Competing Order Parameters and Multicritical Phenomena

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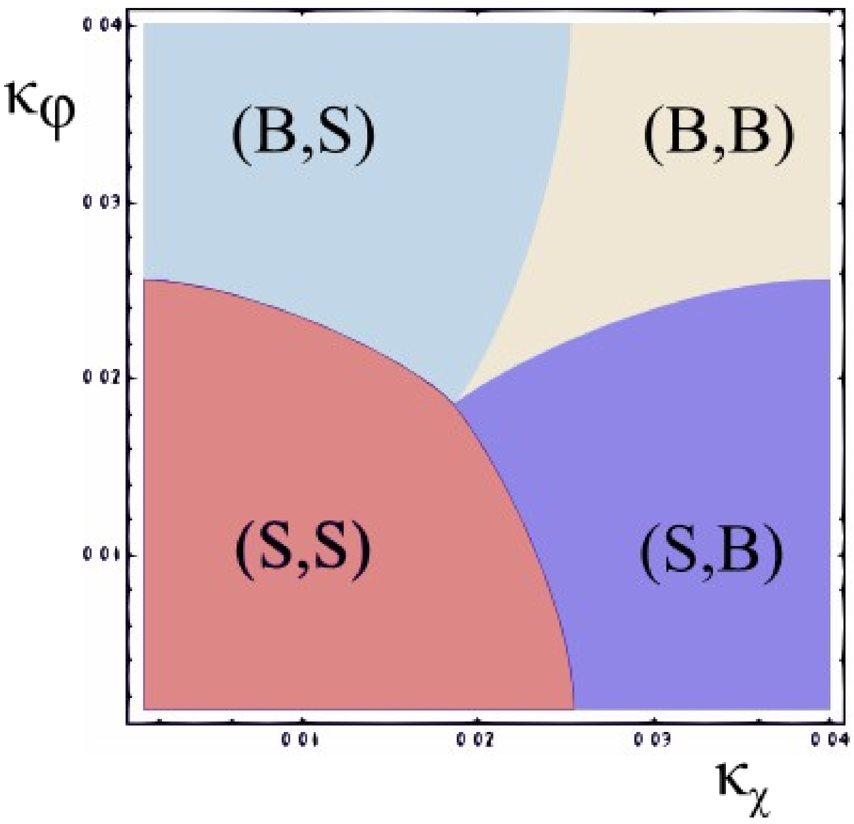
Outline

- Motivation
  - Anisotropic Antiferromagnets
- Method – Truncation
- Phase Structure
  - Fixed Points
  - Stability
  - Local Phase Diagrams

Competing Orders

- Phases characterized by order parameters
- Goal: determine interplay between two order parameters

