Sarma phase in relativistic and non-relativistic systems

Tina Katharina Herbst

In Collaboration with

I. Boettcher, J. Braun, J. M. Pawlowski, D. Roscher,
N. Strodthoff, L. von Smekal and C. Wetterich

arXiv:1409.5232 and **arXiv:1409.5070**

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Lefkada, Greece

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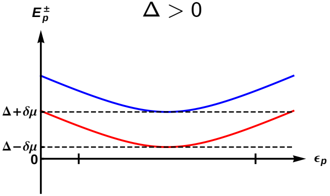
The Sarma Phase

[Sarma (1963)]

homogeneous superfluid phase with gapless fermionic excitations



- 2 fermion species with spin imbalance

$$\delta\mu = \frac{\mu_1 - \mu_2}{2}$$

- Dispersion relation [lowest branches]

$$E_p^{(\pm)} = \sqrt{\varepsilon_p^2 + \Delta^2} \pm \delta\mu$$

[ε_p... microscopic dispersion relation; Δ... pairing gap]

[Boettcher, TKH, Pawlowski, Strodthoff, von Smekal, Wetterich (2014)]

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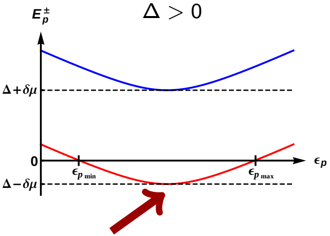

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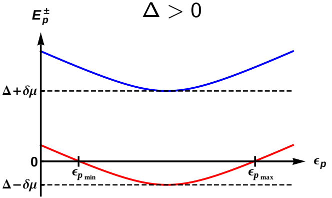

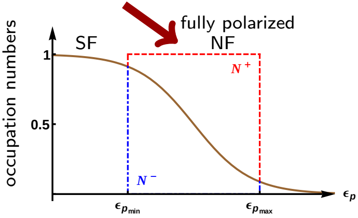

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- Sarma: $\Delta > 0$ and lowest branch below zero
- non-monotonous behavior of occupation numbers
- $T > 0$: Fermi surfaces smeared out
  $\rightarrow$ no sharp distinction
  $\rightarrow$ Sarma *crossover*

[Boettcher, TKH, Pawlowski, Strodthoff, von Smekal, Wetterich (2014)]

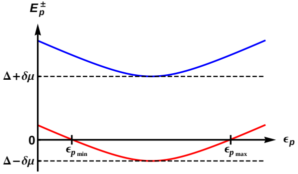
A Simple Criterion for the Sarma Phase

zero-crossing of the lower branch if

$$\delta\mu > \min_p \sqrt{\varepsilon_p^2 + \Delta^2}$$

assume: $\min_p \varepsilon_p = 0 \quad \Longrightarrow \quad \delta\mu_c > \Delta_c$

[NB: not valid on BEC-side, where $\mu < 0$]



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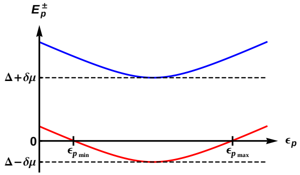

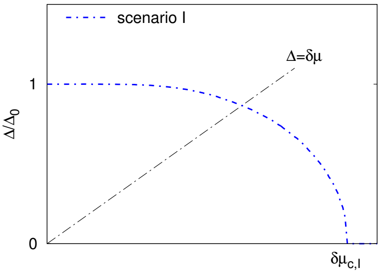

- Second Order Transition:
  Condition always fulfilled

[Boettcher, TKH, Pawlowski, Strodthoff, von Smekal, Wetterich (2014)]

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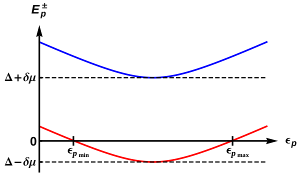

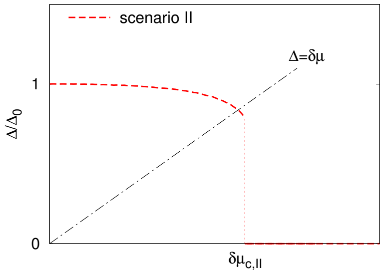

- Second Order Transition:
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- First Order Transition:
  Δ_c vs $\delta\mu_c$ decides
  - $\Delta_c < \delta\mu_c$: Sarma phase

