levels appears. In a certain case, the number of levels may become infinite.

Let us explicitly formulate this result in the simplest case. Consider three spinless neutral

ticle bound states emerge one after the other. At $g = g_0$ (infinite scattering length) their number is infinite. As g grows on beyond g_0, levels leave into continuum one after the other (see fig. 1).

The number of levels is given by the equation

$$N \approx \frac{1}{\pi} \ln(|a|/r_0) \qquad (1)$$

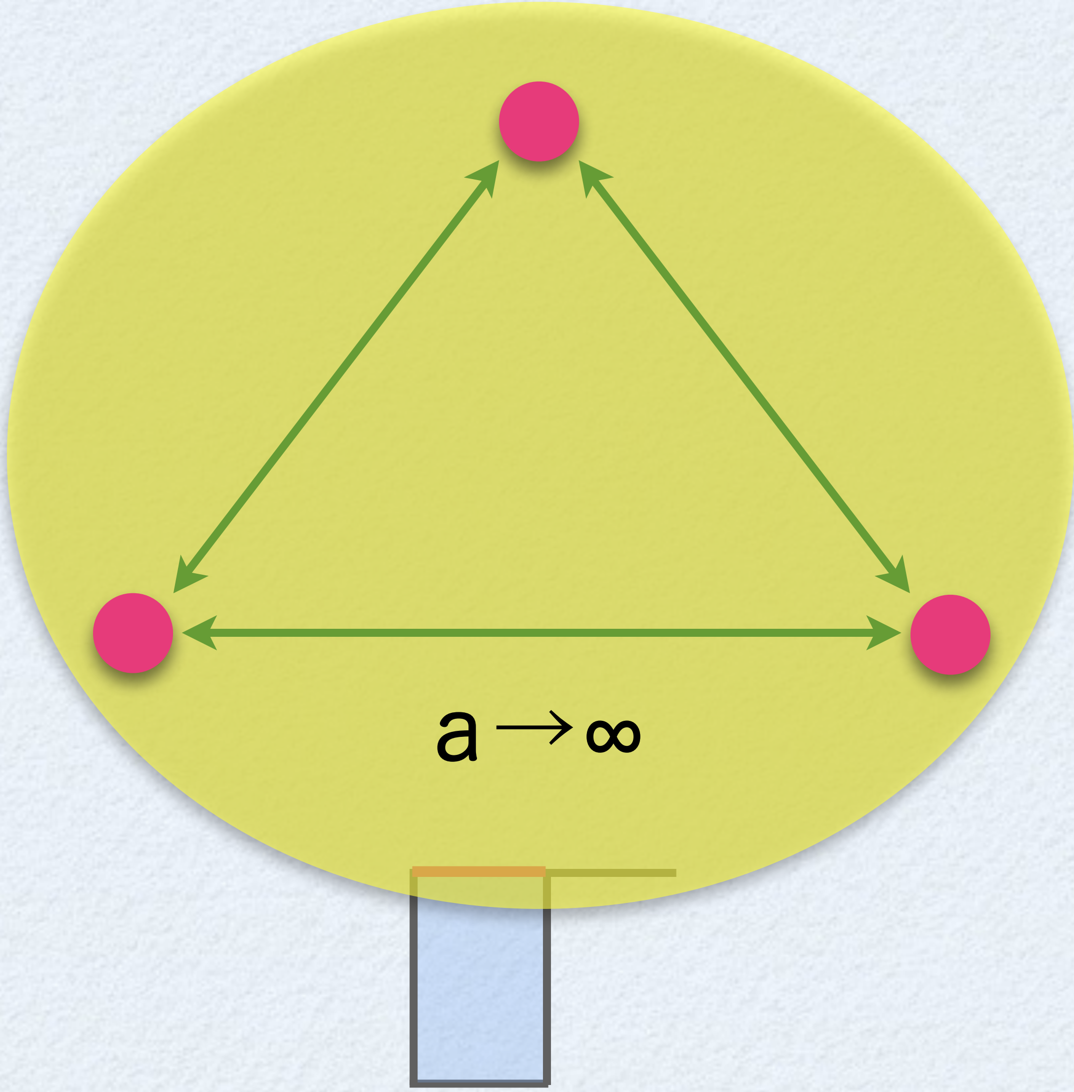
All the levels are of the 0^+ kind; corresponding wave functions are symmetric; the energies $E_N \ll 1/r_0^2$ (we use $\hbar = m = 1$); the range of these bound states is much larger than r_0.

Efimov (1970)

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)



Efimov effect

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Efimov (1970)

Discrete scaling symmetry

Efimov effect

When 2 bosons interact with infinite “a”,
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Efimov (1970)

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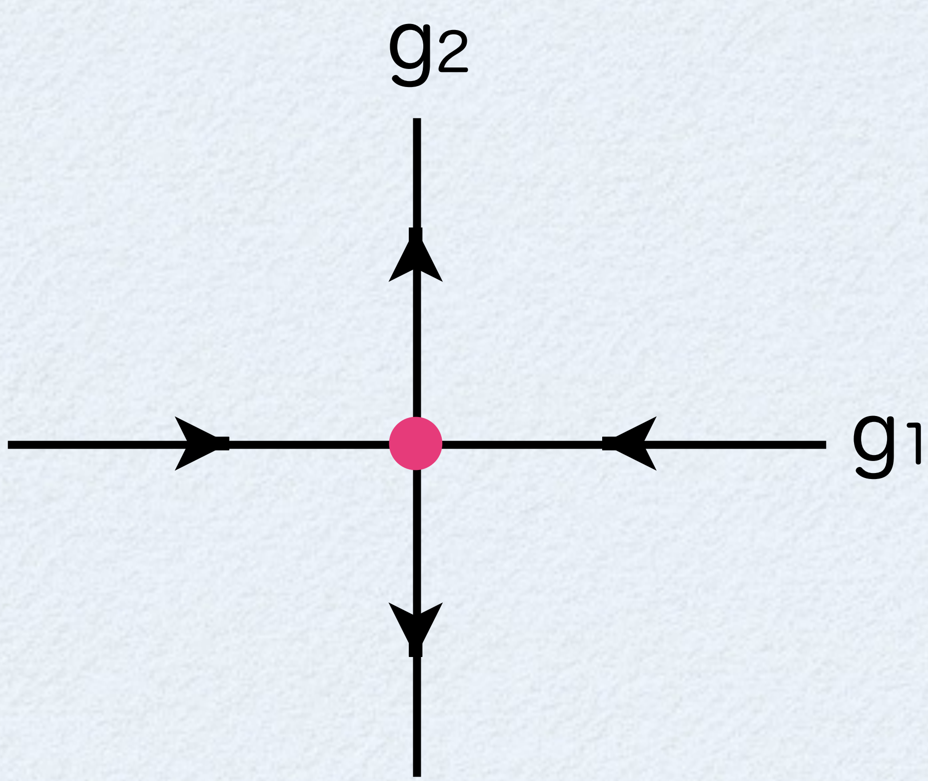
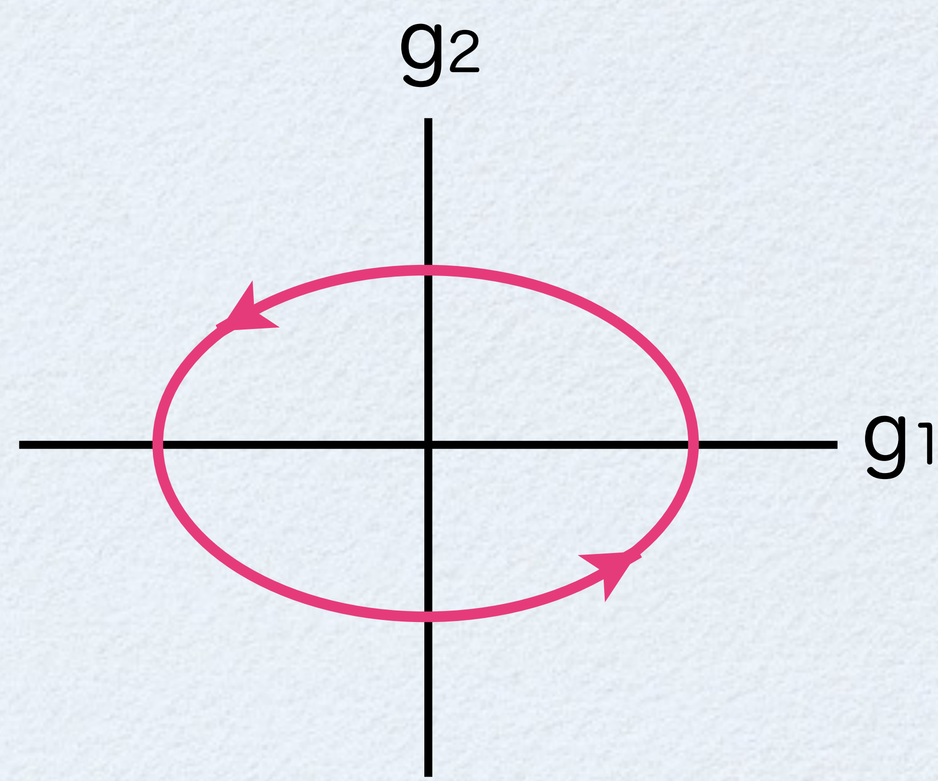
Discrete scaling symmetry

Renormalization group limit cycle

Renormalization group flow diagram in coupling space





RG fixed point

⇒ Scale invariance

E.g. critical phenomena

RG limit cycle

⇒ Discrete scale invariance

E.g. Efimov effect

Rare manifestation in physics !

Renormalization group limit cycle

K. Wilson (1971) considered for strong interactions

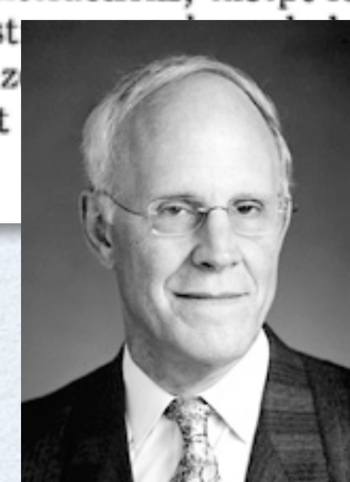
L REVIEW D VOLUME 3, NUMBER 8 15 APRIL 1971

Renormalization Group and Strong Interactions*

KENNETH G. WILSON
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
and
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850†

(Received 30 November 1970)

The renormalization-group method of Gell-Mann and Low is applied to field theories of strong interactions. It is assumed that renormalization-group equations exist for strong interactions which involve one or several momentum-dependent coupling constants. The further assumption that these coupling constants approach fixed values as the momentum goes to infinity is discussed in detail. However, an alternative is suggested, namely, that these coupling constants approach a limit cycle in the limit of large momenta. Some results of this paper are: (1) The e^+-e^- annihilation experiments above 1-GeV energy may distinguish a fixed point from a limit cycle or other asymptotic behavior. (2) If electrodynamics or weak interactions become strong above some large momentum Λ, then the renormalization group can be used (in principle) to determine the renormalized coupling constants of strong interactions, except for $U(3)\times U(3)$ symmetry-breaking parameters. (3) Mass terms in the Lagrangian of st
must break a symmetry of the combined interactions with z
weak interactions can be understood assuming only that
interactions.



QCD is asymptotic free

(2004 Nobel prize)

Renormalization group limit cycle

K. Wilson (1971) considered for strong interactions

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Efimov effect (1970) is its rare manifestation!

