A beyond the local potential approximation study for the dynamical chiral symmetry breaking in effective model of QCD

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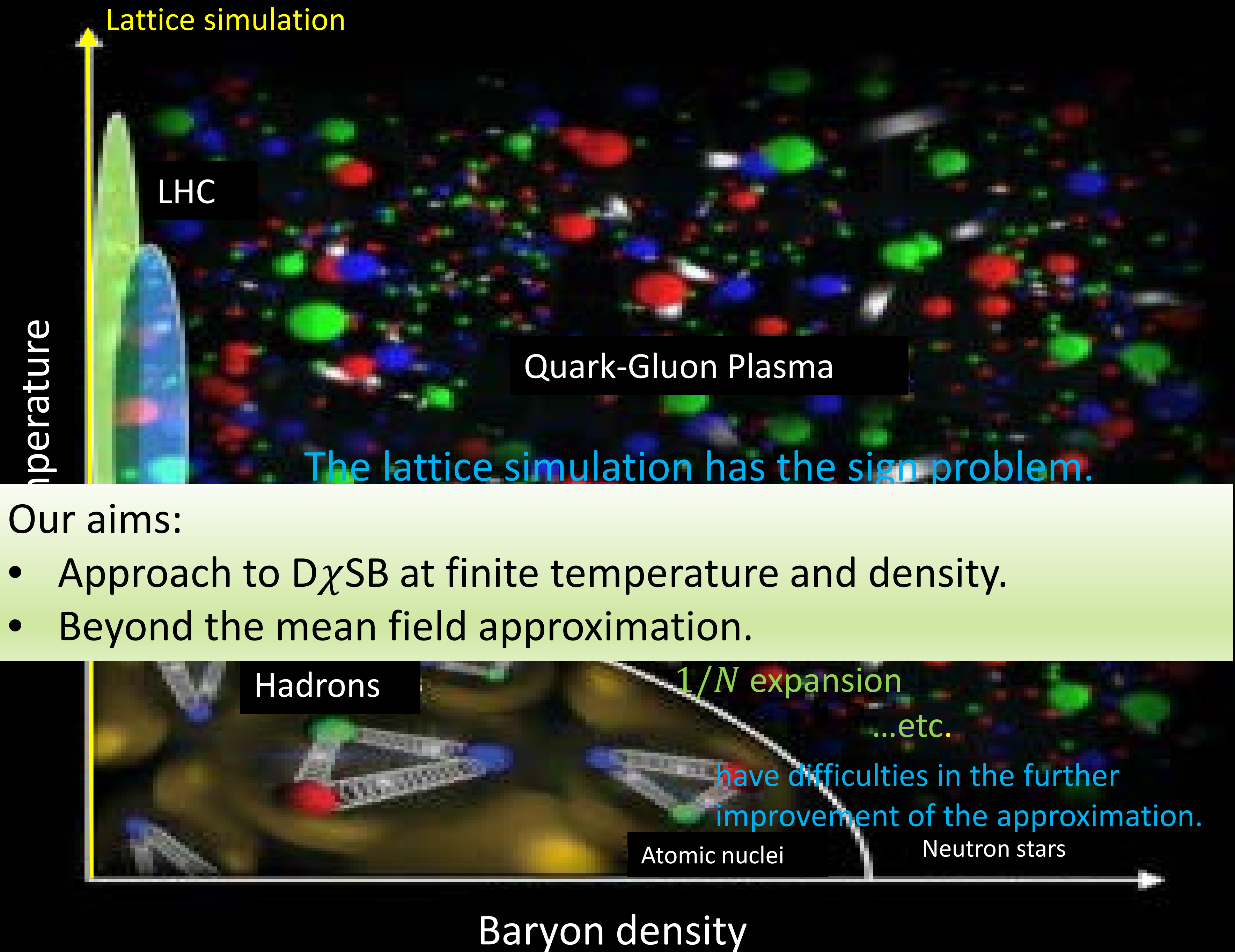
Introduction



Our aims:

- Approach to DχSB at finite temperature and density.
- Beyond the mean field approximation.

