Quantum Critical Points On Graphene's Honeycomb Lattice

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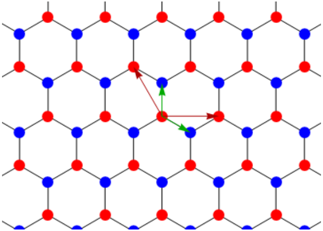
based on: L.J. & I. Herbut, Phys. Rev. B **89**, 205403 (2014)

Half-filled Hubbard model on honeycomb lattice

- Hamiltonian:

$$H = H_0 + H_1$$



- Hopping:

$$H_0 = -t \sum_{\vec{A},i,\sigma=\pm 1} u_\sigma^\dagger(\vec{A}) v_\sigma(\vec{A} + \vec{\delta}_i) + h.c.$$

- Interaction:

$$H_1 = \sum_{\vec{X},\vec{Y},\sigma,\sigma'} n_\sigma(\vec{X}) \left[\frac{U}{2} \delta_{\vec{X},\vec{Y}} + \frac{e^2(1-\delta_{\vec{X},\vec{Y}})}{4\pi|\vec{X}-\vec{Y}|} \right] n_{\sigma'}(\vec{Y})$$

U: on-site repulsion ("Hubbard-U")

e^2: long-range part of Coulomb interaction ("dim'less charge")

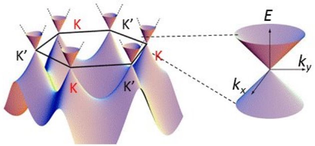
Low-energy effective theory

- Spectrum of H_0:

  $$\varepsilon_{1,2}(\vec{q}) = \pm v_F|\vec{q} - \vec{K}_{1,2}| + \mathcal{O}((\vec{q} - \vec{K}_{1,2})^2)$$

  Dirac points: $\vec{K}_{1,2} = \pm\vec{K}$

  Fermi velocity: $v_F = \frac{\sqrt{3}}{2}ta$



- Low energy:

[Semenoff, PRL 1984]

$$S_0 = \int d\tau d\vec{x} \sum_{\sigma=\uparrow,\downarrow} \bar{\psi}_\sigma(\vec{x},\tau)\gamma_\mu\partial_\mu\psi_\sigma(\vec{x},\tau)$$

where

$$\psi_\sigma(\vec{q}) = \begin{pmatrix} u_\sigma(\vec{K}+\vec{q}) \\ v_\sigma(\vec{K}+\vec{q}) \\ u_\sigma(-\vec{K}+\vec{q}) \\ v_\sigma(-\vec{K}+\vec{q}) \end{pmatrix}, \qquad \{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}\mathbb{1}_4, \ (\mu,\nu = 0,1,2)$$

Interactions

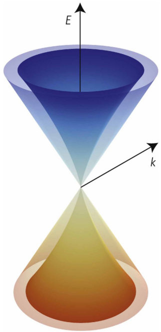
- Charge is RG (marginally) irrelevant:
[Gonzalez, Guinea, Vozmediano, NPB 1994 & PRB 1999]

$$-k\partial_k e^2 \propto -k\partial_k \left(\frac{e_{\text{el}}^2}{v_{\text{F}}} \right) < 0$$

- Lorentz symmetry emergent even with interactions:

dynamical critical exponent $z = 1$

[Herbut, PRL 2006]
[Herbut, Juricic, Vafek, PRB 2009]
[Juricic, Herbut, Semenoff, PRB 2009]



[Elias *et al.*, Nat. Phys. 2011]

- Low-energy effective theory:

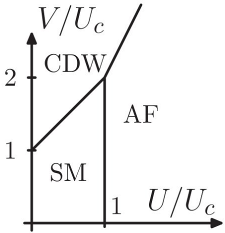
$$S_1 = S_0 + \int d\tau d\vec{x} \sum_x g_x \left(s^x_{\sigma\sigma'} \bar{\psi}_\sigma M_x \psi_{\sigma'} \right)^2$$

with $M_x \in \mathbb{C}^{4\times4}$, $s^x \in \mathbb{C}^{2\times2}$ and compatible with lattice symmetries