Effective field theory

PHYSICAL REVIEW LETTERS

VOLUME 82 18 JANUARY 1999 NUMBER 3

Renormalization of the Three-Body System with Short-Range Interactions

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[1]*Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195*
[2]*TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3*
[3]*Kellogg Radiation Laboratory, 106-38, California Institute of Technology, Pasadena, California 91125*
[4]*Department of Physics, University of Washington, Seattle, Washington 98195*
(Received 9 September 1998)

We discuss renormalization of the nonrelativistic three-body problem with short-range forces. The problem becomes nonperturbative at momenta of the order of the inverse of the two-body scattering length, and an infinite number of graphs must be summed. This summation leads to a cutoff dependence that does not appear in any order in perturbation theory. We argue that this cutoff dependence can be absorbed in a single three-body counterterm and compute the running of the three-body force with the cutoff. We comment on the relevance of this result for the effective field theory program in nuclear and molecular physics. [S0031-9007(98)08276-3]

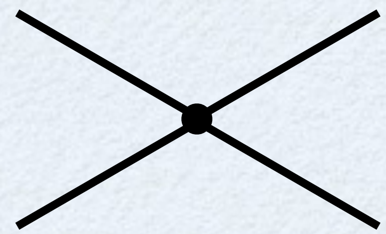
PACS numbers: 03.65.Nk, 11.80.Jy, 21.45.+v, 34.20.Gj

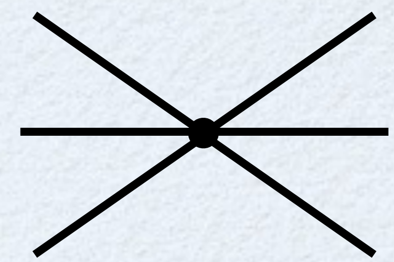
Systems composed of particles with momenta k much

dence can be absorbed in the coefficients of the leading-

Effective field theory

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi + g_2(\psi^\dagger\psi)^2 + g_3(\psi^\dagger\psi)^3$$

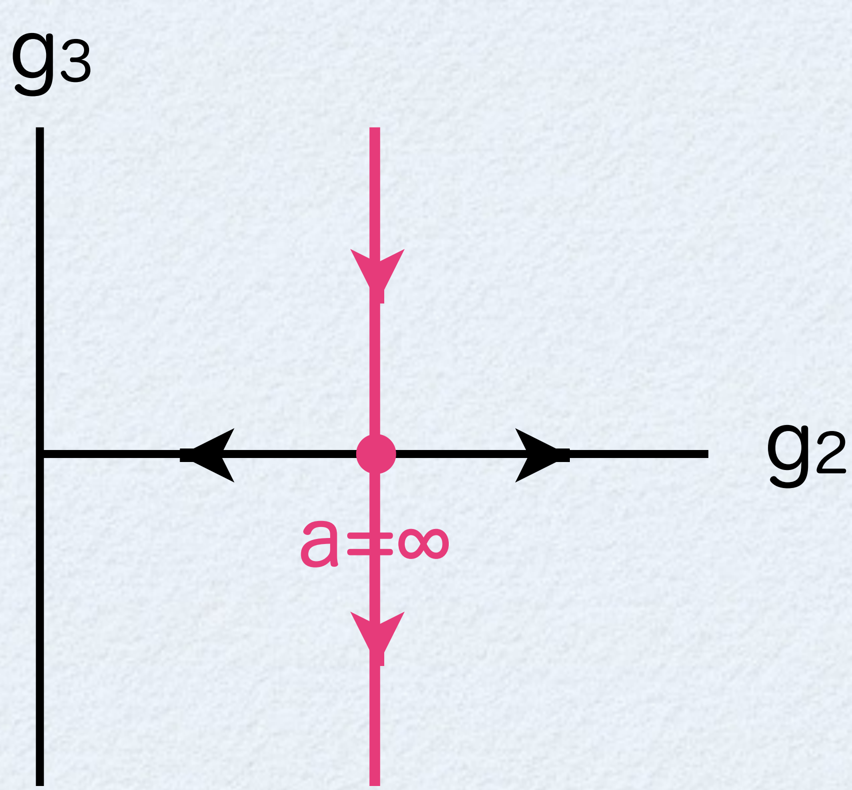




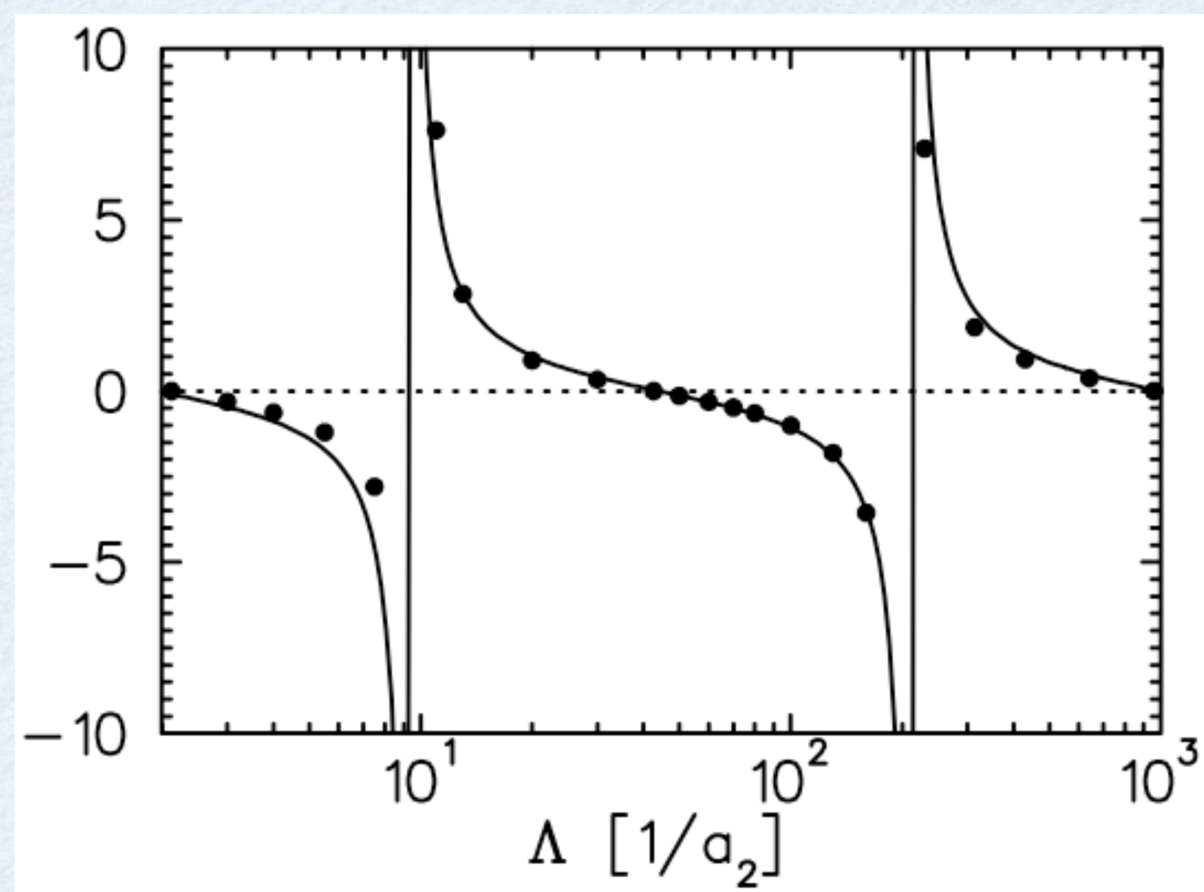
g_2 has a fixed point corresponding to a=∞

What is flow of g_3?

$$g_3(\Lambda) = -\frac{\sin[s_0 \ln(\Lambda/\Lambda_*) - \arctan(1/s_0)]}{\sin[s_0 \ln(\Lambda/\Lambda_*) + \arctan(1/s_0)]}$$



RG limit cycle



Effective field theory

What is flow of g_3?

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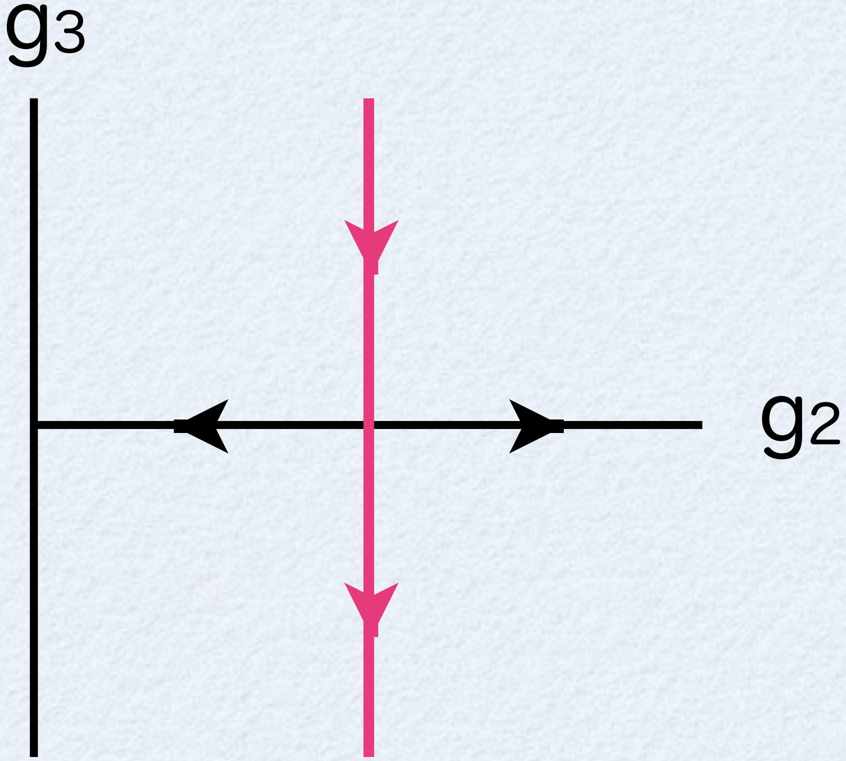
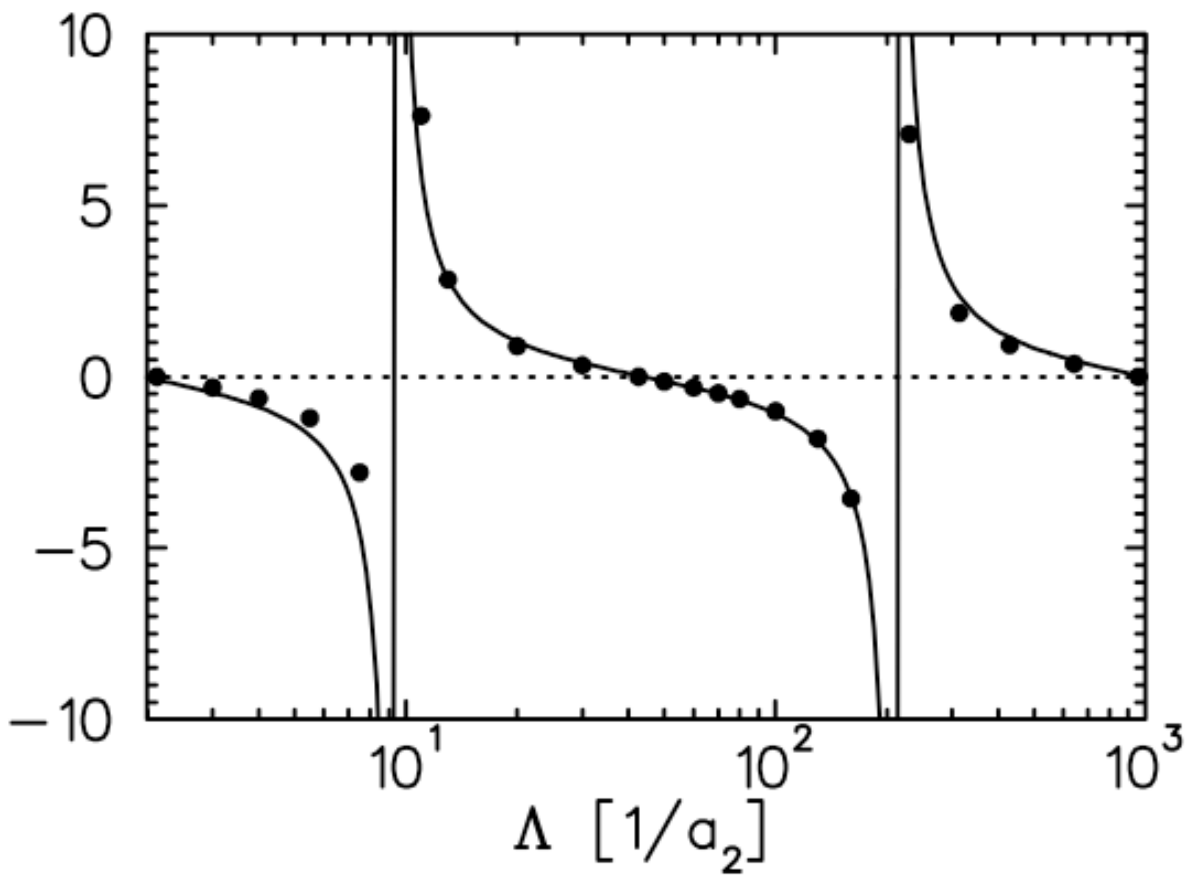
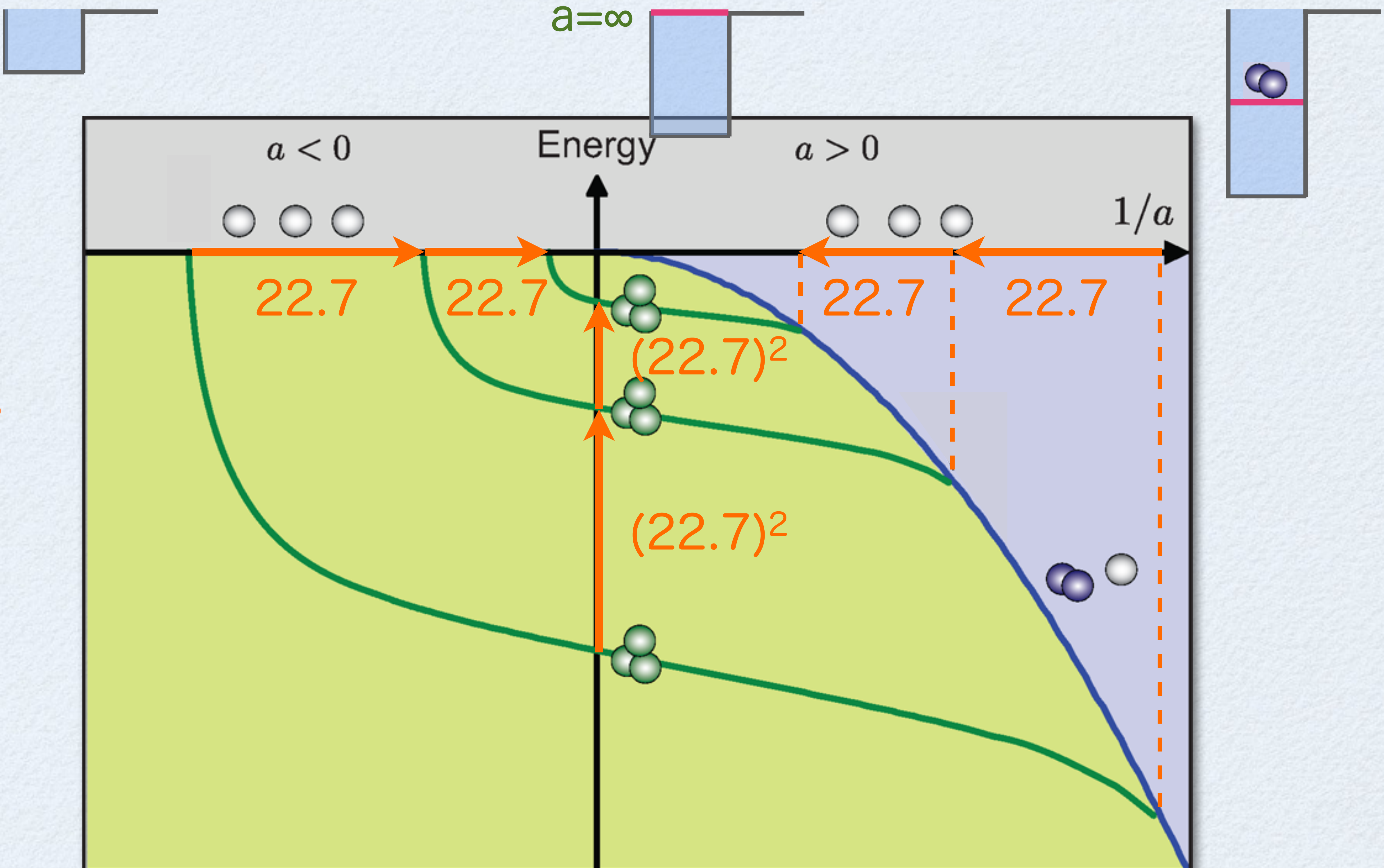




