Fermionic functional renormalization group approach to antiferromagnetically ordered phases

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in collaboration with **Andreas Eberlein** and **Carsten Honerkamp**

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RWTH AACHEN

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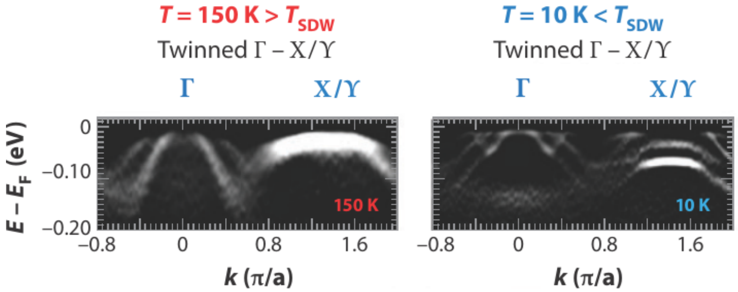
- the DFG for financial support through FOR 723

Introduction – Spin-density wave



ARPES measurements on $BaFe_2As_2$[1]

[1] Lu et al. 2012.

Previous work

fRG studies of spontaneous symmetry breaking

cf. plenary talk by Andreas Eberlein tomorrow

- reduced mean-field models[2]
- fermionic fRG for a singlet superconductor[3]
- channel-decomposed study of singlet superconductors[4]

Our study[5]

- channel-decomposed fRG approach to AF phases

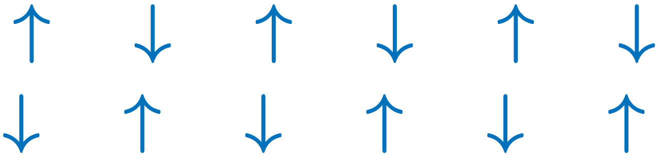
[2] Gersch et al. 2005; Salmhofer et al. 2004; SM and Honerkamp 2012.
[3] Gersch, Honerkamp, and Metzner 2008.
[4] Eberlein and Metzner 2013a,b.
[5] SM, Eberlein, and Honerkamp 2014.

Outline

fRG scheme for Antiferromagnetism
- Parametrization
- Ward identity and relation to mean-field

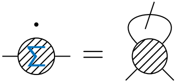
Numerical results for a two-pocket model

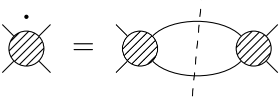
fRG scheme for Antiferromagnetism



One-particle irreducible fRG scheme[7] (in equilibrium)

Katanin truncation[6]



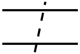


 single scale propagator

$S = \partial_\lambda \; G|_\Sigma$

with flow equations

describing the lowering of the IR cutoff λ



$\partial_\lambda \left[G(p_1) G(p_2) \right] / 2$

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[6]Katanin 2004.

[7]Metzner et al. 2012.

SDW phase – Symmetries

Trigger the symmetry breaking

- apply a small external staggered magnetization

$$\Delta_0 \sum_{x,y} (-1)^{x+y} \, \bar{\Psi}_{x,y} \sigma_z \Psi_{x,y}$$

$$\begin{matrix} \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow \\ \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow \end{matrix}$$

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Broken symmetries

- translational
- spin SU(2)

Residual $\mathrm{U}_z(1)$ spin symmetry

- spin rotation about z axis
  $\Psi \to e^{i\alpha\sigma_z} \, \Psi$

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AF residual composite symmetry

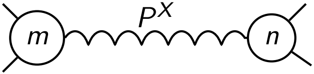
- translation by one site then spin flip
  $\Psi(k) \to e^{ik_x} \, \sigma_x \, \Psi(k)$

Channel decomposition[8]

- decompose the interaction into different channels Φ_X (pairing, CDW, magnetic)
- magnetic channel splits: S_z^2 and $S_x^2 + S_y^2$ differ

[8] Husemann and Salmhofer 2009; Karrasch et al. 2008.

Exchange parametrization[8]



$$\Phi_X(l, q, q') = \sum_{m,n} f_m(q)\, P^X_{m,n}(l)\, f_n(q')$$

- decompose the interaction into different channels Φ_X (pairing, CDW, magnetic)
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- parametrize Φ_X in a Hubbard-Stratonovich spirit

[8] Husemann and Salmhofer 2009.

