Dimensional BCS-BEC crossover

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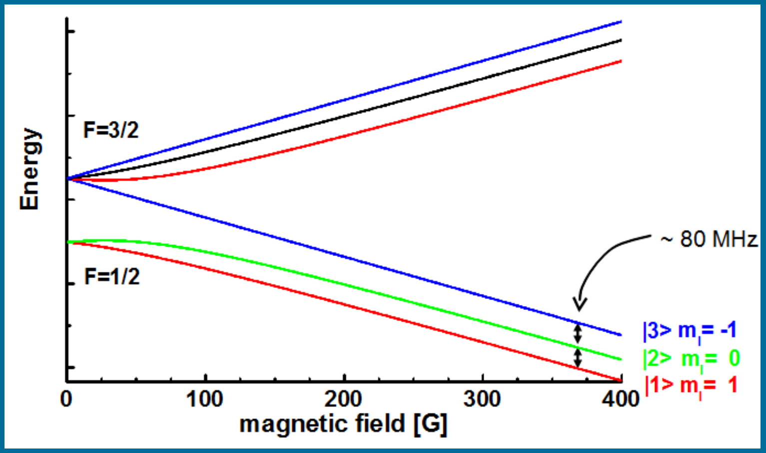
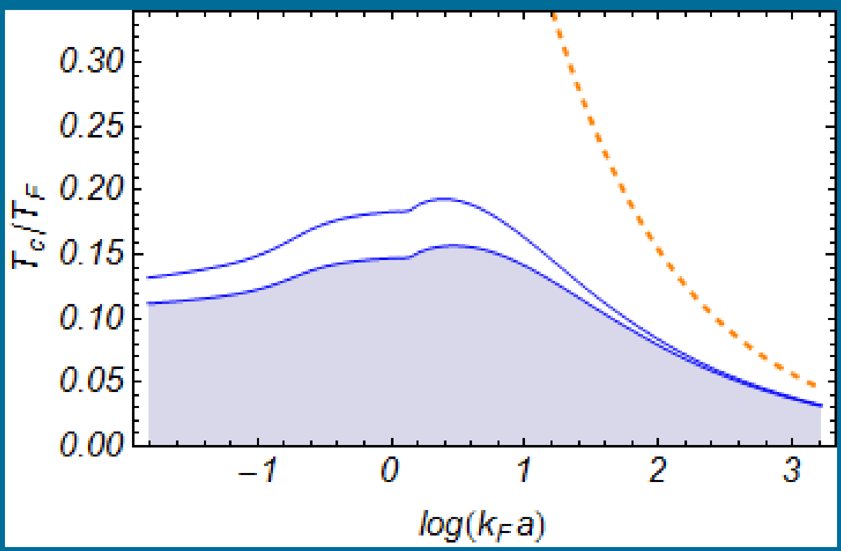
ERG 2014

Outline

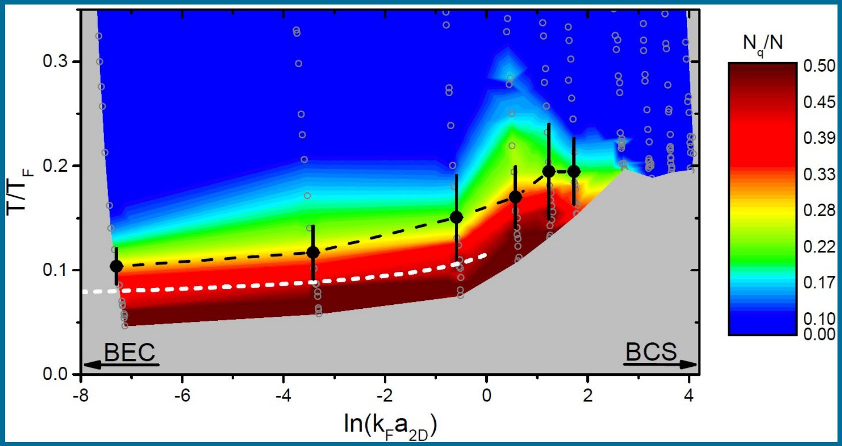


Ultracold atoms



BCS-BEC Crossover

2D Experiments



with...

Theory

C. Wetterich
J. M. Pawlowski

T. K. Herbst
N. Strodthoff
J. Braun (Darmstadt)
D. Roscher (Darmstadt)
L. von Smekal (Giessen/Darmstadt)
T. Enss
M. Scherer
S. Lammers
F. Golibrzuch
S. Diehl (Dresden)
S. Floerchinger (CERN)

Experiment

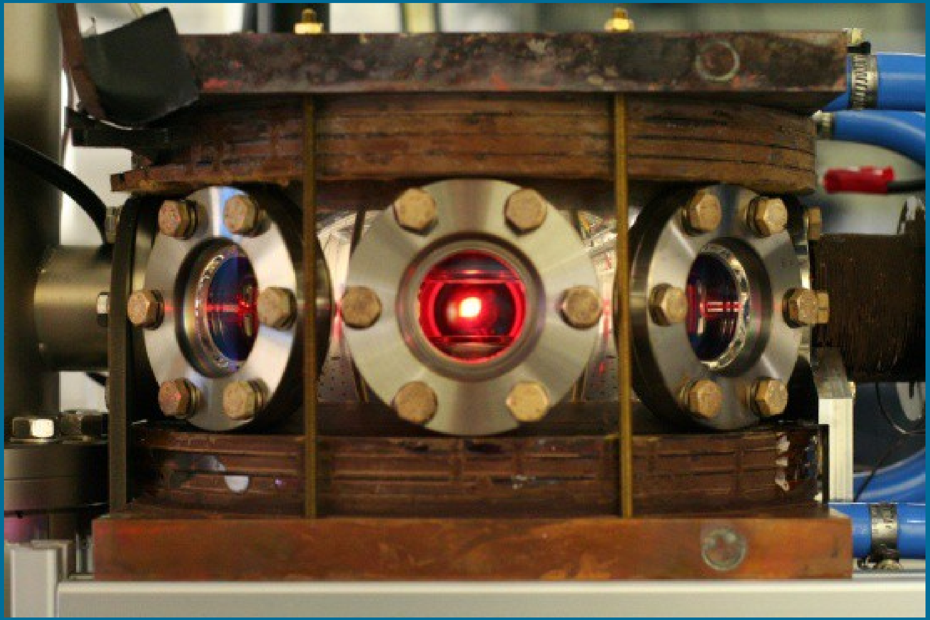
S. Jochim

M. Ries
A. Wenz
G. Zuern
P. Murthy
M. Neidig
T. Lompe
D. Kedar
L. Bayha

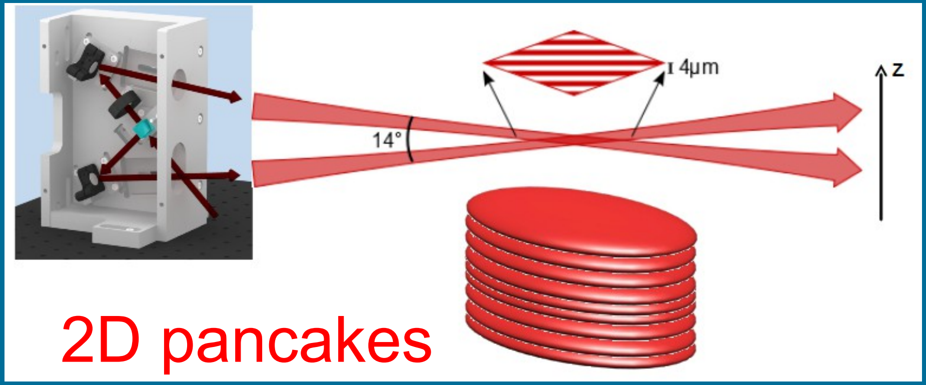
Many-body physics with cold atoms

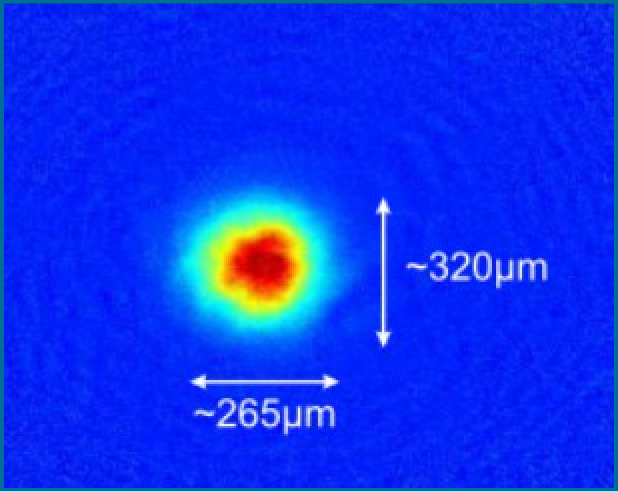
Ultracold atoms:

- 10^6 atoms trapped by light (Hz-kHz)
- 100 nK, $T/T_F = O(1)$

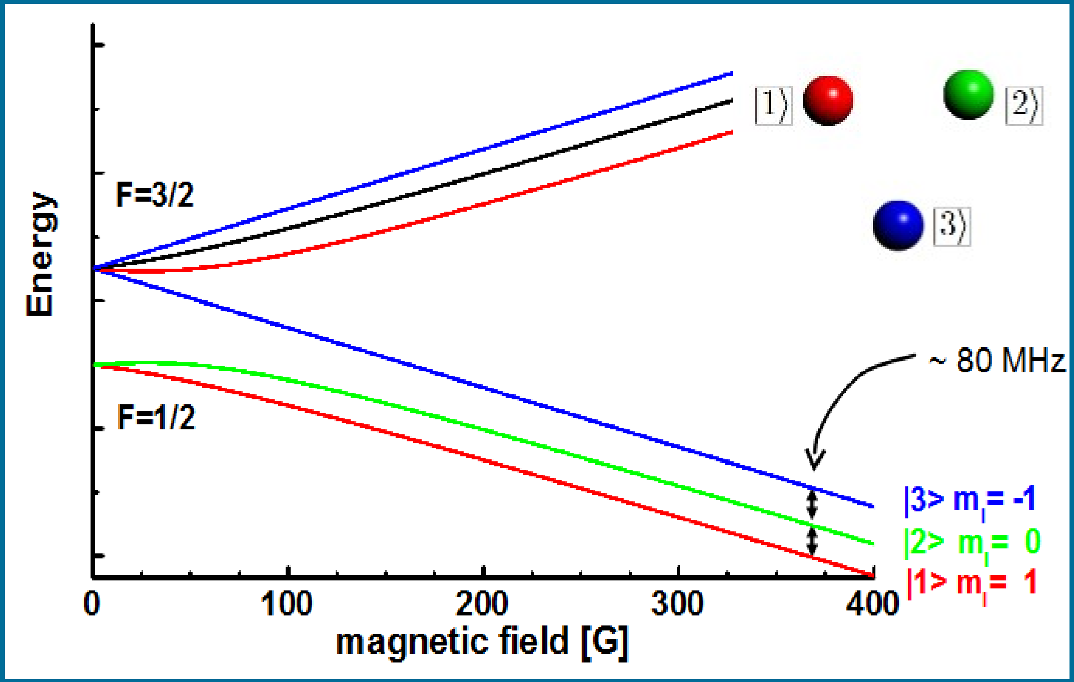


Clean realizations of many-body Hamiltonians.
High experimental control & accessibility.

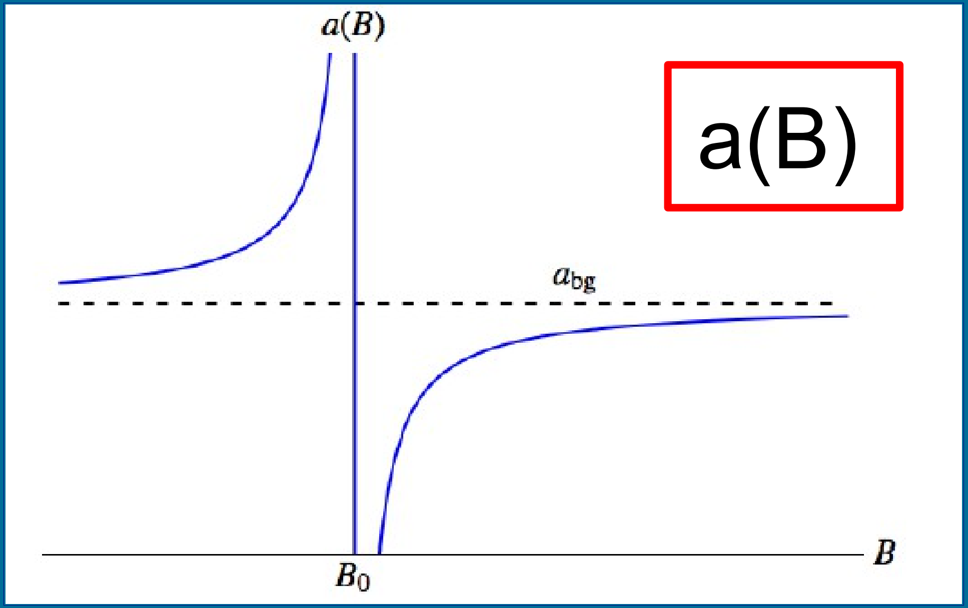




Many-body physics with cold atoms



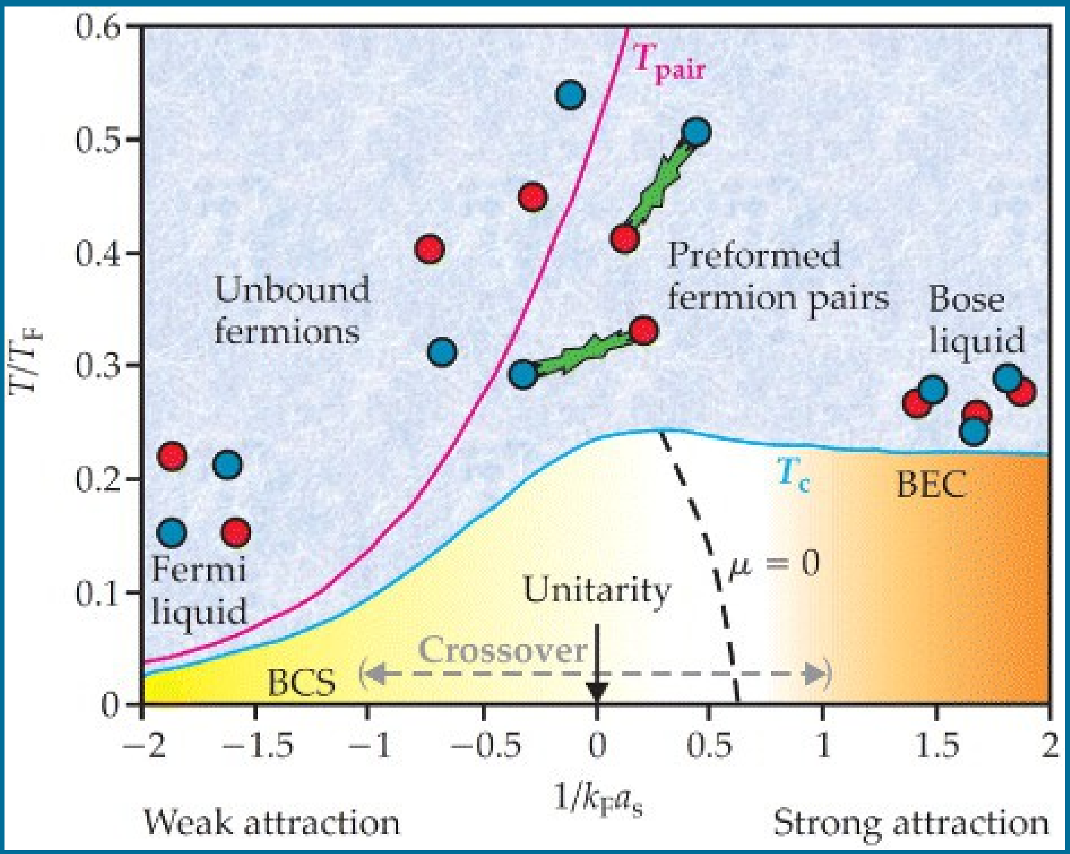
Li-6



Interactions between |1> and |2> atoms encoded in the s-wave scattering length a=a(B)