Analysis Method

Model

Quark-Meson model with O(4) symmetry

$$\Gamma_\Lambda = \int_x \left\{ Z_\psi(\Phi^2)\bar{\psi}(\gamma^\mu \partial_\mu + i\gamma_0 \mu)\psi + \frac{1}{2} Z_\phi(\Phi^2)(\partial_\mu \Phi)^2 + U(\Phi^2) + \frac{\bar{h}(\Phi^2)}{\sqrt{2}} \bar{\psi}(\sigma + i\gamma_5 \tau^i \pi_i)\psi \right\}$$

- $\Phi^2 = \sigma^2 + \vec{\pi}^2$
- LPA: $Z_\psi = 1, Z_\phi = 1$ at all RG scale.
- The fermion field has degrees of freedom of color.

Background field method

- Split up the meson field into a background field ϕ and a fluctuation field φ, i.e. $\Phi = \phi + \varphi$.
- Expand couplings around the background field ϕ:

$$U(\Phi^2) = U(\phi^2) + U''(\phi^2)\varphi^2 + \mathcal{O}(\varphi^4)$$
$$\bar{h}(\Phi^2) = \bar{h}(\phi^2) + \bar{h}''(\phi^2)\varphi^2 + \mathcal{O}(\varphi^4)$$
$$Z_\psi(\Phi^2) = Z_\psi(\phi^2) + Z''_\psi(\phi^2)\varphi^2 + \mathcal{O}(\varphi^4)$$
$$Z_\phi(\Phi^2) = Z_\phi(\phi^2) + Z''_\phi(\phi^2)\varphi^2 + \mathcal{O}(\varphi^4)$$

  - ✓ Couplings depend on the background field ϕ.
  - ✓ Neglect the derivative terms of $\bar{h}$, Z_ϕ, Z_ψ for simplicity.

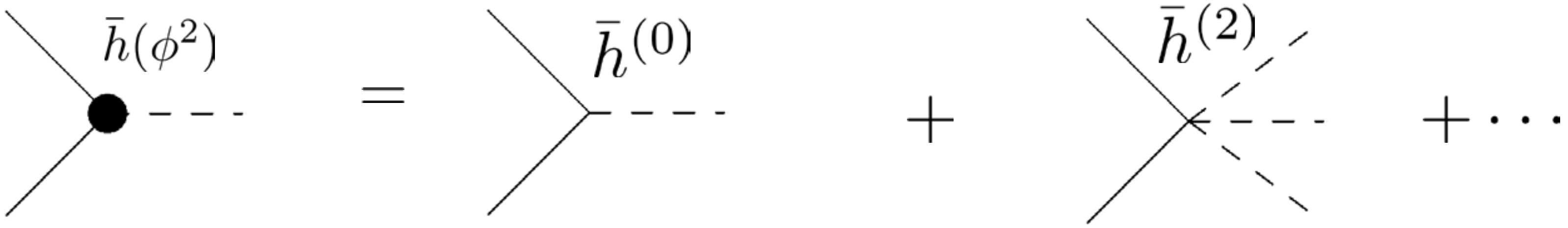
Meaning of $h(\phi^2)$

- "Yukawa coupling $h(\phi^2)$" includes higher order operators:

$$\bar{h}(\phi^2) = \bar{h}^{(0)} + \bar{h}^{(2)}\phi^2 + \bar{h}^{(4)}\phi^4 + \cdots$$

✓Yukawa interactions

$$\frac{\bar{h}(\phi^2)}{\sqrt{2}}\bar{\psi}(\sigma + i\gamma_5\sigma^i\pi_i)\psi = \frac{\bar{h}^{(0)}}{\sqrt{2}}\bar{\psi}(\sigma + i\gamma_5\sigma^i\pi_i)\psi + \frac{\bar{h}^{(2)}}{\sqrt{2}}\phi^2\bar{\psi}(\sigma + i\gamma_5\sigma^i\pi_i)\psi + \cdots$$



RG equations

- Wetterich equation:

$$\partial_t \Gamma_\Lambda = \frac{1}{2}\mathrm{tr}\left[\left(\frac{\delta^2}{\delta\varphi^2}\Gamma_\Lambda + R_\Lambda^\phi\right)^{-1}\cdot(\partial_t R_\Lambda^\phi)\right] - \mathrm{tr}\left[\left(\frac{\delta}{\delta\bar{\psi}}\Gamma_\Lambda\frac{\delta}{\delta\psi} + R_\Lambda^\psi\right)^{-1}\cdot(\partial_t R_\Lambda^\psi)\right]$$

