NUCLEAR CHIRAL THERMODYNAMICS and the FUNCTIONAL RENORMALIZATION GROUP

Wolfram Weise

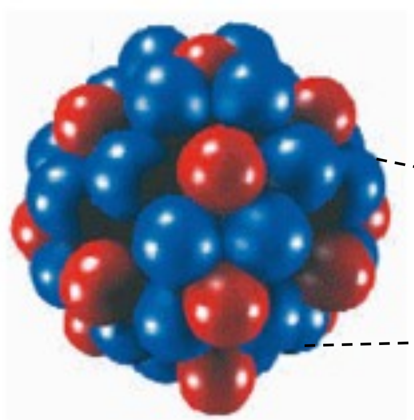
ECT* Trento and **T**echnische **U**niversität **M**ünchen

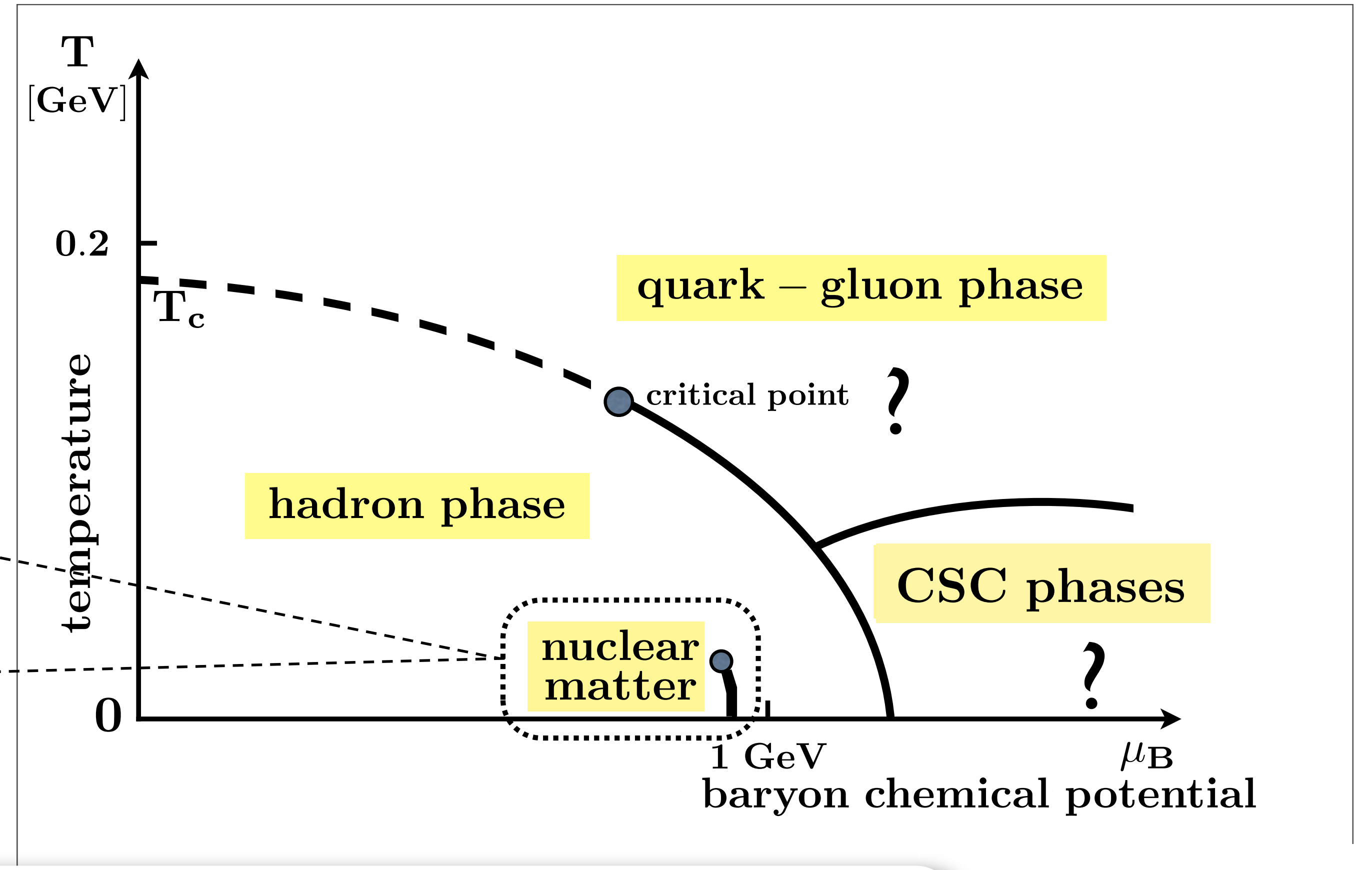
- Introductory glance at the **QCD phase diagram**
- **Chiral models** of the **nuclear equation of state**
- Beyond mean field:
  non-perturbative fluctuations and **FRG** approach
- Symmetric and asymmetric **nuclear matter**
- **Neutron matter** and **neutron stars**
- Density & temperature dependence of **chiral order parameter**

(based on doctoral thesis work of **Matthias Drews**)

NUCLEAR MATTER and QCD PHASES

nuclei





Scales in N = Z **nuclear matter**:

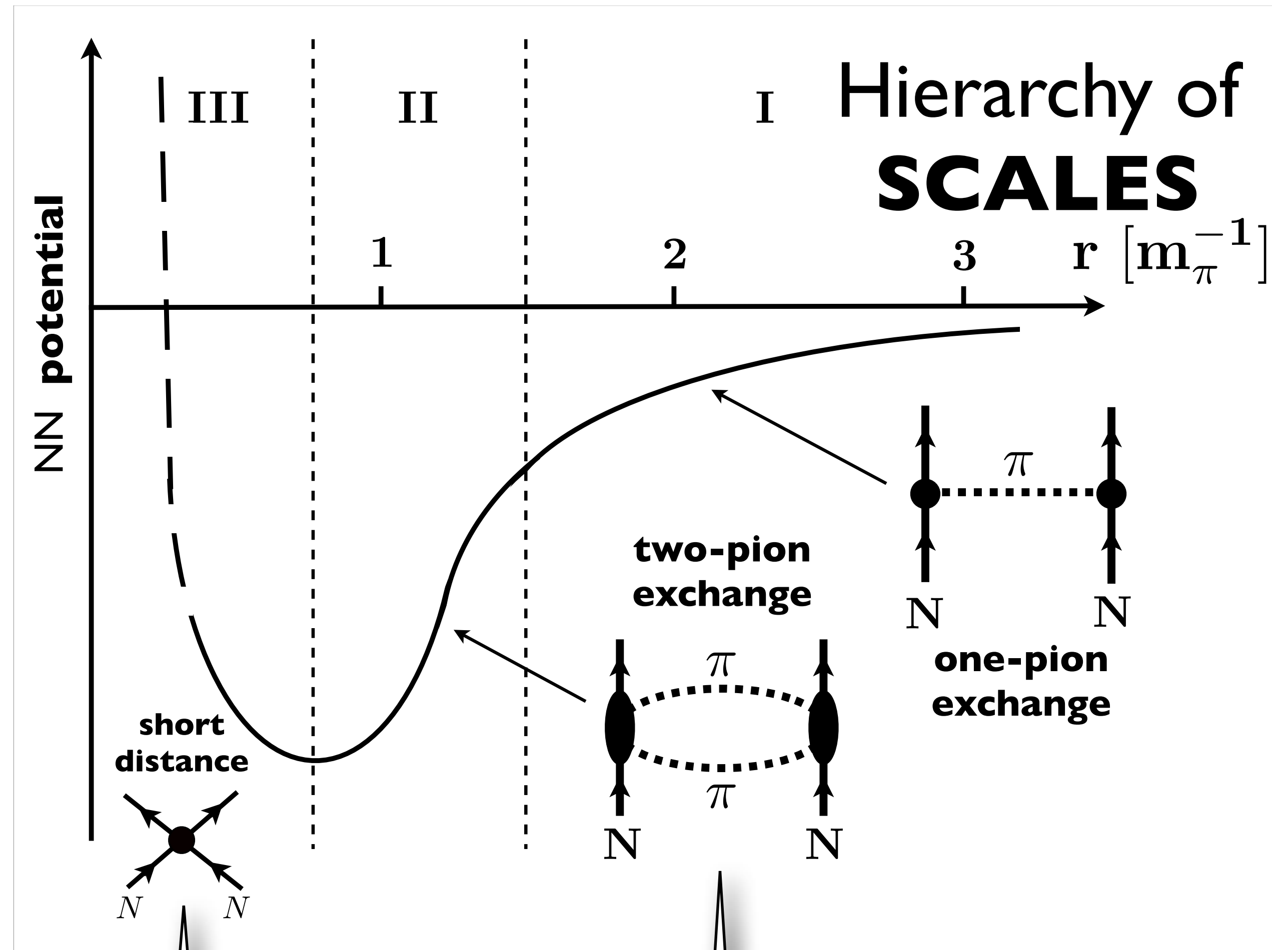
- momentum scale:
  Fermi momentum $k_F \simeq 1.4\ \mathrm{fm}^{-1} \sim 2m_\pi$
- NN distance: $d_{NN} \simeq 1.8\ \mathrm{fm} \simeq 1.3\ m_\pi^{-1}$
- energy per nucleon: $E/A \simeq -16\ \mathrm{MeV}$
- compression modulus: $K = (260 \pm 30)\ \mathrm{MeV} \sim 2m_\pi$

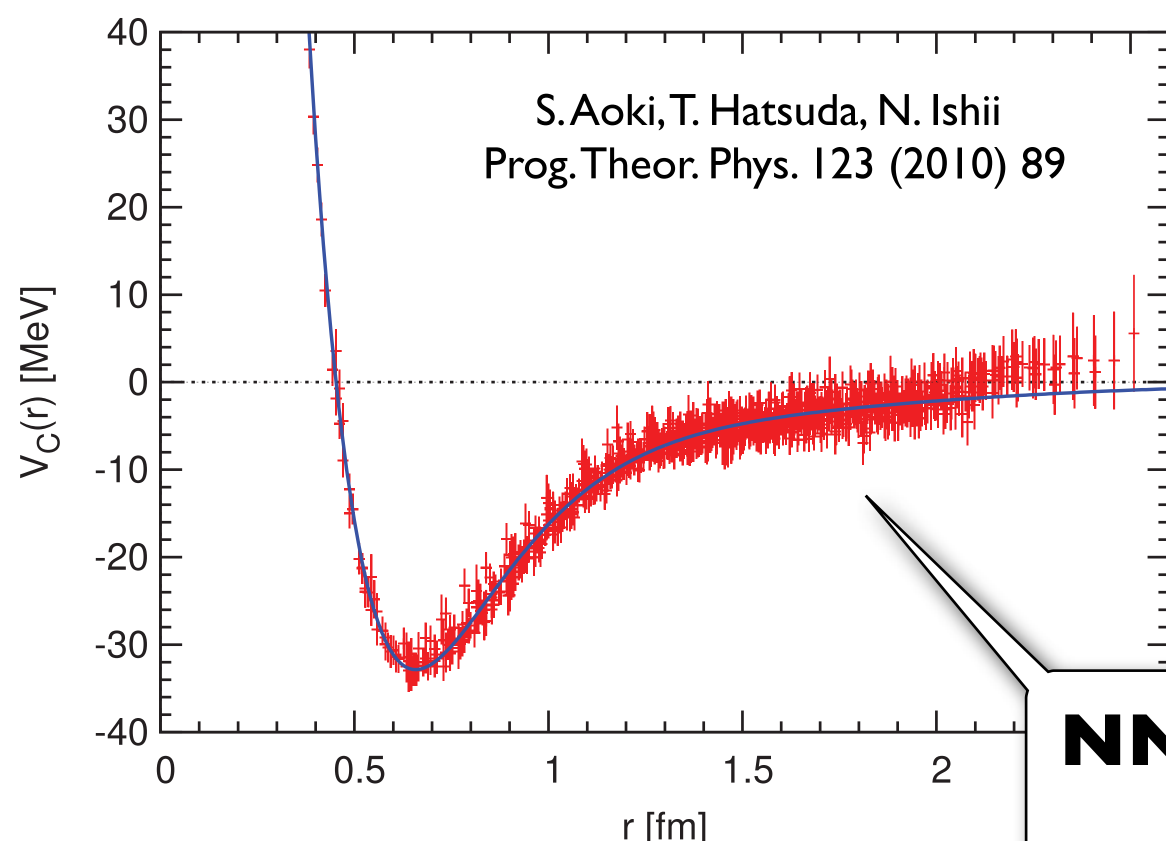
Nuclear Forces

contemporary approaches:

Chiral Effective Field Theory
+
Lattice QCD

Early history: M. Taketani et al. (1951)





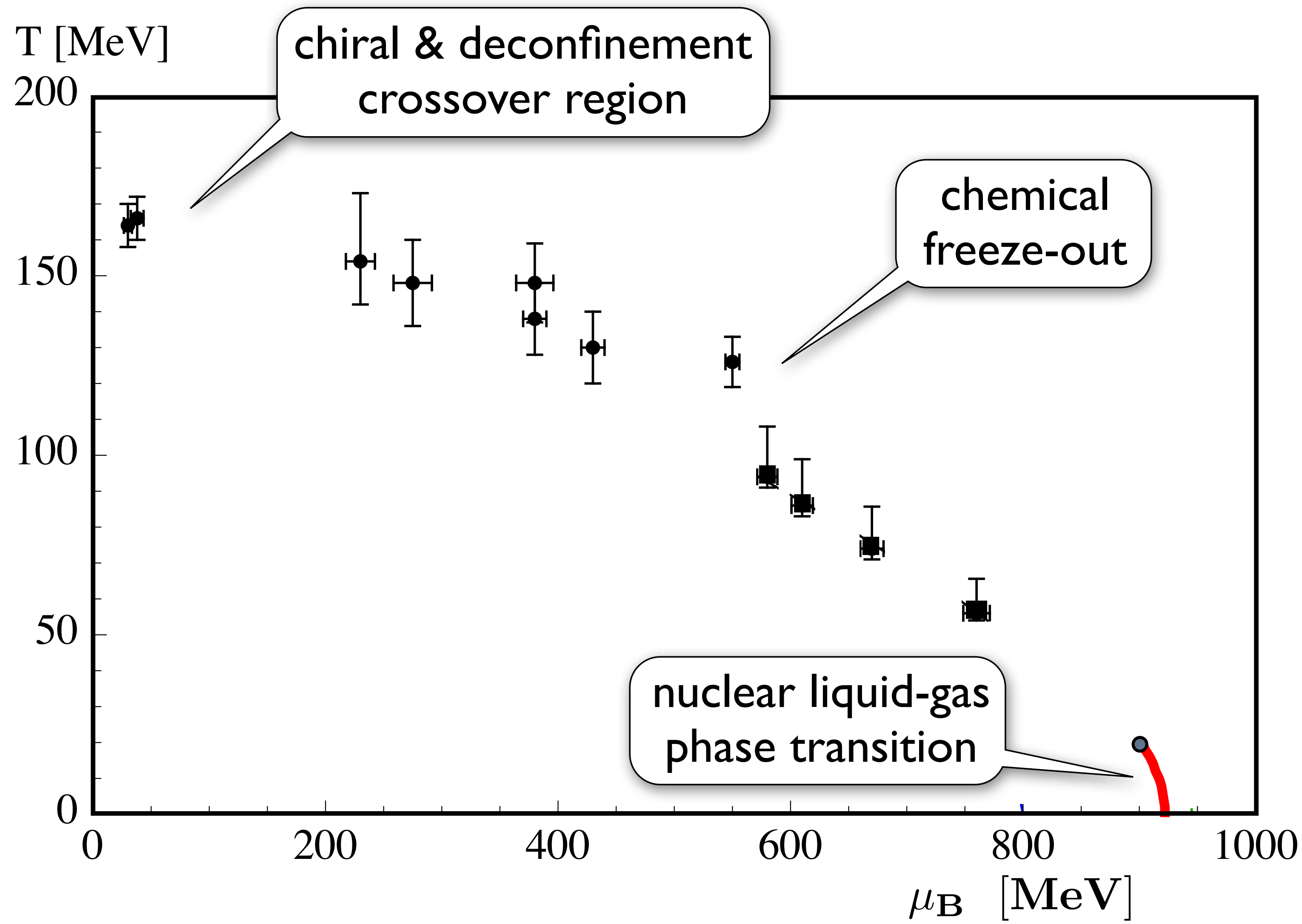
contact terms

explicit treatment of **two-pion exchange**

NN Central Potential from Lattice QCD

PHASE DIAGRAM
and
CHEMICAL FREEZE-OUT

- Empirical freeze-out systematics: thermal hadron production



A. Andronic,
P. Braun-Munzinger,
J. Stachel

Phys. Lett.
B 673 (2009) 142
B 678 (2009) 516

PIONS, NUCLEONS and **NUCLEI** in the context of **LOW-ENERGY QCD**

- **CONFINEMENT** of quarks and gluons in hadrons
- Spontaneously broken **CHIRAL SYMMETRY**
- **LOW-ENERGY QCD** with light (u,d) quarks:
  Effective **F**ield **T**heory of (weakly) interacting
  Nambu-Goldstone Bosons (pions)

- **Chiral EFT** represents QCD at energy/momentum scales

$$Q << 4\pi\, f_\pi \sim 1\, \mathrm{GeV}$$

- **Strategies** at the interface between QCD and **nuclear physics** :

In-medium **Ch**iral **P**erturbation **T**heory
based on **non-linear sigma model**
(with inclusion of nucleons)

expansion of free energy density
in powers of Fermi momentum

Chiral Nucleon-Meson model
based on **linear sigma model**

non-perturbative
Renormalization Group approach

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PART I:

In-medium Chiral Perturbation Theory and the nuclear many-body problem

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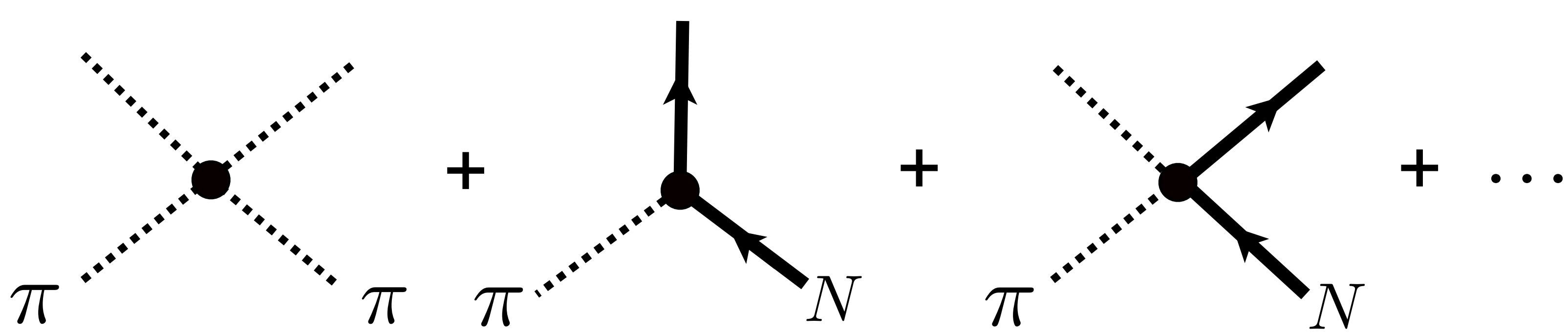
CHIRAL EFFECTIVE FIELD THEORY

- Interacting systems of
  PIONS (light / fast) and **NUCLEONS** (heavy / slow):

$$\mathcal{L}_{eff} = \mathcal{L}_\pi(U, \partial U) + \mathcal{L}_N(\Psi_N, U, \ldots)$$

$$U(x) = exp[i\tau_a \pi_a(x)/f_\pi]$$

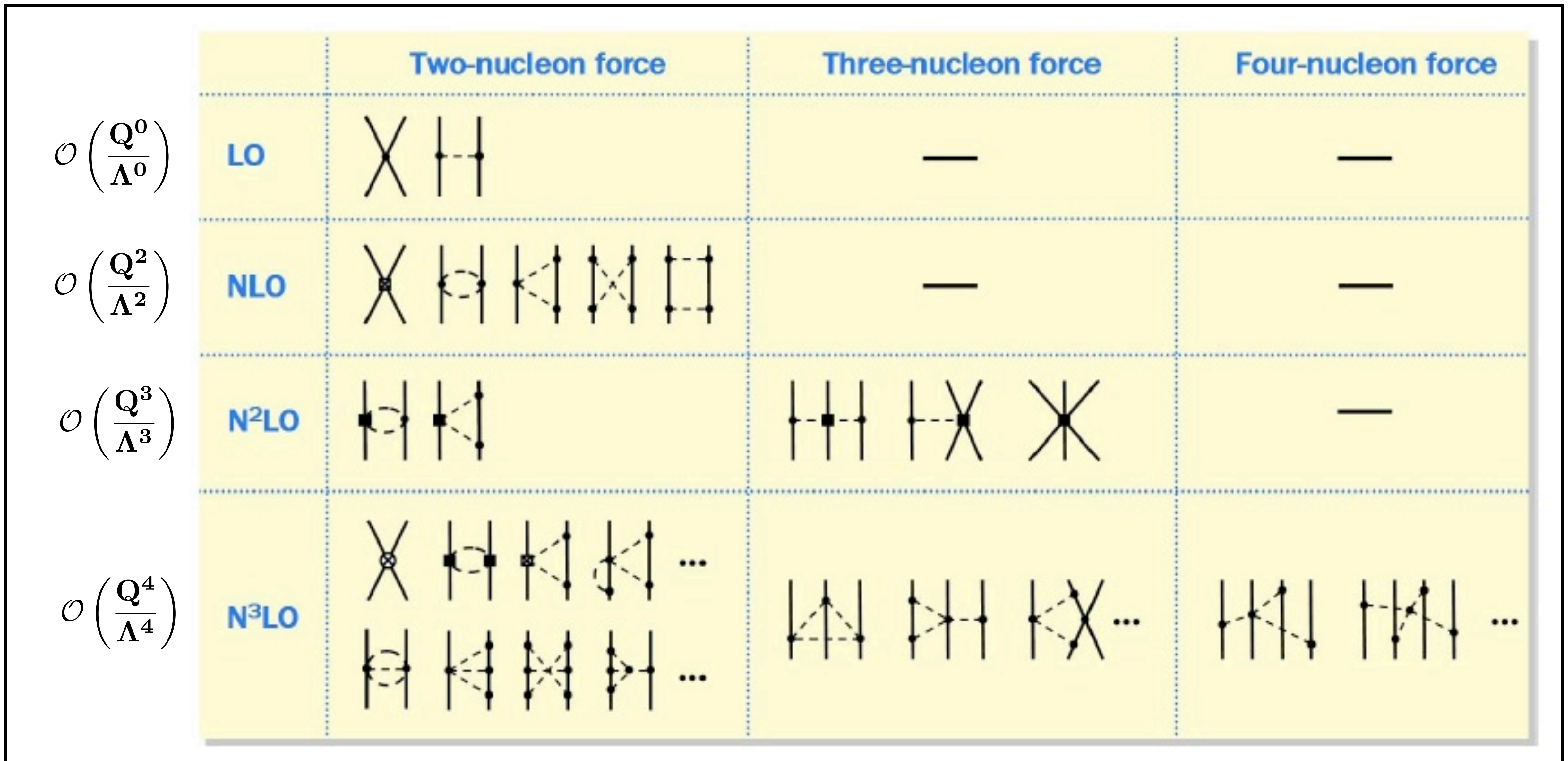
- Construction of Effective Lagrangian: **Symmetries**



short distance dynamics: **contact terms**

NUCLEAR INTERACTIONS from CHIRAL EFFECTIVE FIELD THEORY

Weinberg   Bedaque & van Kolck   Bernard, Epelbaum, Kaiser, Meißner; ...