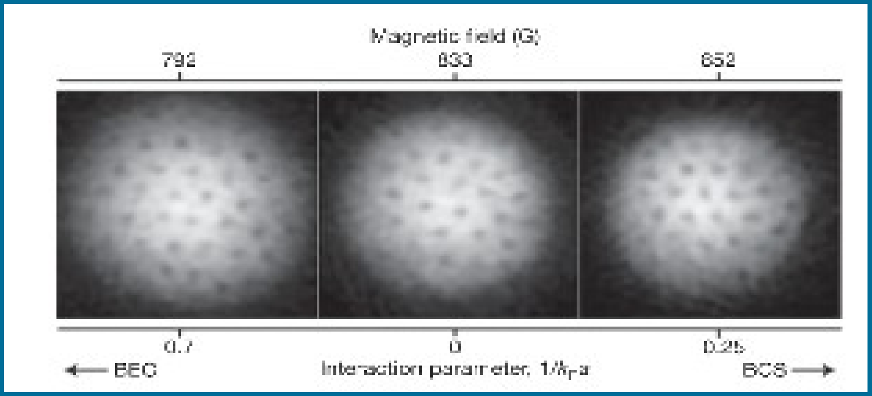
For small a: $\sigma = 4\pi a^2$

BCS-BEC Crossover



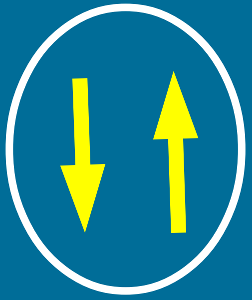
C.A.R. Sà de Melo, Physics Today 61(10), 45 (2008)

superfluidity: vortices



Zwierlein et al., Nature 435 , 1047-1051 (2005)

$$(k_{\mathrm{F}} a)^{-1}$$



$$\sigma = 4\pi a^2$$

BCS-BEC Crossover

Feshbach resonance physics described by two-channel model

$$S = \int_x \Big[\psi_1^* \Big(\partial_\tau - \nabla^2 - \mu_1\Big)\psi_1 + \psi_2^* \Big(\partial_\tau - \nabla^2 - \mu_2\Big)\psi_2 + \phi^* \Big(\partial_\tau - \frac{1}{2}\nabla^2 + m_\phi^2\Big)\phi - h\Big(\phi^*\psi_1\psi_2 + \phi\psi_2^*\psi_1^*\Big)\Big]$$

Yukawa-type model:

- Fermion propagator
- Boson propagator
- Effective potential

Key observables:

- Phase diagram
- Equation of state
- Momentum distribution

$$P(\mu, T, a) = \frac{k_\mathrm{B} T}{V} \log Z_{k=0}$$

Functional Renormalization Group

FRG setup $$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\frac{1}{\Gamma_k^{(2)} + R_k}\partial_k R_k\right)$$

Effective potential: Taylor expansion or resolution on a grid

$$U_k(\rho) = m_k^2(\rho - \rho_{0k}) + \frac{\lambda_k}{2}(\rho - \rho_{0k})^2 + \dots$$

Boson propagator: scale-dependent derivative expansion

$$\Sigma_{\phi k}(P) = Z_k \mathrm{i} p_0 + V_k p_0^2 + A_k p^2$$

Fermion propagator: iterative, frequency and momentum-resolved*

$$\Sigma_\psi(P) \simeq \frac{4C}{-\mathrm{i}p_0 + p^2 - \mu} - \delta\mu$$

*IB, S. Diehl, J. M Pawlowski, C. Wetterich, PRA 87, 023606 (2013)

Functional Renormalization Group

q^2-opt:

$$R_{\phi,k}(Q) = \left(k^2 - \frac{q^2}{2}\right)\theta\left(k^2 - \frac{q^2}{2}\right),$$

$$R_{\psi,k}(Q) = \left[\mathrm{sgn}(q^2-\mu)k^2 - (q^2-\mu)\right]\theta\left(k^2 - |q^2-\mu|\right),$$

Q-exp:

$$R_{\phi,k}(Q) = \left(\mathrm{i}q_0 + \frac{q^2}{2}\right) r\left(\frac{q_0^2 + q^4/4}{c_\phi k^4}\right),$$

$$R_{\psi,k}(Q) = \left(\mathrm{i}q_0 + q^2 - \mu\right) r\left(\frac{q_0^2 + (q^2-\mu)^2}{k^4}\right),$$

$$r(X) = \frac{1}{e^X - 1}$$

IB, J. M. Pawlowski, C. Wetterich, PRA 89, 053630 (2014)

Functional Renormalization Group

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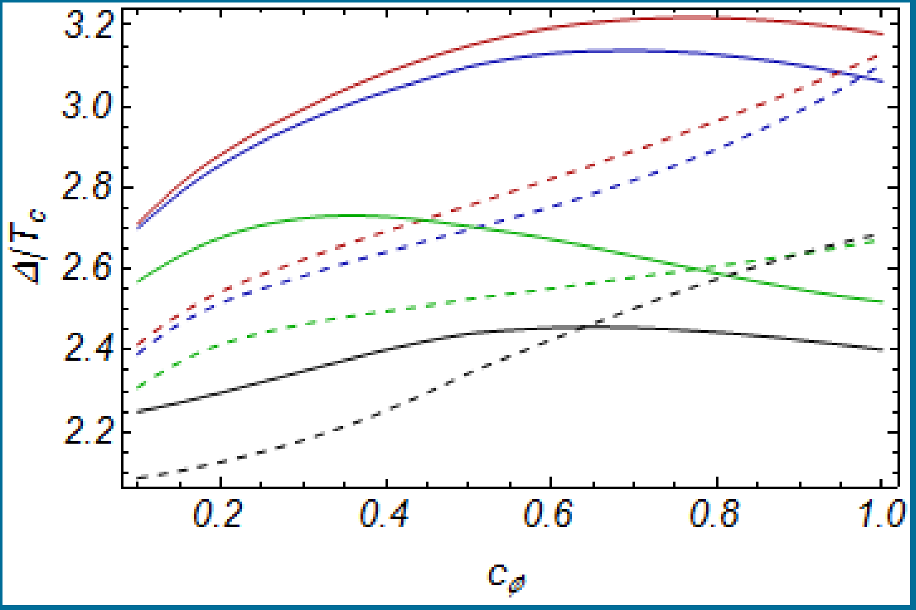
IB, J. M. Pawlowski, C. Wetterich, PRA 89, 053630 (2014)

Functional Renormalization Group

Critical temperature & superfluid gap of the Unitary Fermi Gas

	Δ/μ		T_c/μ	
Truncation	q^2-opt	Q-exp	q^2-opt	Q-exp
FB_0, ϕ^4	1.09	1.244(5)	0.441	0.399(2)
FB_0, ϕ^8	1.13	1.227	0.424	0.380
FB_0, ϕ^{10}	1.16	-	0.427	0.394
FB, ϕ^4	1.05	1.228(10)	0.405	0.389(2)
FB, ϕ^8	-	1.240	-	0.380
FB, ϕ^{10}	-	-	-	0.386



	T_c/μ				
Truncation	F	FB_0	FB	FBM_0	FBM
ϕ^4	0.664	0.381	0.381	0.385	0.383

$$\frac{T_c}{\mu} = 0.38(2), \quad \frac{\Delta}{\mu} = 1.04(15)$$

IB, J. M. Pawlowski, C. Wetterich, PRA 89, 053630 (2014)

Dimensional BCS-BEC Crossover

$$\delta\mu = h = (\mu_1 - \mu_2)/2$$

d

Dimensional BCS-BEC Crossover

$$\delta\mu = h = (\mu_1 - \mu_2)/2$$

d

Phase structure of spin-imbalanced unitary Fermi gases
IB, J. Braun, T. K. Herbst, J. M. Pawlowski, D. Roscher, C. Wetterich - arXiv:1409.5070

see talk by Dietrich Roscher today

Sarma phase in relativistic and non-relativistic systems
IB, T. K. Herbst, J. M. Pawlowski, N. Strodthoff, L. von Smekal, C. Wetterich - arXiv:1409.5232

see talk by Tina K. Herbst today

Dimensional BCS-BEC Crossover

$$\delta\mu = h = (\mu_1 - \mu_2)/2$$

d

Observation of pair condensation in a strongly interacting two-dimensional Fermi gas
M. G. Ries, A. N. Wenz, G. Zuern, L Bayha, IB, D. Kedar, P. A. Murthy, M. Neidig, T. Lompe, S. Jochim - arXiv:1409.5373

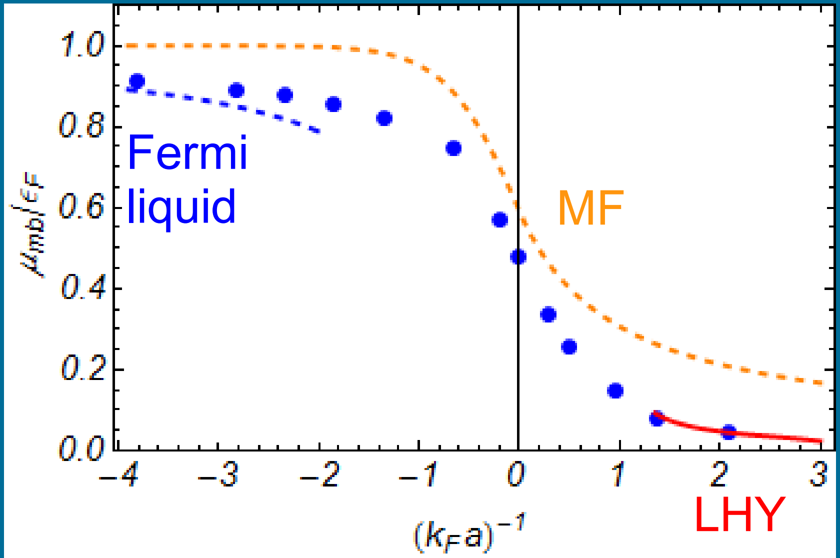
Dimensional BCS-BEC crossover in ultracold Fermi gases
IB, PHD thesis (2014)