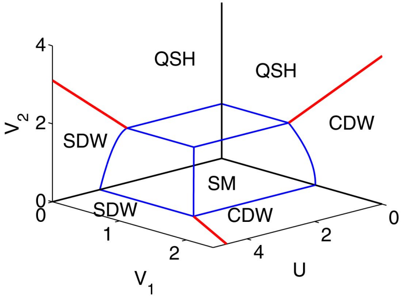
[Herbut, Roy, Juricic, PRB 2009]

Schematic phase diagram

If interactions strong enough: (some) symmetry breaking



[Herbut, PRL 2006]



[Raghu, Qi, Honerkamp, Zhang, PRL 2008]

Transition is probably direct and continuous . . . [Assaad, Herbut, PRX 2013]

. . . but what are the universality class and the critical exponents?

Gross-Neveu-Yukawa effective theories

- Lagrangian in $2 < D < 4$:

$$\mathcal{L} = \bar{\Psi}(\mathbb{1}_2 \otimes \gamma_\mu)\partial_\mu \Psi + \frac{1}{2}\phi_a(\bar{m}^2 - \partial_\mu^2)\phi_a + \bar{\lambda}(\phi_a^2)^2 + \bar{g}\phi_a \bar{\Psi}(\sigma_a \otimes \mathbb{1}_4)\Psi$$

where $\Psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}$ 8-component, $\quad \bar{\Psi} = \Psi^\dagger(\mathbb{1} \otimes \gamma_0)$

(1) "chiral Ising": $a \equiv 0$, $\sigma_0 \equiv \mathbb{1}_2$

$$\langle \phi_0 \rangle \propto \langle \bar{\Psi}\Psi \rangle = \sum_{\vec{k}=\pm\vec{K}+\vec{q}} \sum_{\sigma=\uparrow,\downarrow} \Big(\langle u_\sigma^\dagger u_\sigma \rangle - \langle v_\sigma^\dagger v_\sigma \rangle \Big)$$

$\Rightarrow$ CDW order parameter

(2) "chiral H'berg": $a = 1, 2, 3$, σ_a: Pauli matrices

$$\langle \vec{\phi} \rangle \propto \langle \bar{\Psi}(\vec{\sigma} \otimes \mathbb{1}_4)\Psi \rangle = \sum_{\vec{k}=\pm\vec{K}+\vec{q}} \sum_{\sigma=\uparrow,\downarrow} \Big(\langle u_\sigma^\dagger \vec{\sigma}_{\sigma\sigma'} u_{\sigma'} \rangle - \langle v_\sigma^\dagger \vec{\sigma}_{\sigma\sigma'} v_{\sigma'} \rangle \Big)$$

$\Rightarrow$ AF order parameter

Spontaneous symmetry breaking

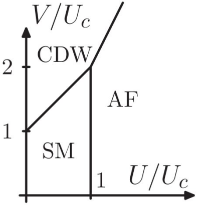
Strong coupling $\frac{\bar{g}^2}{\bar{m}^2}$: expect finite order parameter

χ Ising: $\langle\phi_0\rangle \propto \langle\bar{\Psi}\Psi\rangle \neq 0$ breaks $\mathbb{Z}_2$ sublattice-exchange

- SM to CDW transition (for large V_1)
- in (standard) $\mathbb{Z}_2$–Gross-Neveu universality

…exponents fairly well known

[Rose, Vitale, Wetterich, PRL 2001]
[Hofling, Nowak, Wetterich, PRB 2002]
[Braun, Gies, D. Scherer, PRD 2011]
…



χ H'berg: $\langle\vec{\phi}\rangle \propto \langle\Psi(\vec{\sigma}\otimes\mathbb{1}_4)\Psi\rangle \neq \vec{0}$ breaks $\mathrm{SU}(2)_{\mathrm{sp}}$ to $\mathrm{U}(1)$

- SM to AF transition (for large U)
- $\mathrm{SU}(2)$–Gross-Neveu universality class not yet as well understood

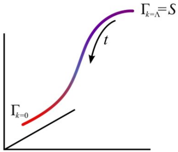
however, ϵ-expansion: [Rosenstein, Yu, Kovner, PLB 1993] [Herbut, Juricic, Vafek, PRB 2009]