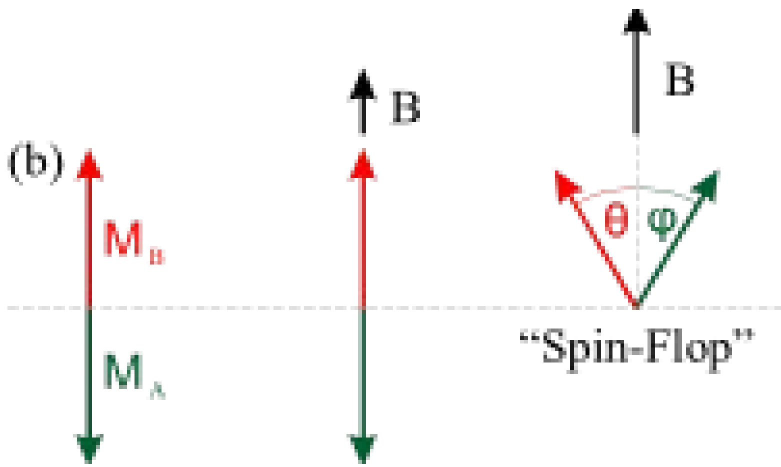


Anisotropic Antiferromagnets

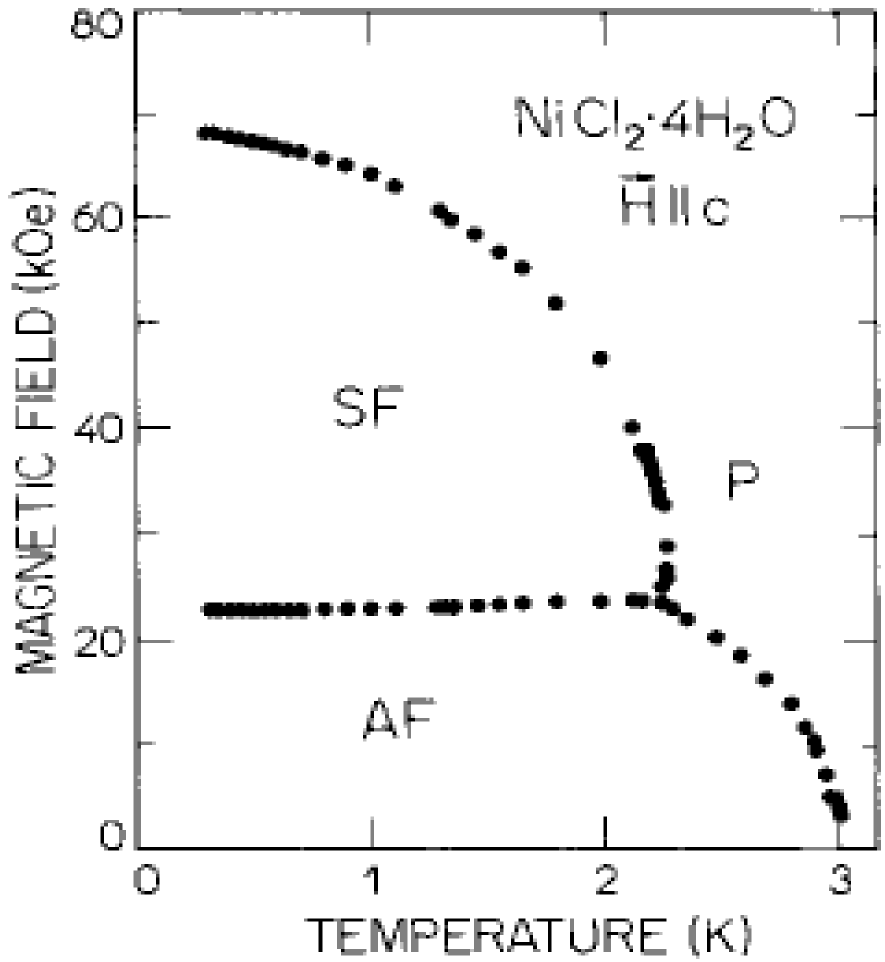
- Anisotropic Heisenberg Model in $d = 3$ dimensions.
- Symmetric under $\mathrm{O}(2) \oplus \mathrm{O}(1)$

$$\mathcal{H} = J \sum_{\langle mn \rangle} \mathbf{S}_m \cdot \mathbf{S}_n + A \sum_{\langle mn \rangle} S_{m,z} S_{n,z} - H \sum_m S_{m,z}$$

Anisotropic Antiferromagnets



- External fields induce a spin-flop phase
- Goal: determine nature of multicritical point



C. C. Becerra, N. F. Oliveira Jr. and A. Paduan-Filho ' 1988

Functional Renormalization Group

- Universality: Explore scalar field theory with $\mathrm{O}(M) \oplus \mathrm{O}(N)$ symmetry in $d = 3$ dimensions
- Method: Functional Renormalization Group

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{STr} \Big(\partial_k R_k \big(\Gamma_k^{(2)} + R_k \big)^{-1} \Big)$$