IB, J. M. Pawlowski, C. Wetterich (in preparation, 2014)

Dimensional BCS-BEC Crossover

Equation of state at T=0

3



Crossover parameter 3D:

$$(k_{\mathrm{F}} a)^{-1} \sim (\sqrt{\mu_{\mathrm{mb}}}\, a)^{-1}$$

d

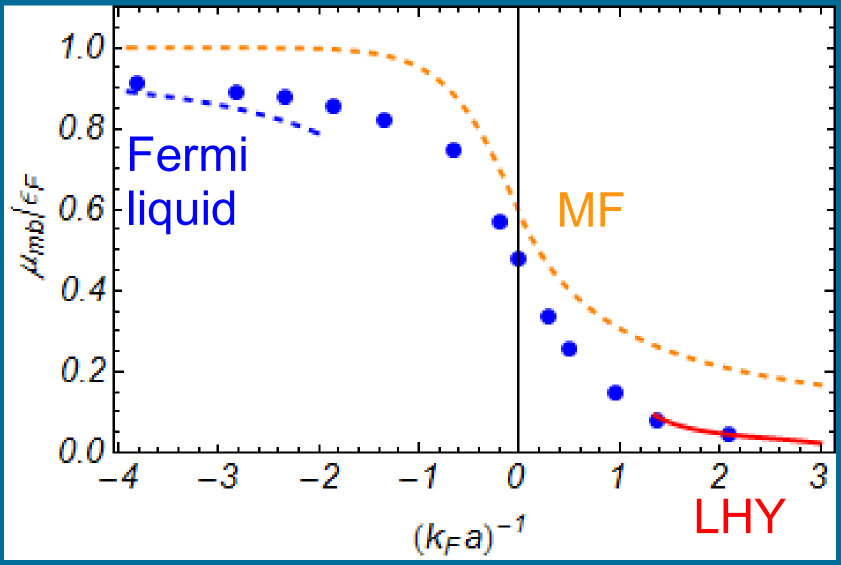
$$\mu_{\mathrm{mb}} = \mu - \frac{\varepsilon_{\mathrm{B}}}{2} \geq 0$$

2

Dimensional BCS-BEC Crossover

Equation of state at T=0

3



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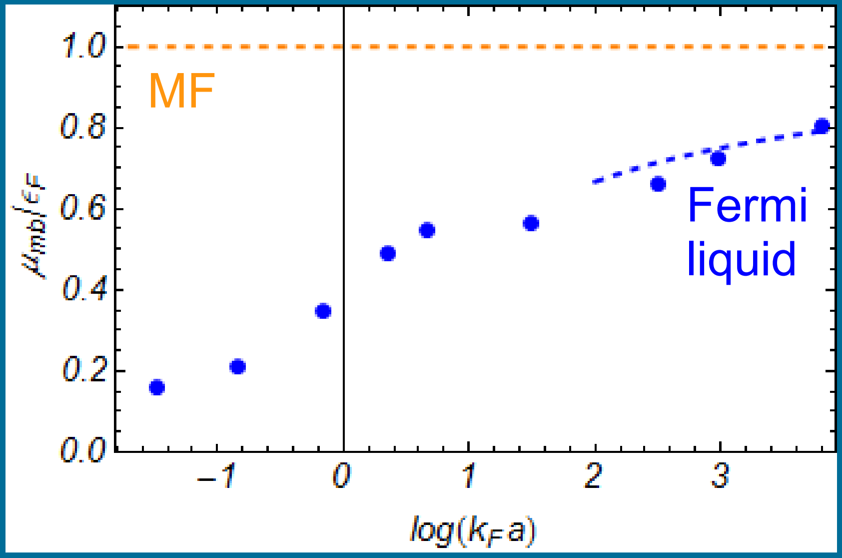

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Crossover parameter 3D:

$$(k_F a)^{-1} \sim (\sqrt{\mu_{mb}}\, a)^{-1}$$

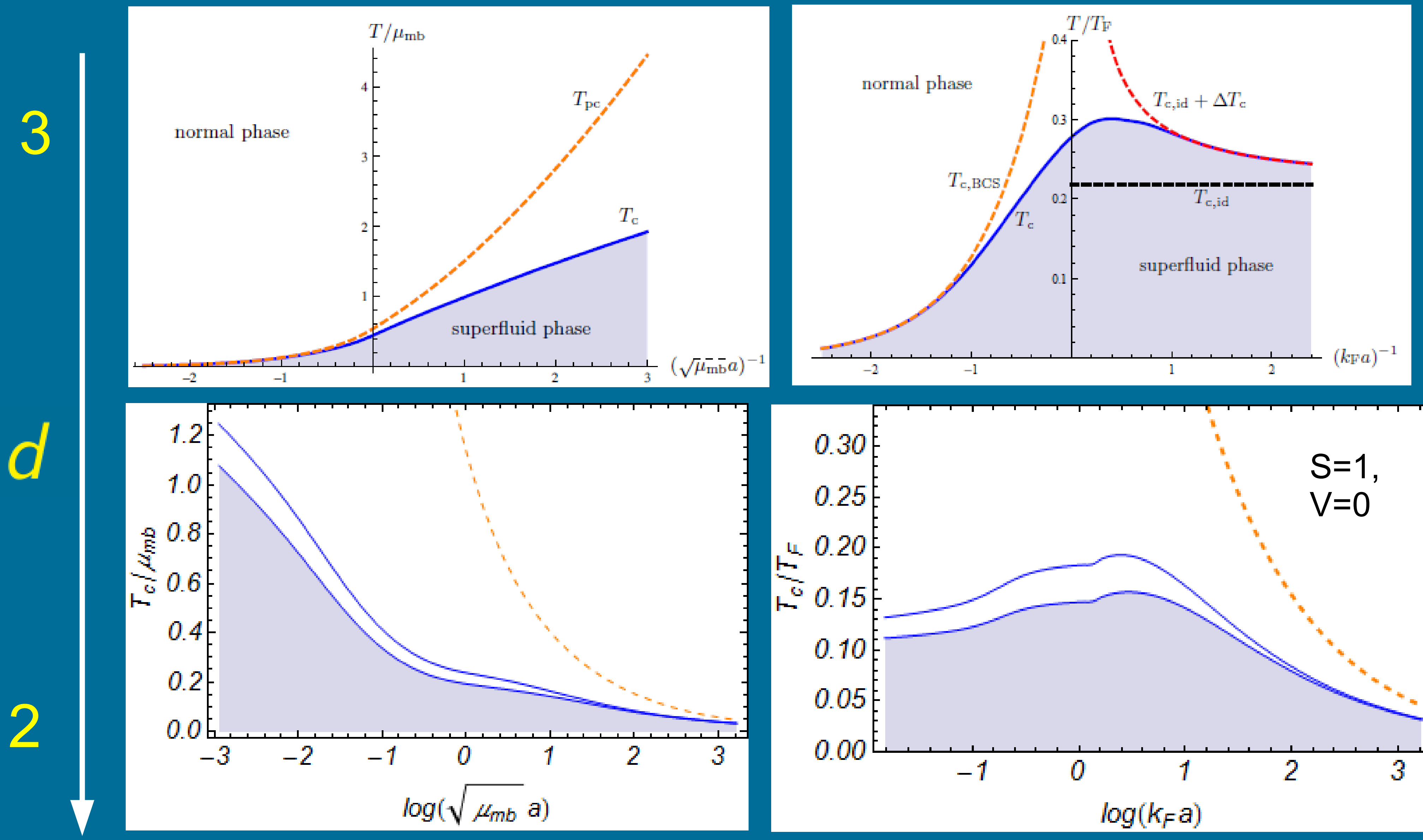
2D:

$$\log(k_F a) \sim \log(\sqrt{\mu_{mb}}\, a)$$

$$\mu_{mb} = \mu - \frac{\varepsilon_B}{2} \geq 0$$

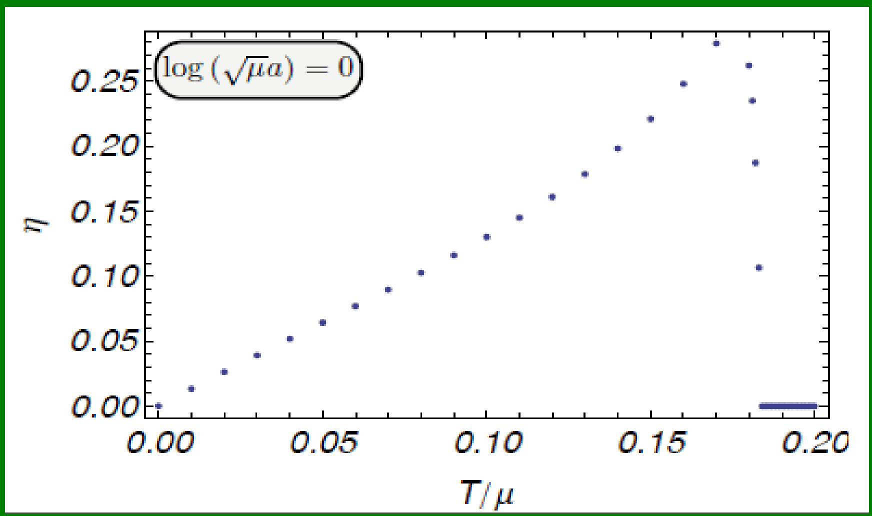
Dimensional BCS-BEC Crossover

Phase diagram



Dimensional BCS-BEC Crossover

BKT scaling is resolved from FRG for all log(kFa):



ram

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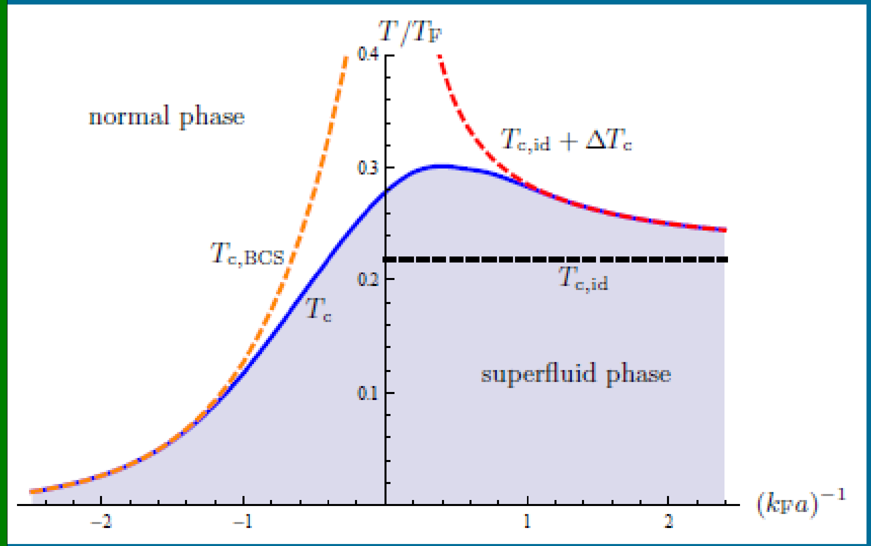

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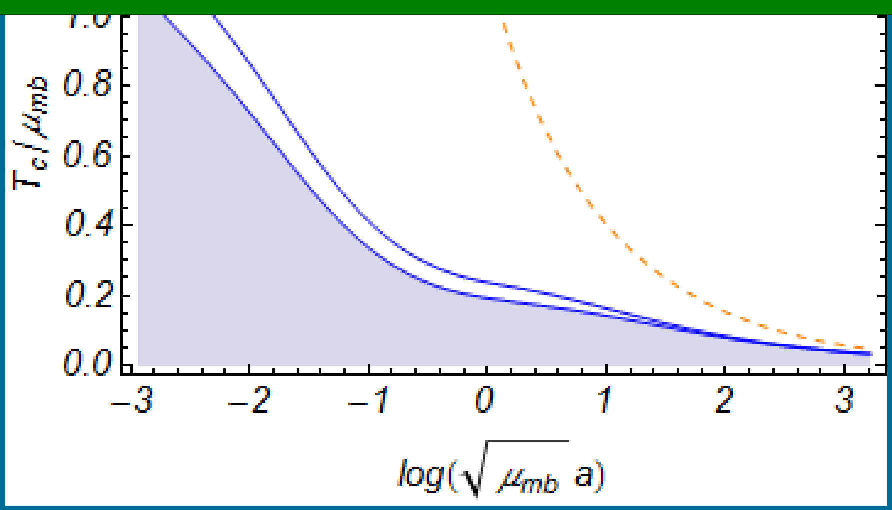
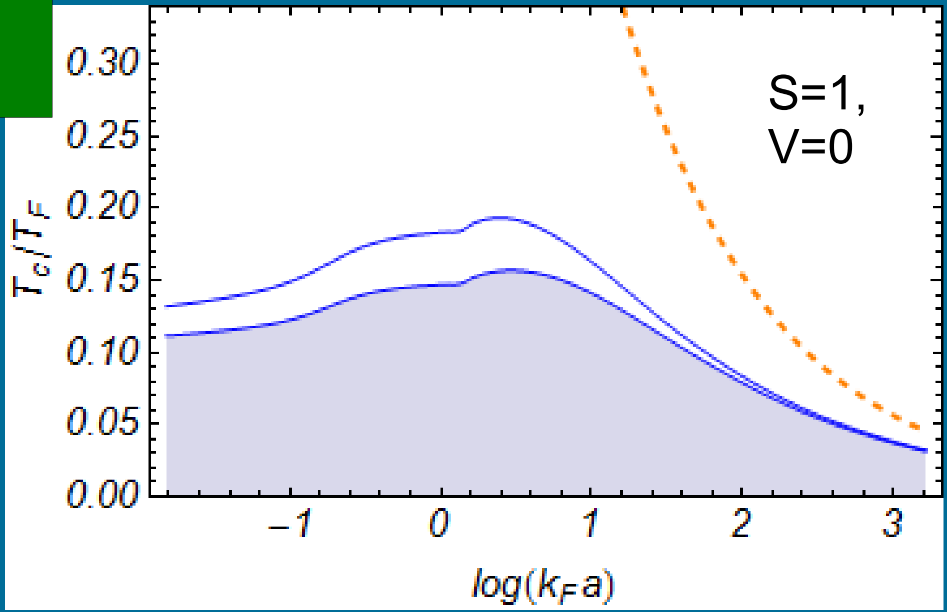
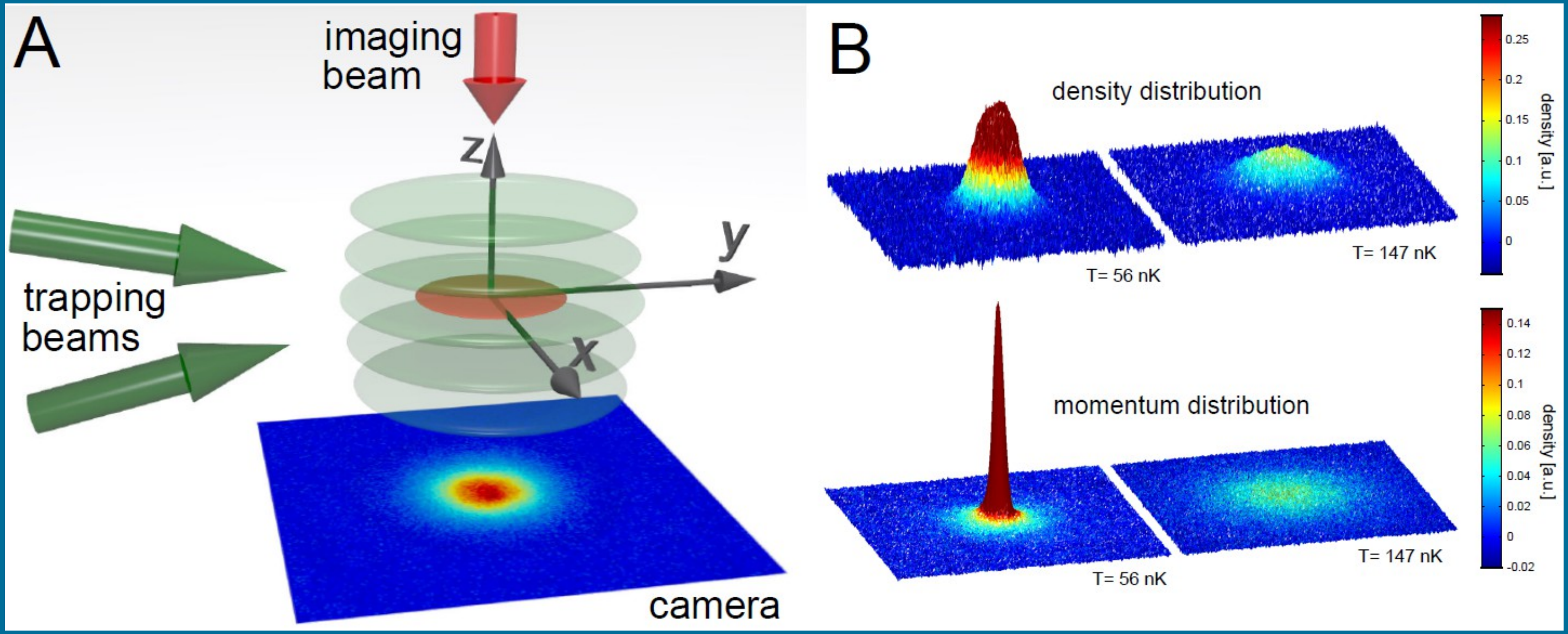




