Effective action for fermion composite operators

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Literature

This talk is based on the papers:

A. Patkos, Mod.Phys.Lett. A27 (2012) 1250212
AJ and A. Patkos, Phys. Rev. D88, 065008 (2013)
AJ, A. Patkos and P. Posfay, arXiv:1406.3195 [hep-th]

Other papers we used:

B. Rosenstein, D. Warr and S.H. Park, Phys. Rev. Lett. 62, 1433 (1989)
H. Gies and C. Wetterich, Phys. Rev. D65:065001 (2002)
J. Jaeckel and C Wetterich, Phys. Rev. D68:025020 (2003)
J. Braun, H. Gies and D.D. Scherer, Phys. Rev. D83:085012 (2011)
...

cf. also A. Eberlein's and W. Metzner's talk

Outlines

Outlines

Motivation

- fundamental matter is fermionic (+ Higgs)
  $\Rightarrow$ bosonic matter are fermionic compounds (bound states)
  describe them without auxiliary bosonic representants
- double representation of the same physical quantity?
  (cf. linear sigma model)
  understand the systematics
- scalar potential $\equiv$ potential for fermionic composite operators
  condensation (Bose-, chiral condensation, superfluidity)
  how can it be consitent with the fermionic nature?

Outlines

consider $\Gamma[\bar{\psi}, \psi, \Phi]$

- Φ bosonic fields – omit for the present discussion
- $\bar{\psi}$, ψ are fermionic (Grassmann) fields; Nambu representation $\Psi = \begin{pmatrix} \psi \\ \bar{\psi}^T \end{pmatrix}$

Wetterich equation for evolution in scale k:

$$\partial_k \Gamma_k[\Psi] = \frac{1}{2} \hat{\partial}_k \, \mathrm{STr} \log \left(\Gamma^{(1,1)}_{k,\Psi\Psi} + R_k \right)^{-1}$$

where

- R_k regulator
- $\hat{\partial}_k$: k derivation acting only on R_k
- $(\Gamma^{(1,1)}_{k,\Psi\Psi})_{ij} = \overrightarrow{\partial}\Psi_i \Gamma_k[\Psi] \overleftarrow{\partial}\Psi_j = \begin{pmatrix} \Gamma_{\psi\psi} & \Gamma_{\psi\bar{\psi}} \\ \Gamma_{\bar{\psi}\psi} & \Gamma_{\bar{\psi}\bar{\psi}} \end{pmatrix}$

Local fermionic potential approximation

We need an Ansatz to be able to handle the Wetterich equation.
Simple approach: $\mathrm{LPA}' = \mathrm{LPA} +$ wave function renormalization:

$$\Gamma_k[\Psi] \to \int d^d x \, \left[Z_k \bar\psi \partial\!\!\!/ \psi + U_k(\Psi_x) \right].$$

How should we interpret it in the fermionic case?

- $\psi_i^2(x) = 0$ because of the fermionic nature
  $\Rightarrow \quad (\bar\psi C \psi)^n = 0$ for large enough n!
  $\Rightarrow \quad U_k(\Psi)$ is a finite polynomial?

caveats

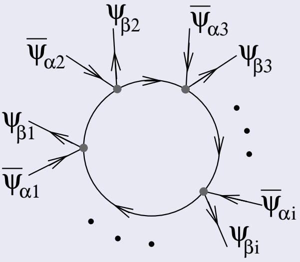
- observed condensation of fermion composites
- after bosonization (Hubbard-Stratonovich trf.) $U_b(\Phi)$ is allowed in any form!
- $\psi(x_1), \ \psi(x_2) \quad \Rightarrow \quad x_1 = x_2$ very unlikely...