…and also aimed at in simulations: [Sorella, Otsuka, Yunoki, Sci. Rep. 2012] [Assaad, Herbut, PRX 2013]

We investigate both by means of FRG …

…and use χ Ising to test validity of nonperturbative approximation

FRG approach



- Wetterich equation:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr}\left[\partial_k R_k (\Gamma_k^{(2)} + R_k)^{-1}\right]$$

- LPA' truncation:

$$\Gamma_k = \int d^D x \left[Z_{\Psi,k} \bar{\Psi}(\mathbb{1} \otimes \gamma_\mu)\partial_\mu \Psi - \frac{1}{2} Z_{\phi,k} \phi_a \partial_\mu^2 \phi_a + U_k(\rho) + \bar{g}_k \phi_a \bar{\Psi}(\sigma_a \otimes \mathbb{1}_4)\Psi \right], \qquad \left[\rho(x) \equiv \frac{1}{2}\phi_a \phi_a\right]$$

χ Ising: $a \equiv 0$ OP $\phi_a \equiv \phi_0 \in \mathbb{R}$

χ H'berg: $a = 1, \dots, 3$ OP $(\phi_a) = \vec{\phi} \in \mathbb{R}^3$

- includes all (perturbatively) relevant operators

[...also many more!]

- becomes exact near $D = 4$

[...we show it also becomes exact near $D = 2$ (for sharp cutoff)]

RG flow near $D = 4$

Flow equations complicated nonlinear functions ...

... but simplify considerably when $D = 4 - \epsilon$ with $\epsilon \ll 1$!

- β functions: [L.J. & Herbut, PRB **89**, 205403 (2014)]

$$\partial_t \lambda = (-\epsilon + 2\eta_\phi)\lambda + 4(S+9)\lambda^2 - 2g^4$$

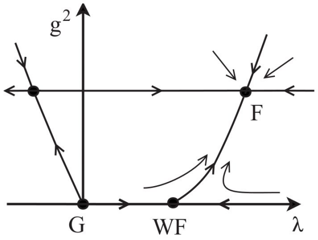
$$\partial_t g^2 = (-\epsilon + \eta_\phi + 2\eta_\psi)g^2 - 2(S-1)g^4$$

- anomalous dimensions:

$$\eta_\phi = 4g^2 \qquad \eta_\psi = \frac{S+1}{2} g^2$$

(χ Ising: $S = 0$, χ H'berg: $S = 2$)

$\Rightarrow$ exactly the flow equations in $(4 - \epsilon)$-expansion (one-loop)!

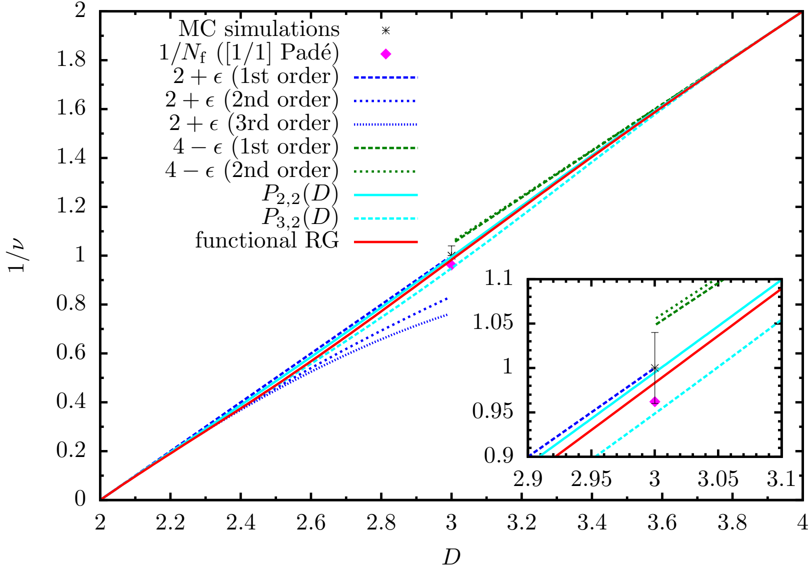


[Herbut, Juricic, Vafek, PRB 2009]