2D experiment

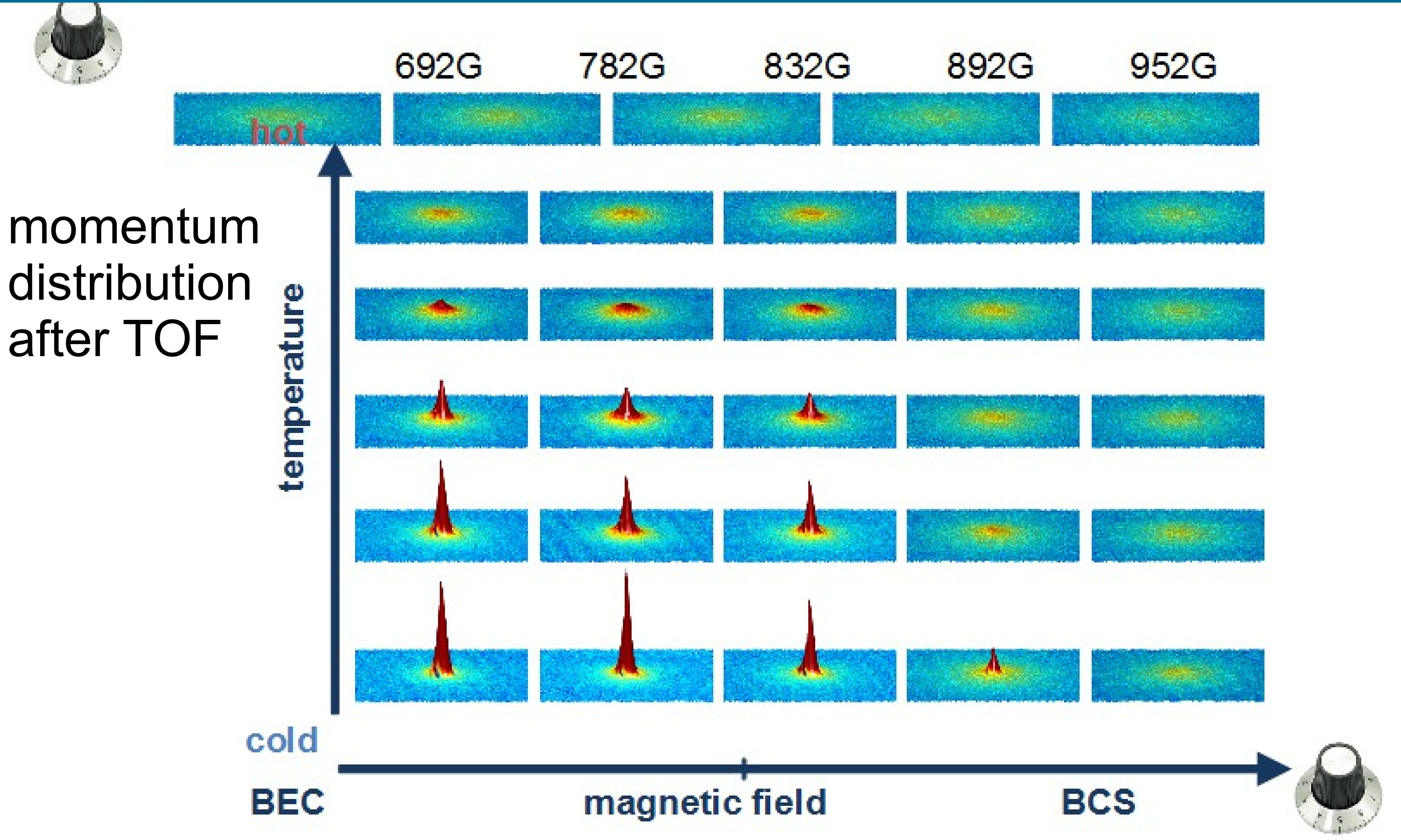


Jochim group @ PI Heidelberg

- in-situ profiles and momentum distribution after TOF expansion
- trap aspect ratio ωx/ωz=1/300 => effectively 2D

2D experiment

Superfluid/condensation transition from enhanced occupation of low-momentum states