- **Systematically organized HIERARCHY**

IN-MEDIUM CHIRAL PERTURBATION THEORY

- **Small scales**: energy, momentum, m_π, $k_F << 4\pi f_\pi \sim 1\,\mathrm{GeV}$

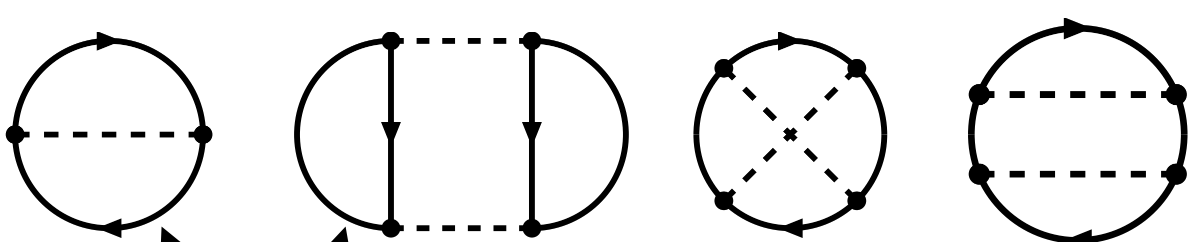
- **Loop expansion** of (**I**n-**M**edium) **Ch**iral **P**erturbation **T**heory

  ↕

  Systematic expansion of **ENERGY DENSITY** $\mathcal{E}(k_F)$ in **powers** of **Fermi momentum** [modulo functions $f_n(k_F/m_\pi)$]

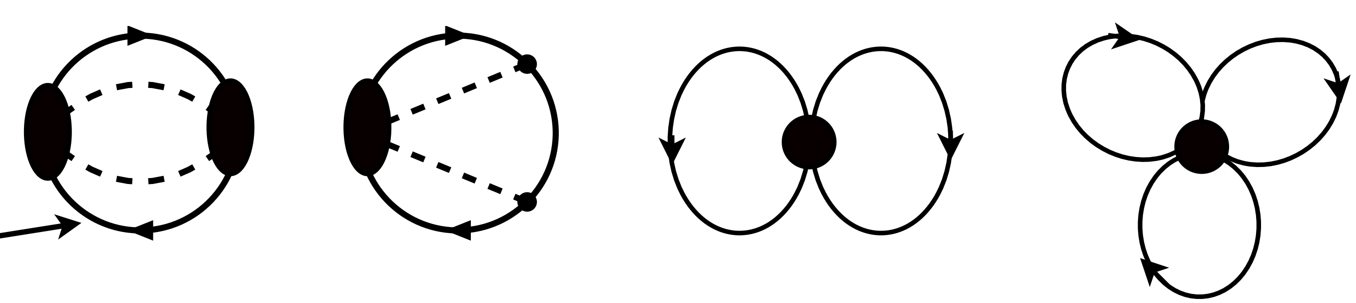
  (works for $k_F << 4\pi\, f_\pi \sim 1\,\mathrm{GeV}$)

- Nuclear **thermodynamics**: compute **free energy density**



(3-loop order)

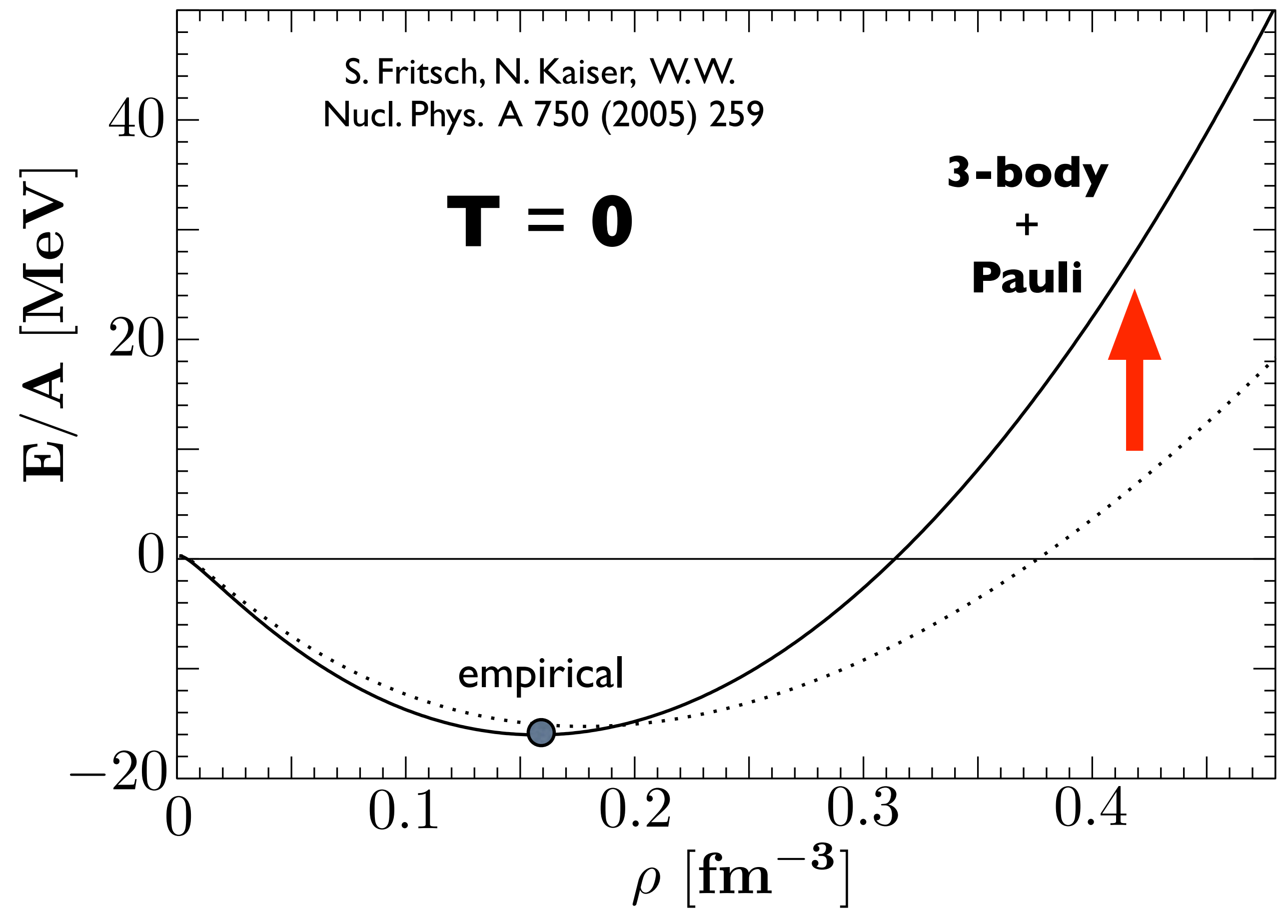
N. Kaiser, S. Fritsch, W.W. (2002-2004)



in-medium nucleon propagators incl. Pauli blocking

NUCLEAR MATTER

- **In-medium ChPT**
  3-loop $(\pi, \mathrm{N}, \Delta)$
- **Input** parameters:
  two contact terms
- basically:
  analytic calculation
- **Output**:



  - Binding & saturation
    $\mathbf{E_0/A = -16\,MeV}\ ,\ \rho_0 = \mathbf{0.16\,fm^{-3}}\ ,\ \mathbf{K = 290\,MeV}$
  - Realistic (complex, momentum dependent) single-particle potential
    ... satisfying Hugenholtz - van Hove and Luttinger theorems (!)
  - Asymmetry energy: $\mathbf{A(k_F^0) = 34\,MeV}$
  - Fermi Liquid Theory:
    Quasiparticle interaction and Landau parameters

J.W. Holt, N. Kaiser, W.W.
Nucl. Phys. A 870 (2011) 1,
Nucl. Phys. A 876 (2012) 61,
Phys. Rev. C 87 (2013) 014338

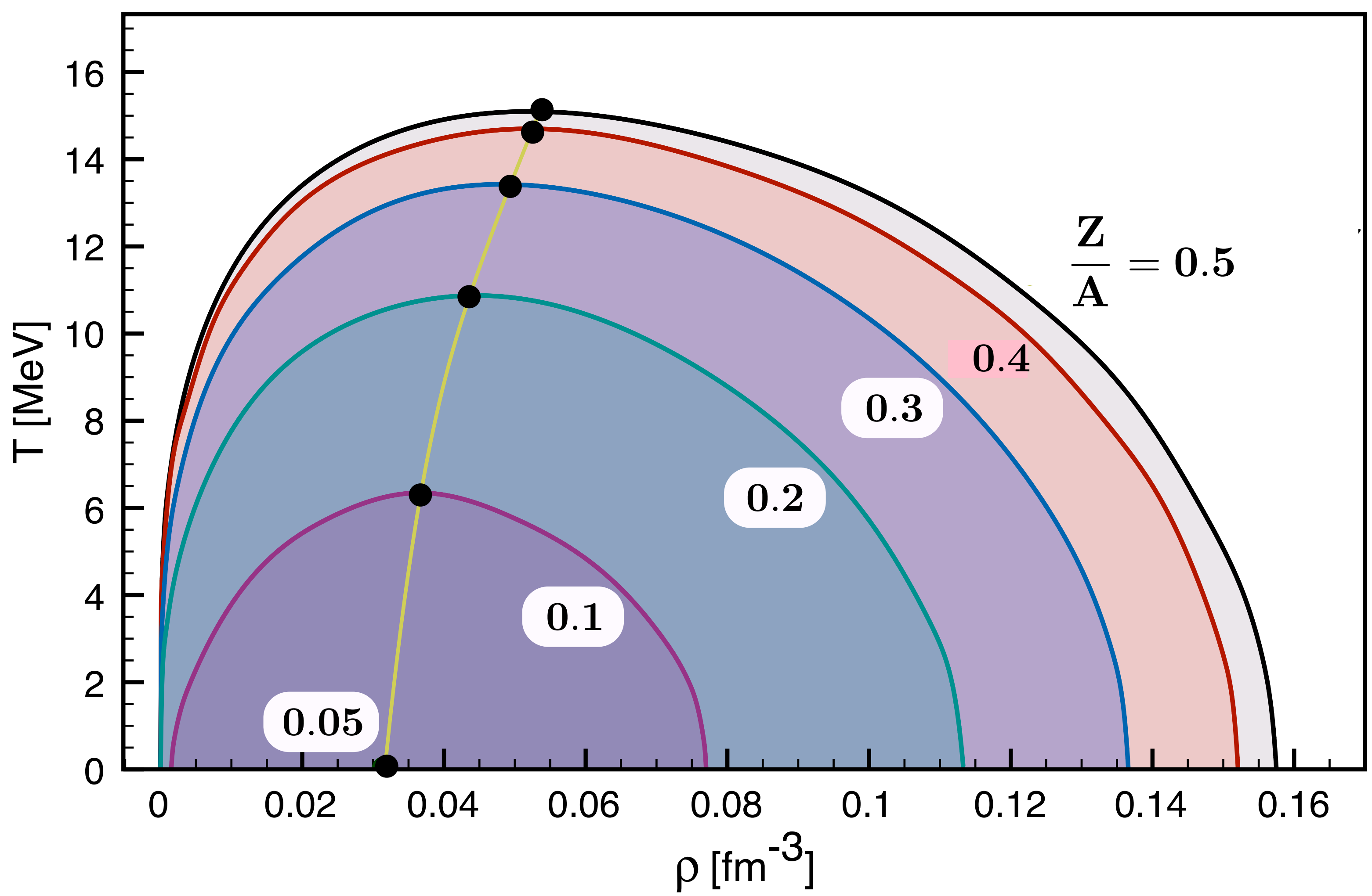
C. Wellenhofer, J.W. Holt, N. Kaiser, W.W.
Phys. Rev. C 89 (2014) 064009