Critical point in $2 < D < 4$: Correlation-length exponent

[L.J. & Herbut, PRB **89**, 205403 (2014)]

χ Ising: [SM-CDW transition]



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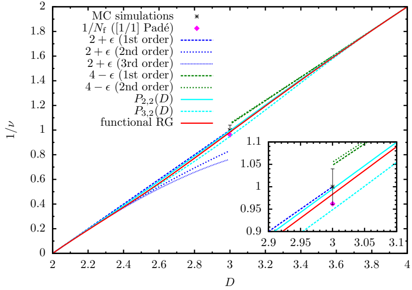


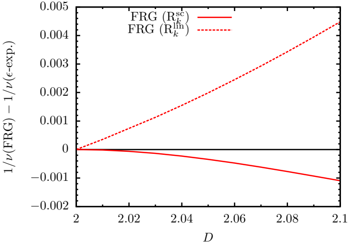
$D = 4 - \epsilon$: exact

$D = 2 + \epsilon$: exact (sharp cutoff)

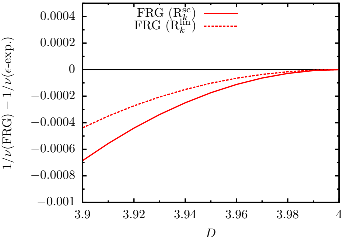
(linear cutoff: slightly not so!)

$D = 3$: agrees within $\Delta\nu/\nu \simeq$ 3–7 % with best available results

$D \geq 2$:



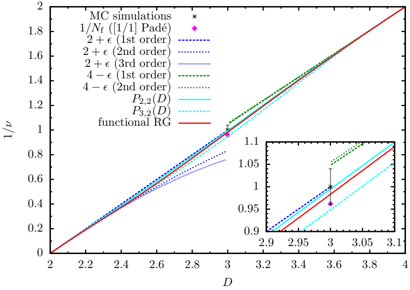
$D \leq 4$:



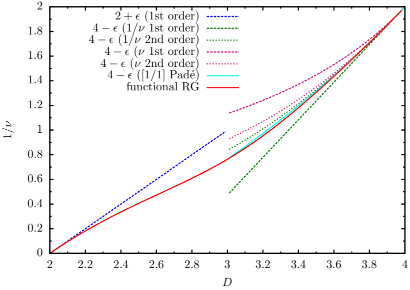
Critical point in $2 < D < 4$: Correlation-length exponent

[L.J. & Herbut, PRB **89**, 205403 (2014)]

χ Ising: [SM-CDW transition]



χ H'berg: [SM-AF transition]



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[L.J. & Herbut, PRB **89**, 205403 (2014)]

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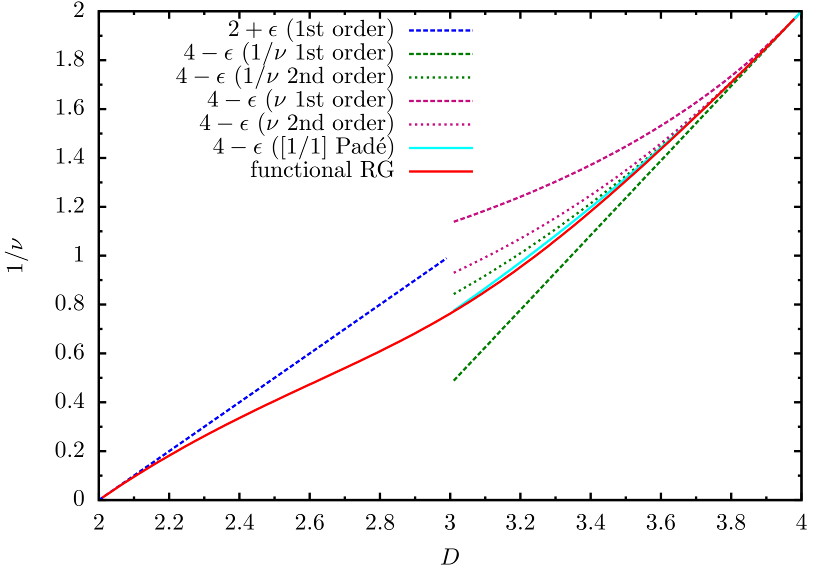
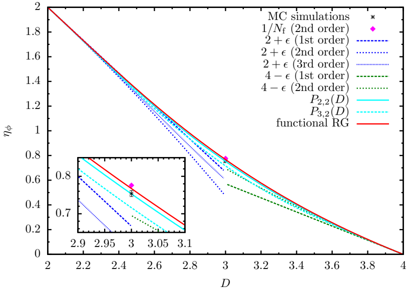