C.Wetterich ' 1993

- Phase diagrams: Solving the renormalization group equation from an initial scale $k = \Lambda$ to $k = 0$
- Fixed Points: Zeros of Flow equation

Functional Renormalization Group

Truncation of the effective action:

$$\Gamma[\phi,\chi] = \int \mathrm{d}^d x \; (\frac{1}{2} Z_\phi \phi_a(-\nabla^2)\phi_a + \frac{1}{2} Z_\chi \chi_a(-\nabla^2)\chi_a + U(\rho_\phi,\rho_\chi))$$

Symmetry invatiants:

$$\rho_\phi = \frac{1}{2}\phi_a\phi_a \;,\; \rho_\chi = \frac{1}{2}\chi_a\chi_a \;,\; a = 1,...,M \;,\; b = 1,...,N$$

Expansion of the effective potential: coupling constants

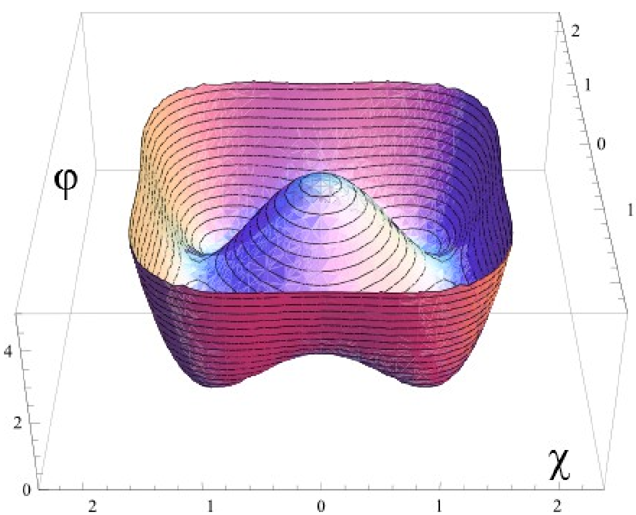
$$U(\rho_\phi,\rho_\chi) = \sum_{1\leq i+j\leq \mathrm{ord}} \frac{\lambda_{ij}}{i!j!}(\rho_\phi - \rho_{0\phi})^i(\rho_\chi - \rho_{0\chi})^j$$

Order parameters: $\rho_{0\phi}, \rho_{0\chi}$ or in dimensionless form κ_ϕ, κ_χ

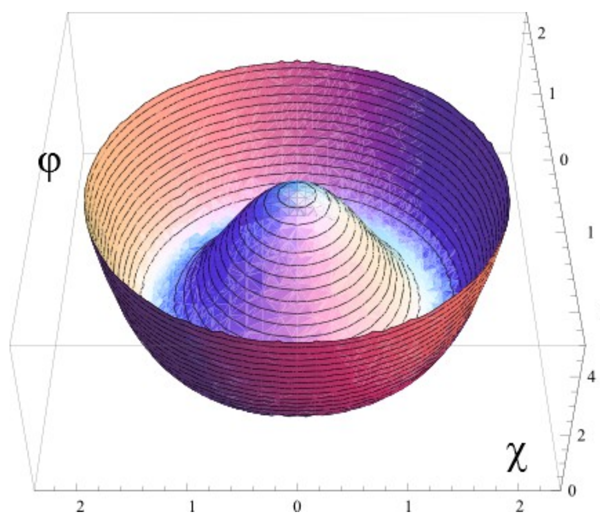
Classes of Potentials

Hessian Determinant at expansion point $\Delta = \lambda_{20}\lambda_{02} - \lambda_{11}^2$