- Use 3d optimized cut-off function:

$$R_\Lambda^\phi = \mathbf{p}^2\left(\frac{\Lambda^2}{\mathbf{p}^2} - 1\right)\theta(1 - \frac{\mathbf{p}^2}{\Lambda^2}) \qquad R_\Lambda^\psi = \not{\mathbf{p}}\left(\frac{\Lambda}{|\mathbf{p}|} - 1\right)\theta(1 - \frac{\mathbf{p}^2}{\Lambda^2})$$

- Obtain four RG equations:

$$U' \equiv \frac{\partial U}{\partial \phi^2}$$

$$\partial_t \mathcal{A}_\Lambda(\phi^2) = \beta_{\mathcal{A}}(\phi^2, U, U', U'', Z_\phi, Z_\psi, h; \Lambda, T, \mu)$$

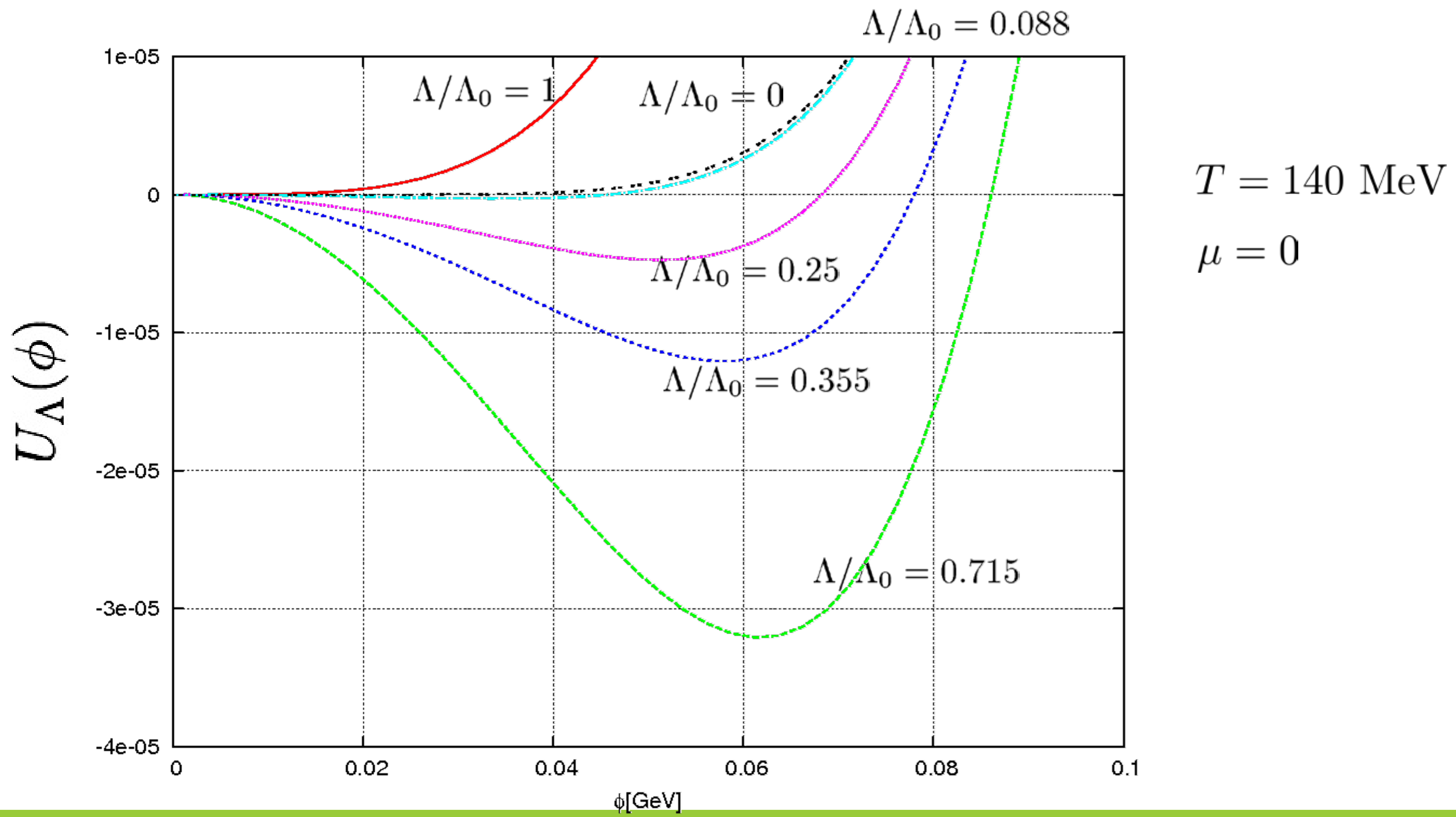
$$\mathcal{A} \in \{U(\phi^2), h(\phi^2), Z_\phi(\phi^2), Z_\psi(\phi^2)\}$$

