# Long-range and anisotropic generalizations of the Kardar-Parisi-Zhang equation

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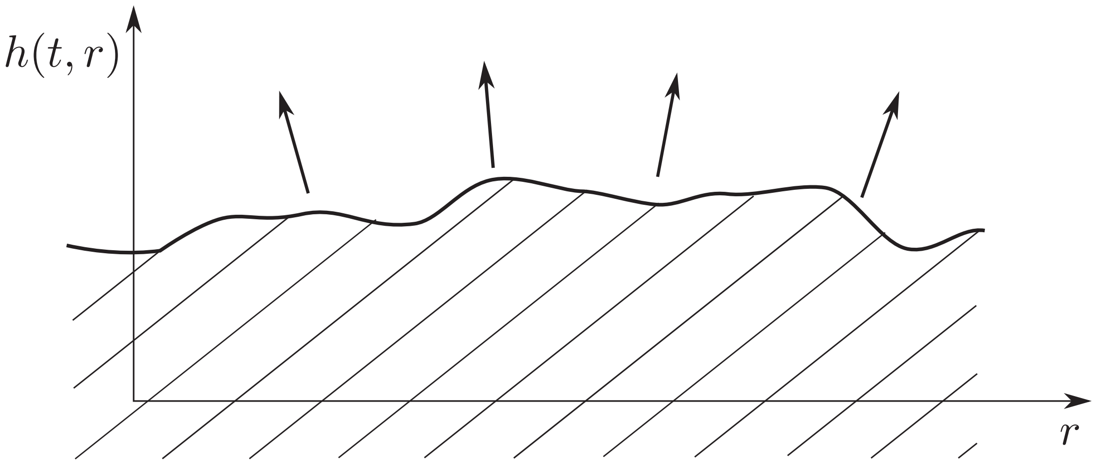
# Experiment

Observations:

- stochastic growth process
- rough structure of the interface
- scale invariance

Can we develop a theoretical understanding of random growth processes having the same features?

## Theoretical description




Dynamical scaling

$$C(t, \vec{r}) = \langle h(t, \vec{r}) h(0,0) \rangle_c \sim r^{2\chi} F(t/r^z)$$

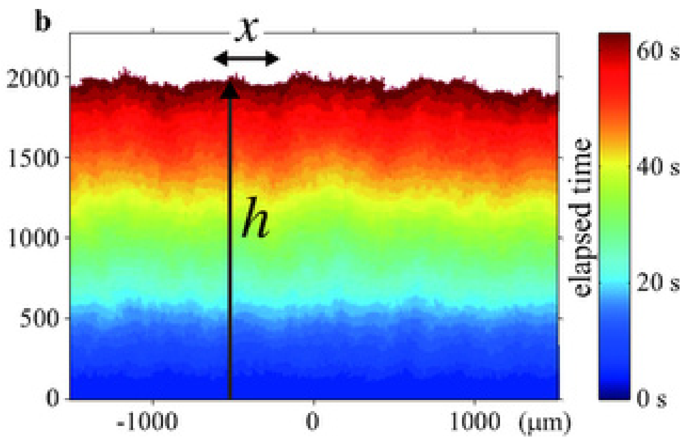
## Model

$$\frac{\partial h(t,\vec{x})}{\partial t} = \nu\,\nabla^2 h(t,\vec{x}) \;+\; \frac{\lambda}{2}\left(\nabla h(t,\vec{x})\right)^2 \;+\; \eta(t,\vec{x})$$