[Boettcher, TKH, Pawlowski, Strodthoff, von Smekal, Wetterich (2014)]

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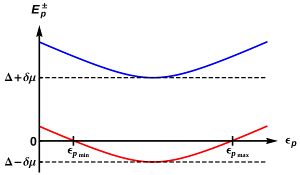

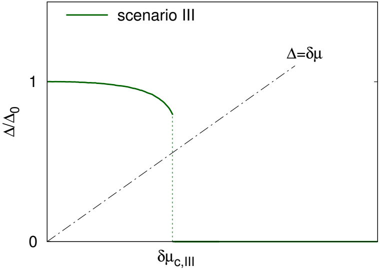

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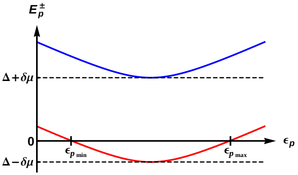

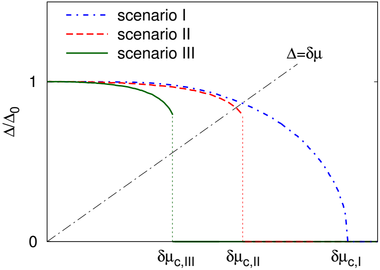

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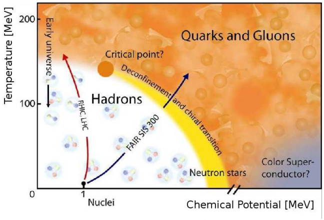
Talk Outline

A Relativistic System:

Quark-Meson Model at Finite Isospin Chemical Potential



[www.gsi.de]

[Remember Monday's talks ? e.g. W. Weise, B.-J. Schaefer, N. Strodthoff]

[F. Rennecke, A. Juricic, N. Khan, J. Luecker, ...]

Quark-Meson Model with Isospin Chemical Potential

[Kamikado, Strodthoff, von Smekal, Wambach (2013)]

- 2 flavors of quarks, $\psi = (u, d)^T$, coupled to mesons, $\sigma, \vec{\pi} = (\pi_0, \pi_+, \pi_-)$
- $\mu_q = \mu_B/3$: imbalance between quarks and antiquarks
- μ_I: imbalance between up and down quarks

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- μ_I: imbalance between up and down quarks
- BCS-BEC crossover analogy
  - $|\mu_I| > m_\pi/2$: pions condense in a Bose condensate
  - $|\mu_I| \gg m_\pi$: Cooper-pairing of quarks and antiquarks

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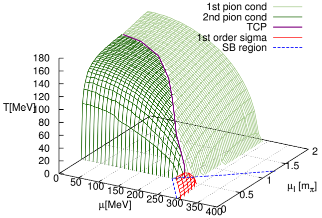

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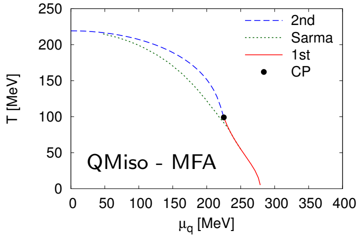


- fix $\mu_I = m_\pi > m_\pi/2$
  (pion condensation possible)
- vary μ_q
- order parameter: $\Delta^2 \sim \pi_+\pi_-$
- mean field approximation: no bosonic fluctuations

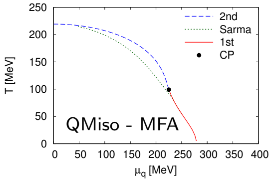
Including Fluctuations: Functional Renormalization Group

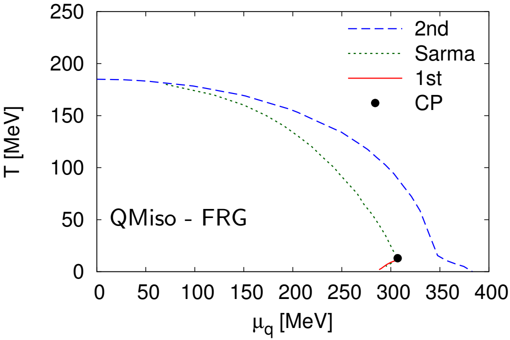
- bosonic fluctuations included (in LPA)

$$\Rightarrow \partial_t U_k(\chi, \Delta)$$

- solve on 2-dimensional grid
  - chiral SB ($\sim \chi$) & pion condensation ($\sim \Delta$)
  - resolve first and second order transitions