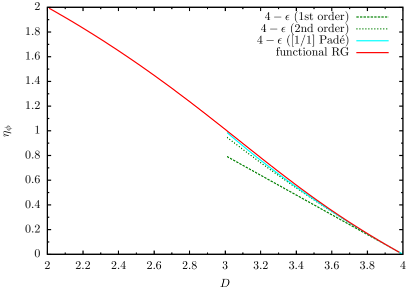
Critical point in $2 < D < 4$: Anomalous dimension η_ϕ

[L.J. & Herbut, PRB **89**, 205403 (2014)]

χ Ising: [SM-CDW transition]



χ H'berg: [SM-AF transition]



$D = 4 - \epsilon$: exact

$D = 2 + \epsilon$: exact (sharp cutoff)

$D = 3$: agrees within $\Delta\eta/\eta_\phi \simeq$ 3–6 %
with best available results

. . . although $\eta_\phi \simeq \mathcal{O}(1)$ large!

Conclusions

(1) QCPs on honeycomb lattice in two distinct (new) universality classes:

χ Ising (SM-CDW) vs. χ H'berg (SM-AF)

(2) FRG in LPA' desribes critical behavior surprisingly well . . .

. . . agreement in mid-single-digit percent range (χ Ising)

(3) Similar accuracy in χ H'berg?

. . . however: β(FRG) $\simeq$ 1.3 vs. β(QMC) $\simeq$ 0.8 ?

[Sorella, Otsuka, Yunoki, Sci. Rep. 2012]

(4) χ universality classes ideal testing ground for nonperturbative approximation schemes

. . . within FRG and beyond

Symmetries

- Sublattice-exchange symmetry ($u \leftrightarrow v$):

  $$\mathbb{Z}_2 : \quad \Psi \mapsto (\mathbb{1} \otimes \gamma_2)\Psi, \qquad \phi_a \mapsto -\phi_a$$

  (and $q_x \mapsto -q_x,\ q_y \mapsto q_y$)

- Rotations in spin space:

  $$\mathrm{SU(2)_{sp}} : \quad \Psi \mapsto e^{i\theta\vec{n}\cdot(\vec{\sigma}\otimes\mathbb{1}_4)}\Psi, \quad \phi_0 \mapsto \phi_0, \quad \vec{\phi} \mapsto R\vec{\phi} \quad [R \in \mathrm{O(3)}]$$

- Charge conservation:

  $$\mathrm{U(1)_{ch}} : \quad \Psi \mapsto e^{i\theta}\Psi, \quad \phi_a \mapsto \phi_a$$

- Translational invariance ("chiral" symmetry): [Herbut, Juricic, Roy, PRB 2009]

  $$\mathrm{U(1)}_\chi : \quad \Psi \mapsto e^{i\theta(\mathbb{1}_2\otimes\gamma_{35})}\Psi, \quad \gamma_{35} \equiv -i\gamma_3\gamma_5$$

- χ Ising model: elevated chiral symmetry (not in χ H'berg!)

  $$\mathrm{SU(2)}_\chi : \quad \Psi \mapsto e^{i\theta\vec{n}(\vec{\sigma}\otimes\gamma_{35})}\Psi, \quad \phi_0 \mapsto \phi_0$$

- Together:

  χ Ising: $\mathbb{Z}_2 \times \mathrm{U(2)} \times \mathrm{U(2)}$

  χ H'berg: $\mathbb{Z}_2 \times \mathrm{U(2)} \times \mathrm{U(1)}$