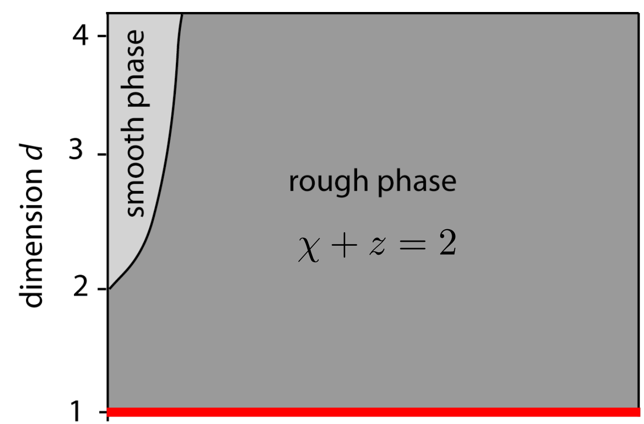
KPZ equation

$$\left\langle \eta(t,\vec{x})\eta(t',\vec{x}')\right\rangle = 2\,D\,\delta^d(\vec{x}-\vec{x}')\,\delta(t-t')$$

Kardar, Parisi, Zhang, PRL (1986).




Takeuchi, Sano, Sasamoto, Spohn, Sci. Rep. (2011).




$$g = \lambda^2 D/\nu^3$$

exactly solvable (since 2010)
see e.g. Corwin, Random Matr. (2012).

## The upper critical dimension

$d_c$ finite

Mézard, Parisi, J Phys. I (1991): $d_c = 2$

Le Doussal, Wiese, PRB (2003): $d_c = 2.5$

Garel, Orland, PRB (1997): $d_c = 2$

Colaiori, Moore, PRL (2001): $d_c = 4$

Halpin-Healy, PRL (1989): $d_c = 4$

Fogedby, PRL (2005): $d_c = 4$

$d_c$ infinite

Marinari, Pagnani, Parisi, PRE (2013)

Tang, Forrest, Wolf, PRA (1992)

Ala-Nissila, et al., J Stat Phys (1992)

Castellano, et al., PRL (1998)

Oliveira, Alves, Ferreira., PRE (2013)

Study variants of the KPZ equation

- long-range correlated noise: $d_c = 4$ Janssen, Täuber, Frey, Eur. Phys. J. B (1999)
- anisotropy: $d_c \geq 3$ Täuber, Frey, Europhys. Lett. (2002)

## Method

nonperturbative renormalization group (NPRG)

$$\partial_\kappa \Gamma_\kappa = \frac{1}{2}\mathrm{Tr}\left[\partial_\kappa R_\kappa (\Gamma_\kappa^{(2)} + R_\kappa)^{-1}\right]$$

Wetterich, Phys. Lett. B (1993).

ansatz

$$\Gamma_\kappa[\varphi,\tilde\varphi] = \int_{\mathbf{x}} \left\{ \tilde\varphi f_\kappa^\lambda(\text{-}\tilde D_t^2, \text{-}\nabla^2) D_t\varphi - \frac{1}{2}\left[\nabla^2\varphi f_\kappa^\nu(\text{-}\tilde D_t^2, \text{-}\nabla^2)\tilde\varphi + \tilde\varphi f_\kappa^\nu(\text{-}\tilde D_t^2, \text{-}\nabla^2)\nabla^2\varphi\right] - \tilde\varphi f_\kappa^D(\text{-}\tilde D_t^2, \text{-}\nabla^2)\tilde\varphi \right\}$$

$$\tilde D_t = \partial_t - \lambda(\nabla\varphi)\cdot\nabla$$

form strongly constrained by symmetries: Galilei (gauged), shift symmetry (time gauged)

truncated at quadratic order in the response field

arbitrary momentum/frequency dependence

Blaizot, Méndez-Galain, Wschebor, Phys Lett B (2006)

$$R_\kappa(\mathbf{q}) = r\left(\frac{q^2}{\kappa^2}\right)\begin{pmatrix} 0 & \nu_\kappa q^2 \\ \nu_\kappa q^2 & -2D_\kappa \end{pmatrix}$$

$$r(x) = \alpha/(\exp(x) - 1)$$

correlation and scaling functions

$$C(\varpi, p) = \langle h(\varpi,p)h(-\varpi,-p)\rangle_c \sim \frac{1}{p^{d+2+\chi}}\mathring{F}(\tau)$$