Solving RG equations

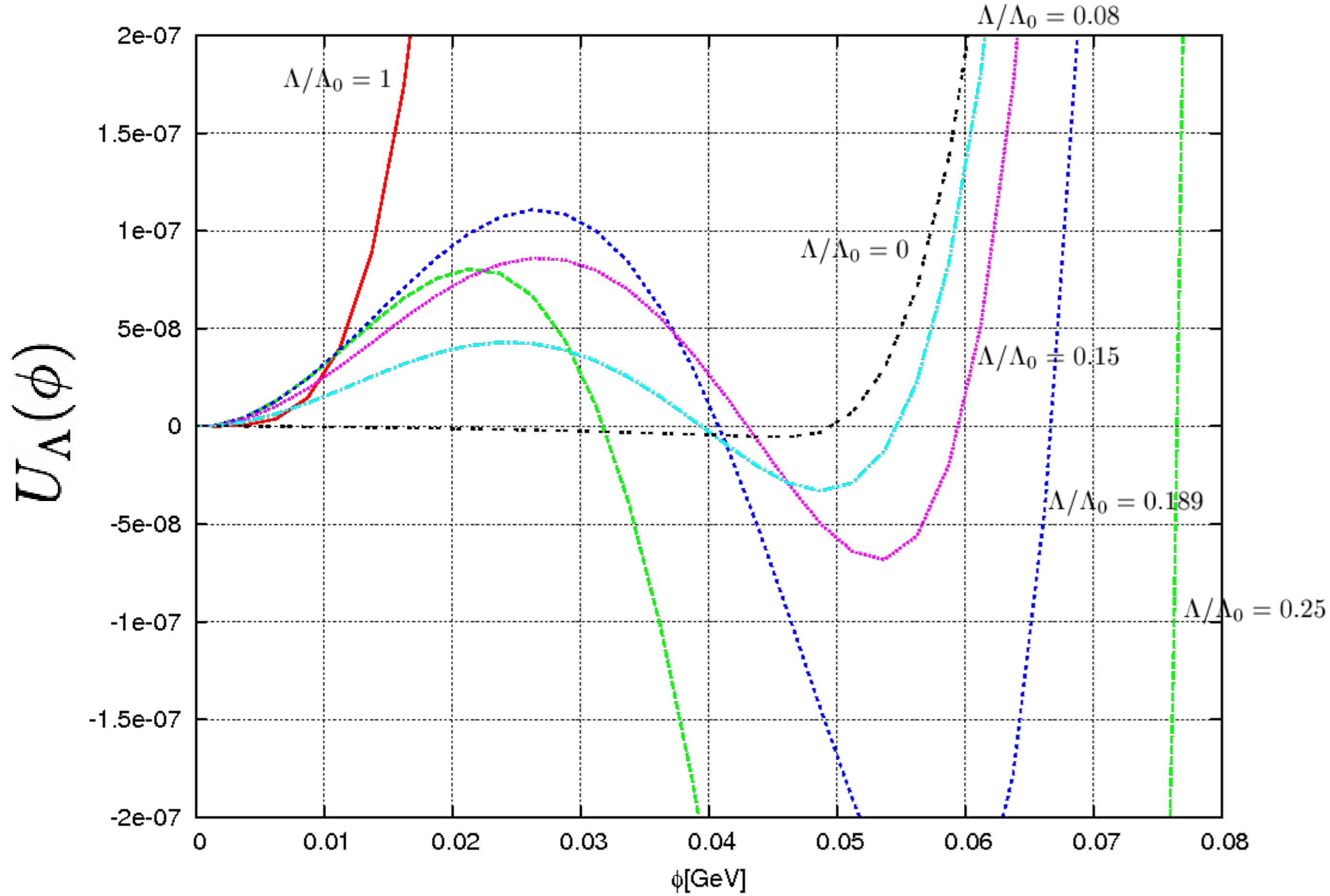
- How to solve the RG equations?
  - ✓We solve numerically RG equations as the coupled partial differential equations.
- In the numerical calculations, we use the grid method:
  - ✓The derivatives U' and U'': the 7-point formula
  - ✓The derivative by scale : fourth-order Runge-Kutta method
- Setting initial values:
  - ✓The field renormalization factors $Z_\psi = Z_\phi = 1$ at $\Lambda = \Lambda_0$.
  - ✓The initial values of $\bar{h},\ \Lambda_0, U(\phi^2)$ yield a vacuum pion decay constant of $f_\pi \sim 87$ MeV.
  - ✓Chiral limit