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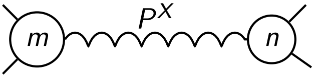


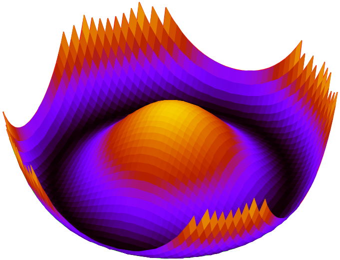
$$\Phi_X(l, q, q') = \sum_{m,n} f_m(q)\, P^X_{m,n}(l)\, f_n(q') \approx P^X(l)$$

- decompose the interaction into different channels Φ_X (pairing, CDW, magnetic)
- magnetic channel splits: S_z^2 and $S_x^2 + S_y^2$ differ
- parametrize Φ_X in a Hubbard-Stratonovich spirit
- only retain constant fermion-boson vertices

[8] Husemann and Salmhofer 2009.

Goldstone theorem

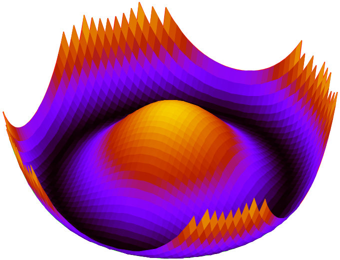
Potential for order parameter



$\Delta_0 \neq 0$ corresponds to a tilt

Goldstone theorem

Potential for order parameter



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Radial mode

- massive bosons
- regular S_z^2 interaction for fermions
- 'Radial vertex'

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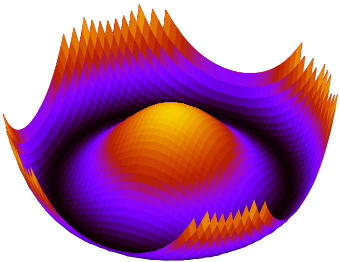


$\Delta_0 \neq 0$ corresponds to a tilt

Radial mode

- massive bosons
- regular S_z^2 interaction for fermions
- 'Radial vertex'

Angular mode(s)

- *massless* bosons
- singular $S_x^2 + S_y^2$ interaction for fermions
- 'Goldstone vertex'

Approximations – Overview

Essential quantities retained

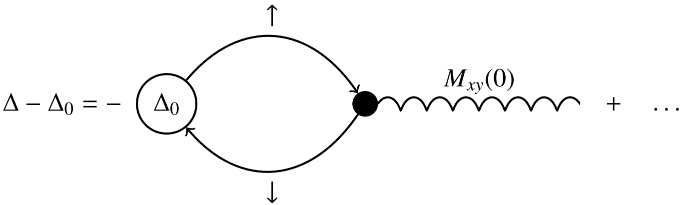
- AF gap $\Delta(k)$
- Radial, Goldstone, CDW and singlet pairing interactions
  (exchange propagators $M_z(l)$, $M_{xy}(l)$, $N(l)$ and $D(l)$)

Approximations – Overview

Essential quantities retained

- AF gap $\Delta(k)$
- Radial, Goldstone, CDW and singlet pairing interactions
  (exchange propagators $M_z(l)$, $M_{xy}(l)$, $N(l)$ and $D(l)$)
- anomalous interactions would be needed
  if the normal self-energy was included

SU(2) Ward identity



Spontaneous symmetry breaking

- limit $\Delta_0 \to 0$
- $M_{xy}(0) \to \infty$ as required by the Goldstone theorem

Measure for the quality of approximations

Random phase approximation

Neglect
the coupling
between
different interaction channels.

Random phase approximation

The mean-field gap equation

$$\Delta = U \int d\mathbf{k} \, \frac{\Delta}{\sqrt{\epsilon(\mathbf{k})^2 + \Delta^2}}$$

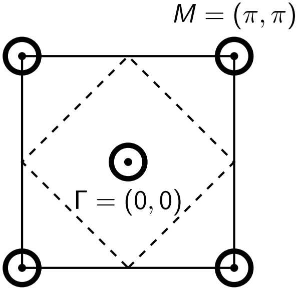
is recovered

and the WI is fulfilled exactly.

Numerical Results for a Two-Pocket Model

Two-pocket model[9] in 2D

Fermi surface

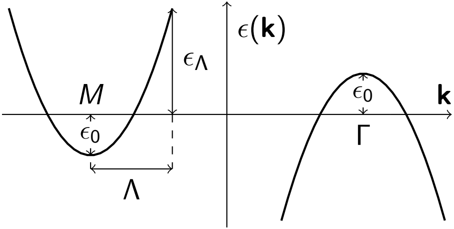


Interaction

featureless, of strength U

Dispersion

(along BZ diagonal)



Simplifications

- circular symmetry
- constant DOS

[9] Chubukov, Efremov, and Eremin 2008.