Recent review: J.W. Holt, N. Kaiser, W.W. Prog. Part. Nucl. Phys. 73 (2013) 35

PHASE DIAGRAM of NUCLEAR MATTER

- Trajectory of **CRITICAL POINT** for **asymmetric matter**

as function of proton fraction Z/A

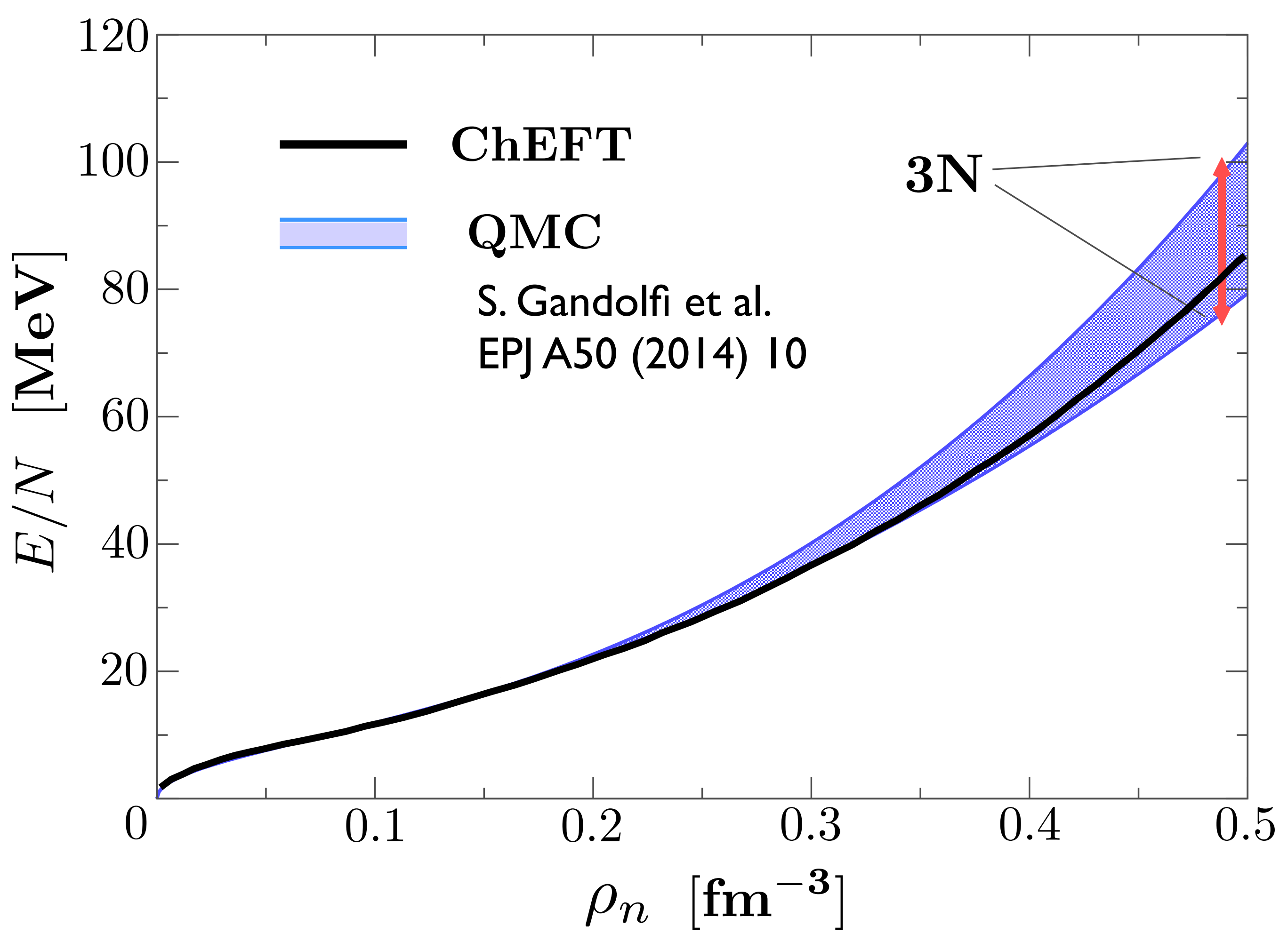


S. Fiorilla, N. Kaiser, W.W.
Nucl. Phys. A880 (2012) 65

. . . determined almost completely by
isospin dependent (one- and two-) **pion** exchange dynamics

NEUTRON MATTER

- In-medium chiral effective field theory (3-loop) with resummation of short distance contact terms (large nn scattering length, $a_s = 19$ fm)



- Neutron matter behaves almost (but not quite) like a unitary Fermi gas

- Bertsch parameter

$$\xi = \frac{\bar{E}}{E_{\text{Fermi gas}}} \simeq 0.5$$

J.W. Holt, N. Kaiser, W.W.
Phys. Rev. C 87 (2013) 014338

- agreement with sophisticated many-body calculations

(e.g. recent **Q**uantum **M**onte **C**arlo computations)

PART II:

Chiral Nucleon-Meson Model
and
Functional Renormalization Group

Mesons, Nucleons, Nuclear Matter and Functional Renormalization Group

- **Chiral nucleon - meson model** $\psi = (\psi_p, \psi_n)^T$

$$\mathcal{L} = \bar{\psi} i \gamma_\mu \partial^\mu \psi + \frac{1}{2} \partial_\mu \sigma \, \partial^\mu \sigma + \frac{1}{2} \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}$$
$$- \bar{\psi} \Big[g(\sigma + i \gamma_5 \, \boldsymbol{\tau} \cdot \boldsymbol{\pi}) + \gamma_\mu (g_\omega \, \omega^\mu + g_\rho \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu) \Big] \psi$$
$$- \frac{1}{4} F^{(\omega)}_{\mu\nu} F^{(\omega)\mu\nu} - \frac{1}{4} \boldsymbol{F}^{(\rho)}_{\mu\nu} \cdot \boldsymbol{F}^{(\rho)\mu\nu}$$
$$+ \frac{1}{2} m_V^2 \left(\omega_\mu \omega^\mu + \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu \right) - \mathcal{U}(\sigma, \boldsymbol{\pi})$$

- **Effective potential** constructed to reproduce standard nuclear thermodynamics around equilibrium

- **Mean field** calculations
  S. Floerchinger, Ch. Wetterich : Nucl. Phys. A 890-891 (2012) 11

- **Mesonic** and **nucleonic particle-hole** fluctuations treated non-perturbatively using **FRG**

M. Drews, T. Hell, B. Klein, W.W. Phys. Rev. D 88 (2013) 096011
M. Drews, W.W. arXiv:1404.0882

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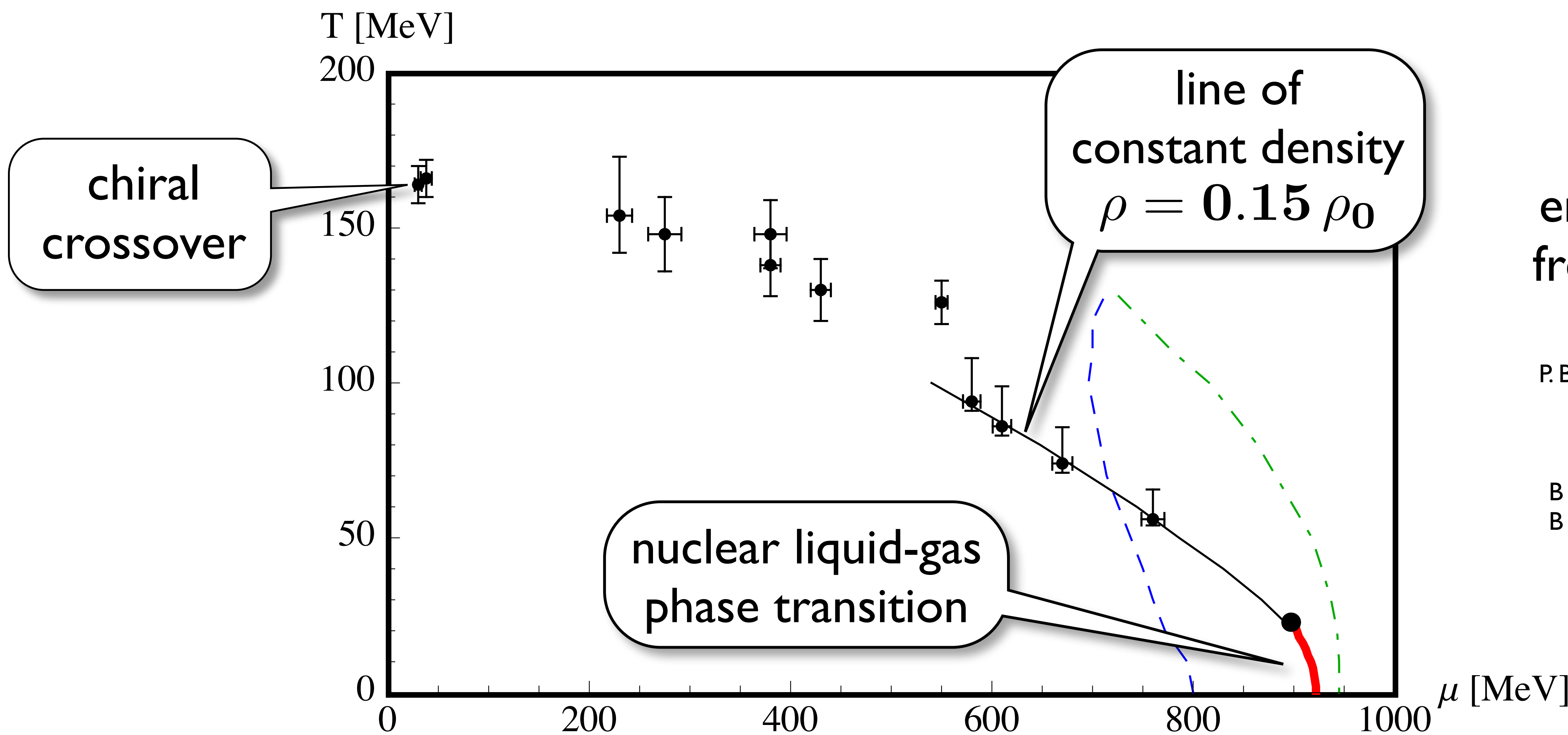
CHEMICAL FREEZE-OUT

S. Floerchinger, Ch. Wetterich : Nucl. Phys. A 890-891 (2012) 11

- **Chiral nucleon - meson model** in mean-field approximation



empirical freeze-out

A. Andronic,
P. Braun-Munzinger,
J. Stachel

Phys. Lett.
B 673 (2009) 142
B 678 (2009) 516

- **Chemical freeze-out** in **baryonic matter** at T < 100 MeV is **not** associated with (**chiral**) phase transition or rapid crossover

Fixing the input: some comments

- **Potential** $\mathcal{U}(\sigma,\boldsymbol{\pi}) = \mathcal{U}_0(\chi) - m_\pi^2 f_\pi(\sigma - f_\pi)$

  chiral invariant part parametrized in powers of $\chi = \frac{1}{2}(\sigma^2 + \boldsymbol{\pi}^2)$

  explicit chiral symmetry breaking

- **Scalar** ("sigma") **field**
  has mean-field (chiral **order parameter**) and fluctuating pieces.
  σ **mass**: NOT to be confused with pole in I = 0 s-wave pion-pion T matrix.
  Nucleon mass: $m_N^2 = 2g\,\chi$ ... in vacuum: $m_N = g\,f_\pi$

- **Vector fields** encode short-distance NN dynamics,
  treated as self-consistently determined background mean fields (non-fluctuating)
  (not to be identified with physical ω and ρ mesons)

  Effective chemical potentials $\mu_{n,p}^{\mathrm{eff}} = \mu_{n,p} - g_\omega\,\omega_0 \pm g_\rho\,\rho_0^3$

  Relevant quantities: $G_\rho = \frac{g_\rho^2}{m_V^2}\,,\; G_\omega = \frac{g_\omega^2}{m_V^2}$ $\longleftrightarrow$ contact terms in ChEFT

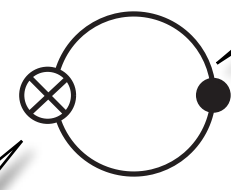
- **Parameters**: 2 coefficients in $\mathcal{U}_0$, $\mathbf{m}_\sigma \simeq \mathbf{0.8\,GeV}$, $\mathbf{G}_\rho \sim \mathbf{G}_\omega/4 \simeq \mathbf{1\,fm^2}$
  determined by tuning nuclear matter properties and symmetry energy

Chiral nucleon - meson model beyond mean-field
- Renormalization Group strategies -

M. Drews, T. Hell, B. Klein, W.W. Phys. Rev. D 88 (2013) 096011

- Fluctuations: Wetterich's RG flow equations

effective action **full propagator**

$$k\,\frac{\partial \Gamma_k}{\partial k} =$$  $$= \frac{1}{2}\,\mathrm{Tr}\,\frac{k\,\frac{\partial R_k}{\partial k}}{\Gamma_k^{(2)} + R_k}$$