Numerical Results

RG evolution of potential in LPA without running of Yukawa coupling



RG evolution of potential in LPA without running of Yukawa coupling

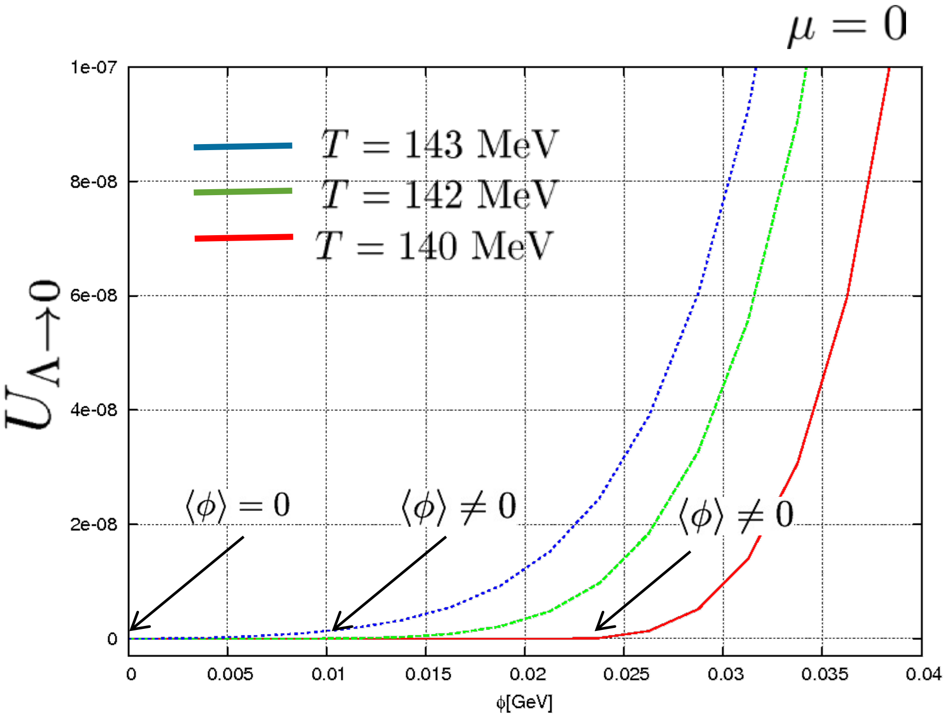


$T = 45$ MeV

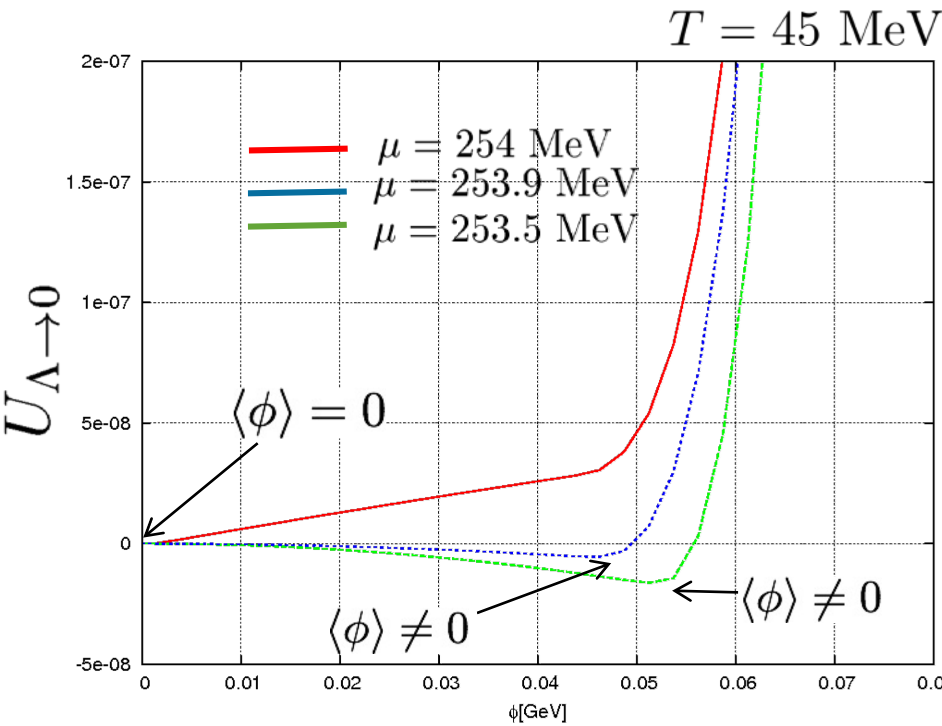
$\mu = 253.9$ MeV

Phase transition

2nd order



1st order