Expansion of the exact effective potential for generic background:

$$\Gamma_k[\Psi] = \sum_{n\,\mathrm{even}} \frac{1}{n!} \sum_{indices} \int_{x_i} \Gamma^{(n)}_{k;i_1\ldots i_n}(x_1,\ldots,x_n)\Psi_{i_1}(x_1)\ldots\Psi_{i_n}(x_n)$$

- the proper vertices $\Gamma^{(n)} \neq 0$
- in functional sense
  Γ_k is not a finite polynomial!

n-point function



for 4-fermion interaction

Assumption behind the LPA

propagator varies in spacetime much slower than vertices

- assume that $\Gamma_k^{(n>2)}$ localized within L resolution (compositeness) scale (or sufficiently small outside):

  $\Gamma_k^{(n)}(x, \ldots, x_n) \neq 0$ only for $|x - x_{i>1}| < L$ (ie. $x_{i>1} \in \Delta V_x$)

- assume the most important configurations for the k-evolution are slowly varying on this scale, ($L\partial\Psi \approx 0$)

The n-th term

$$\int dx \int_{x_i \in \Delta V_x} \prod_{i>1} dx_i \, \Gamma^{(n)}(x, \ldots, x_n)\Psi(x)\Psi(x_2)\ldots\Psi(x_n)$$

use average value

$x \neq x_{i>1}$, but to compute correlations we can use $\Psi(x_i) \approx \Psi(x)$

$\Rightarrow \int dx U^{(n)}(x)\Psi^n(x)$

just for notation! $\Rightarrow$ like wave fnct. renormalization...

Fermionic vs. bosonic potential

Compare the fermionic LPA' with the bosonized potential version.
Simplest case with just one scalar field:

$$\Gamma_k[\Psi] = \int d^d x \, \left[Z_k \bar{\psi} \partial\!\!\!/ \psi + U_k(\Psi) \right]$$

$$\Gamma_k^{aux}[\Psi, \sigma] = \int d^d x \, \left[Z_k \bar{\psi} \partial\!\!\!/ \psi + U_{k,aux}(\sigma) + \sigma \bar{\psi} \psi \right]$$

Constant background, use EoM: $\bar{\psi}\psi = -U'_{k,aux}(\sigma)$ to arrive at:

$$U_k(\Psi) = U_{k,aux}(\sigma) - \sigma U'_{k,aux}(\sigma)$$

Legendre transformation.

$\Rightarrow$ if asymptotically $U_{k,aux}$ increases faster than σ, then the $U_{k,aux} < \sigma U'_{k,aux}$, so asymptotically $U_k(\Psi) < 0$

inverse asymptotic behaviour

with the convention $\bar{\psi} \to i\bar{\psi}$ we would get positive potential

Evaluation of the tracelog

We have to calculate $\mathrm{Tr}\log(\Gamma^{(2)} + R)$ $\Rightarrow$ trace of a supermatrix.
Representation

$$\begin{pmatrix} \Gamma_{\psi\psi} & \Gamma_{\psi\bar\psi} \\ \Gamma_{\bar\psi\psi} & \Gamma_{\bar\psi\bar\psi} \end{pmatrix} = \begin{pmatrix} 1 & C \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & B \\ A & 0 \end{pmatrix} \begin{pmatrix} 1 & \bar C \\ 0 & 1 \end{pmatrix}$$

From consistency:

$$A = \Gamma_{\bar\psi\psi}, \quad B = M\Gamma_{\psi\bar\psi}, \quad C = \Gamma_{\psi\psi}\Gamma^{-1}_{\bar\psi\psi}, \quad \bar C = \Gamma^{-1}_{\psi\bar\psi}\Gamma_{\bar\psi\bar\psi}$$

$$M = 1 - \Gamma_{\psi\psi}\Gamma^{-1}_{\bar\psi\psi}\Gamma_{\bar\psi\bar\psi}\Gamma^{-1}_{\psi\bar\psi}.$$

Therefore

$$\mathrm{Tr}\log\Gamma^{(1,1)}_{k,\Psi\Psi} = \mathrm{Tr}\log\Gamma_{\bar\psi\psi} + \mathrm{Tr}\log\Gamma_{\psi\bar\psi} + \mathrm{Tr}\log M$$