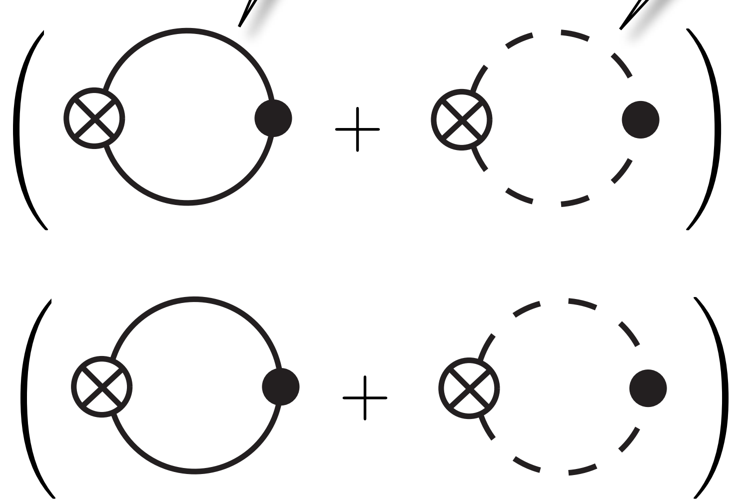
regulator: $R_k(\boldsymbol{p}^2) = (k^2 - \boldsymbol{p}^2)\,\theta(k^2 - \boldsymbol{p}^2)$

C. Wetterich:
Phys. Lett. B 301 (1993) 90

D.F. Litim, J.M. Pawlowski:
JHEP 0611 (2006) 026
J.-P. Blaizot, A. Ipp,
R. Mendez-Galain, N. Wschebor:
NP A784 (2007) 376

- Thermodynamics

nucleons **pions**

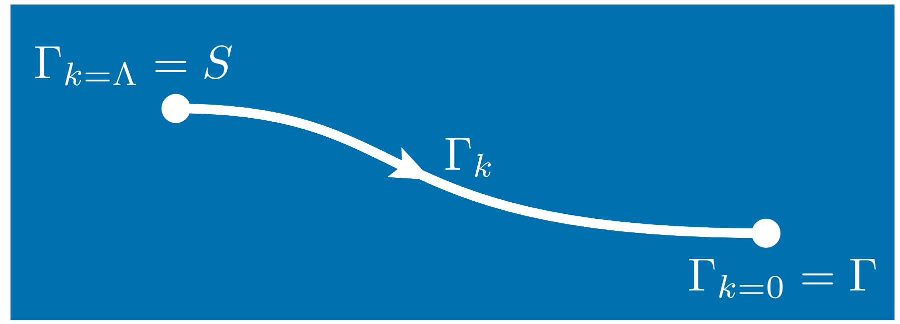
$$k\,\partial_k \bar{\Gamma}_k(T,\mu) =$$ 

$$\Big|_{T,\,\mu_p,\,\mu_n} \qquad - \ (\ldots)\Big|_{T=0,\,\mu=\mu_0\,(=m_N - E_0/A)}$$

Flow equations in practice

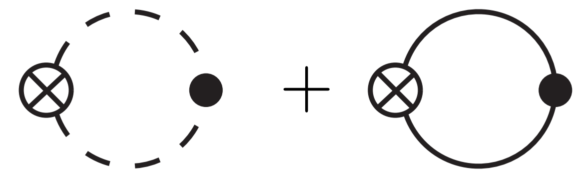
UV scale:

$\boldsymbol{\Lambda = 1.4\,\mathrm{GeV}}$



"full" effective action & effective potential $\mathbf{U_{k=0}}$

$$k\frac{\partial U_{k,\chi}}{\partial k}(T, \mu_p, \mu_n, \chi, \omega_0, \rho_0^3) =$$



$$= \frac{k^5}{12\pi^2}\left\{\frac{1+2n_{\mathrm{B}}(E_\sigma)}{E_\sigma} + \frac{3\left[1+2n_{\mathrm{B}}(E_\pi)\right]}{E_\pi} - 4\sum_{i=n,p}\frac{1-n_{\mathrm{F}}(E_{\mathrm{N}}-\mu_{i,\mathrm{eff}})}{E_{\mathrm{N}}}\right\}$$

$$E_\pi^2 = k^2 + U_k'(\chi)\,,\; E_\sigma^2 = k^2 + U_k'(\chi) + 2\chi\, U_k''(\chi)\,,$$

$$U_k'(\chi) = \frac{\partial U_k(\chi)}{\partial \chi}\,, \quad E_{\mathrm{N}}^2 = k^2 + 2g^2\chi\,,$$

$$\mu_{n,p}^{\mathrm{eff}}(k) = \mu_{n,p} - g_\omega\,\omega_0(k) \pm g_\rho\,\rho_0^3(k)\,,$$

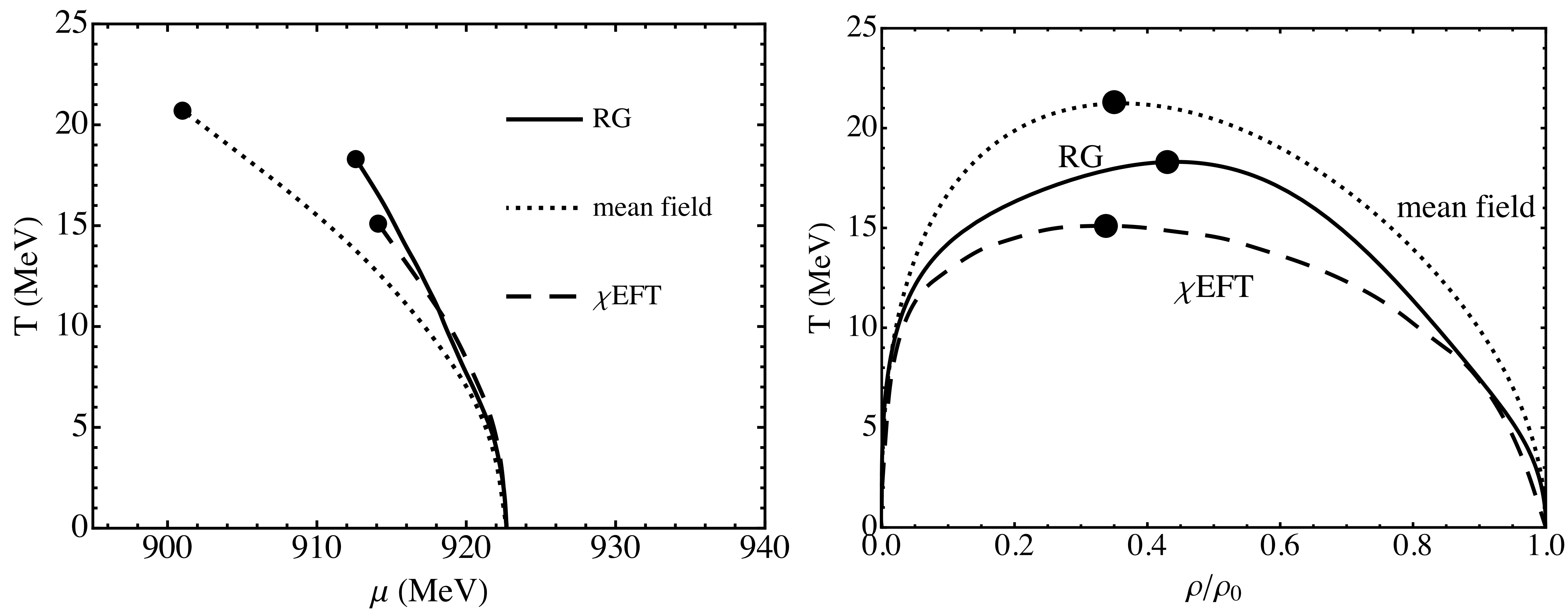
$$n_{\mathrm{B}}(E) = \frac{1}{\mathrm{e}^{E/T}-1}\,,$$

$$n_{\mathrm{F}}(E) = \frac{1}{\mathrm{e}^{E/T}+1}\,.$$

. . . plus vector field equations, then full system of equations solved on a grid.

Results : Liquid - Gas Transition
- symmetric nuclear matter -

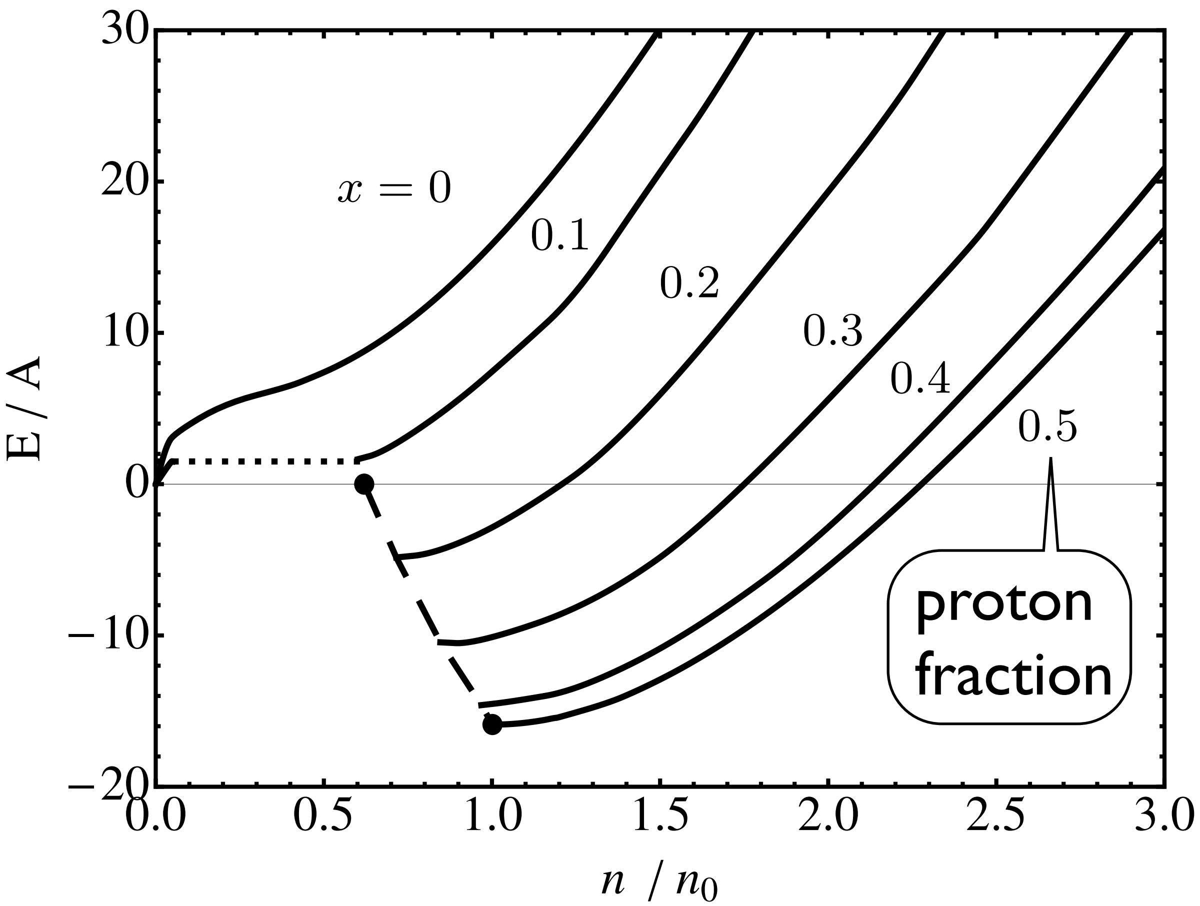
M. Drews, T. Hell, B. Klein, W. W.
Phys. Rev. D 88 (2013) 096011