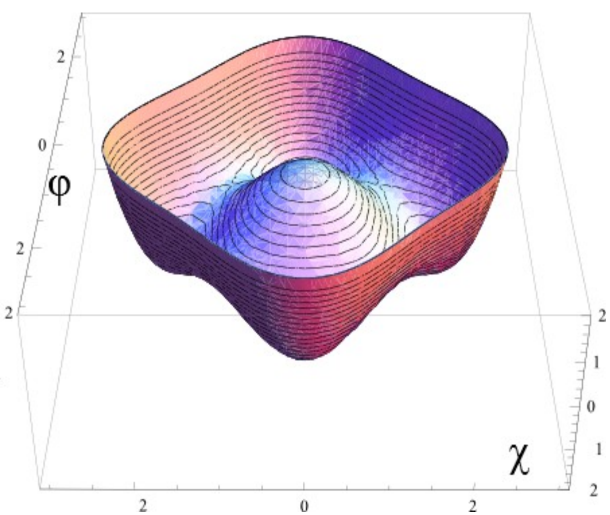
$\Delta > 0$



$\Delta = 0$



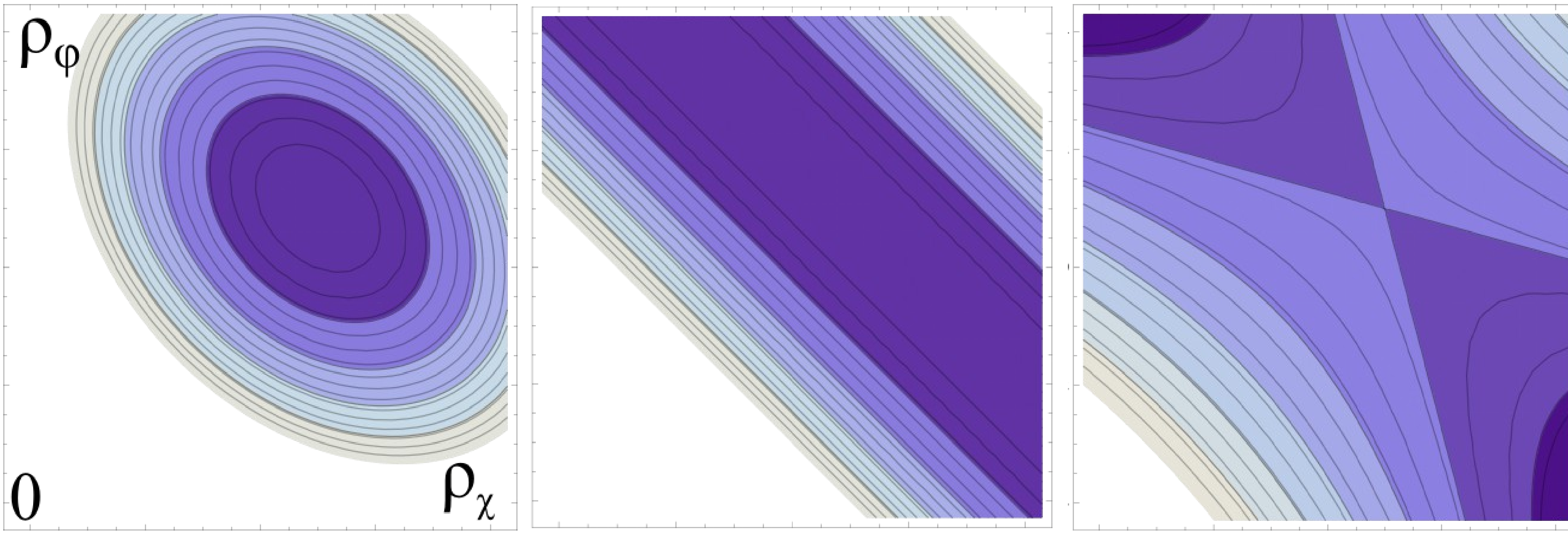
$\Delta < 0$



- Measure of distance from symmetry enhancement
- Renormalization Group flow cannot cross $\Delta = 0$