Efimov effect at a≠∞



Ferlaino & Grimm, Physics (2010)

Discrete scaling symmetry

Why 22.7 ?

Just a numerical number given by

22.6943825953666951928602171369...

log(22.6943825953666951928602171369...)

= 3.1221174311042196807309173243 8...

= π / 1.0062378251027814890640668123 4...

= π / s_0

$$\frac{2\pi}{s_0}\frac{\sinh(\frac{\pi}{6}s_0)}{\cosh(\frac{\pi}{2}s_0)}=\frac{\sqrt{3}\pi}{4}$$

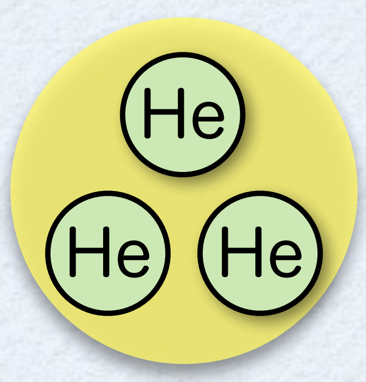
22.7 = exp(π / 1.006...)

Where Efimov effect appears ?

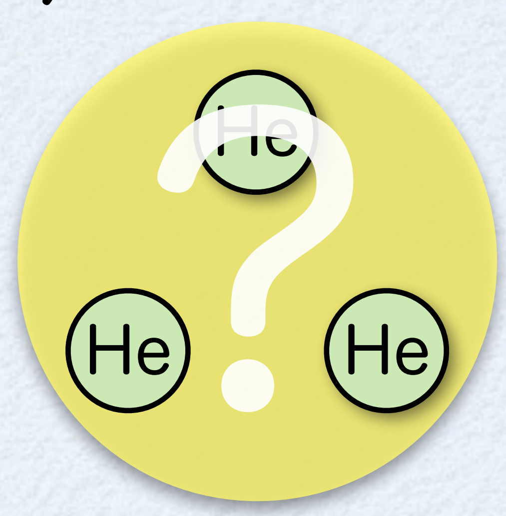
× Originally, Efimov considered
^3H nucleus (≈3n) and ^{12}C nucleus (≈3α)

△ ^4He atoms ($a \approx 1\times10^{-8}$ m $\approx 20 r_0$) ?

2 trimer states were predicted
1 was observed (1994)



E_b = 125.8 mK

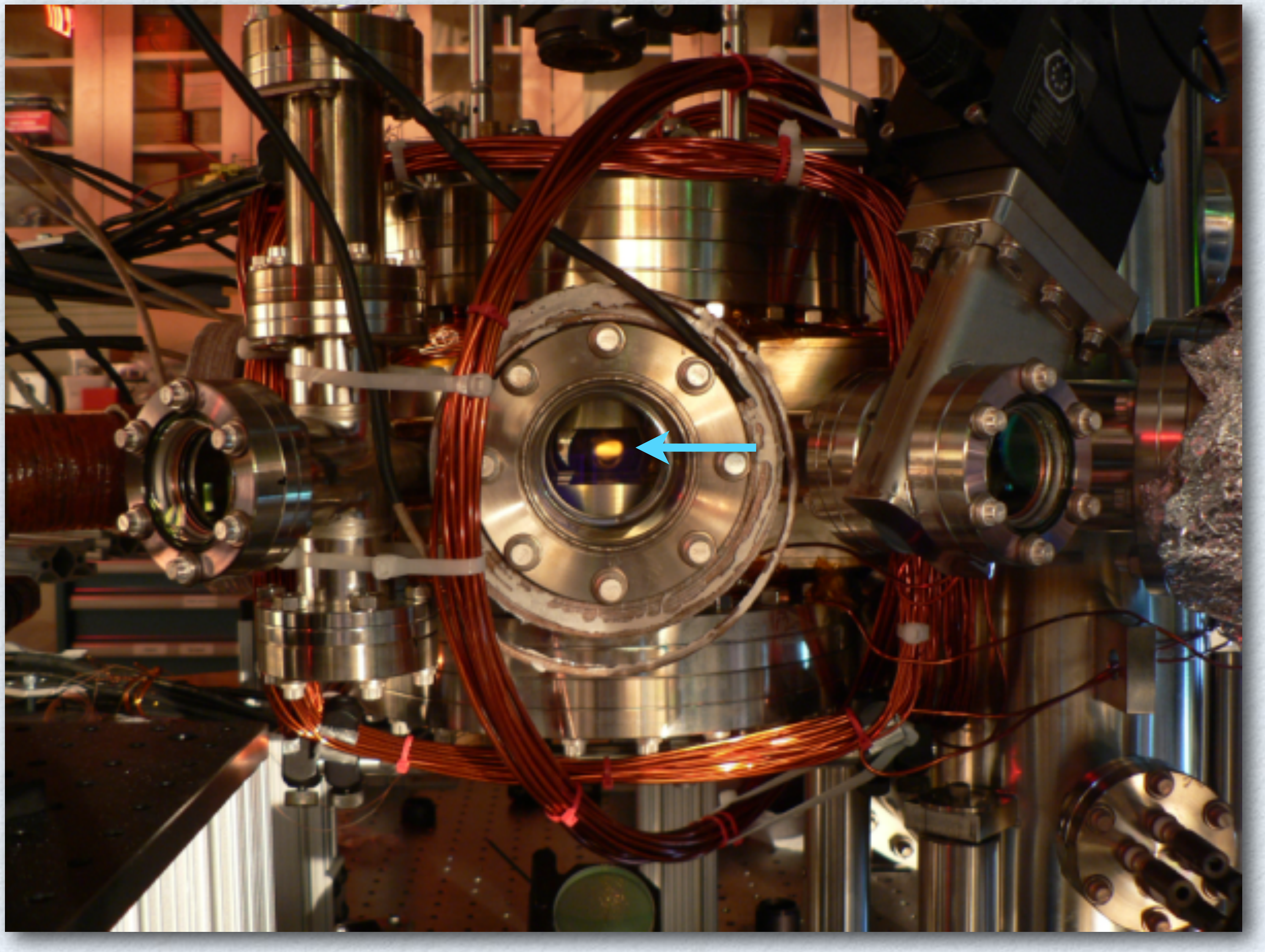


(E_b = 2.28 mK)

Ultracold atoms !