Chiral phase diagram in LPA case

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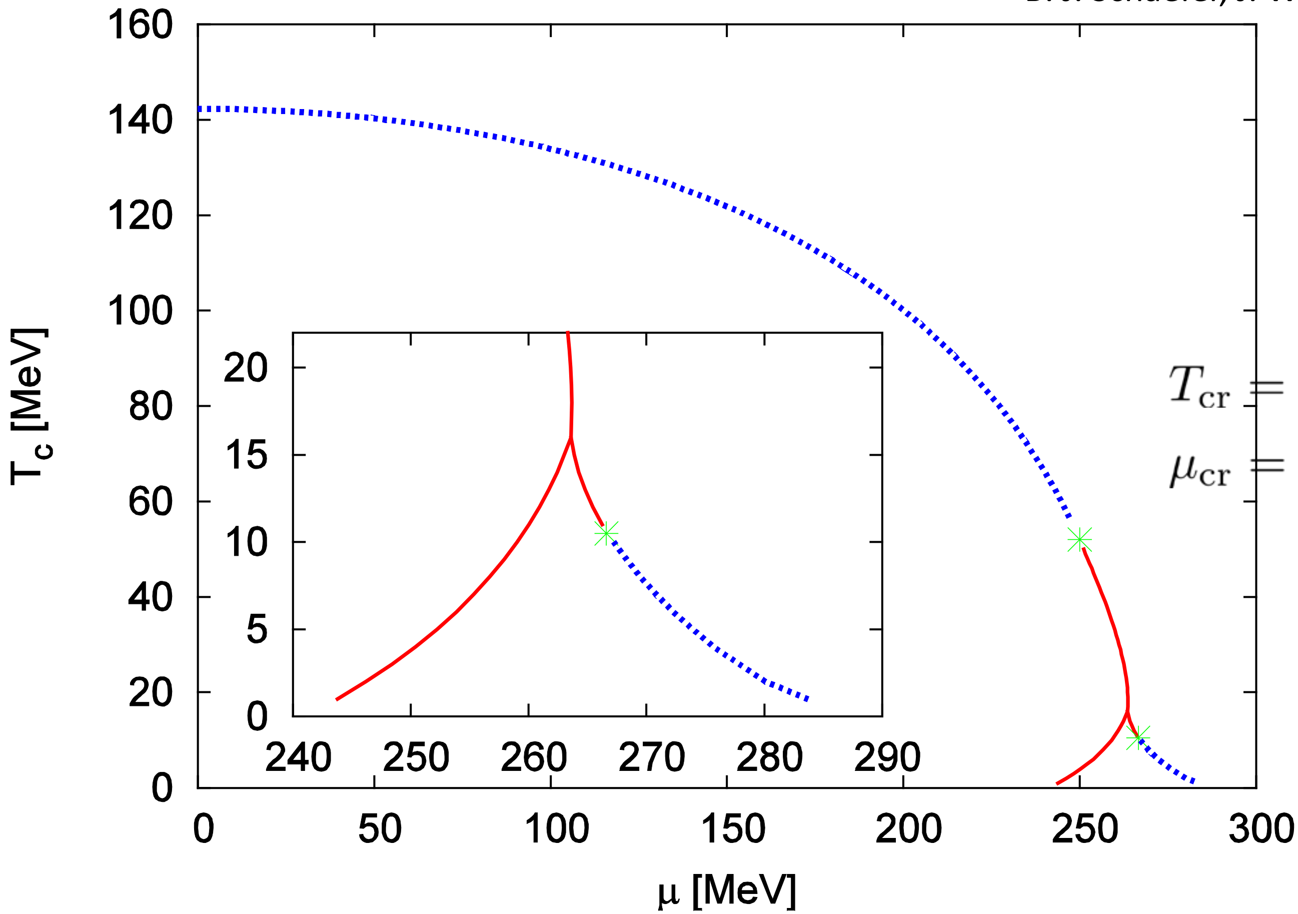
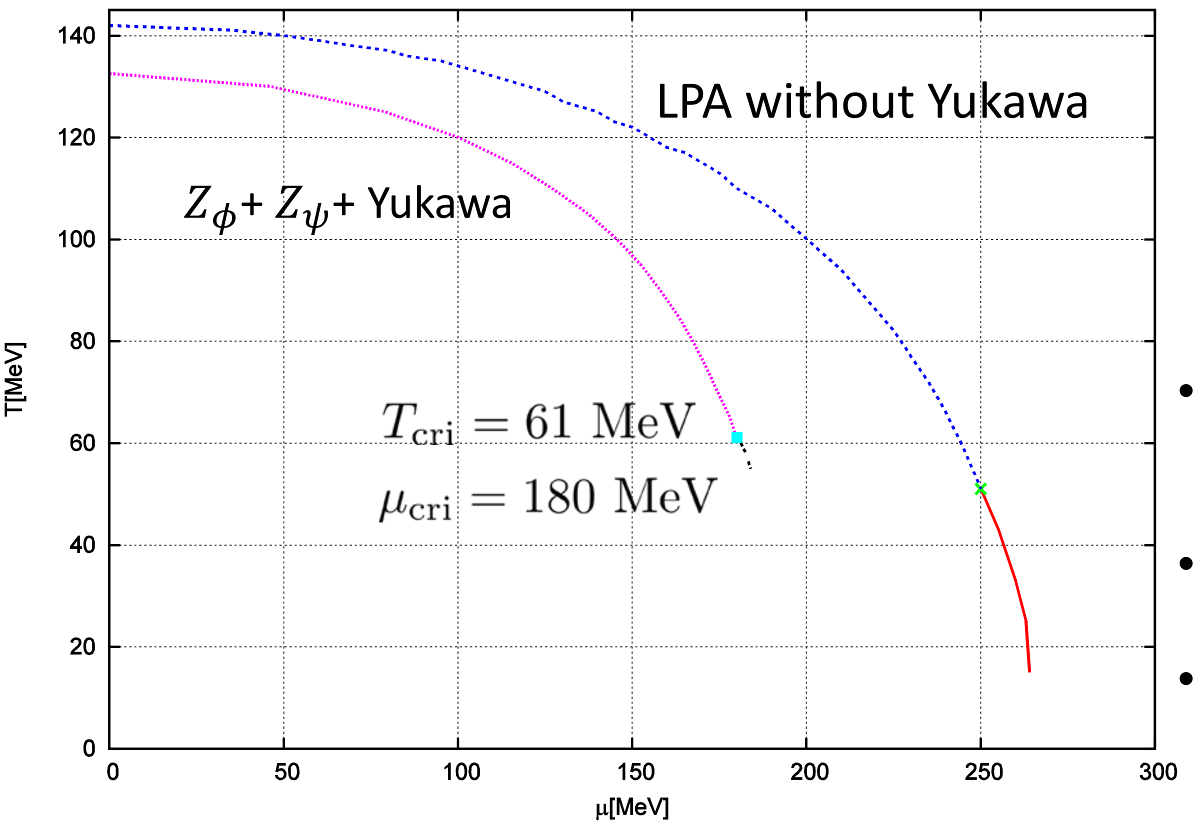