Classes of Potentials

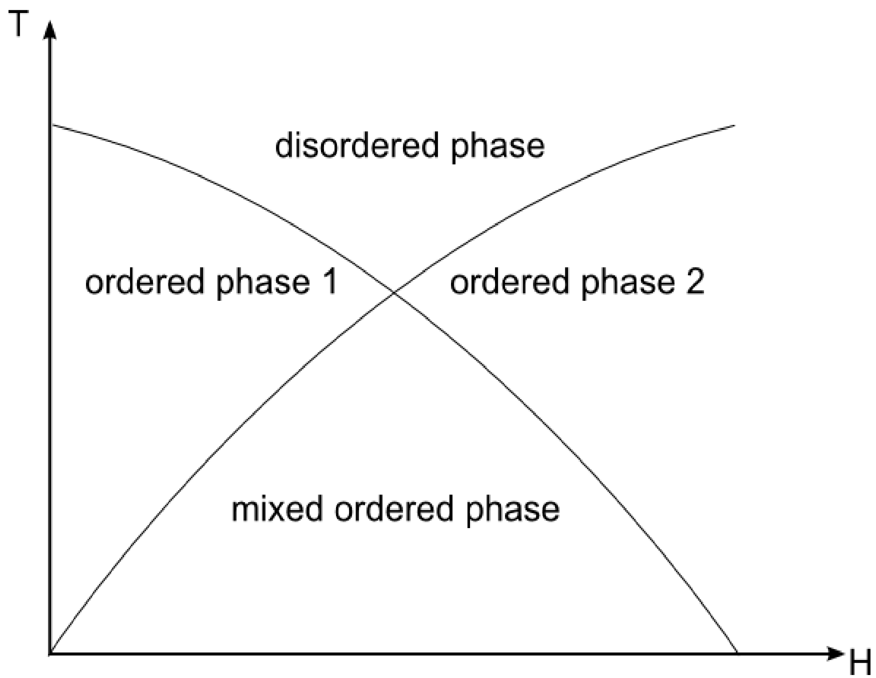
$\Delta > 0$ $\Delta = 0$ $\Delta < 0$



- $\Delta > 0$ Continuous PT: Minimum moves to either axis
- $\Delta < 0$ 1st order PT: relative height of Minima changes