Outlines

Gross-Neveu model and representation through invariants

$\Gamma[\Psi]$ is a c-number $\Rightarrow$ depends on fermion bilinears
symmetries $\Rightarrow$ action depends on invariants

Gross-Neveu model: matter content $\psi_i,\ i = 1 \dots N_f$

$$S[\Psi] = \int d^d x \left[\sum_{i=1}^{N_f} \bar{\psi}_i \partial\!\!\!/ \psi_i + \frac{g}{2N_f} I \right]$$

where $I = \left(\sum_{i=1}^{N_f} \bar{\psi}_i \psi_i \right)^2$.

- chiral symmetry: $\psi \to -\gamma_5 \psi,\ \bar{\psi} \to \bar{\psi}\gamma_5$
- $O(N_f)$ flavour symmetry

Ansatz for the effective action

$$\Gamma_k[\Psi] = \int d^d x \left[Z_k \sum_{i=1}^{N_f} \bar{\psi}_i \partial\!\!\!/ \psi_i + U(I) \right]$$

- could depend on other invariants (eg. $(\bar{\psi}\gamma_\mu\psi)^2$)
- assumption on dependence on I only $\Rightarrow$ self-consistent!

Computation overview

General structure of the expressions: $\Gamma^{(2)} \sim G_0^{-1} - \#\Psi \otimes \Psi^T$

$\Rightarrow$ Inverse, tracelog can be computed

- no flavour mixing $\Rightarrow$ use background $\psi = (\zeta, \ldots, \zeta)$
- use static background

The most complicated expressions:

$$\mathrm{Tr}\log\Gamma_{\bar{\psi}\psi} = \mathrm{Tr}\log G_0^{-1} - \int_q \log\left(1 + \tilde{U}N_f(\zeta^T G_0 \zeta)\right)$$

$$\mathrm{Tr}\log M = \int_q \log\left(1 - \frac{\tilde{U}^2 N_f(\zeta^T G_0\zeta)(\zeta^T \hat{G}_0 \zeta)}{(1+\tilde{U}N_f(\zeta^T G_0 \zeta))(1+\tilde{U}N_f(\zeta^T \hat{G}_0\zeta))}\right)$$

where

$$\tilde{U} = 2U' + 4IU'', \quad G_0^{-1} = Z\not{q}(1 + r_k(q)) + m$$
$$\hat{f}(q) = f(-q), \qquad m = 2U'\bar{\psi}\psi$$

Cancellations! ... also in more complicated setup

Finally we get a considerably simple expression:

$$\mathrm{STr}\log\Gamma = -2\,\mathrm{Tr}\log G_0^{-1} + \int_q \log\left(1 - \frac{2m\bar{\psi}\psi}{Z^2 P_k^2(q) + m^2}\right).$$

Evaluating the integrals using Litim's regulator we find

$$\boxed{\partial_k U_k = k^{d+1} Q_d \left[\frac{4N_f + 1}{Z^2k^2 + 4I\,U'^2} - \frac{1}{Z^2k^2 + 4IU'(U' + \tilde{U})}\right]}$$

where $Q_d^{-1} = (4\pi)^{d/2}\Gamma(\frac{d}{2}+1)$.

Fermionic & bosonic type terms! (without explicit bosons)

Outlines

Fixed point analysis

Rescaling:

$$x = \frac{k^{2(1-d-\eta)} I}{(4Q_d N_f)^2} \quad y = \frac{k^{-d} U(I)}{4Q_d N_f} \quad t = \log k$$

we find

$$\partial_t y = -dy + (2d-1)y - \frac{1 - 1/(4N_f)}{1 + 4xy'^2} + \frac{1}{4N_f} \frac{1}{1 + 12xy'^2 + 16x^2 y' y''}$$

For fixed points: $\partial_t y = 0$

polynomial Ansatz $y_*(x) = \sum_n \frac{1}{n} \ell_{*n} x^n$

This yields:

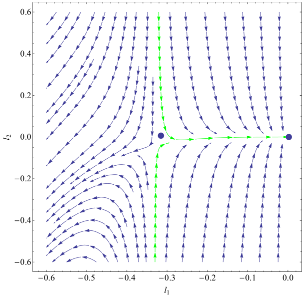
$$0 = (d-2)\ell_{*1} + \left(1 - \frac{1}{2N_f}\right) 4\ell_{*1}^2$$

$$0 = \left[-\frac{d}{n} + 2(d-1) + 8\ell_{*1}\left(1 - \frac{n}{2N_f}\right)\right] \ell_{n*} + \mathcal{F}(\ell_{*i<n})$$

Solvable: $\ell_{*1} \to \ell_{*2} \to \cdots \to \ell_{*n} \to \ldots$.

Flow

For $2 < d < 4$: one nontrivial fixed point for any N_f:
Flow pattern $d = 3$

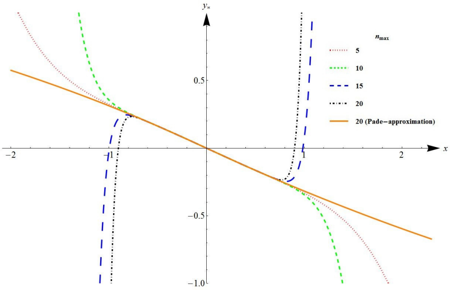


- critical exponents (agree with bosonized version for $N_f = \infty$)

$$\Theta_n = d - 2n\left(1 + \frac{(n-1)(n-2)}{2N_f - 1}\right)$$

- for $d = 3$ only $\Theta_{n=1} = 1$ is relevant.
- $d = 2$ only Gaussian fixed point
  $d = 4$ non-Gaussian FP$\to \infty$ for $N_f = \infty$.

- nth order potential: no convergence for larger x values.
- exact asymptotics can be obtained: $y_{*as} \sim x^{\frac{d}{2(d-1)}}$
  $\Rightarrow$ Padé resummation
- Physical regime $x > 0$
  $\Rightarrow$ $U < 0$ as we expected.
- physical point is at $\Psi = 0$.



$N_f = 2$ potentials

Resummation:

$$y_*(x) = (1+x^2)^{\frac{d}{4(d-1)}} \lim_{N\to\infty} \mathrm{Pade}_N^N \left[\frac{\sum_{n=1}^{2N} \frac{1}{n} \ell_{*n} x^n}{(1+x^2)^{\frac{d}{4(d-1)}}} \right],$$

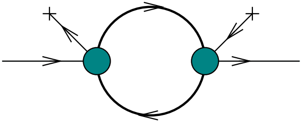
Pade_N^N: resum polynomials with degree $2N$ to ratio of polynomials with degree N.

Wave function renormalization

Formula for determining the wave function renormalization

$$\partial_k Z_k \frac{\delta(0)}{(2\pi)^d} = \frac{1}{N_f} \frac{d}{dq^2} \left\{ -i \,\mathrm{Tr}\, \not{q} \overrightarrow{\partial}_{\bar{\psi}(-q)} \partial_k \Gamma_k \overleftarrow{\partial}_{\psi(q)} \right\}$$

Diagrammatically we have:



- proportional to the fermionic background $\bar{\psi}\psi$
- at the physical point $\bar{\psi}\psi = 0$
- no IR divergences occur

Result

$$Z = 0$$

Outlines

Conclusions

Working with fermionic potential is sensible and feasible
(neglecting $(\bar{\psi}\psi)^n \sim$ neglecting σ^n)

- sensible to speak about $U(\Psi)$ fermionic potential
  $\Rightarrow$ resolution scale, compositeness scale
- fermionic tracelog can be worked out explicitly
- fixed point properties of 3D Gross-Neveu model: one nontrivial fixed point, all critical exponents can be found (agree with the bosonized results for $N_f = \infty$)
- fixed point potential stabilizes using Padé resummation
- wave function renormalization for GN model in LPA' is 1.