Implementation

fRG setup

- Additive frequency cutoff

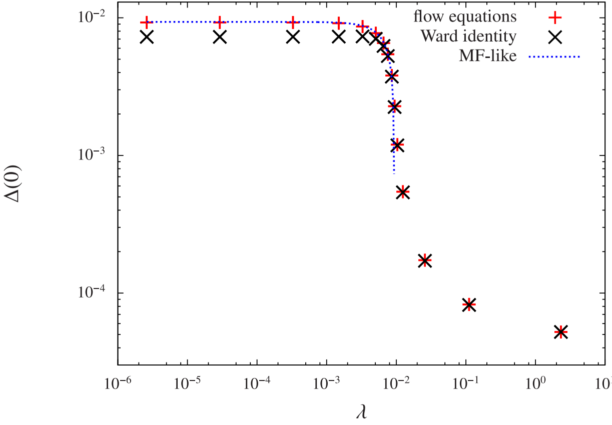
$$ik_0 \to i\,\mathrm{sign}(k_0)\,\sqrt{k_0^2 + \lambda^2}$$

- Momentum parametrization

$$P(l) = \frac{1}{m_P(l_0)\,[1 + n_P(l_0)\,\mathbf{l}^2]}$$

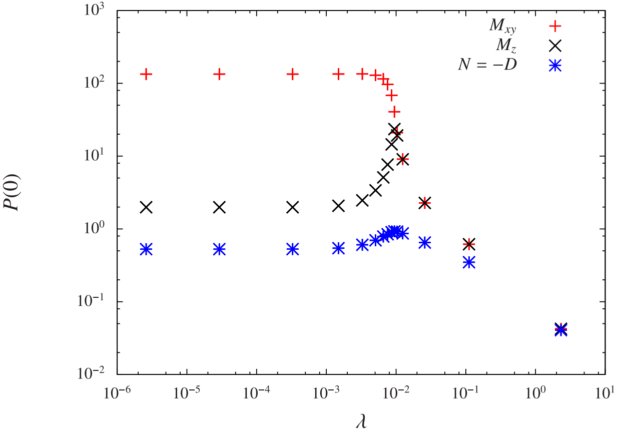
- m_P and n_P live on fixed frequency grid
interpolated elsewhere

Flow of the gap



Parameters: $U = 1.0$, $\epsilon_\Lambda = 0.58$, $\epsilon_0 = 3.0 \cdot 10^{-2}$

Flow of the exchange propagators



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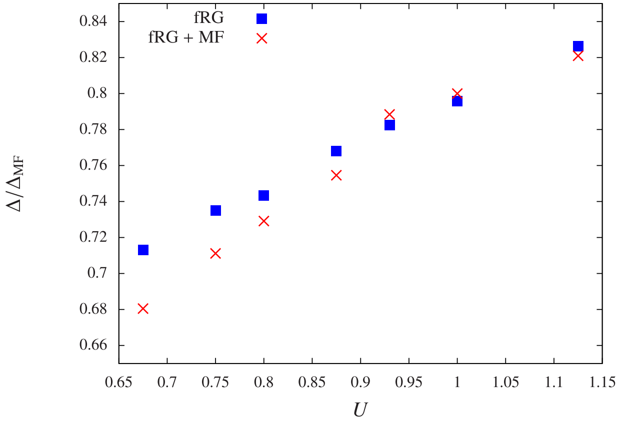
Comparison to fRG+MF[10]



static approximation

Parameters: $\epsilon_\Lambda = 0.58$, $\epsilon_0 = 3.0 \cdot 10^{-2}$

[10] Wang, Eberlein, and Metzner 2014.

Summary

Our fRG approach

- works at least on a qualitative level
- predicts a reduction of the mean-field gap

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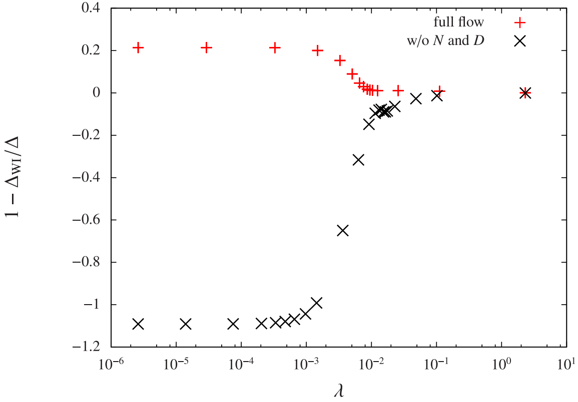
Outlook

- Purely fermionic fRG for AF+dSC

Importance of CDW and pairing fluctuations



- CDW and pairing channels decouple at RPA level.
- Beyond mean-field, however they are crucial.