Higher spins, momenta expansion and the fRG

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ERG 2014, Lefkada, September 26, 2014

A systematic truncation scheme for the fRG

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One more Hamiltonian fRG

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Polchinski's Equation

Polchinski's equation

$$\dot{S}[\phi] = \frac{1}{2}\int_{xy} \frac{\delta S[\phi]}{\delta\phi(x)}\dot{C}(x-y)\frac{\delta S[\phi]}{\delta\phi(y)} - \frac{1}{2}\int_{xy} \frac{\delta}{\delta\phi(x)}\dot{C}(x-y)\frac{\delta S[\phi]}{\delta\phi(y)}$$

local Lagrangian truncation

$$S_\Lambda[\phi] = \int_x \mathcal{L}(x, \phi(x), \partial\phi(x), \partial^2\phi(x), ...)$$

it comprehends also some nonlocal terms (by Taylor series about x)

Notations

$$\phi_M \equiv \phi_{\mu_1\mu_2\ldots\mu_n}(x) \equiv \partial_{\mu_n}\ldots\partial_{\mu_2}\partial_{\mu_1}\phi(x)$$

$$M \equiv (\mu_1, \mu_2, \ldots\mu_n), \;\; n \in \mathbb{N} \qquad\qquad (-)^M \equiv (-1)^n$$

$M = ()$ corresponds to $\phi_M(x) = \phi(x)$ and $n = 0$

we use Einstein's summation convention

$$\frac{\delta S}{\delta\phi(x)} = (-)^M \partial_M \frac{\partial\mathcal{L}}{\partial\phi_M}(x) \equiv \frac{\partial\mathcal{L}}{\partial\phi}(x) - \partial_{\mu_1}\frac{\partial\mathcal{L}}{\partial\phi_{\mu_1}}(x) + \partial_{\mu_1}\partial_{\mu_2}\frac{\partial\mathcal{L}}{\partial\phi_{\mu_1\mu_2}}(x) + \ldots$$

From S to $\mathcal{L}$

Right hand side of Polchinski's equation

$$\frac{(-)^N}{2}\left\{\int_{xy}\frac{\partial\mathcal{L}}{\partial\phi_M}(x)\dot{C}_{M+N}(x-y)\frac{\partial\mathcal{L}}{\partial\phi_N}(y)-\dot{C}_{M+N}(0)\int_x\frac{\partial^2\mathcal{L}}{\partial\phi_M\partial\phi_N}(x)\right\}$$

no explicit derivatives act on $\mathcal{L}$!

From $\mathcal{L}$ to $\mathcal{H}$

Replace derivatives with fields at any scale

introduce tensor fields π^M (no $M = ()$ from here on)

Covariant Ostrogradsky formalism

$$\mathcal{H}(x, \phi(x), \pi^M(x)) = \underset{\phi_M}{\text{ext}} \left\{ \pi^M(x)\phi_M(x) - \mathcal{L}(x, \phi(x), \phi_M(x)) \right\}$$

that is

$$\pi^M(x) = \frac{\partial \mathcal{L}}{\partial \phi_M}(x) \quad , \quad \phi_M(x) = \frac{\partial \mathcal{H}}{\partial \pi^M}(x)$$

equations of motion

$$(-)^M \partial_M \pi^M(x) = \frac{\partial \mathcal{H}}{\partial \phi}(x)$$

$$\phi_M(x) = \frac{\partial \mathcal{H}}{\partial \pi^M}(x)$$

Flow Eq. for $\mathcal{H}$

Flow equation of the Hamiltonian density

$$\int_x \dot{\mathcal{H}}(x) = -\frac{1}{2}\int_{xy} \frac{\partial \mathcal{H}}{\partial \phi}(x)\dot{C}(x-y)\frac{\partial \mathcal{H}}{\partial \phi}(y) - \frac{1}{2}\dot{C}(0)\int_x \frac{\partial^2 \mathcal{H}}{\partial \phi \partial \phi}(x)$$

$$+\int_{xy} \pi^M(x)\dot{C}_M(x-y)\frac{\partial \mathcal{H}}{\partial \phi}(y) - \frac{1+(-)^M}{2}\dot{C}_M(0)\int_x \frac{\partial^2 \mathcal{H}}{\partial \pi^L \partial \phi}(x)\left(\frac{\partial^2 \mathcal{H}}{\partial \pi^\cdot \partial \pi^\cdot}\right)^{-1\,LM}(x)$$

$$-\frac{(-)^N}{2}\int_{xy} \pi^M(x)\dot{C}_{M+N}(x-y)\pi^N(y) \;+\; \frac{(-)^N}{2}\dot{C}_{M+N}(0)\int_x \left(\frac{\partial^2 \mathcal{H}}{\partial \pi^\cdot \partial \pi^\cdot}\right)^{-1\,MN}(x)$$

no derivative of the fields ϕ and π^M appears!