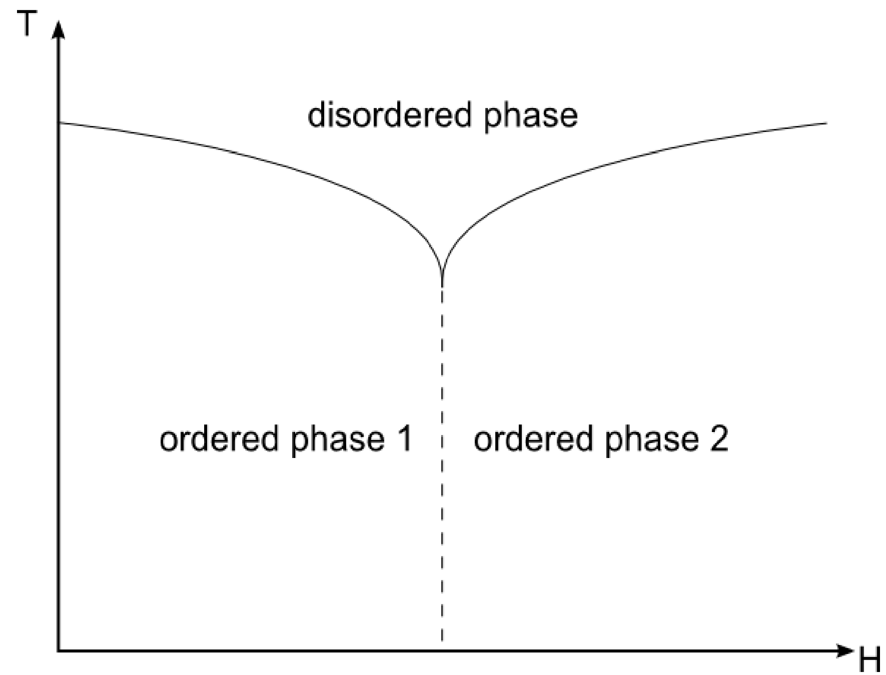
General Phase Structure

$\Delta > 0$



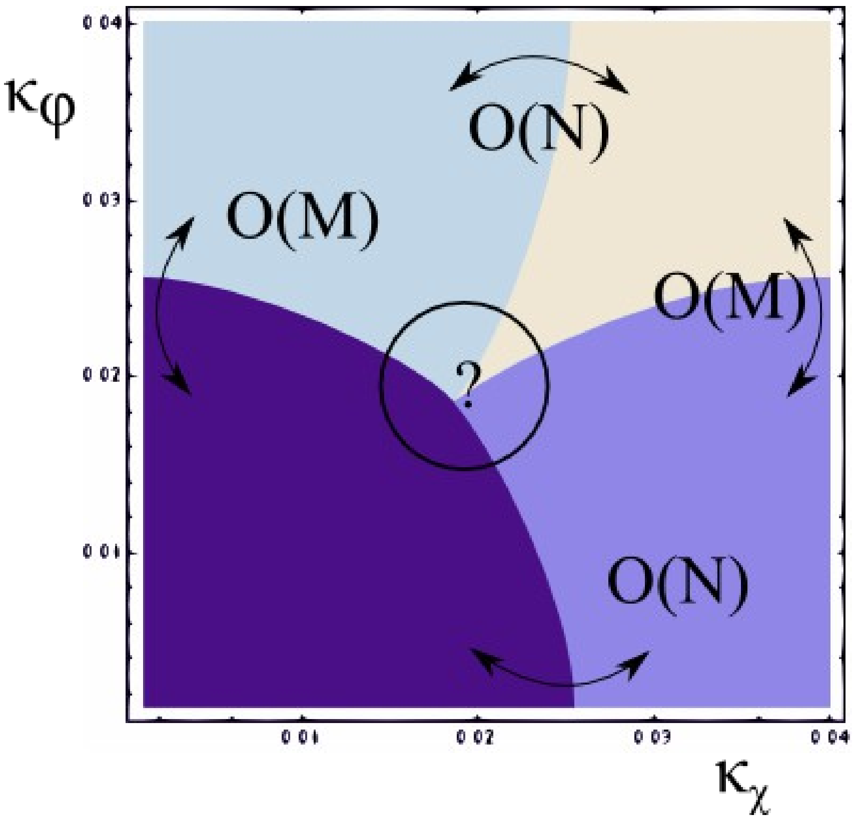
- Tetracritical phase diagram
- Rich critical physics at MCP

$\Delta < 0$



- Bicritical phase diagram
- No coexistence phase

Continuous Phase Transitions $\Delta > 0$



For every initial potential in any order of truncation we need to tune 2 couplings to reach the MCP

- MCP RG equivalent to a FP with 2 relevant couplings
- Identify stable fixed point by # of positive critical exponents = 2

Fixed Points:

Isotropic FP: Symmetry enhancement to WF $\mathrm{O}(M+N)$

Decoupled FP: Decoupling to WF $\mathrm{O}(M)$ and $\mathrm{O}(N)$

Biconical FP: NEW

Isotropic Fixed Point

M, N	y_1	y_2	y_3	η_ϕ	η_χ
$1, 1$	2.08	1.45	-0.034	0.043	0.043
$1, 2$	1.91	1.36	0.095	0.041	0.041
$1, 3$	1.77	1.29	0.203	0.037	0.037
$2, 2$	1.77	1.29	0.203	0.037	0.037

- IFP stable for $\mathrm{O}(1) \oplus \mathrm{O}(1)$ symmetric models
- In only these models a small asymmetry between ϕ and χ will be cured by the RG flow close to the MCP