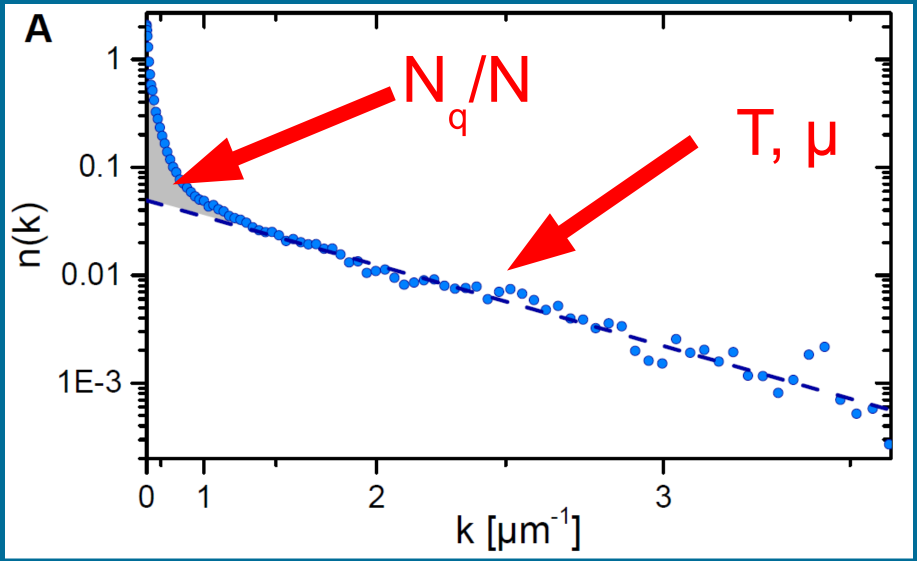
2D experiment

- 2D scattering length a from magnetic field B

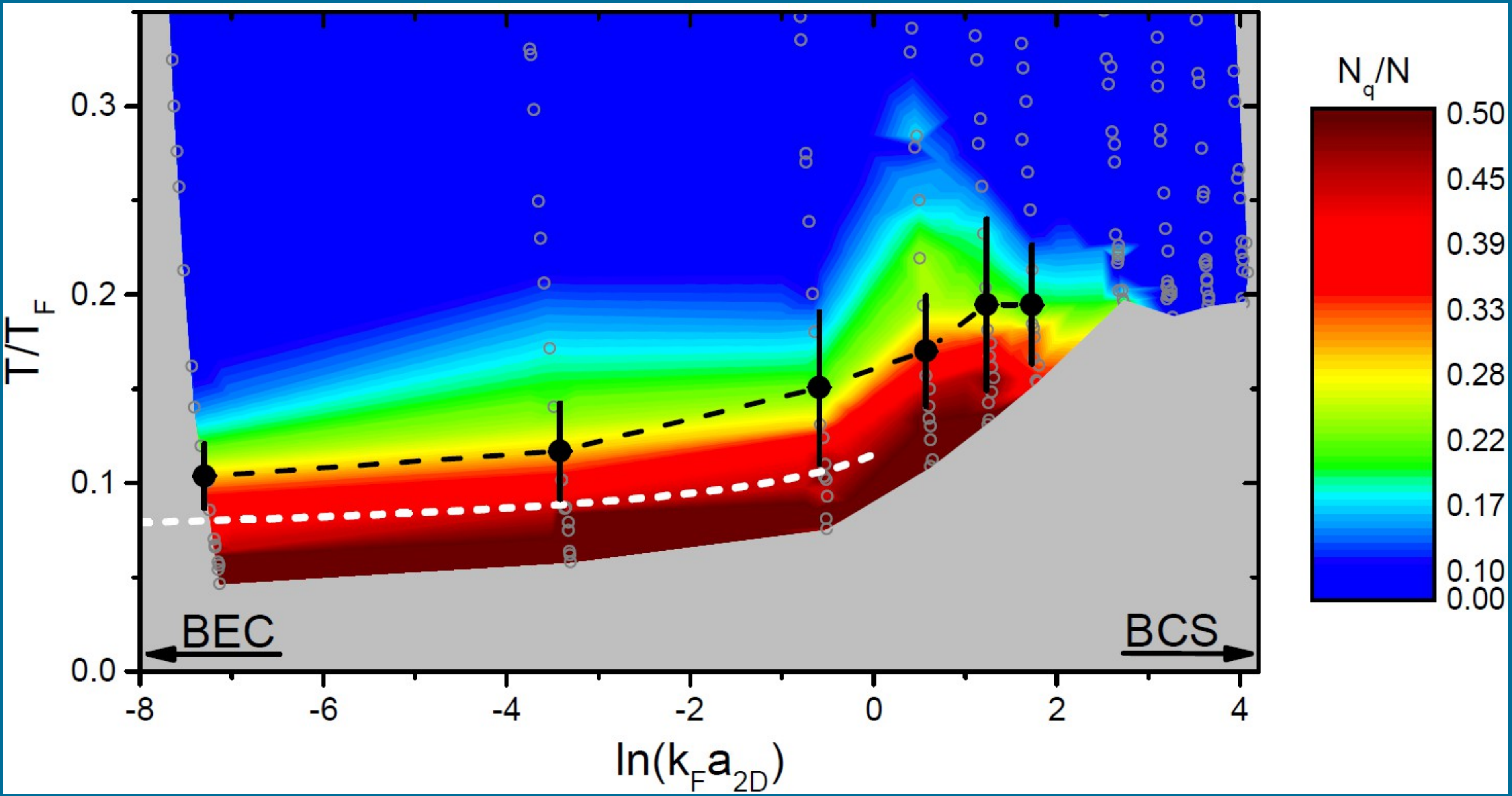
$$a_{2D} = \ell_z \sqrt{\frac{\pi}{0.905}} \exp\left(-\sqrt{\frac{\pi}{2}}\frac{\ell_z}{a_{3D}(B)}\right)$$

- Boltzmann fit gives T and μ
- Density n from in-situ image

=> Full thermodynamic characterization n(μ,T,a) !

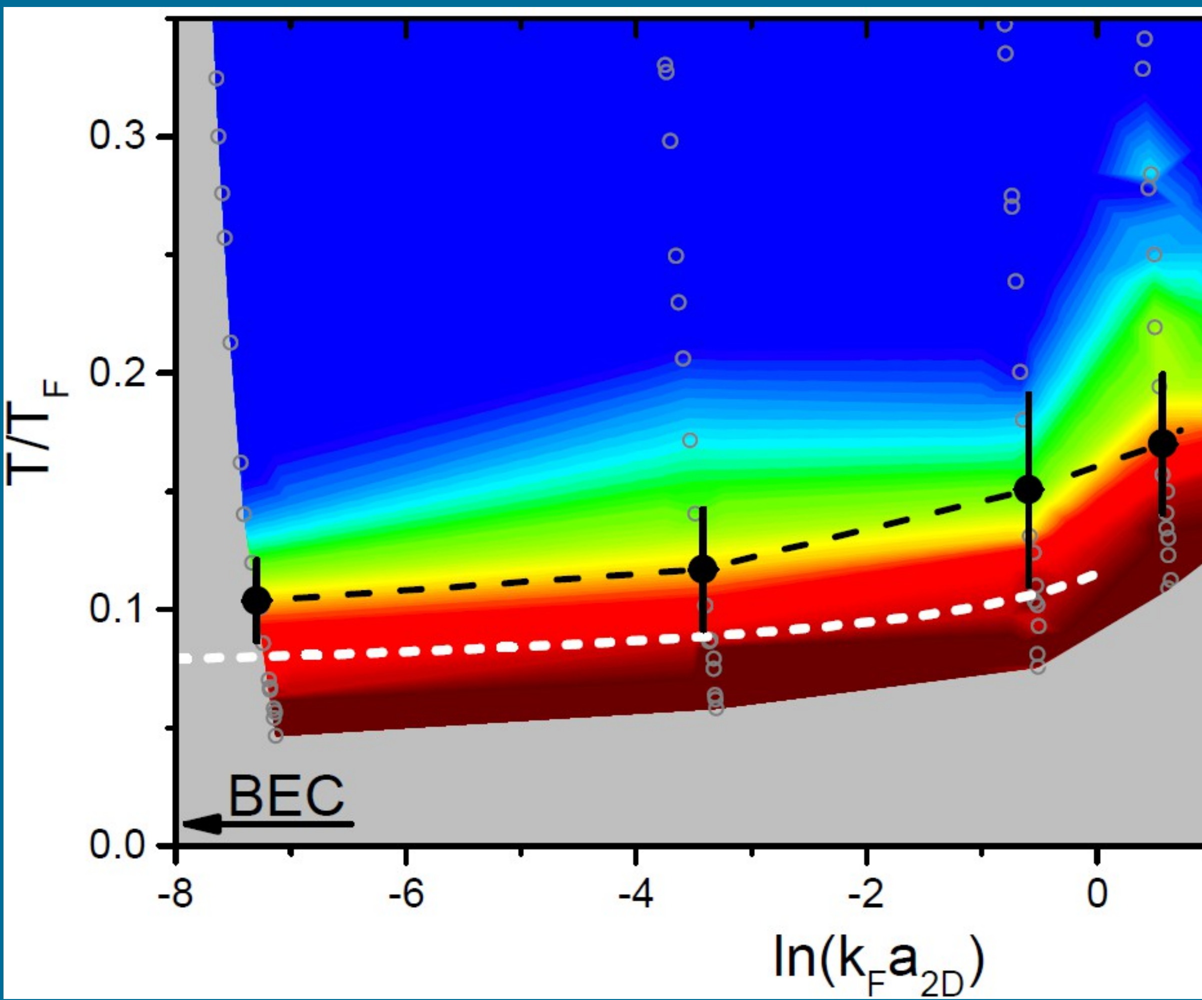


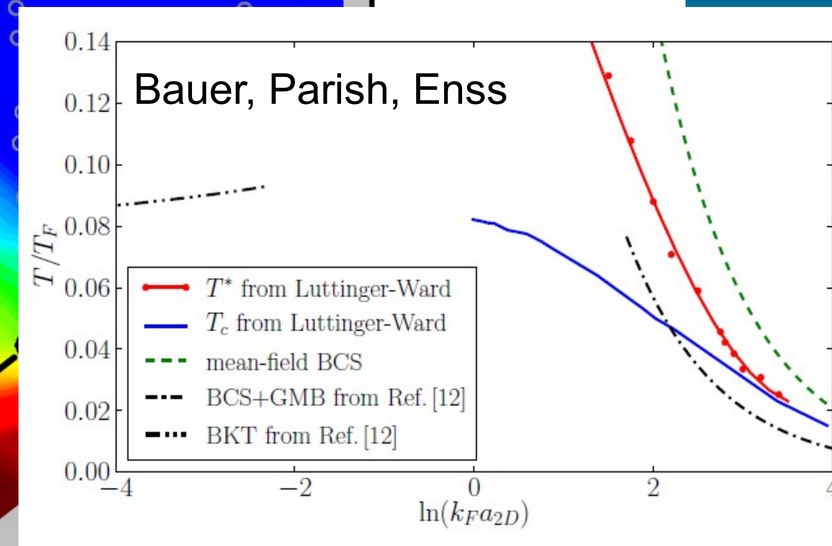
2D experiment

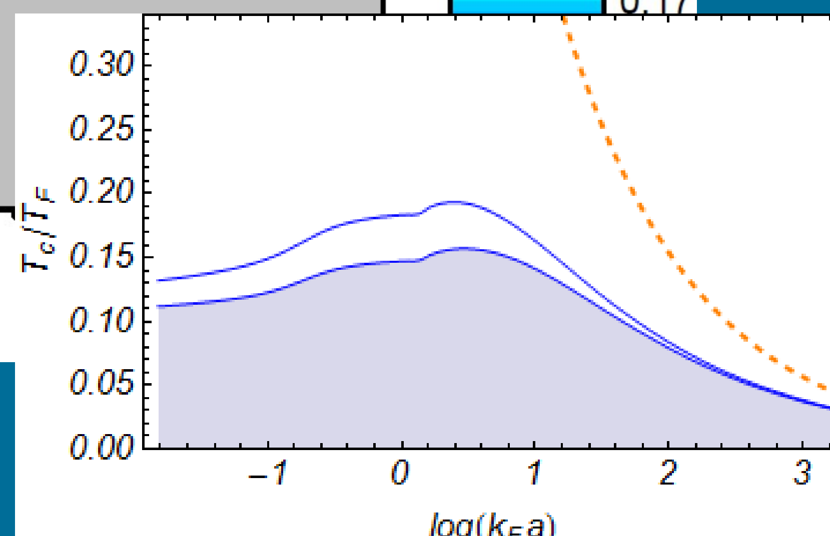


BKT verified from „infinite“ correlation length and algebraic decay of correlations below Tc