Uniform Hamiltonian Approximation

$\mathcal{H}$ does not explicitly depend on x

$$\dot{\mathcal{H}} = -\frac{1}{2}\dot{\hat{C}}(0)\left(\frac{\partial \mathcal{H}}{\partial \phi}\right)^2 - \frac{1}{2}\dot{C}(0)\frac{\partial^2 \mathcal{H}}{\partial \phi \partial \phi}$$
$$-\frac{1+(-)^M}{2}\dot{C}_M(0)\frac{\partial^2 \mathcal{H}}{\partial \pi^L \partial \phi}\left(\frac{\partial^2 \mathcal{H}}{\partial \pi \cdot \partial \pi \cdot}\right)^{-1\,LM}$$
$$+\frac{(-)^N}{2}\dot{C}_{M+N}(0)\left(\frac{\partial^2 \mathcal{H}}{\partial \pi \cdot \partial \pi \cdot}\right)^{-1\,MN}$$

Partial differential eq. for a function of infinitely many higher-spin fields

Momenta Expansion

n-th order: neglect momenta with rank bigger than n

Zeroth order = full dependence on ϕ = LPA

First order = full dependence on π^μ and ϕ

In generic d: $\mathcal{H}\left(\varpi \equiv \pi^\mu \pi_\mu / 2,\ \phi\right)$

First Order Momenta Expansion

By dropping all higher momenta the flow equation simplifies to

$$\dot{\mathcal{H}} = \frac{K_0}{\Lambda^{2-\eta}} \left(\frac{\partial \mathcal{H}}{\partial \phi}\right)^2 + \Lambda^{d-2+\eta} I_0 \frac{\partial^2 \mathcal{H}}{\partial \phi \partial \phi} - \frac{\Lambda^{d+\eta}}{d} I_1 \mathrm{tr} \left(\frac{\partial^2 \mathcal{H}}{\partial \pi \cdot \partial \pi \cdot}\right)^{-1}$$

Rescaling $\mathcal{H}$, ϖ and ϕ we remove regulator dependence: $K_0 = I_0 = I_1 = 1$
(not scheme dependence)

Critical Behavior

Dimensionless renormalized fields

$$\mathcal{H} \to \Lambda^d \mathcal{H} \quad , \quad \varpi \to \Lambda^{2d_\pi} \varpi \quad , \quad \phi \to \Lambda^{d_\phi} \phi$$

Demanding $[\pi^\mu \phi_\mu] = d$: $\quad d_\phi = (d-2+\eta)/2 \qquad d_\pi = (d-\eta)/2$

$$\dot{\mathcal{H}} = d\mathcal{H} - (d-\eta)\,\varpi\mathcal{H}^{(1,0)} - \left(\frac{d-2+\eta}{2}\right)\phi\mathcal{H}^{(0,1)}$$
$$+\mathcal{H}^{(0,1)\,2} + \mathcal{H}^{(0,2)} - \frac{1}{d}\left(\frac{d-1}{\mathcal{H}^{(1,0)}} + \frac{1}{\mathcal{H}^{(1,0)} + 2\varpi\mathcal{H}^{(2,0)}}\right)$$

Special solutions: $\mathcal{H}(\varpi,\phi) = \mathcal{T}(\varpi) - \mathcal{V}(\phi)$

Separable FP

Two equations coupled through η

$$-d\mathcal{V} + \left(\frac{d-2+\eta}{2}\right)\phi\mathcal{V}' + \mathcal{V}'^2 - \mathcal{V}'' = -c$$

$$d\mathcal{T} - (d-\eta)\varpi\mathcal{T}' - \frac{1}{d}\left(\frac{d-1}{\mathcal{T}'} + \frac{1}{\mathcal{T}' + 2\varpi\mathcal{T}''}\right) = c$$

$d = 3$ First equation shows three families of FP solutions $\forall\eta$:
Gaußian, Wilson-Fisher, High−Temperature

Y. A. Kubyshin, R. Neves, R. Potting (2001);
H. Osborn and D. E. Twigg (2009);
C. Bervillier (2013)

FP $\mathcal{T}$

Boundary conditions:

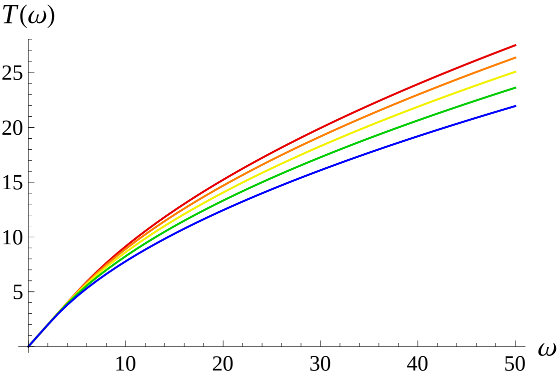
$\mathcal{T}'(0) = \zeta_0,$	equivalent to $\eta = \partial_t \log Z_\phi$
$\mathcal{T}''(0) = -\frac{d\eta}{d+2}\zeta_0^3$	from the FP equation

One FP solution $\forall \eta \geq 0$

$\mathcal{T}$ is linear in ϖ only for $\eta = 0$

FP $\mathcal{T}$

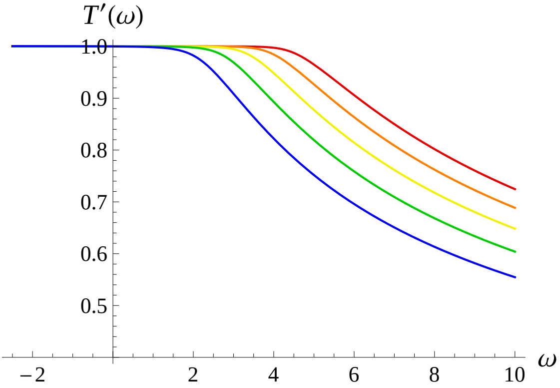
$d = 3$



η varying from $\eta = 10^{-7}$ to $\eta = 10^{-3}$

FP $\mathcal{T}$

$d = 3$



η varying from $\eta = 10^{-7}$ to $\eta = 10^{-3}$

Linear perturbations

Linearizing around $\mathcal{T}(\varpi) = \zeta_0 \varpi$

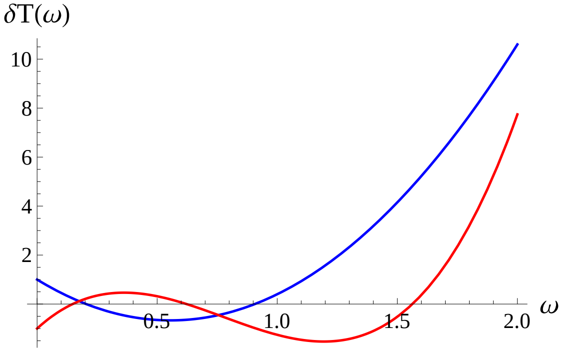
$$\delta\mathcal{T}(\varpi) \propto {}_1F_1\left(-\frac{d-\lambda}{d-\eta}; \frac{d}{2}; \frac{d}{2}\zeta_0^2 (d-\eta)\varpi\right)$$

eigenvalues: $\lambda_n = d - n(d-\eta)$ for $n = 0, 1, 2 \cdots$

eigenperturbations: polynomia of order n

Linear perturbations

$\eta = 0$



$n = 2$, $n = 3$

Linear perturbations

Linearizing around the full $\eta > 0$ FP solution.

For any λ the perturbations are polynomial at the origin

$$\delta g(\varpi) \underset{\varpi \to 0}{\sim} e^{a(d,\eta)\zeta_0^2 \varpi} \, {}_1F_1\left(b(d,\eta,\lambda);\, 1+\frac{d}{2};\, c(d,\eta)\zeta_0^2 \varpi\right)$$

and behave as

$$\delta g(\varpi) \underset{\varpi \to +\infty}{\sim} \sqrt{\varpi}$$

Eigenvalues are not quantized !?

Determination of η

Computation of $\eta = -\partial_t \log Z_\phi$

$$Z_\phi = \left[\frac{d}{dp^2}\frac{\delta^2 S^I}{\delta\hat{\phi}(-p)\delta\hat{\phi}(p)}\right]_{p=0,\phi=\phi_{\min}}$$

$$= \frac{1}{d}\delta_{\mu\nu}\int_x \left[\left(\frac{\partial^2\mathcal{H}}{\partial\pi_\cdot\partial\pi_\cdot}\right)^{-1\,(\mu)(\nu)}(x) + 2\frac{\partial^2\mathcal{H}}{\partial\pi^N\partial\phi}(x)\left(\frac{\partial^2\mathcal{H}}{\partial\pi_\cdot\partial\pi_\cdot}\right)^{-1\,N(\mu,\nu)}(x)\right]_{\phi=\phi_{\min}}$$

Determination of η

good reason for the failure: no spin 2

or maybe it's fault of the separability assumption?

or it's just correct that η is a free parameter?

but what about the linear perturbations?