Ultracold atom experiments

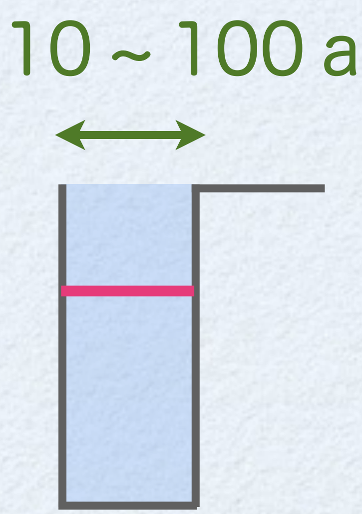
Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

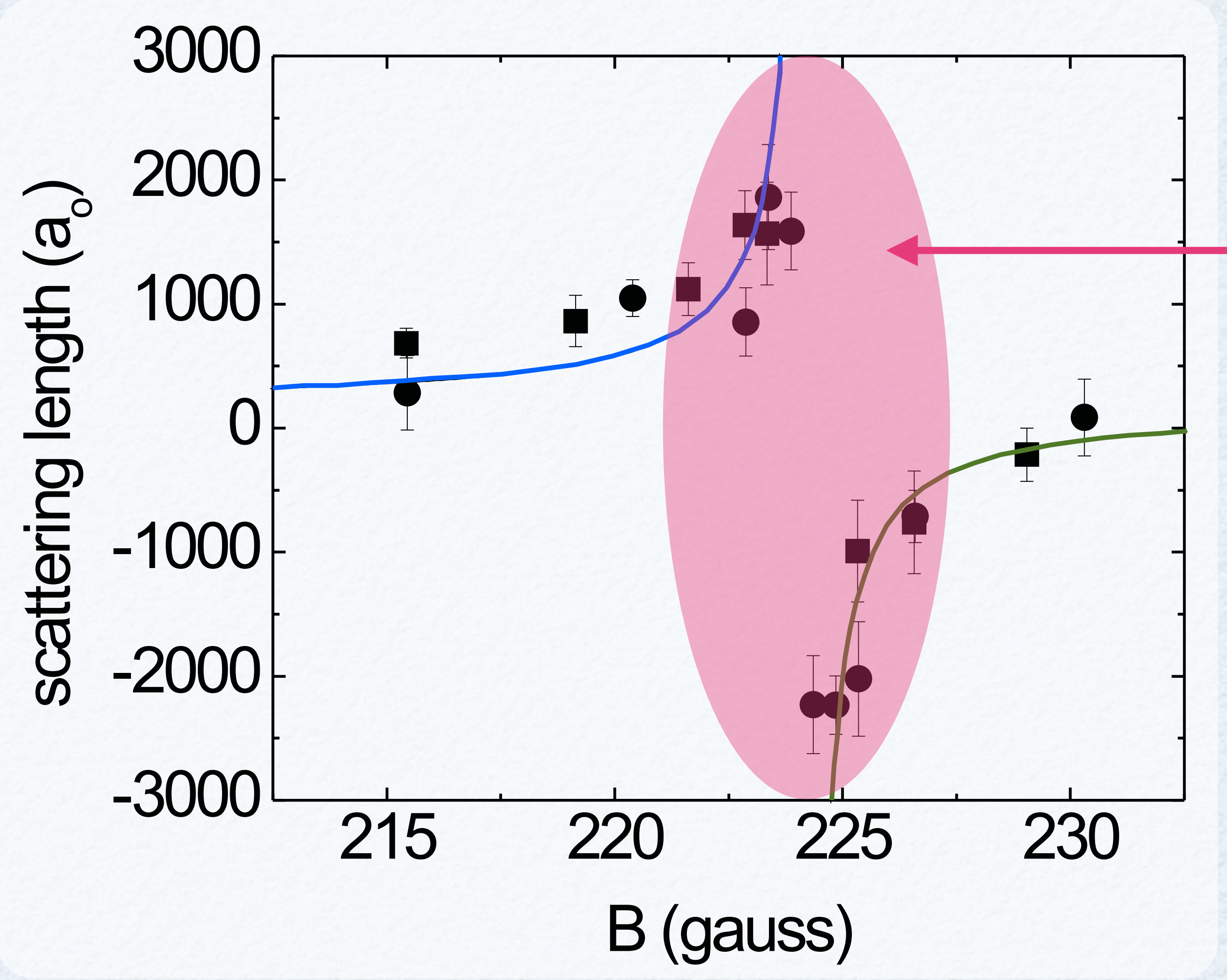


Ultracold atom experiments

Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

✓ Interaction strength by Feshbach resonances

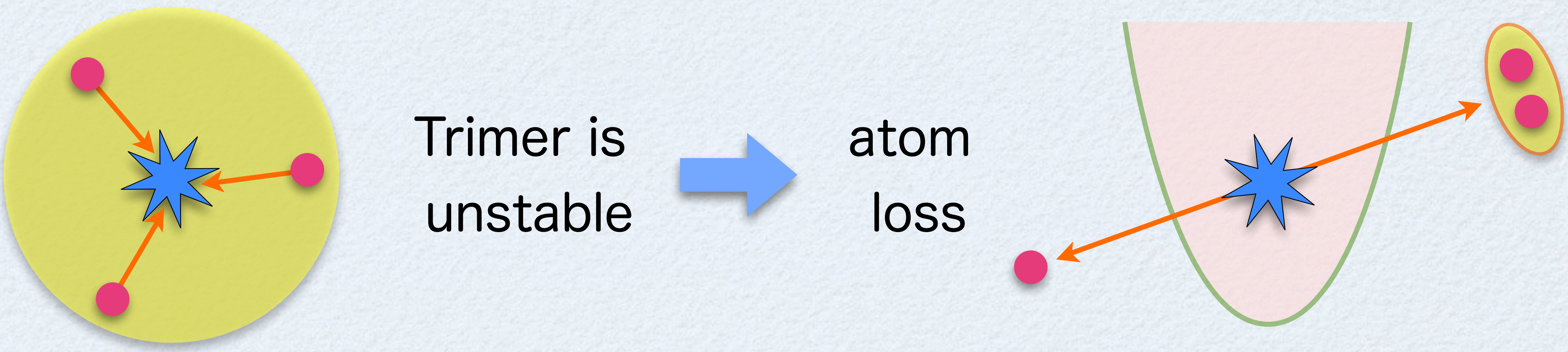




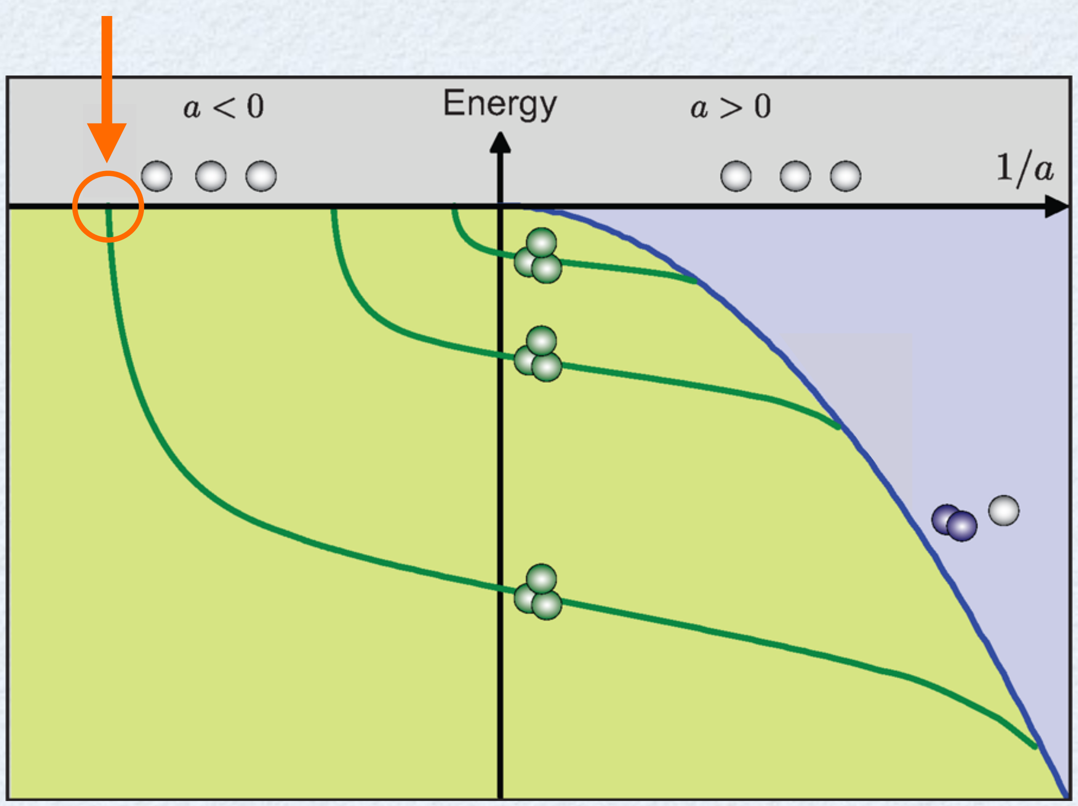
C.A. Regal & D.S. Jin PRL90 (2003)

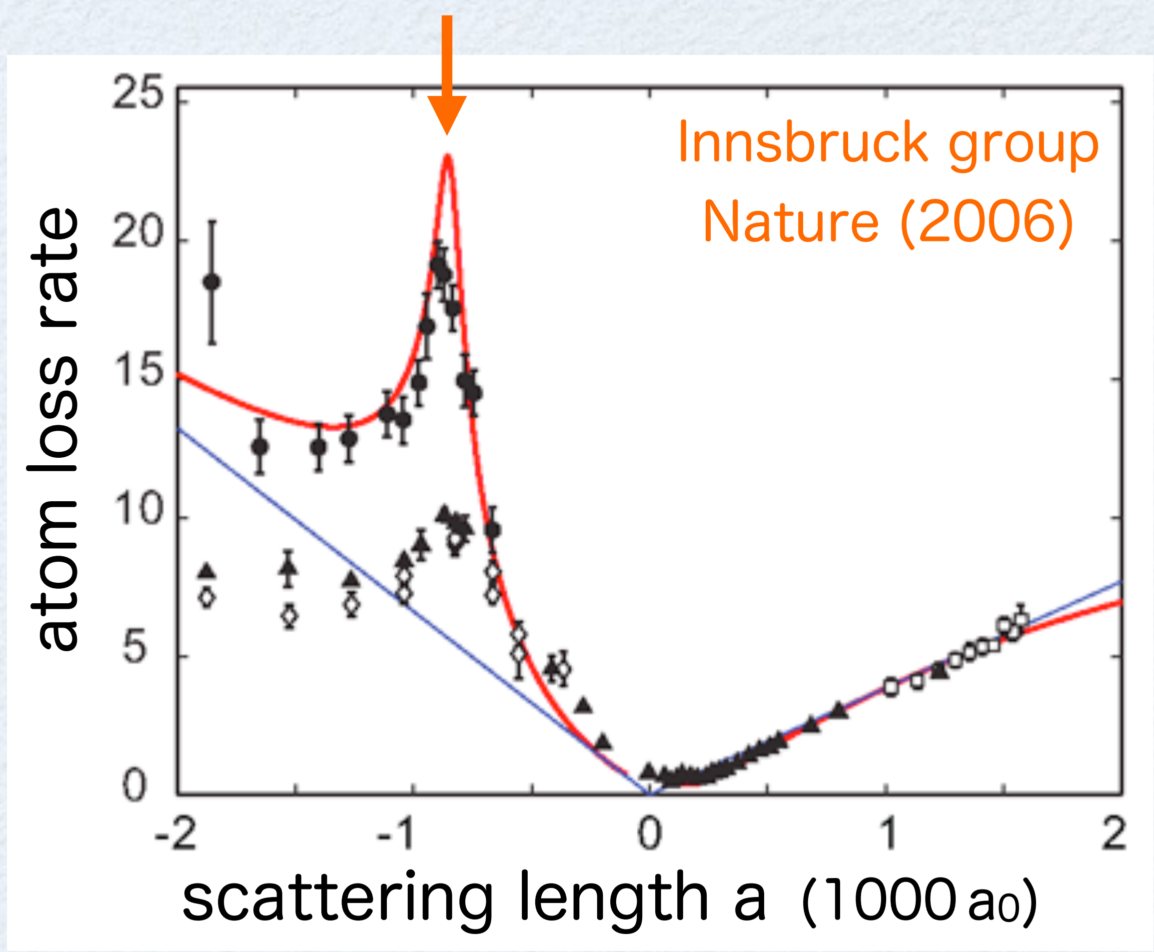
Ultracold atom experiments

First experiment by Innsbruck group for ^{133}Cs (2006)