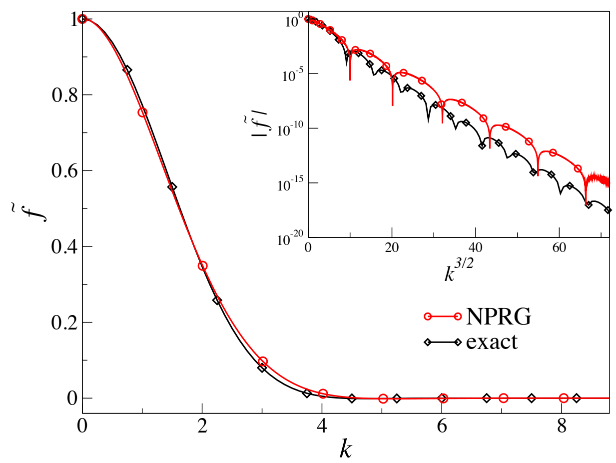
$$\mathring{F}(\tau) = \frac{\hat\zeta^D(\tau)}{(\tau\,\hat\zeta^\lambda(\tau))^2 + \hat\zeta^\nu(\tau)^2},$$

$$f_*^X(\varpi,p) \sim p^{-\eta_*^X}\,\zeta^X(\varpi/p^z)$$

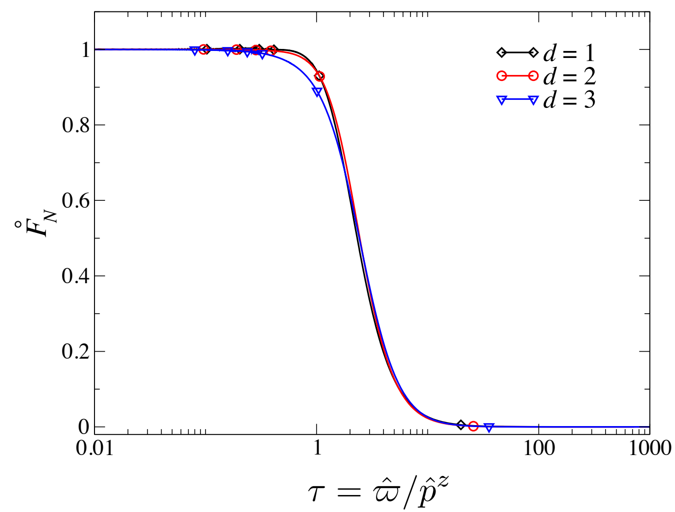
Canet, Delamotte, Wschebor PRE (2011).
TK, Canet, Wschebor PRE (2012).

# Scaling functions and critical exponents

*d* = 1 vs. exact result




d ≥ 1




exact result from Prähofer, Spohn J. Stat. Phys. (2004).

roughness exponent $\chi$ for different dimensions *d* and comparison to literature results

| $d$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\chi$ (NPRG) | 1/2 | 0.373(1) | 0.179(4) | – |
| $\chi$ (Literature)* | 1/2 | 0.379(15) | 0.300(12) | 0.246(7) |

*average over simulation and RG studies

Canet, Delamotte, Wschebor PRE (2011).
TK, Canet, Wschebor PRE (2012).

## Long-range correlated noise

$$\langle \eta(t,\vec{x})\eta(t',\vec{x}') \rangle = 2\, R^D(\vec{x}-\vec{x}')\,\delta(t-t')$$

$$R^D(\vec{x}-\vec{x}') \sim |\vec{x}-\vec{x}'|^{2\rho-d}, \quad \rho \le d/2$$ spacially correlated noise