2D experiment







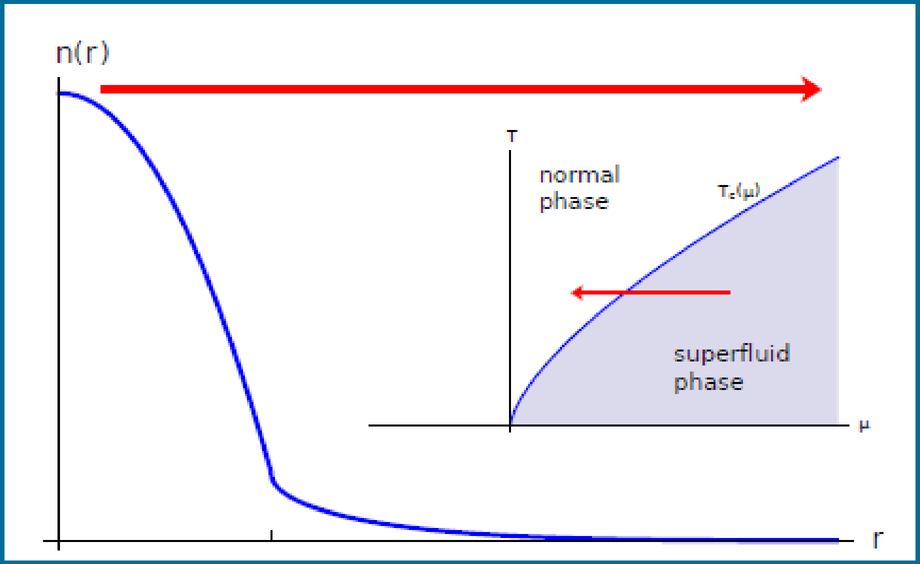
Understanding of the phase diagram within reach!

Equation of state determination

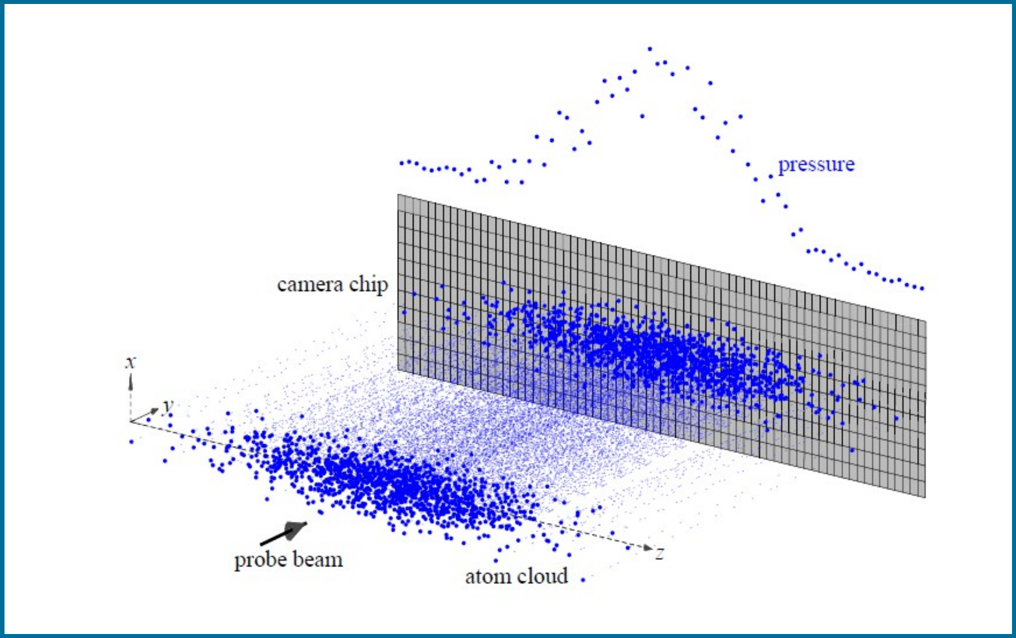
Equation of state n(μ,T,a)
from in-situ images

$$P(\mu, T) \to P(\mu - V_{\mathrm{ext}}(\vec{x}), T)$$

local density approximation



$$P(\mu_z, T) = \frac{m\omega_r^2}{2\pi}\overline{n}(z),$$



Nascimbene, Navon, Chevy, Salomon
New Journal of Physics 12 (2010) 103026

Equation of state determination

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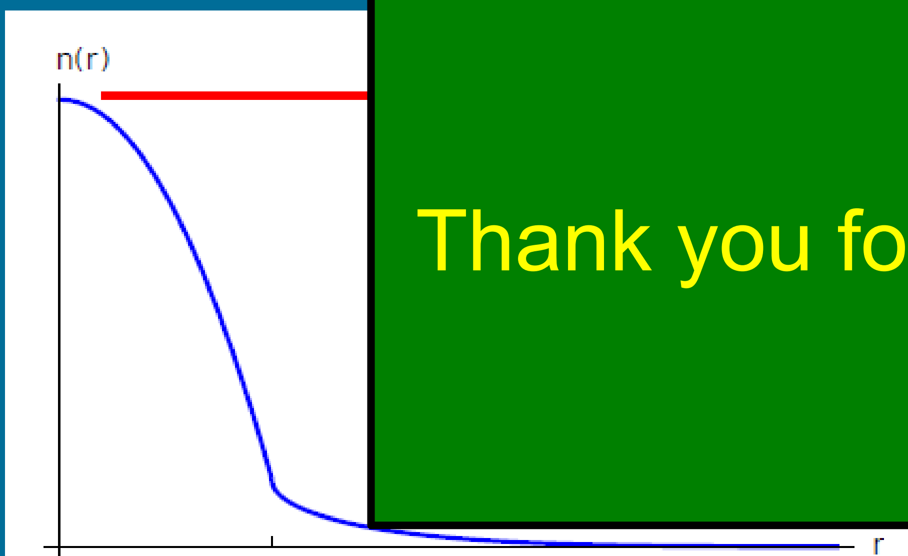
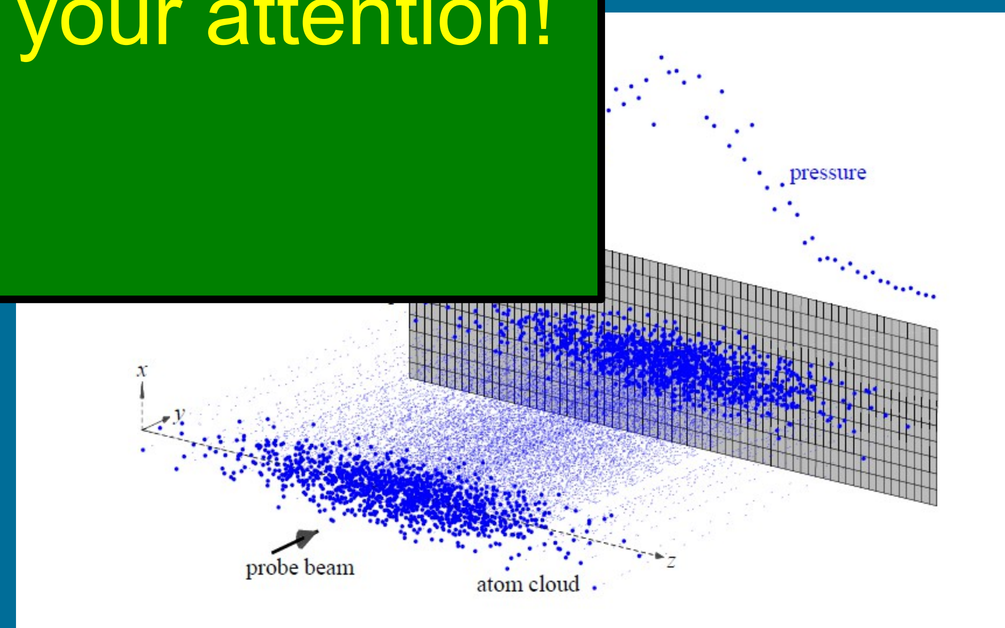
roximation



Thank you for your attention!



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