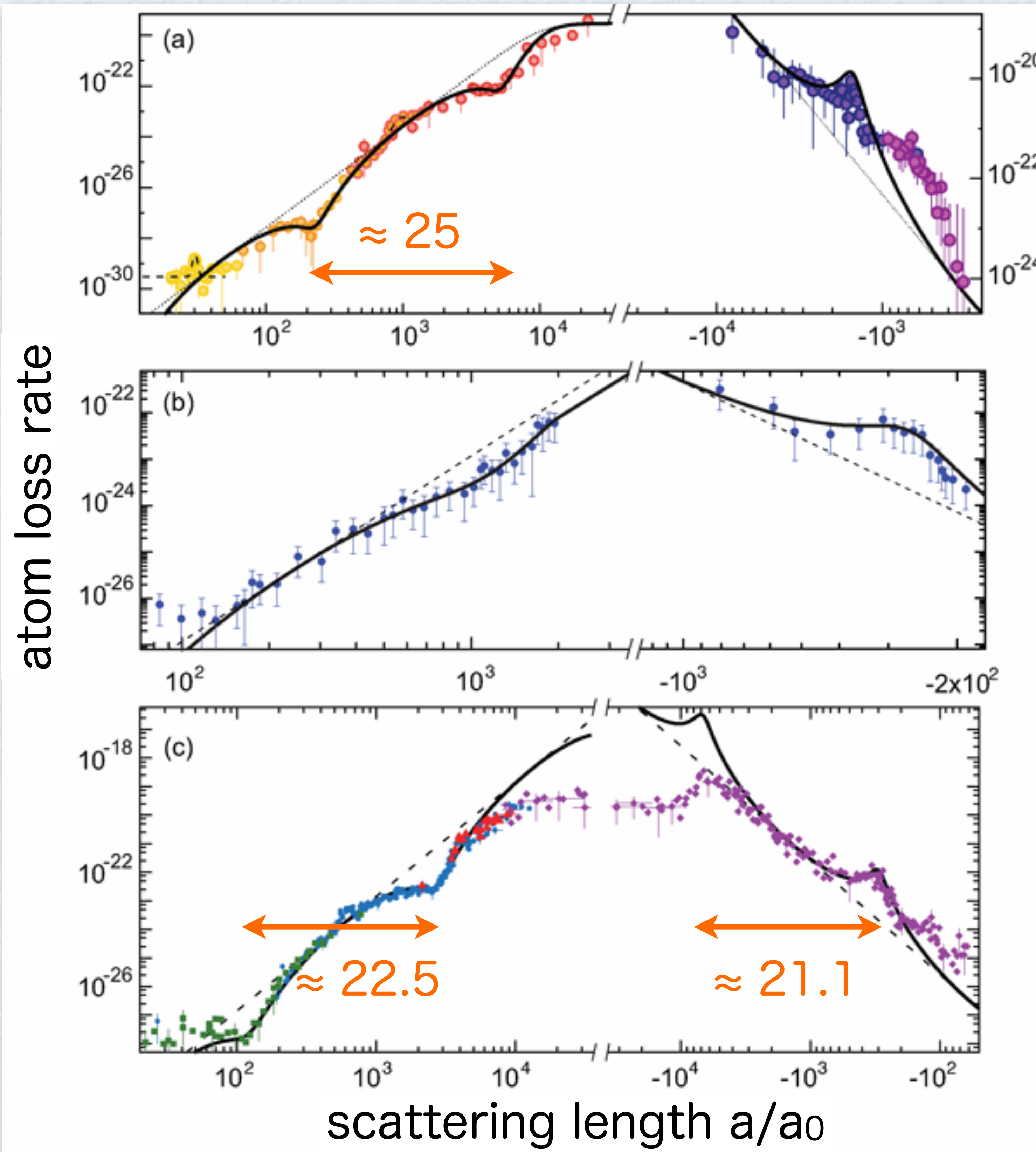
signature of trimer formation





Ultracold atom experiments

F. Ferlaino & R. Grimm, Physics 3, 9 (2010)



Florence group
for ^{39}K (2009)

Bar-Ilan University
for ^{7}Li (2009)

Rice University
for ^{7}Li (2009)

Discrete scaling
& Universality !

New discovery

1. Universality in physics
2. What is the Efimov effect ?
3. Super Efimov effect

Few-body universality

Efimov effect

- 3 bosons
- 3 dimensions
- s-wave resonance

exponential scaling

$$E_n \sim e^{-2\pi n}$$

Super Efimov effect

New !

- 3 fermions
- 2 dimensions
- p-wave resonance

"doubly" exponential

$$E_n \sim e^{-2e^{3\pi n/4}}$$

PRL **110,** 235301 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

Yusuke Nishida,[1] Sergej Moroz,[2] and Dam Thanh Son[3]
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[2]*Department of Physics, University of Washington, Seattle, Washington 98195, USA*
[3]*Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA*
(Received 18 January 2013; published 4 June 2013)



Few-body universality

Efimov effect

- 3 bosons
- 3 dimensions
- s-wave resonance

exponential scaling

$$E_n \sim e^{-2\pi n}$$

Super Efimov effect

New !

- 3 fermions
- 2 dimensions
- p-wave resonance

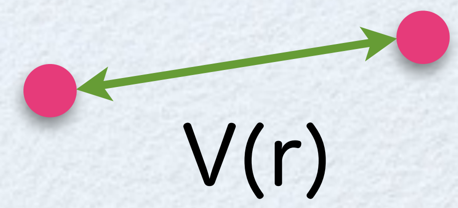
"doubly" exponential

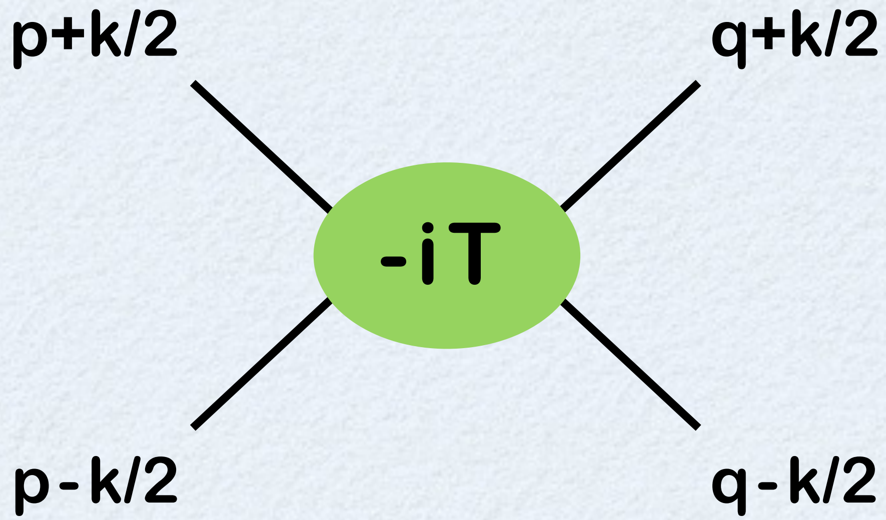
$$E_n \sim e^{-2e^{3\pi n/4}}$$

- Low-energy EFT for 2D p-wave scattering
- (conventional) RG analysis ⇐ Exact !
- "Non-relativistic" and "few-body" problem

P-wave scattering in 2D

Two fermions with short-range potential





⇒ Effective range expansion

Cf. H.-W. Hammer & D. Lee
Ann. Phys. 325, 2212 (2010)

$$-iT = \frac{2i}{m}\frac{\vec{p}\cdot\vec{q}}{-\frac{1}{a} - \frac{m\varepsilon}{\pi}\ln\left(-\frac{\Lambda^2}{m\varepsilon}\right) + \sum_{n=2}^{\infty} c_n\,(m\varepsilon)^n}$$

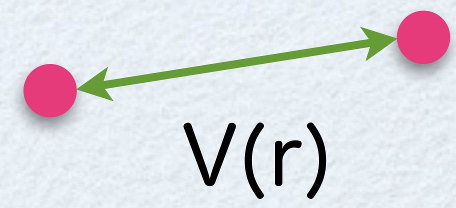
scattering "length"

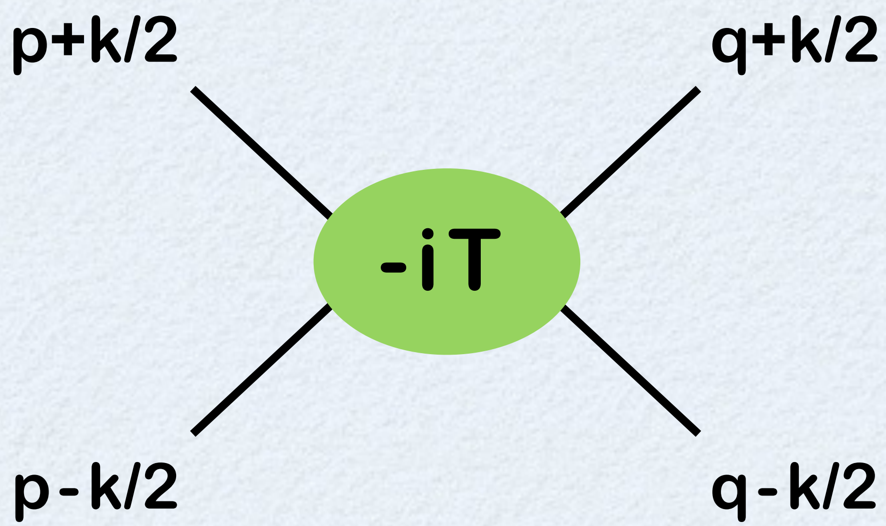
effective "range"

collision energy $\varepsilon = E - \frac{k^2}{4m} + i0^+$

P-wave scattering in 2D

Two fermions with short-range potential





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Cf. H.-W. Hammer & D. Lee
Ann. Phys. 325, 2212 (2010)

$$-iT = \frac{2i}{m} \frac{\vec{p} \cdot \vec{q}}{-\frac{1}{a} - \frac{m\varepsilon}{\pi} \ln\left(-\frac{\Lambda^2}{m\varepsilon}\right) + \sum_{n=2}^{\infty} c_n (m\varepsilon)^n}$$

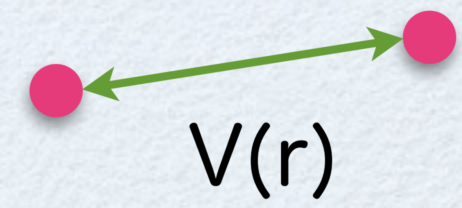
resonance
($a \to \infty$)

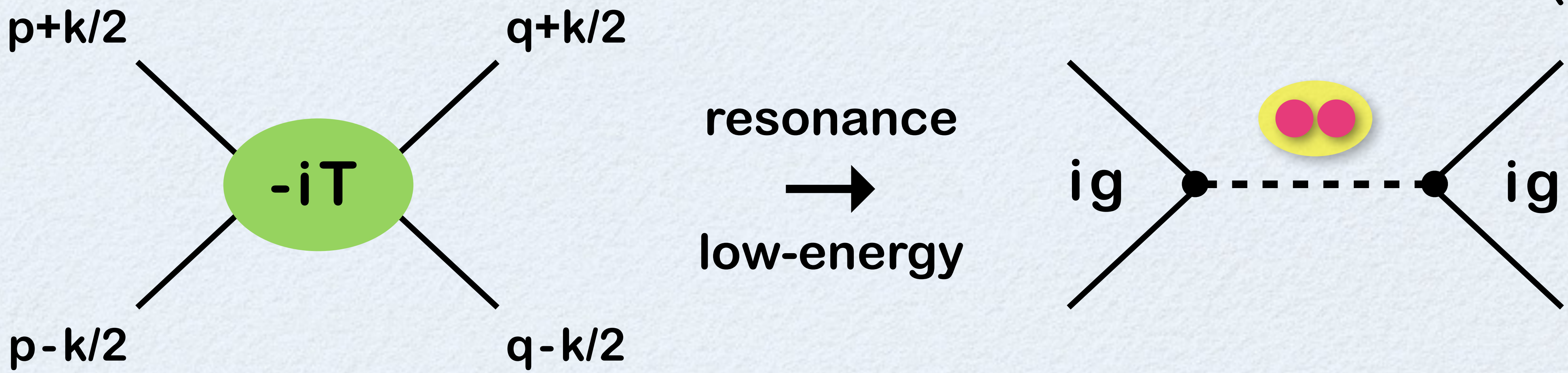
low-energy
($\varepsilon \to 0$)

collision energy $\varepsilon = E - \frac{k^2}{4m} + i0^+$

P-wave scattering in 2D

Two fermions with short-range potential

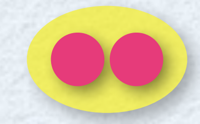




⇒ Effective range expansion

$$-iT \to -\frac{2\pi\,\vec{p}\cdot\vec{q}}{m^2 \ln\left(-\frac{\Lambda^2}{m\varepsilon}\right)} \times \frac{i}{E - \frac{k^2}{4m} + i0^+}$$