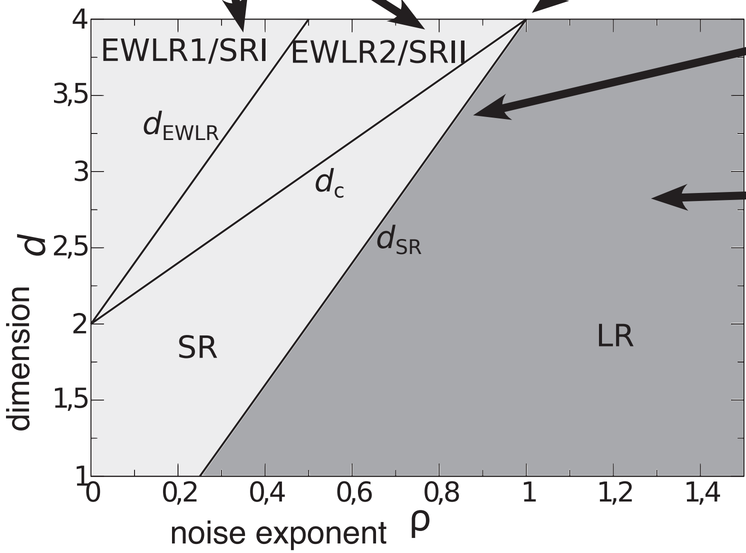
Weak-coupling phase diagram

Janssen, Täuber, Frey, Eur. Phys. J. B (1999).

different short range phases?

$d_c = 4$ upper critical dimension

transition line estimated




exact exponents in the long-range dominated regime

$$\chi_{LR} = \frac{2-d+2\rho}{3}$$

$$z_{LR} = \frac{4+d-2\rho}{3}$$

# Long-range correlated noise (NPRG)

Approximations

$$\hat{f}^X_\kappa(\omega, q) \to \hat{f}^X_\kappa(q) \quad \text{(NLO)}$$

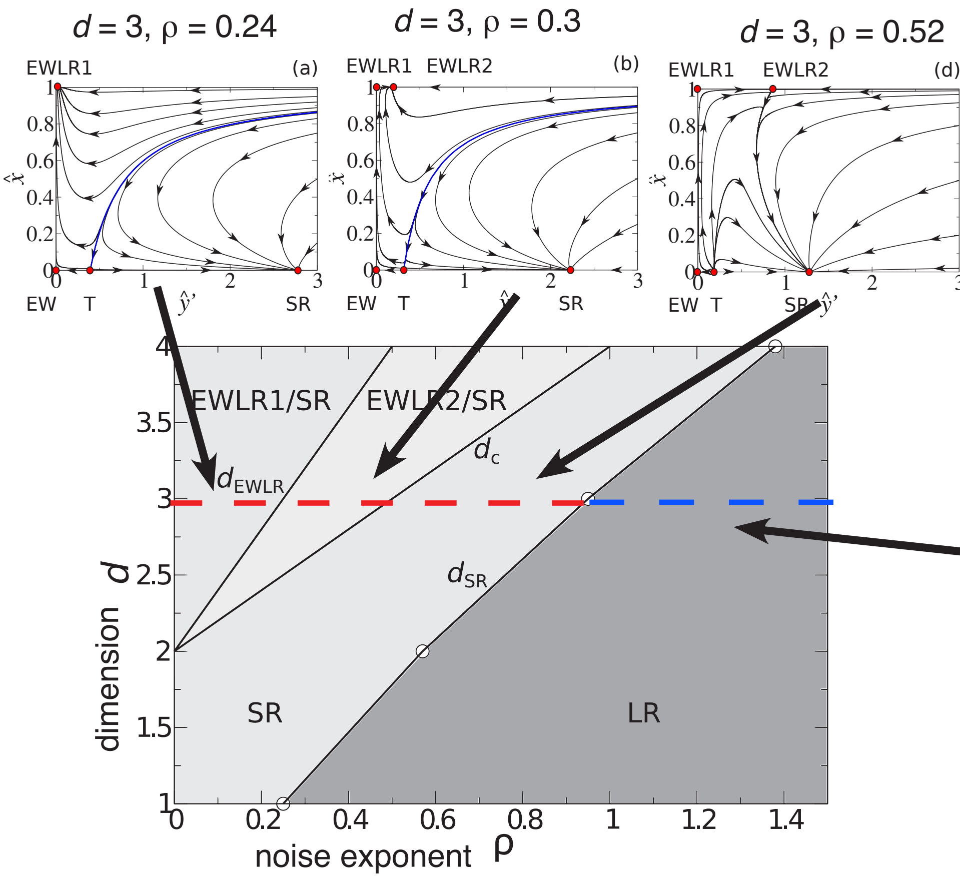
$$\hat{f}^X_\kappa(\omega, q) \to 1 \quad \text{(LPA´)}$$