[Kamikado, Strodthoff, von Smekal, Wambach (2013)]





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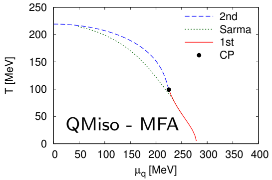
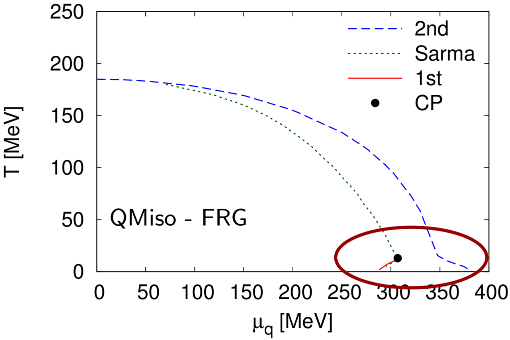


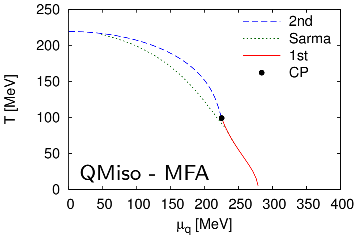


- fluctuations strongly modify phase structure
- two transition branches at low T
  (first and second order)
- Sarma phase down to $T = 0$

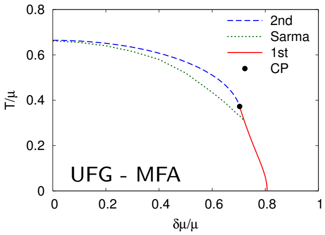
A Potential Non-Relativistic Analog:

The Imbalanced Unitary Fermi Gas

[cf. talks by I. Boettcher, D. Roscher, ...]



very similar!



Unitary Fermi Gas

ultracold two-component fermions close to a broad Feshbach resonance

- 2 fermion species, $\psi = (\psi_1, \psi_2)^T$
- bosonization in particle-particle channel: $\phi \sim \psi_1 \psi_2$ [diatomic molecule; Cooper pair]
- unitary regime: s-wave scattering length diverges, $a^{-1} = 0$; strongly coupled

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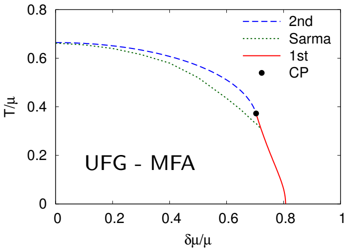


- $\mu > 0$: condensation
- order parameter: $\Delta^2 \sim \phi\phi^*$
- mean field approximation: no bosonic fluctuations
- phase structure very similar to relativistic model

Including Fluctuations: Functional Renormalization Group

- bosonic order-parameter fluctuations included

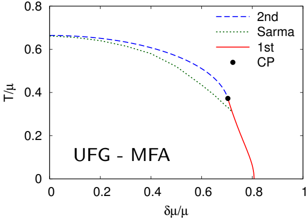
$$\Rightarrow \partial_t U_k(\Delta)\,,\ \partial_t g^2 = \eta_\phi g^2$$

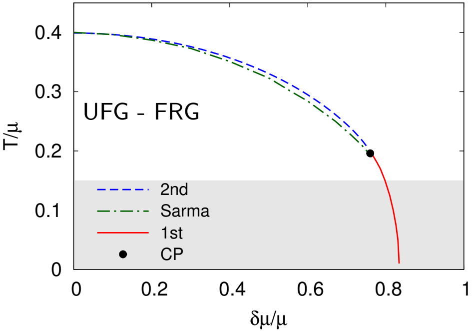
[g... Feshbach coupling]

- solve flow equation on a grid

[Boettcher, Braun, TKH, Pawlowski, Roscher, Wetterich (2014)]

$\rightarrow$ **talk by Dietrich Roscher**





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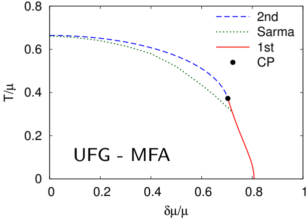