$= (ig)^2\, p \cdot q$ propagator of dimer

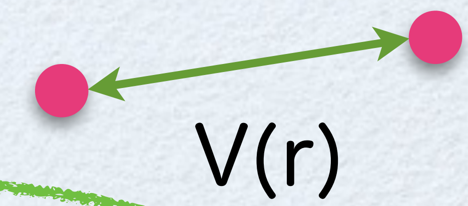


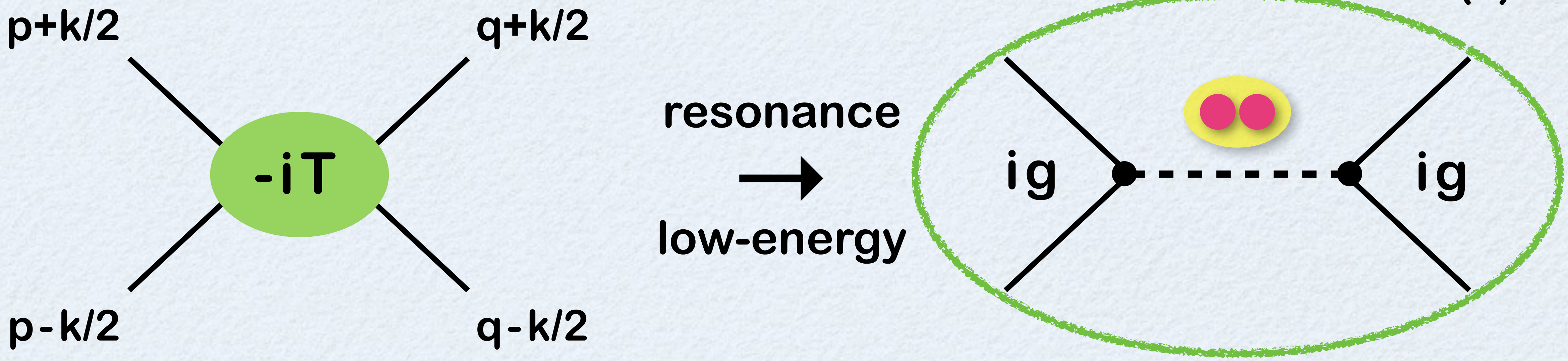
“running” coupling

(logarithmic decrease toward low-energy $p/\Lambda \to 0$)

P-wave scattering in 2D

Two fermions with short-range potential





⇒ Low-energy effective field theory

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\boldsymbol{\nabla}^2}{2m} \right) \psi + \sum_{\pm} \Big[\phi_\pm^\dagger \left(i\partial_t + \frac{\boldsymbol{\nabla}^2}{4m} \right) \phi_\pm$$

$$+ g\, \phi_\pm^\dagger \psi \,(-i)\, (\nabla_x \pm i \nabla_y)\, \psi + \text{h. c.} \Big]$$

dimer field $\Phi_\pm$ couples to two fermions ψ

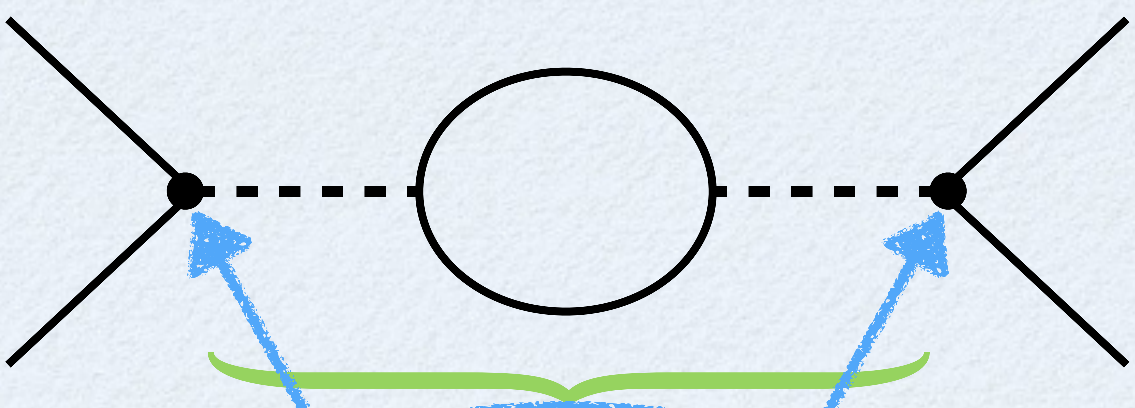
with orbital angular momentum L=±1

RG in 2-body sector

Low-energy effective field theory

$$\mathcal{L} = \psi^\dagger\left(i\partial_t + \frac{\boldsymbol{\nabla}^2}{2m}\right)\psi + \sum_{\pm}\left[\phi_\pm^\dagger\left(i\partial_t + \frac{\boldsymbol{\nabla}^2}{4m}\right)\phi_\pm + g\,\phi_\pm^\dagger\psi\,(-i)\,(\nabla_x \pm i\nabla_y)\,\psi + \text{h.c.}\right] + \cdots$$

marginal coupling (g) — irrelevant ($\cdots$)



$$\frac{1 - \frac{g^2}{\pi}\ln\frac{\Lambda}{e^{-s}\Lambda}}{E - \frac{k^2}{4m} + i0^+}$$

($e^{-s}\Lambda < p < \Lambda$ integrated out)

RG equation $\displaystyle \frac{dg}{ds} = -\frac{g^3}{2\pi}$

$$\Rightarrow\quad g^2(s) = \frac{1}{\frac{1}{g^2(0)} + \frac{s}{\pi}} \to \frac{\pi}{s}$$

logarithmical decrease toward low-energy $s \to \infty$

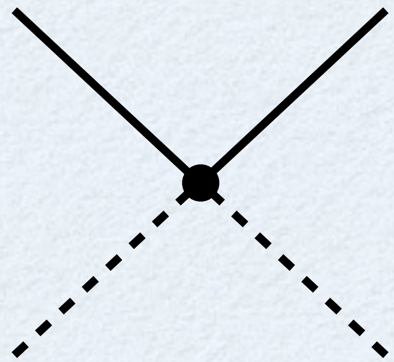
RG in 3-body sector

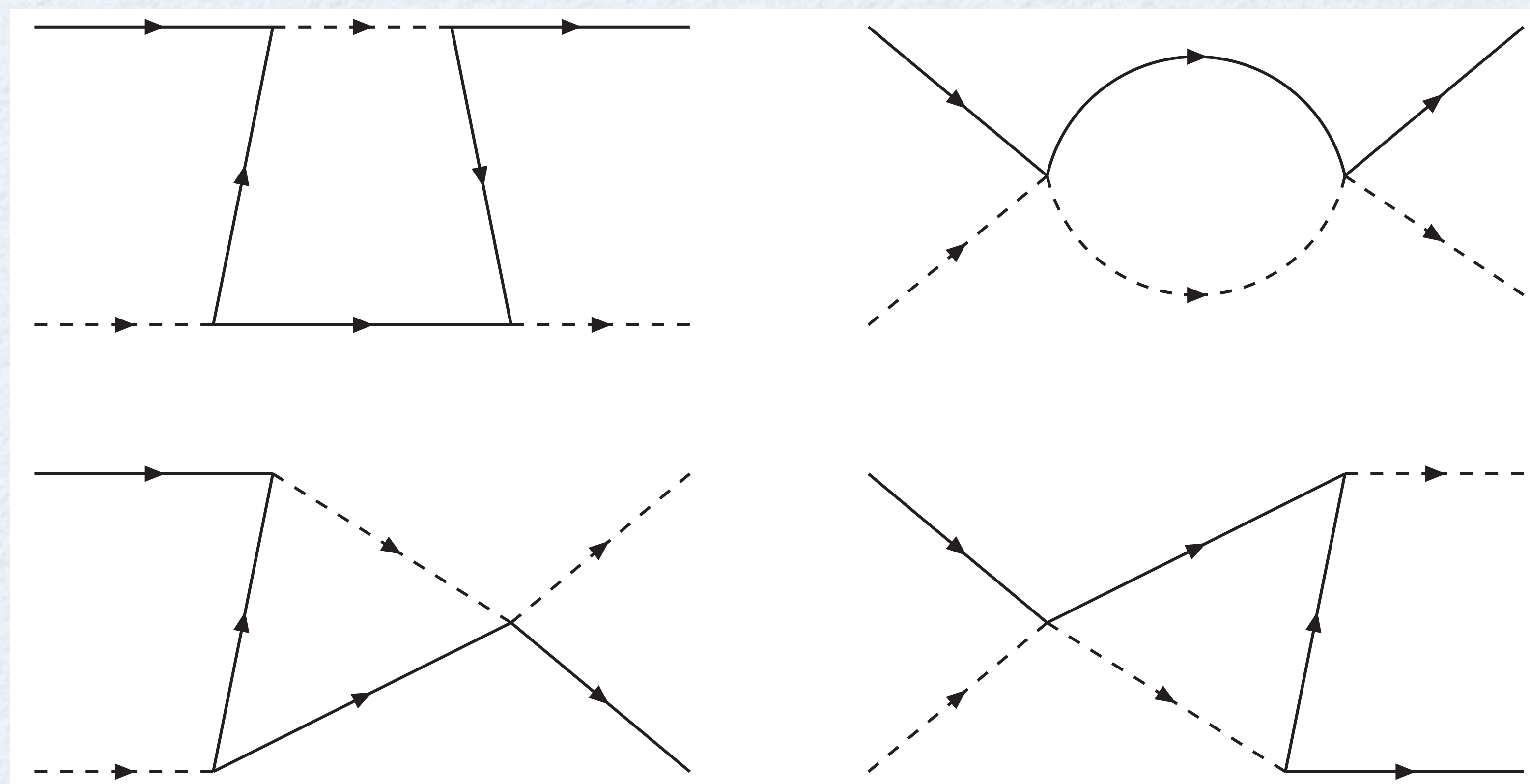
3-body problem ⇔ fermion+dimer scattering

$$\mathcal{L}_{3\text{-body}} = v_3 \sum_{a=\pm} \psi^\dagger \phi_a^\dagger \phi_a \psi + \cdots$$

irrelevant

marginal coupling renormalized by





⇒ RG equation

$$\frac{dv_3}{ds} = \frac{16}{3\pi} g^4 - \frac{11}{3\pi} g^2 v_3 + \frac{2}{3\pi} v_3^2$$

RG in 3-body sector

3-body problem ⇔ fermion+dimer scattering

$$\mathcal{L}_{3\text{-body}} = v_3 \sum_{a=\pm} \psi^\dagger \phi_a^\dagger \phi_a \psi + \cdots$$