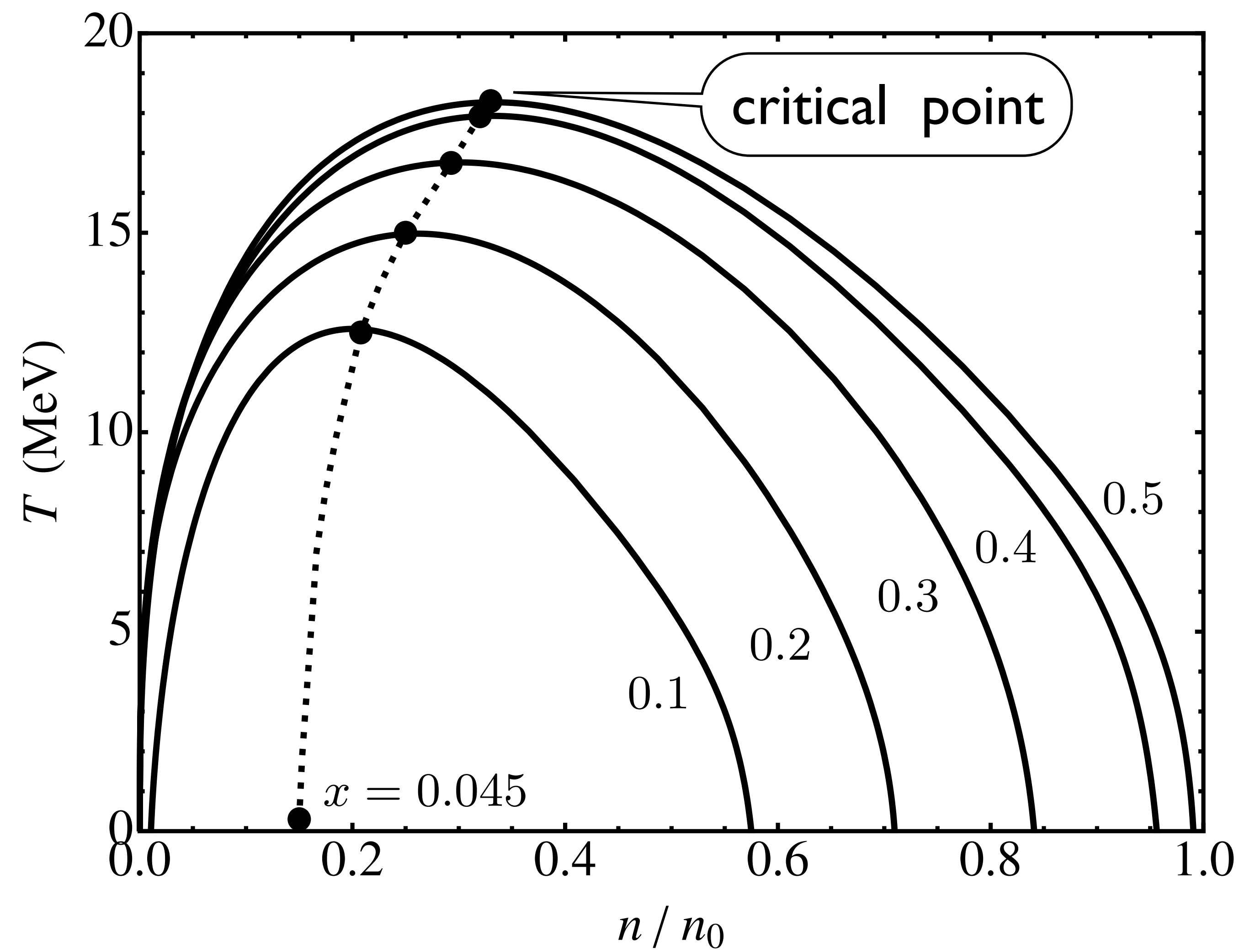
- note: close correspondence between (perturbative) in-medium ChEFT and (non-perturbative) FRG results

Asymmetric nuclear matter in the chiral RG approach

M. Drews, W.W. (2014)



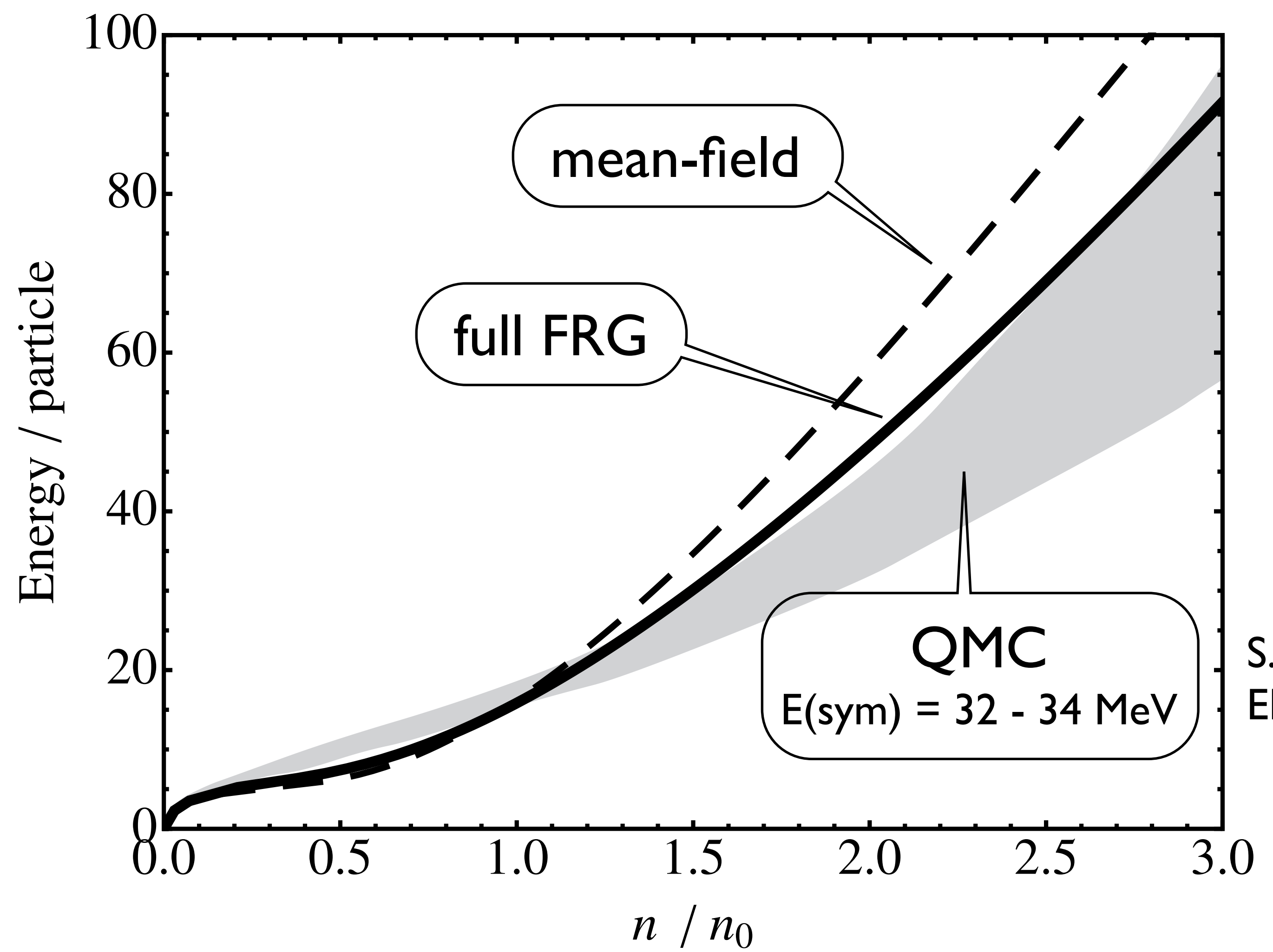
Liquid-gas phase transition:
evolution of coexistence regions
from symmetric to asymmetric nuclear matter



- **FRG results**
(non-perturbative)
are remarkably similar to
in-medium **Chiral EFT calculations** (perturbative)

Neutron matter in the **chiral RG approach**

M. Drews, W.W.
arXiv:1404.0882

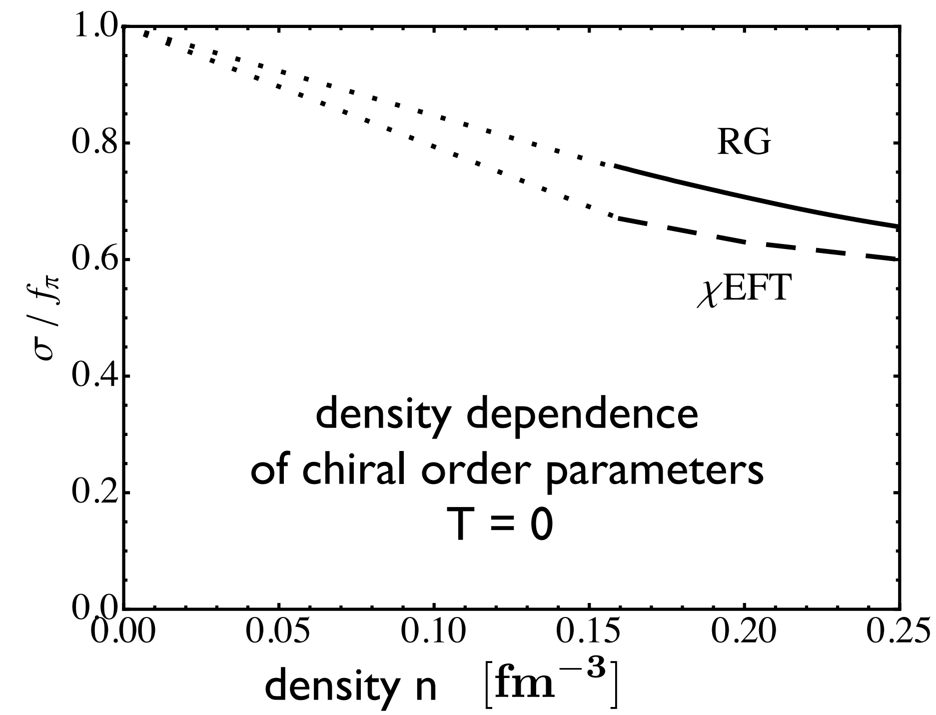


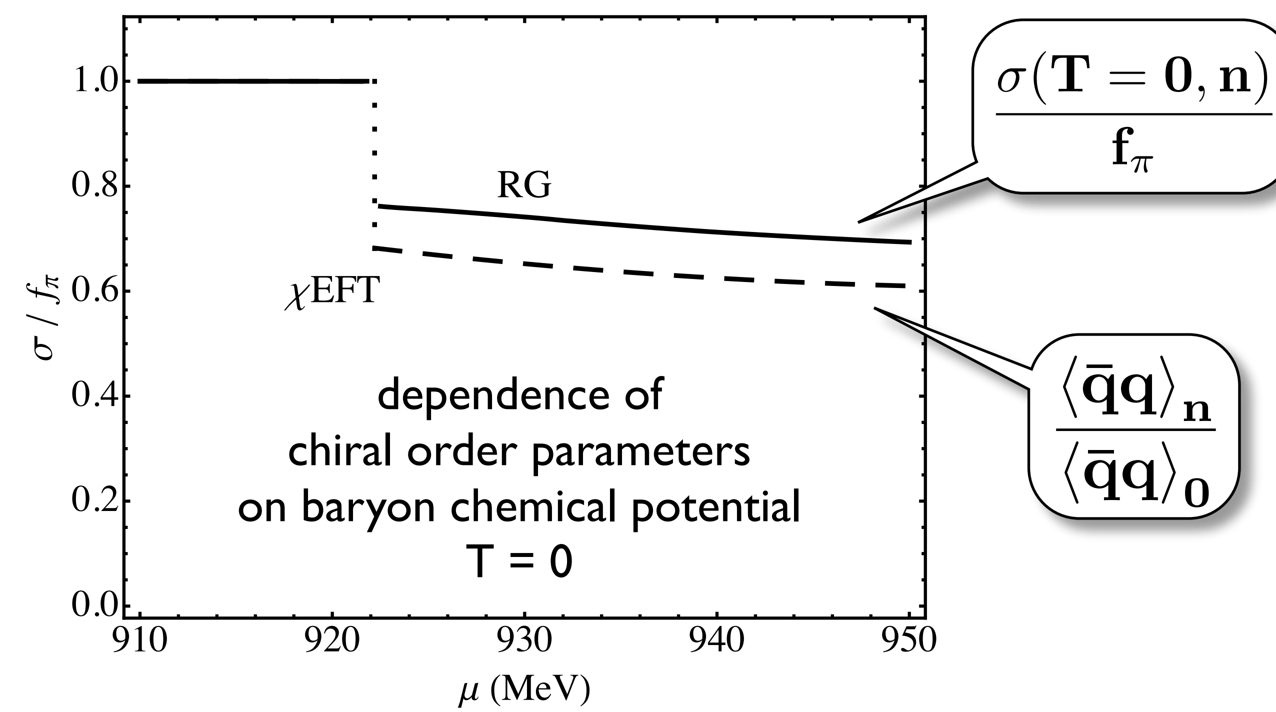
S. Gandolfi et al.
EPJ A50 (2014) 10

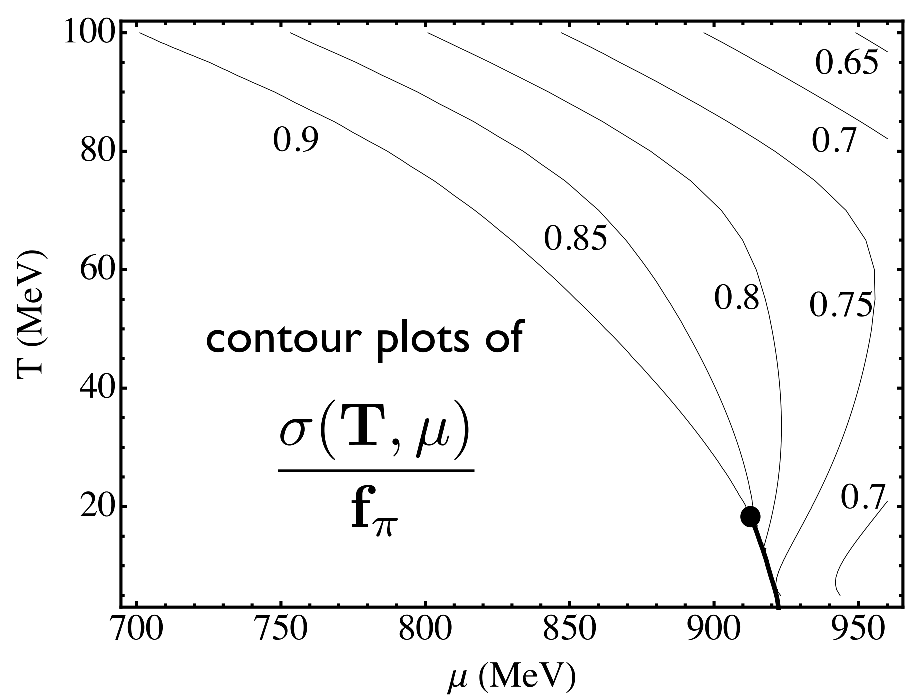
- Single additional parameter:
  coupling strength of isovector-vector field / contact term
  fixed by symmetry energy E(sym) = 32 MeV