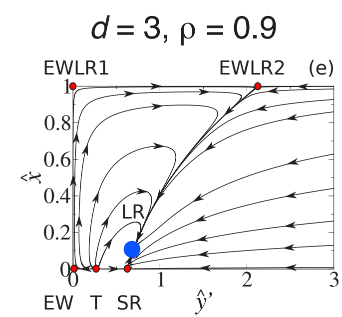
long-range noise

$$\partial_s \hat{x}_\kappa = \hat{x}_\kappa (1 - \hat{x}_\kappa)(\eta^D_\kappa - 2\rho)$$

KPZ coupling

$$\partial_s \hat{y}_\kappa = \hat{y}_\kappa (2\hat{x}_\kappa(\eta^D_\kappa - 2\rho) + d - 2 + 3\eta^\nu_\kappa - \eta^D_\kappa)$$




TK, Canet, Delamotte, Wschebor, PRE (2014).

## Anisotropy

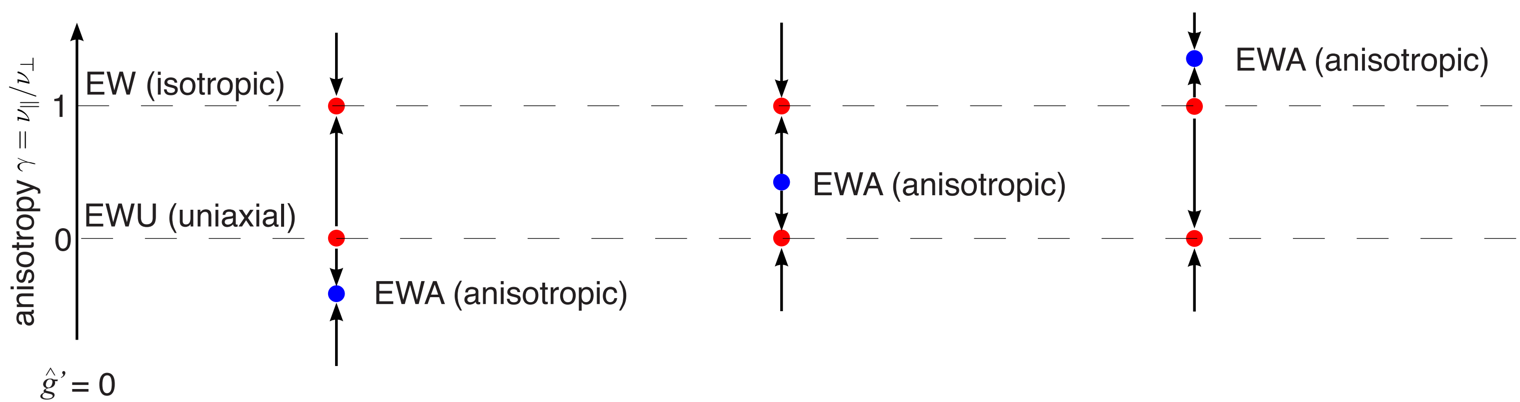
Anisotropic Kardar-Parisi-Zhang equation