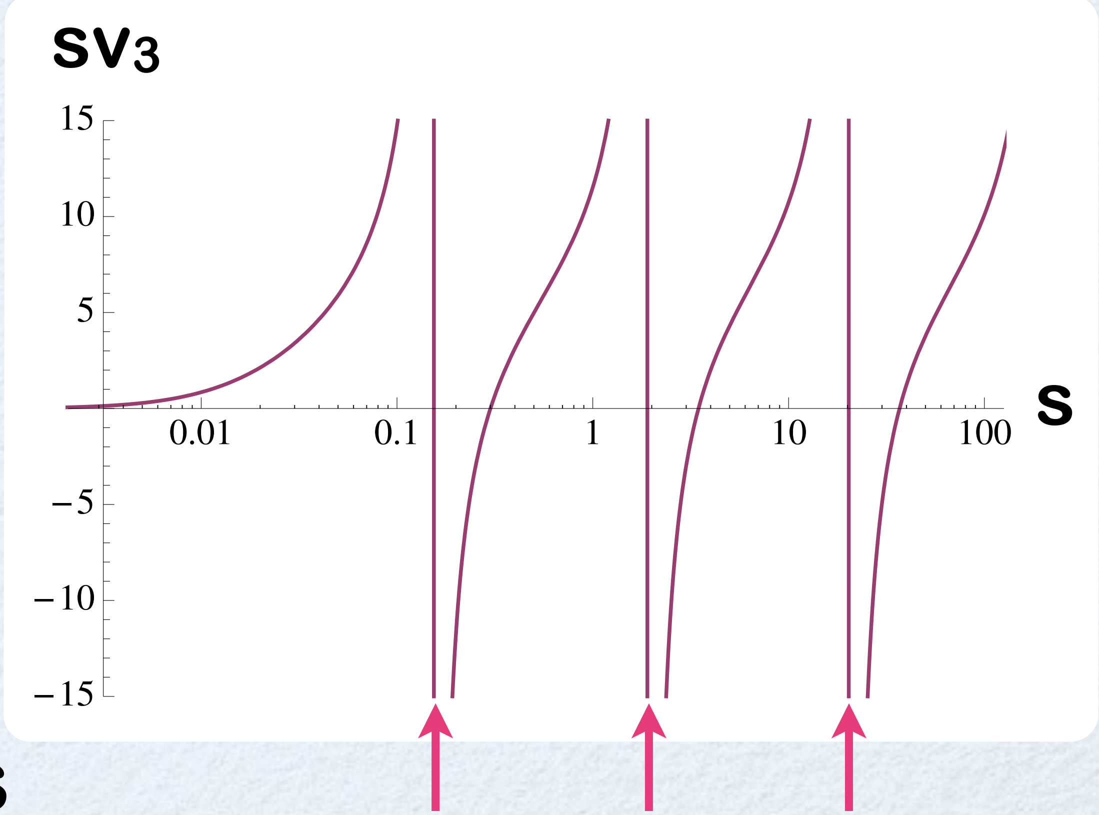
irrelevant

marginal coupling @ low-energy limit s→∞

$$v_3(s) \to \frac{2\pi}{s}\left\{1 - \cot\left[\frac{4}{3}(\ln s - \theta)\right]\right\}$$

diverges at $\ln s = \frac{3\pi n}{4} + \theta$

$\ln s$: $\ln\ln \Lambda/\kappa$; θ: non-universal



=> characteristic energy scales

$$E_n \propto \frac{\Lambda^2}{m} e^{-2e^{3\pi n/4+\theta}}$$

Super Efimov effect !

Model confirmation

$$H = \int \frac{dk}{(2\pi)^2} \frac{k^2}{2m} \psi_k^\dagger \psi_k$$

$$- v_0 \sum_{a=\pm} \int \frac{dk\,dp\,dq}{(2\pi)^6} \psi^\dagger_{\frac{k}{2}+p} \chi_a(p) \psi^\dagger_{\frac{k}{2}-p} \times \psi_{\frac{k}{2}-q} \chi_{-a}(q) \psi_{\frac{k}{2}+q}$$

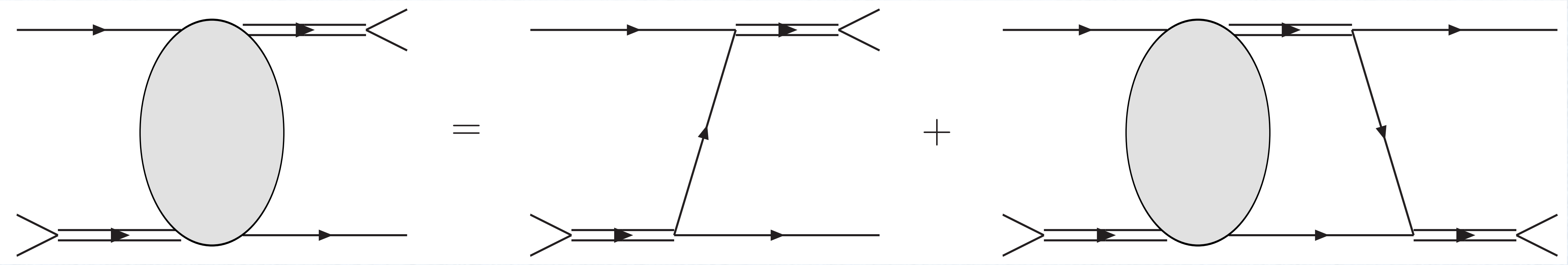
resonance (a→∞)

$$\chi_\pm(p) = (p_x \pm i p_y)\, e^{-p^2/(2\Lambda^2)}$$

Spinless fermions with a separable potential

3-body binding energies $\lambda_n = \ln\ln(mE_n/\Lambda^2)^{-1/2}$

=> solve STM equation numerically



Model confirmation

$$H = \int \frac{dk}{(2\pi)^2} \frac{k^2}{2m} \psi_k^\dagger \psi_k$$

$$- v_0 \sum_{a=\pm} \int \frac{dk\,dp\,dq}{(2\pi)^6} \underbrace{\psi^\dagger_{\frac{k}{2}+p} \chi_a(p) \psi^\dagger_{\frac{k}{2}-p}}_{} \times \underbrace{\psi_{\frac{k}{2}-q} \chi_{-a}(q) \psi_{\frac{k}{2}+q}}_{}$$

resonance (a→∞)

$$\chi_\pm(p) = (p_x \pm i p_y)\, e^{-p^2/(2\Lambda^2)}$$

Spinless fermions with a separable potential

3-body binding energies $\lambda_n = \ln\ln(mE_n/\Lambda^2)^{-1/2}$

n	λ_n	$\lambda_n - \lambda_{n-1}$
0	0.5632	—
1	2.770	2.207
2	5.078	2.308
3	7.430	2.352
4	9.785	2.355
5	12.141	2.356
∞	—	2.35619 ← $3\pi/4$

Super Efimov effect !

⇒ doubly exponential scaling $mE_n/\Lambda^2 \propto e^{-2e^{3\pi n/4+\theta}}$

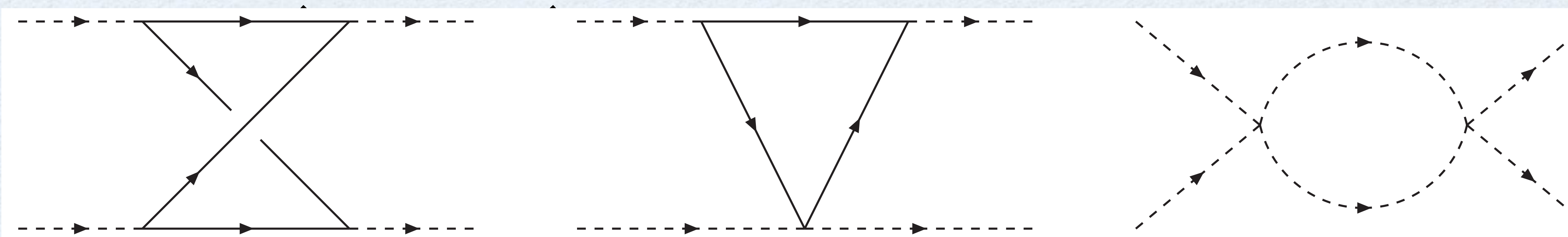
RG in 4-body sector

4-body problem ⇔ dimer+dimer scattering

$$\mathcal{L}_{4\text{-body}} = \sum_{a=\pm}\left[v_4 \phi_a^\dagger \phi_{-a}^\dagger \phi_{-a}\phi_a + v_4' \phi_a^\dagger \phi_a^\dagger \phi_a \phi_a\right] + \cdots$$

irrelevant

marginal couplings renormalized by



⇒ RG equations

$$\frac{dv_4}{ds} = -\frac{8}{\pi}g^4 + \frac{2}{\pi}g^2 v_3 - \frac{2}{\pi}g^2 v_4 + \frac{2}{\pi}v_4^2$$

$$\frac{dv_4'}{ds} = -\frac{4}{\pi}g^4 + \frac{2}{\pi}g^2 v_3 - \frac{2}{\pi}g^2 v_4' + \frac{2}{\pi}v_4'^2$$