$T_{\rm cr} = 52$ MeV

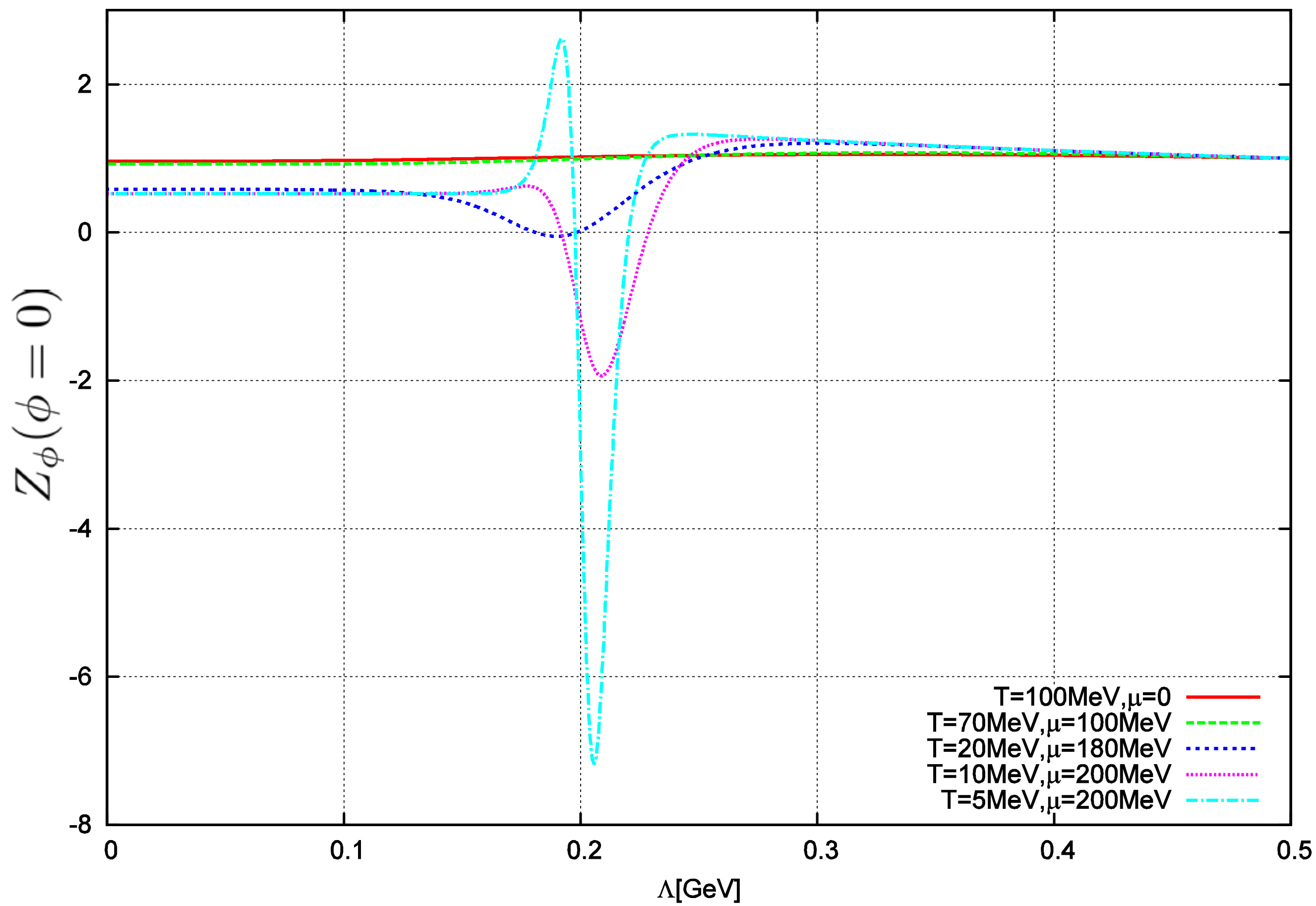
$\mu_{\rm cr} = 251$ MeV

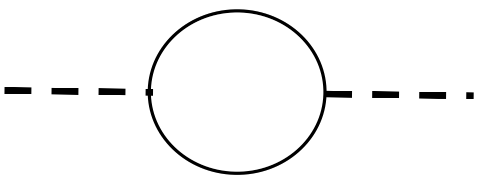
Phase diagram in LPA vs. beyond



- The chiral restoration temperature and density become lower than LPA case.
- In our method, we could find the critical end point.
- However, we could not evaluate low temperature/high density region.

Anomalous dimension for boson





$$\partial_t Z_\phi = (\text{fermion loop})$$

The beta function of Z_ϕ doesn't depend on Z_ϕ.

Summary

- We investigate Quark-Meson model with O(4) at finite temperature and density.
  - ✓ LPA
    - $T_{\mathrm{cri}} = 52$ MeV $\mu_{\mathrm{cri}} = 251$ MeV

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  - ✓ Beyond LPA
    - $T_{\mathrm{cri}} = 61$ MeV $\mu_{\mathrm{cri}} = 180$ MeV
- We want to improve the analysis methods.
  - ✓ How to investigate the low-temperature and high density region?
- Extend Quark-Meson model to bosonized NJL model.