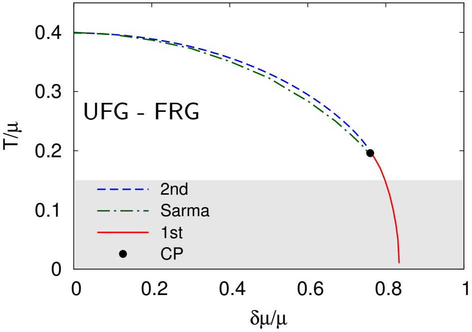

Fluctuations:

- T_c down
- critical imbalance $\delta\mu_c(T = 0)$ grows
- agreement with experiment & Monte Carlo

[Ku et al. (2012), Navon et al. (2013)]

[Goulko and Wingate (2010)]

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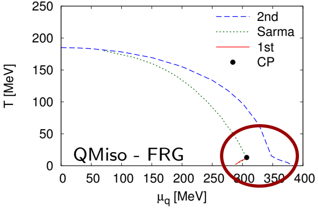
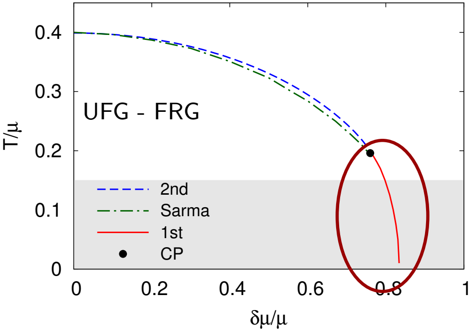
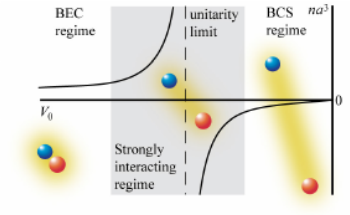




Differences to the Relativistic System:

- NO splitting of transition line
- NO Sarma phase at $T = 0$
- Sarma phase SHRINKS

Sarma Phase in the BCS-BEC Crossover



[Gubbels, Stoof (2012)]

Sarma Phase away from Unitarity

- Estimate possible: relativistic system slightly on BCS-side
- $\rightarrow$ study full imbalanced BCS-BEC crossover ($a^{-1} \neq 0$) at $T = 0$

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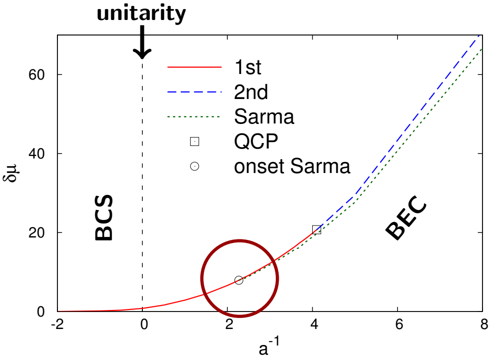

[cf. Sheehy, Radzihovsky (2006), Parish, Marchetti, Lamacraft, Simons (2007)]

MFA:

- Sarma phase at $T = 0$ occurs . . .
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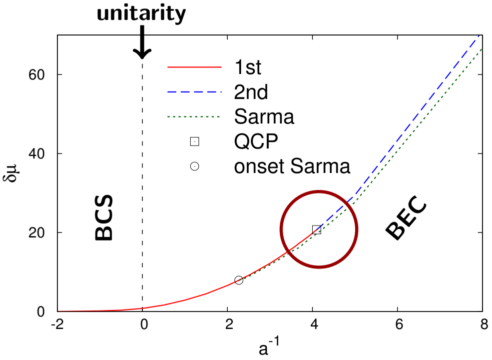

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  $\rightarrow$ if transition of second order, there is always a Sarma phase!

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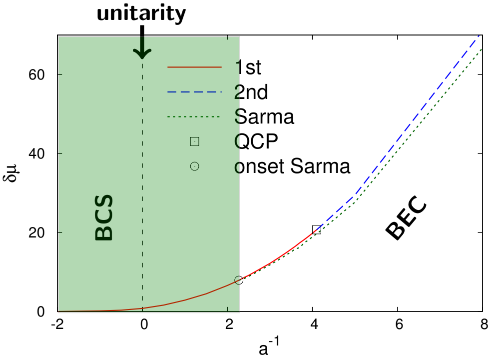

MFA:

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- ... but on BEC-side of the crossover
- Quantum Critical Point
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- Impact of fluctuations ?