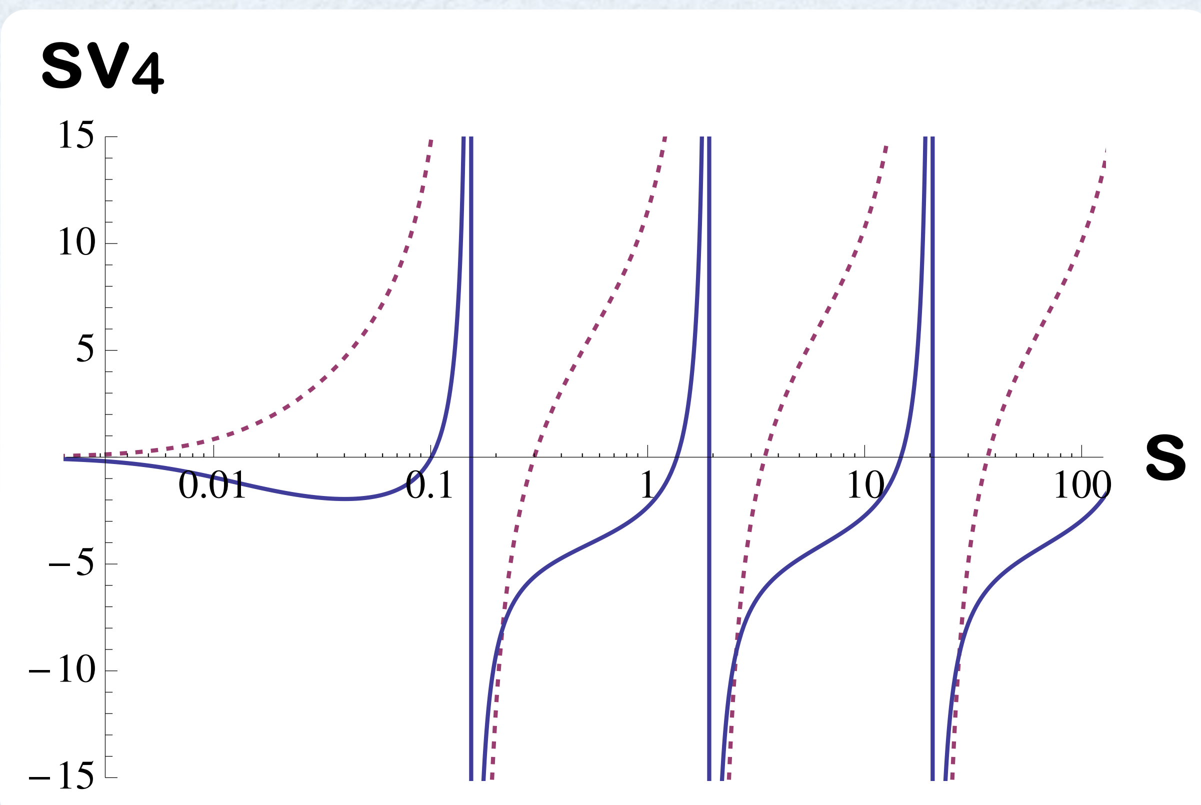
RG in 4-body sector

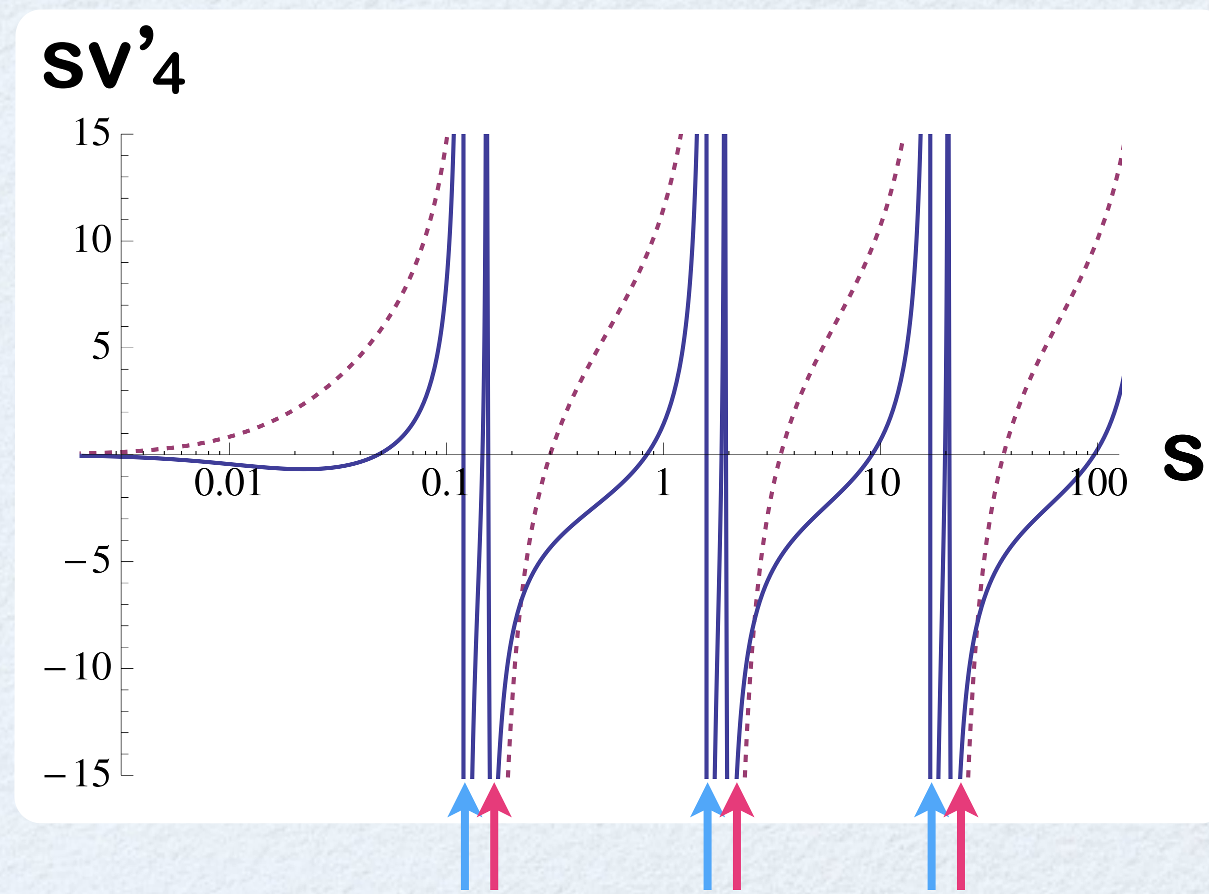
4-body problem ⇔ dimer+dimer scattering

$$\mathcal{L}_{4\text{-body}} = \sum_{a=\pm}\left[v_4 \phi_a^\dagger \phi_{-a}^\dagger \phi_{-a}\phi_a + v_4' \phi_a^\dagger \phi_a^\dagger \phi_a \phi_a\right] + \cdots$$

marginal couplings

irrelevant





L=±2 tetramers attached to every trimer

with resonance energy $E_n \sim e^{-2e^{3\pi n/4+\theta-0.188}}$

Efimov vs super Efimov

Efimov effect

- 3 bosons
- 3 dimensions
- s-wave resonance

exponential scaling

$$E_n \sim e^{-2\pi n}$$

Super Efimov effect

- 3 fermions
- 2 dimensions
- p-wave resonance

"doubly" exponential

$$E_n \sim e^{-2e^{3\pi n/4}}$$

PRL **110**, 235301 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

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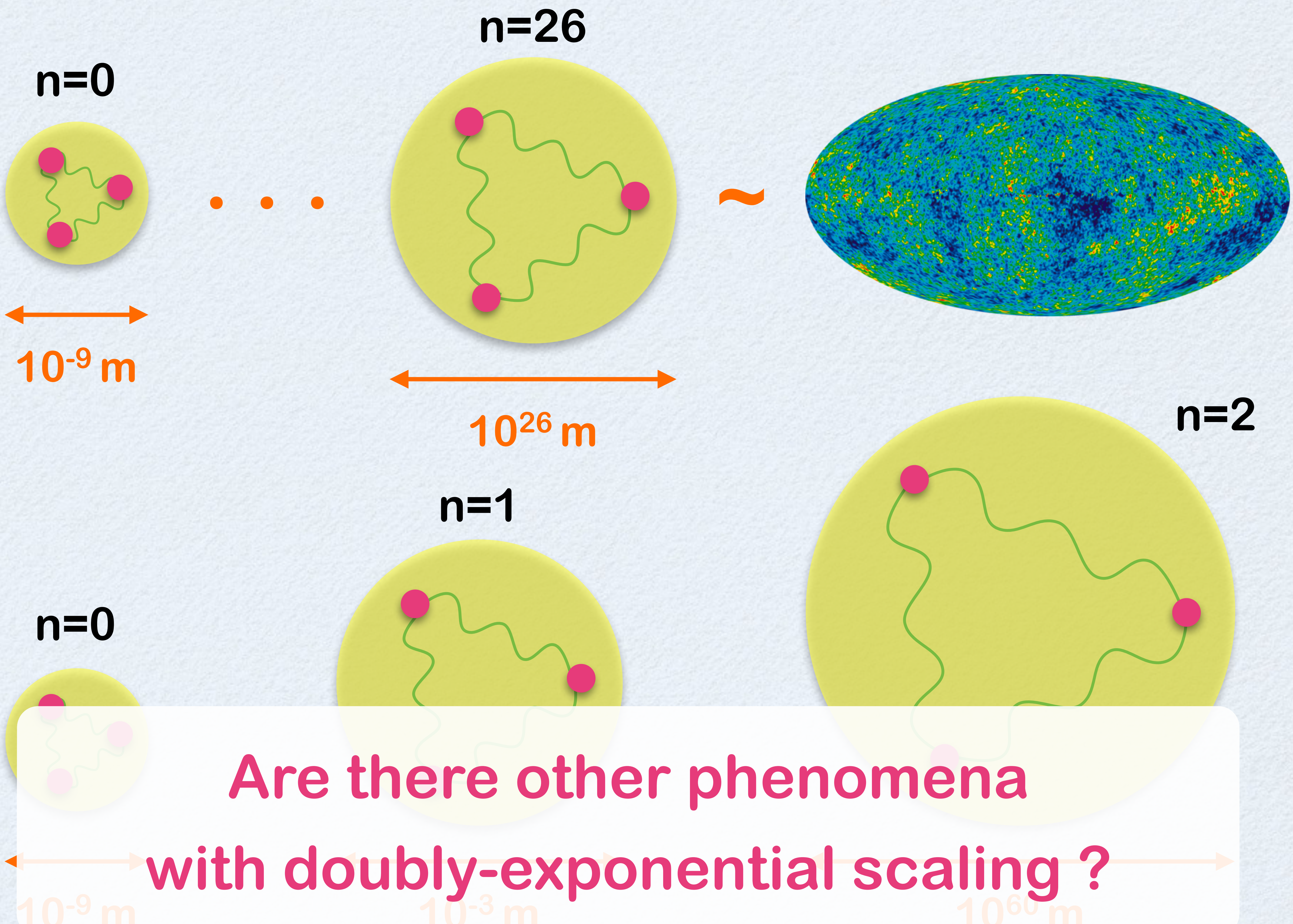
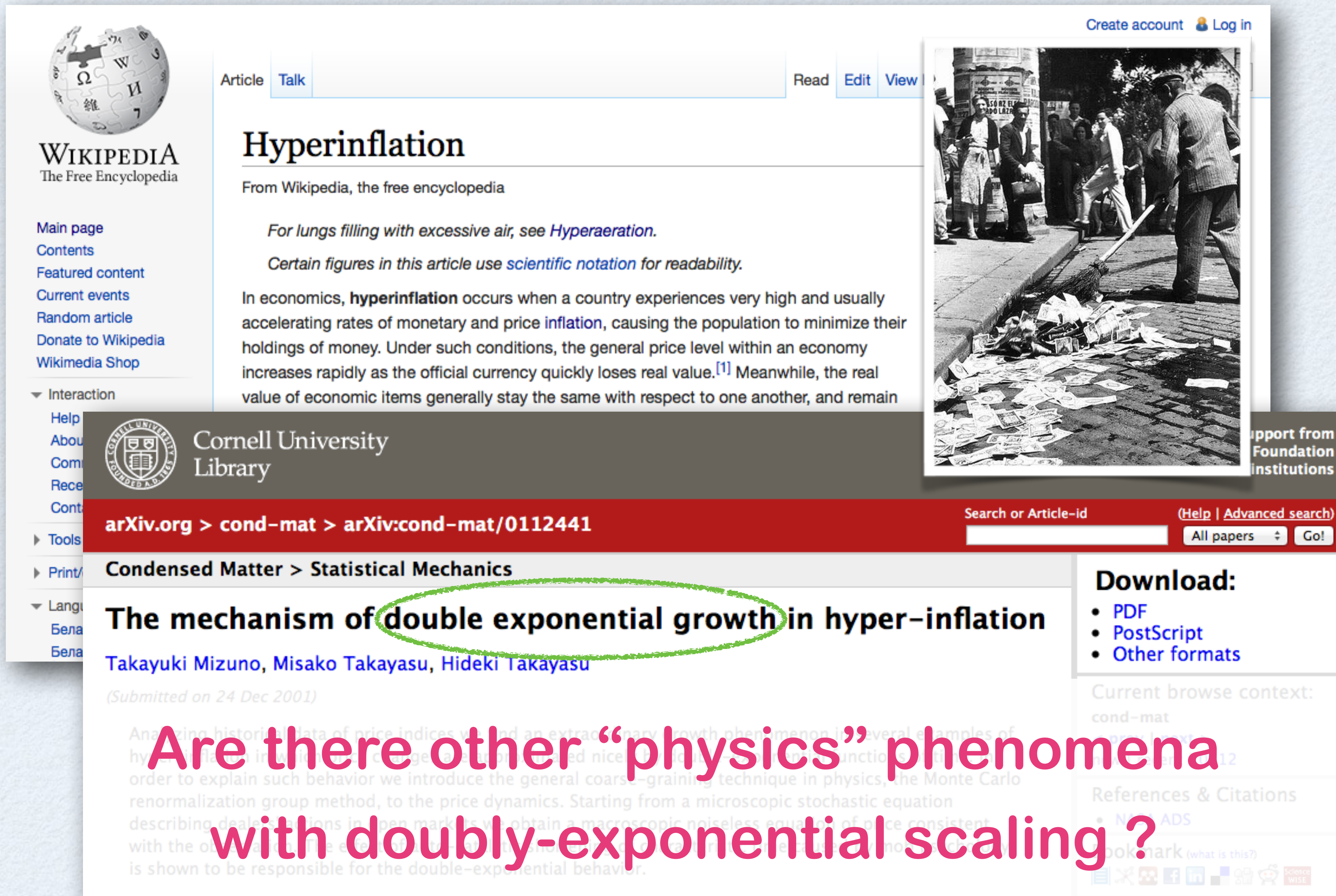
Efimov vs super Efimov



Are there other phenomena with doubly-exponential scaling ?

Efimov vs super Efimov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Hyperinflation

From Wikipedia, the free encyclopedia

For lungs filling with excessive air, see Hyperaeration.

Certain figures in this article use scientific notation for readability.

In economics, **hyperinflation** occurs when a country experiences very high and usually accelerating rates of monetary and price inflation, causing the population to minimize their holdings of money. Under such conditions, the general price level within an economy increases rapidly as the official currency quickly loses real value.[1] Meanwhile, the real value of economic items generally stay the same with respect to one another, and remain

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The mechanism of double exponential growth in hyper-inflation

Takayuki Mizuno, Misako Takayasu, Hideki Takayasu

(Submitted on 24 Dec 2001)

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Are there other "physics" phenomena with doubly-exponential scaling ?

Summary

Efimov effect

- 3 bosons
- 3 dimensions
- s-wave resonance

exponential scaling

$$E_n \sim e^{-2\pi n}$$

Super Efimov effect

New !

- 3 fermions
- 2 dimensions
- p-wave resonance

"doubly" exponential

$$E_n \sim e^{-2e^{3\pi n/4}}$$

- "Exact" RG analysis => super limit cycle
- The first of doubly-exponential scaling ?
- Extension to mass-imbalanced mixtures