(Conventional notation $\nu_i = y_i^{-1}$)

Decoupled Fixed Point

M, N	y_1	y_2	y_3	η_ϕ	η_χ
$1, 1$	1.56	1.56	0.080	0.044	0.044
$1, 2$	1.56	1.45	-0.027	0.044	0.043
$1, 3$	1.56	1.36	-0.112	0.044	0.041
$2, 2$	1.45	1.45	-0.135	0.043	0.043

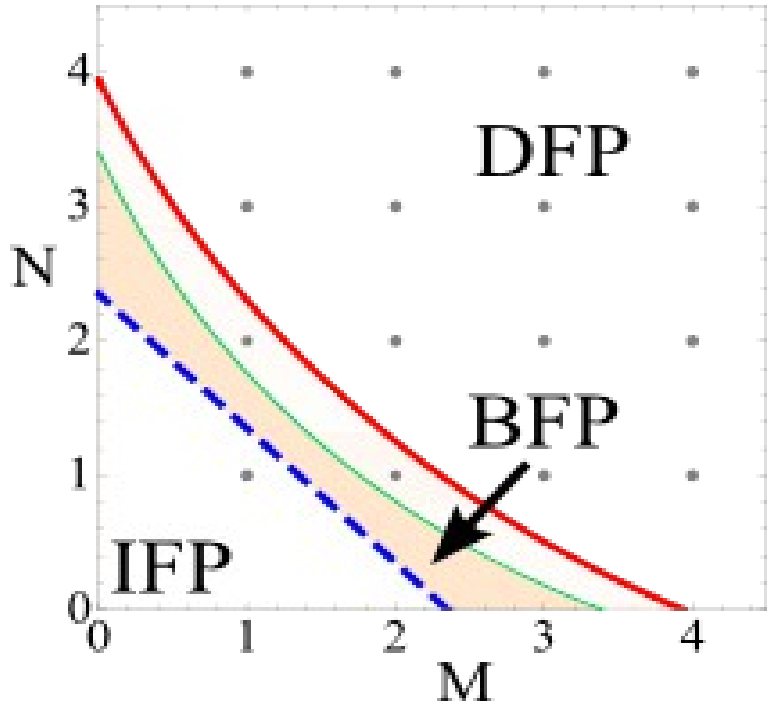
- DFP stable for models with $M + N \geq 3$
- In these models the RG flow decouples both fields near the MCP

Biconical Fixed Point

M, N	y_1	y_2	y_3	η_ϕ	η_χ
$1, 1$	1.93	1.54	0.053	0.055	0.055
$1, 2$	1.59	1.42	0.035	0.046	0.045
$1, 3$	1.72	1.27	0.166	0.045	0.041
$2, 2$	1.72	1.27	0.194	0.043	0.043

- New fixed point – cannot be inferred from WF fixed points
- In our truncation not stable for any physical symmetry group
- Might be stable for $\mathrm{O}(2) \oplus \mathrm{O}(1)$ (P.Calabrese, A.Pelissetto, E.Vicari ' 2002)
- Regulator analysis shows inaccuracy of $\Delta y_3 \lesssim 0.07$

Stability of Fixed Points



Red: FRG+Scaling relation (A.Eichhorn, D.Mesterhazy, M. Scherer ' 2013)