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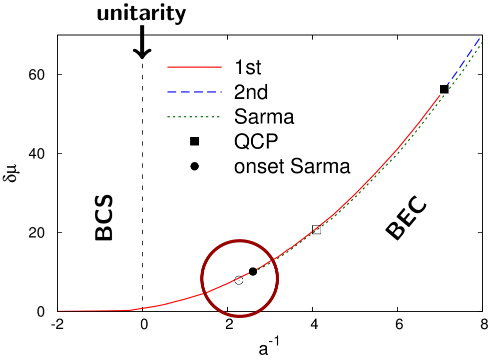

FRG:

- transition line barely changed
- Sarma onset moves *right*

[MFA: open symbols; FRG: full symbols]

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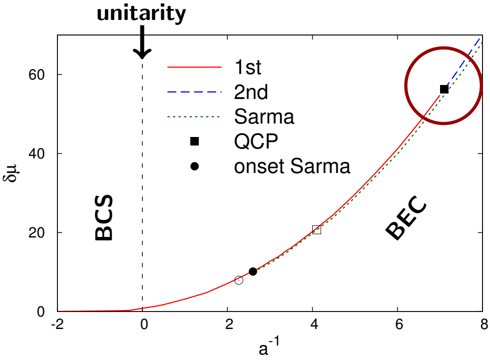

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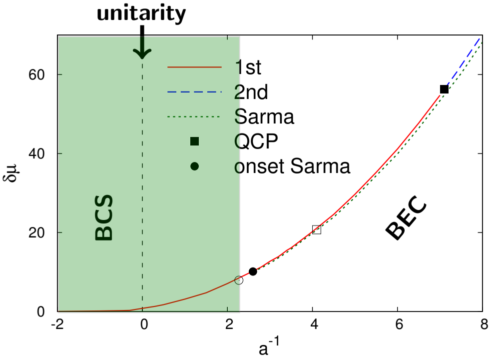

FRG:

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- NO Sarma on BCS-side!

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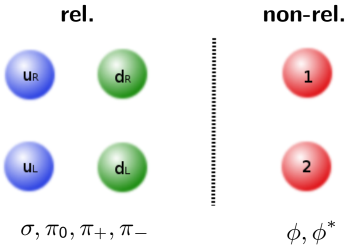
A Second Look at the Two Systems



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- d.o.f. do not match !

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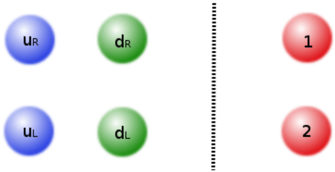
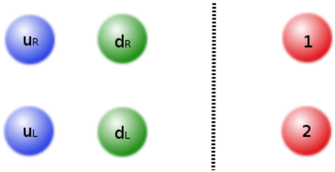


$\sigma, \pi_0, \pi_+, \pi_-$ ϕ, ϕ^*

- rel. system: additional $SU(2)_L \times SU(2)_R$ **chiral symmetry**
- d.o.f. do not match !
- MF phase structure agrees $\rightarrow$ discrepancies in fermionic sector subleading
- large difference beyond MFA $\rightarrow$ bosons and their fluctuations crucial !

A Second Look at the Two Systems

rel. | **non-rel.**



$\sigma, \pi_0, \pi_+, \pi_-$ | ϕ, ϕ^*

- rel. system: additional $SU(2)_L \times SU(2)_R$ **chiral symmetry**
- d.o.f. do not match !
- MF phase structure agrees $\rightarrow$ discrepancies in fermionic sector subleading
- large difference beyond MFA $\rightarrow$ bosons and their fluctuations crucial !