$$\frac{\partial h(t,\vec{x})}{\partial t} = \nu_{\|}\, \nabla_{\|}^2 h(t,\vec{x}) + \nu_{\perp}\, \nabla_{\perp}^2 h(t,\vec{x}) + \frac{\lambda_{\|}}{2}\left(\nabla_{\|} h(t,\vec{x})\right)^2 + \frac{\lambda_{\perp}}{2}\left(\nabla_{\perp} h(t,\vec{x})\right)^2 + \eta(t,\vec{x})$$

$$\vec{x} = (\vec{x}_{\|}, \vec{x}_{\perp})^T, \quad d = d_{\|} + d_{\perp}$$

Wolf, PRL (1991)

Weak-coupling fixed points




Depending on the total dimension and the splitting $d = d_{\|} + d_{\perp}$, the isotropic KPZ equation may become unstable against anisotropic perturbations.

Täuber, Frey, Europhys. Lett. (2002)

## Anisotropy (NPRG)

Ansatz

$$\Gamma_\kappa[\varphi,\tilde\varphi]=\int_{\mathbf{x}}\Big\{\tilde\varphi f_\kappa^\lambda(\nabla)D_t\varphi-\ (\nabla_\parallel^2\varphi)f_\kappa^{\nu\parallel}(\nabla)\tilde\varphi-(\nabla_\perp^2\varphi)f_\kappa^{\nu\perp}(\nabla)\tilde\varphi-\tilde\varphi f_\kappa^D(\nabla)\tilde\varphi\Big\}$$

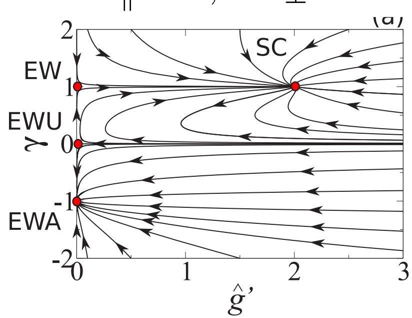
$\hat f_\kappa^X(\hat q_\parallel,\hat q_\perp)$ (NLO)

$\hat f_\kappa^X(\hat q_\parallel,\hat q_\perp)\to 1$ (LPA´)

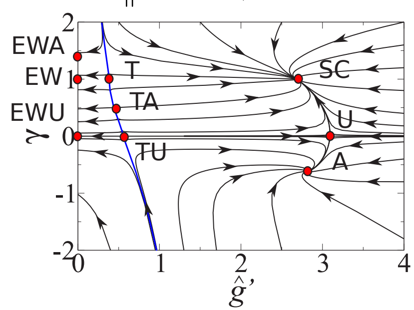
$$D_t\varphi=\partial_t\varphi-\frac{\lambda_\parallel}{2}(\nabla_\parallel\varphi)^2-\frac{\lambda_\perp}{2}(\nabla_\perp\varphi)^2$$

RG trajectories (LPA´)

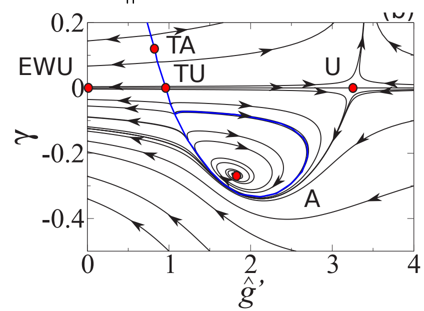
$d_\parallel=1,\quad d_\perp=1$




$d_\parallel=2.5,\quad d_\perp=0.5$




$d_\parallel=2.05,\quad d_\perp=0.95$




New anistropic (A) and uniaxial (U) fixed points

The isotropic strong coupling (SC) fixed point is always stable.

## Conclusion

NPRG approach to the KPZ problem (including long-range noise and anisotropy)

naive extrapolation from weak- to strong-coupling behavior not possible

strong-coupling behavior changes qualitatively the phase diagram

no hint for an upper critical dimension

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