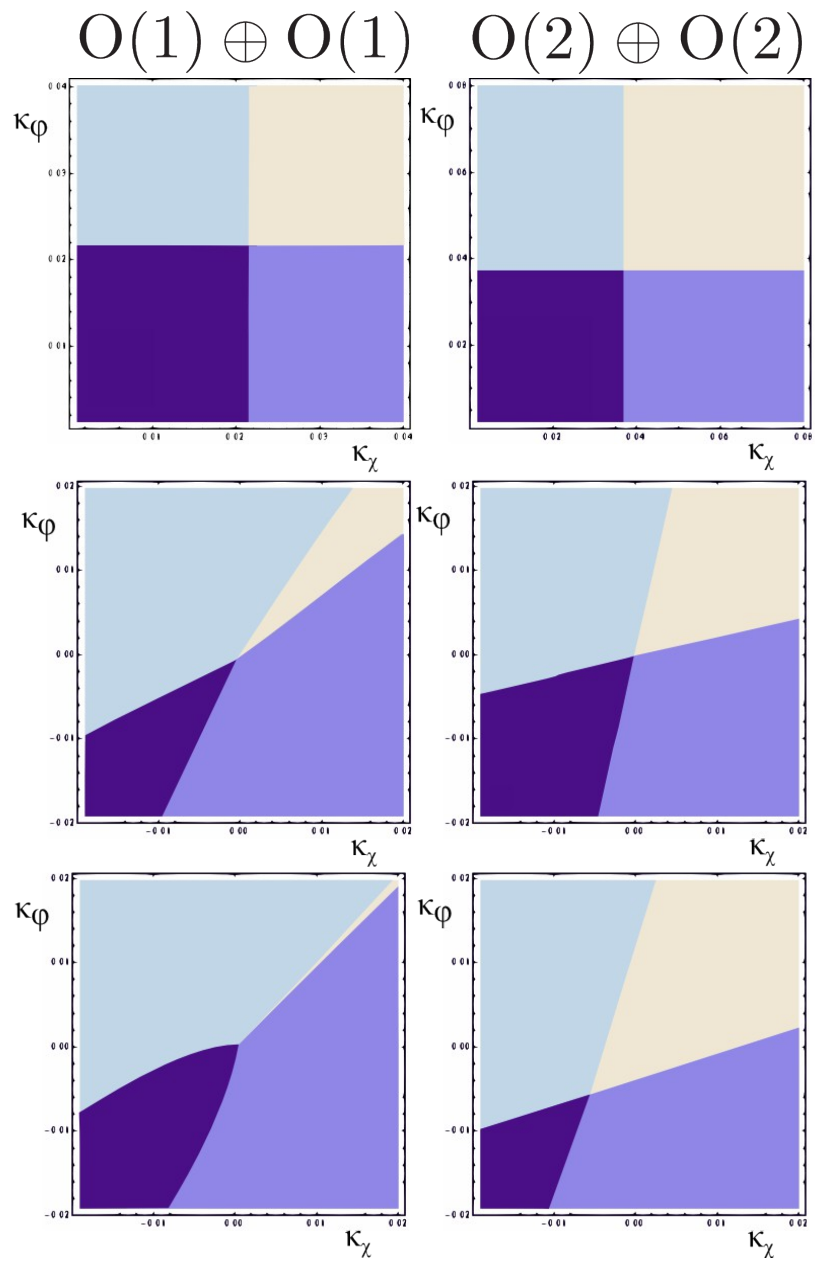
Green: Best unbiased calculation ($\rho^7, \eta \neq 0$)

Effects of Fixed Points on Phase Diagrams

- Initial Potential: at DFP,

$$\lambda_{11} = \epsilon > 0$$

- $O(1) \oplus O(1)$ diagram converges to bicriticality (IFP stable)
- $O(2) \oplus O(2)$ Diagram stays tetracritical (DFP stable)



Global Phase Diagram

Local Phase Diagram Zoom factor: 10^{-5}

Local Phase Diagram Zoom factor: 10^{-9}

Conclusion

- We calculated properties of phase diagrams near MCP for all models exhibiting $\mathrm{O}(M) \oplus \mathrm{O}(N)$ symmetry in $d = 3$ dimensions.
- Only $\mathrm{O}(1) \oplus \mathrm{O}(1)$ symmetric models restore an enhanced symmetry near the MCP.
- Other models with $M + N \geq 3$ (?) decouple close to the MCP.