A Proposition

non-relativistic system with four fermion species and interactions

$$\hat{H} \sim \lambda \Big[(\psi_1\psi_2)^\dagger \psi_1\psi_2 + (\psi_3\psi_2)^\dagger \psi_3\psi_2 + (\psi_1\psi_4)^\dagger \psi_1\psi_4 + (\psi_3\psi_4)^\dagger \psi_3\psi_4 \Big]$$

- same $SU(2) \times SU(2)$ symmetry as the rel. system
- phase structure likely similar
- Sarma phase at $T = 0$ possible

Take-home Messages

- rel. and non-rel. systems for BCS-BEC crossover similar on MF-level ...
- ...but very different beyond
- bosonic d.o.f. and their fluctuations essential
- rel. system: 'non-trivial' phase structure at low T (Sarma!)
- non-rel. system:
  - good agreement with experiment and QMC for UFG
  - Sarma phase at low T only on BEC-side
- proposition for a non-rel. system that might be more similar to the rel. one

Where to go from here

- full imbalanced BCS-BEC crossover & QCP
- mass imbalance [Braun, Roscher]
- lower dimensions [Boettcher]
- ...

Stay Tuned & Thanks !

Backup: Quark-Meson Model

[Kamikado, Strodthoff, von Smekal, Wambach (2013)]

$$\begin{aligned}\mathcal{L}_{QMiso} &= \bar{\psi}\left(\not{\partial} + g(\sigma + i\gamma^5\vec{\pi}\vec{\tau}) - \gamma_0\mu_q - \gamma_0\tau_3\mu_I\right)\psi \\ &+ \tfrac{1}{2}(\partial_\nu\sigma)^2 + \tfrac{1}{2}(\partial_\nu\pi_0)^2 + U(\chi,\rho) - c\sigma \\ &+ \tfrac{1}{2}(\partial_\nu + 2\mu_I\delta^0_\nu)\pi_+(\partial_\nu - 2\mu_I\delta^0_\nu)\pi_- \,,\end{aligned}$$

- 2 flavors
- quark (μ_q) and isospin (μ_I) chemical potentials
- $SU(2)_L \times SU(2)_R \times U(1)_V$ symmetry
- chiral symmetry breaking: $\chi \sim \langle\bar{\psi}\psi\rangle$
- pion condensation: $\Delta^2 = g^2\rho = g^2\pi_+\pi_-$

Backup: FRG for the BCS-BEC Crossover

[Gurarie, Radzihovski (2007); Zwerger (2012); Diehl, Wetterich (2007)]

$$\begin{aligned}\mathcal{L}_{UFG} &= \sum_{\sigma=1,2} \psi_\sigma^* \Big(\partial_\tau - \frac{\nabla^2}{2M_\sigma} - \mu_\sigma\Big)\psi_\sigma + g\Big(\phi^*\psi_1\psi_2 + \text{h.c.}\Big) \\ &+ \phi^*\Big(Z_\phi\partial_\tau - A_\phi\frac{\nabla^2}{4M}\Big)\phi + \nu_\Lambda\phi^*\phi\,.\end{aligned}$$

- 2 species of fermions, $\sigma = 1, 2$
- bosonization: $\phi \sim \psi_1\psi_2$ (particle-particle channel)
- spin imbalance by different chemical potentials μ_1, μ_2
- $\nu_\Lambda \sim a^{-1}$ fine tuned to fix scattering length
- condensation: $\Delta^2 = g^2\rho = g^2\phi^*\phi$

Renormalization of the Propagators

$$\begin{aligned}P_{\psi\sigma,k}(iq_0, \vec{q}) &= iq_0 + q^2 - \mu_\sigma\,, \\ P_{\phi,k}(iq_0, \vec{q}) &= A_{\phi,k}\left(iq_0 + \frac{q^2}{2}\right)\,.\end{aligned}$$

Backup: Renormalization and the Sarma Condition

Fluctuations modify chemical potentials and can thus influence the Sarma criterion, $\Delta = \delta\mu$.

- polaron/FRG studies of balanced UFG: fluctuations increase μ
- estimate: $\mu_{\sigma,\mathrm{eff}} \simeq \mu_\sigma + 0.6\,\mu_{\bar\sigma}$ [$\mu_{\bar\sigma}$ . . . chem. pot. of other species]
- for imbalance: $\delta\mu_{\mathrm{eff}} = (\mu_{1,\mathrm{eff}} - \mu_{2,\mathrm{eff}})/2 \simeq 0.4\,\delta\mu$
- $\rightarrow$ Sarma criterion even less likely fulfilled
- here: unren. Sarma criterion not fulfilled at $T = 0 \Rightarrow$ ren. criterion not fulfilled either