Chiral Order Parameters

- Comparison of chiral effective field theory and model FRG results







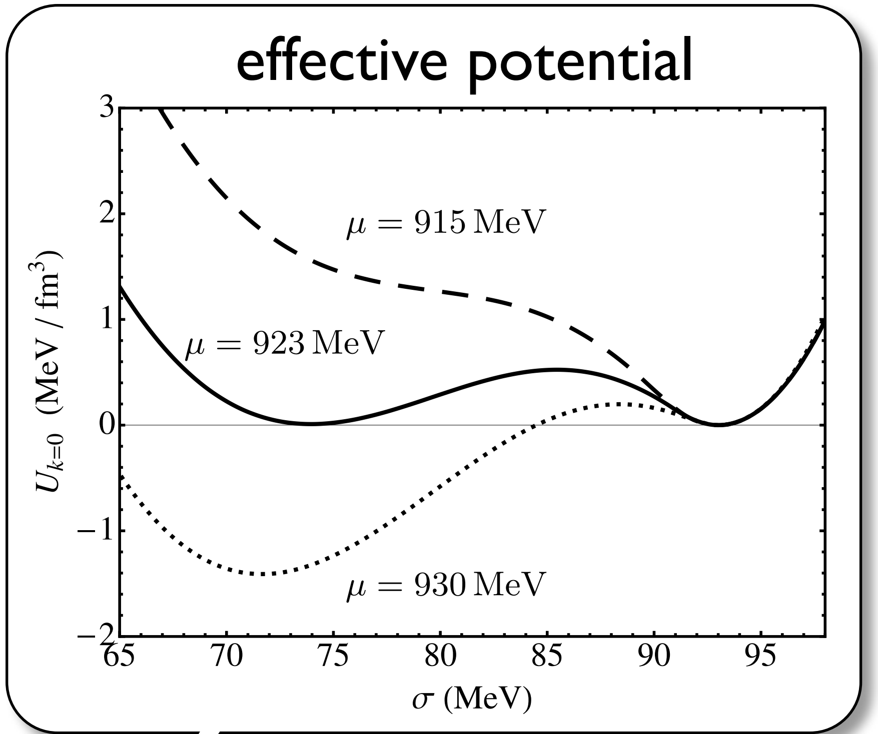
- **No** tendency towards **chiral** phase transition for baryon chemical potentials $\mu \lesssim 1\ \mathrm{GeV}$ and temperatures $\mathbf{T} \lesssim 100\ \mathrm{MeV}$

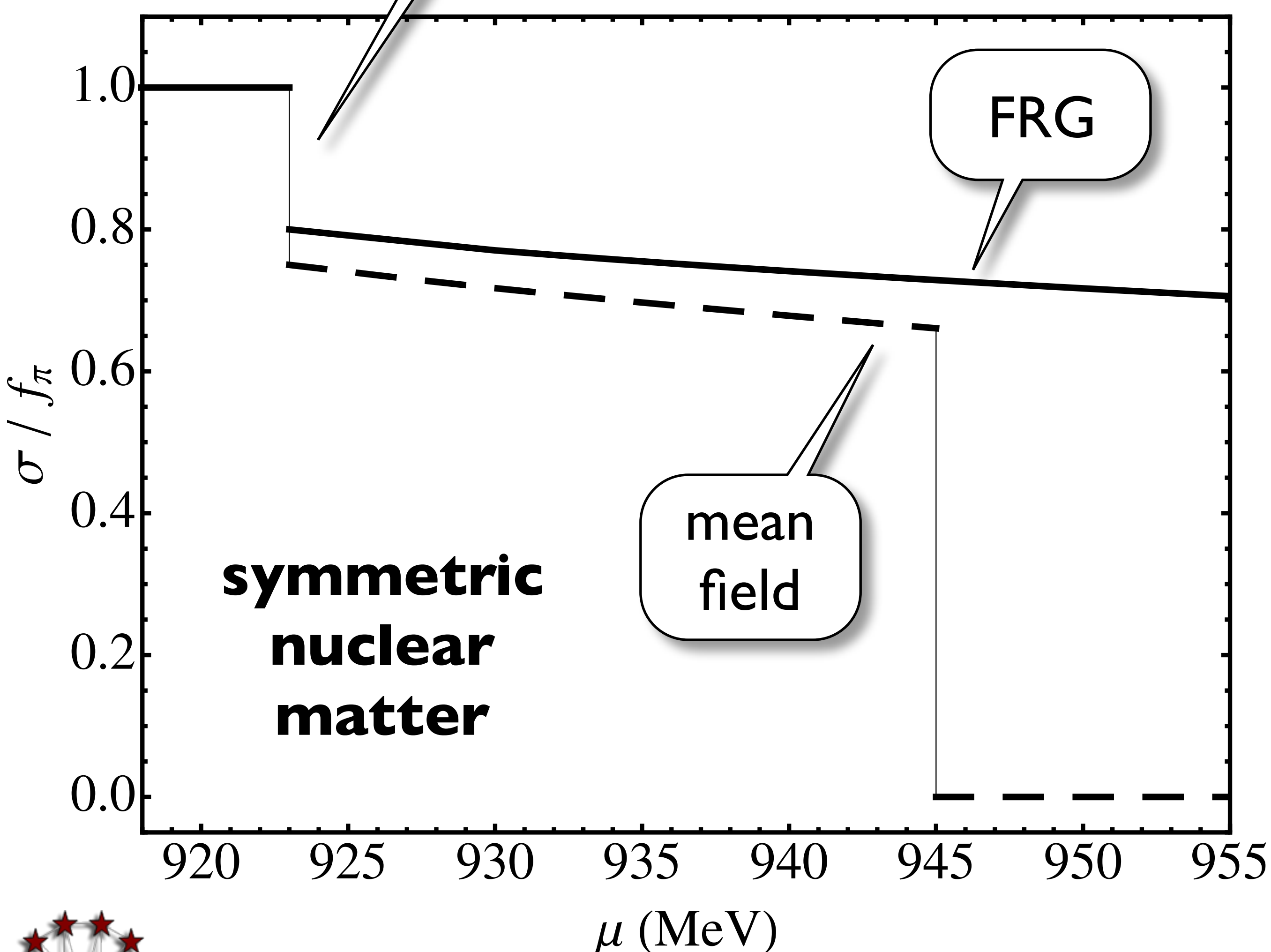
M. Drews, T. Hell, B. Klein, W.W.
Phys. Rev. D 88 (2013) 096011

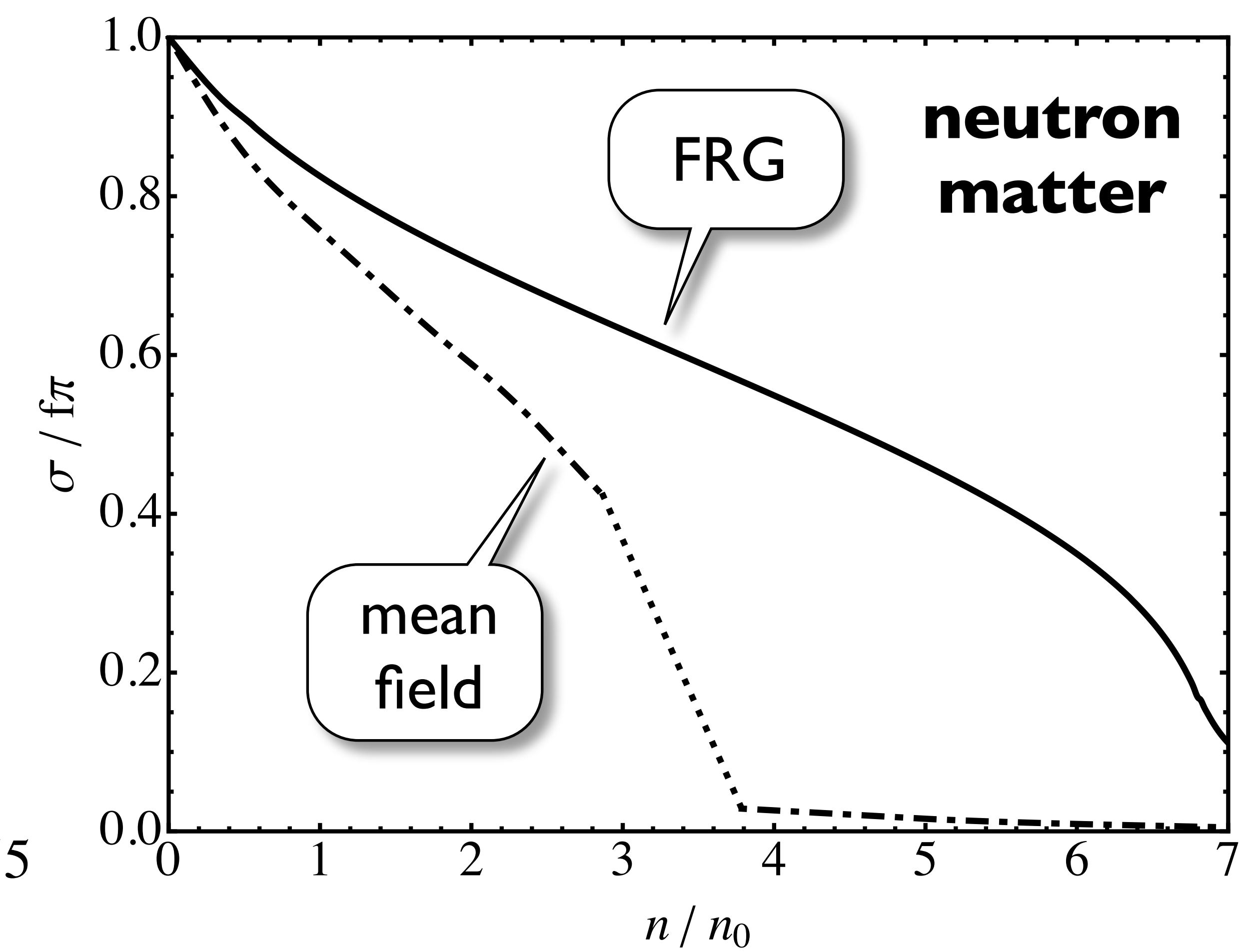
Chiral Order Parameter

- important role of **fluctuations**:

disappearance of
first-order chiral transition
seen in mean-field approximation

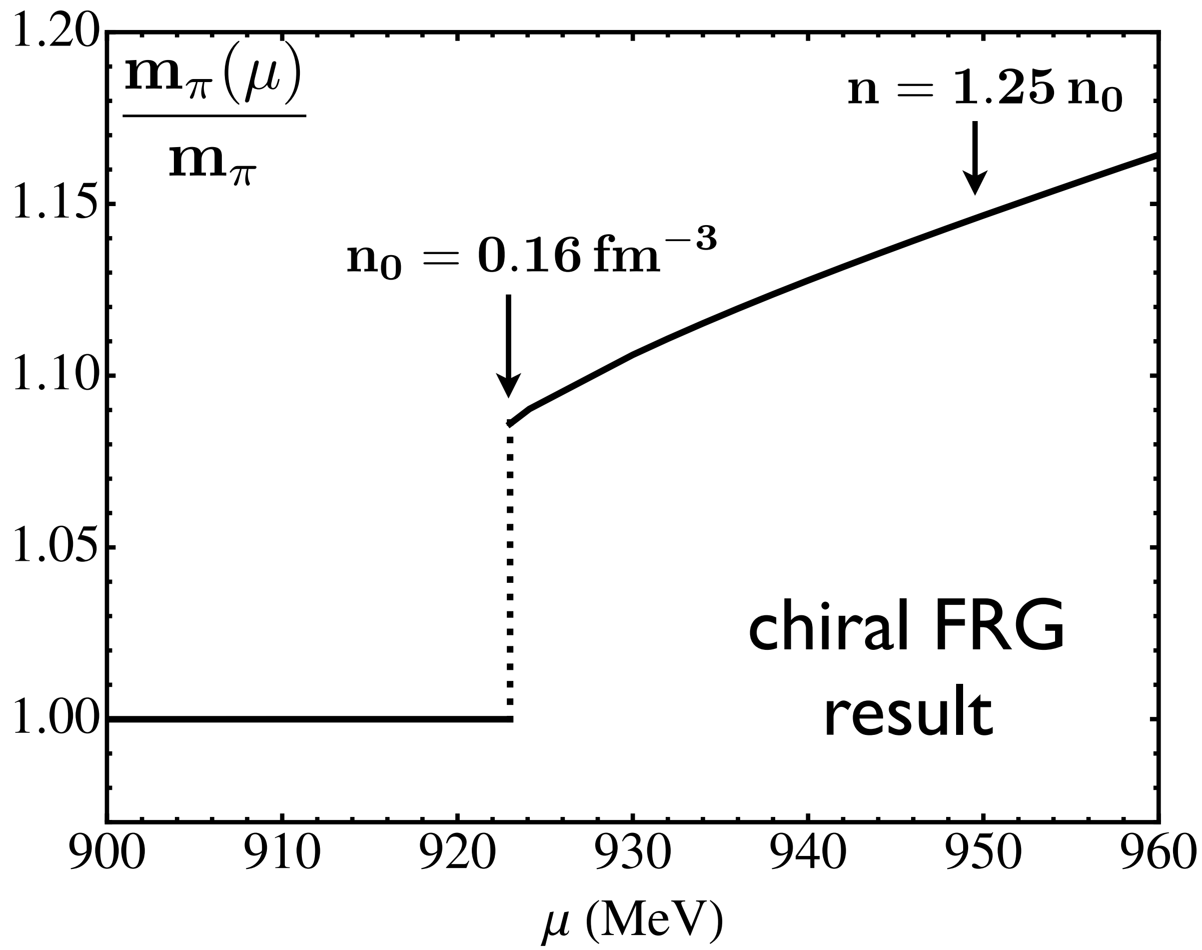


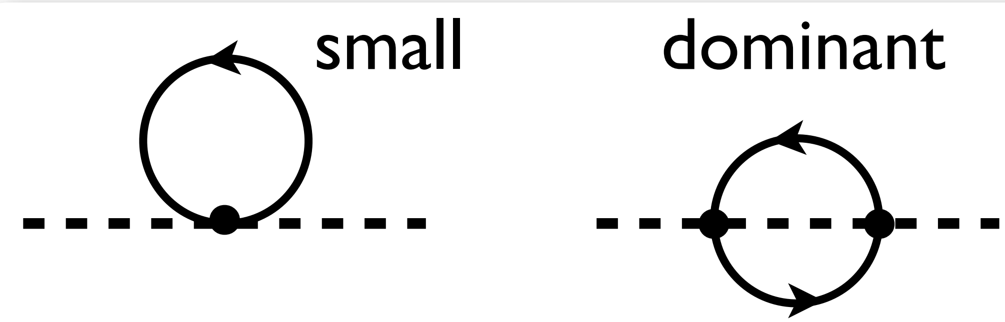




In-medium pion mass

- Contact with phenomenology :
  compare with s-wave pion-nuclear optical potential from pionic atoms





$$U(n) = -\frac{2\pi}{m_\pi}\left[b_0 - (b_0^2 + 2b_1^2)\left\langle\frac{1}{r}\right\rangle\right]\cdot n$$

$$\frac{m_\pi(n)}{m_\pi} \simeq 1 + \frac{U(n)}{m_\pi} \simeq 1.1\,\frac{n}{n_0}$$

- Good agreement of **FRG** calculation with empirical **in-medium pion mass shift**, both in sign and magnitude

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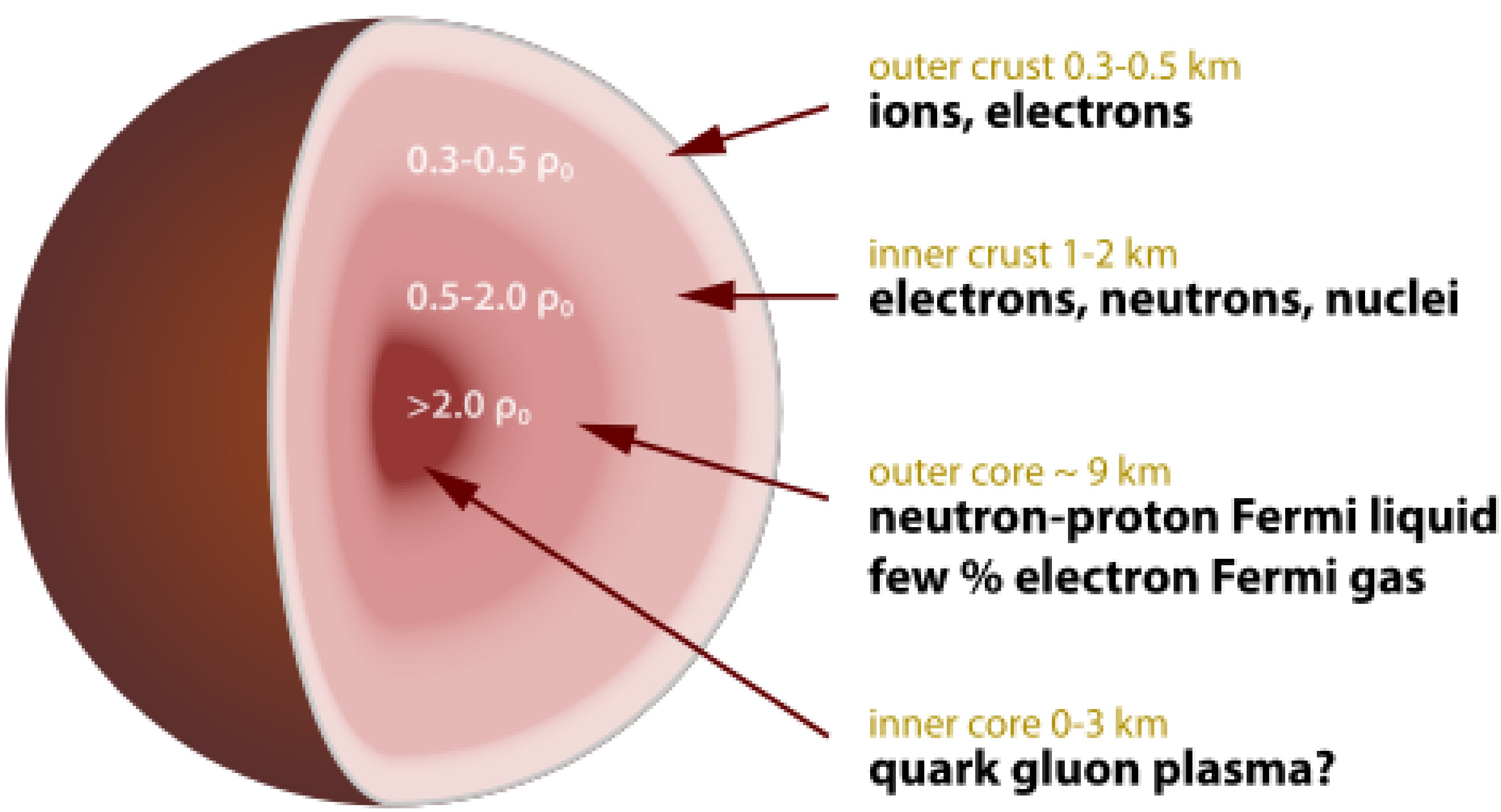
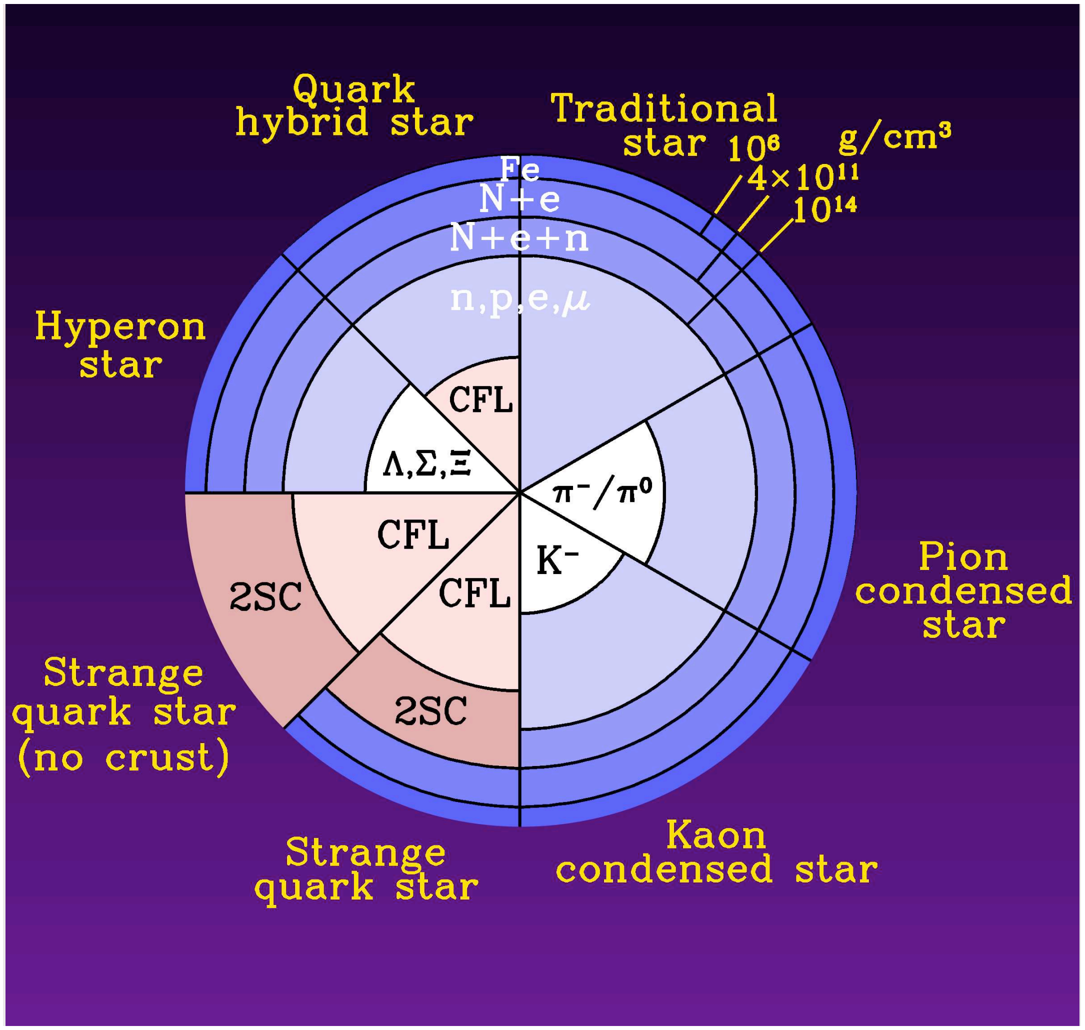
PART III:

Short digression on Neutron Stars



NEUTRON STARS and the EQUATION OF STATE of DENSE BARYONIC MATTER

J. Lattimer, M. Prakash: Astrophys. J. 550 (2001) 426
Phys. Reports 442 (2007) 109



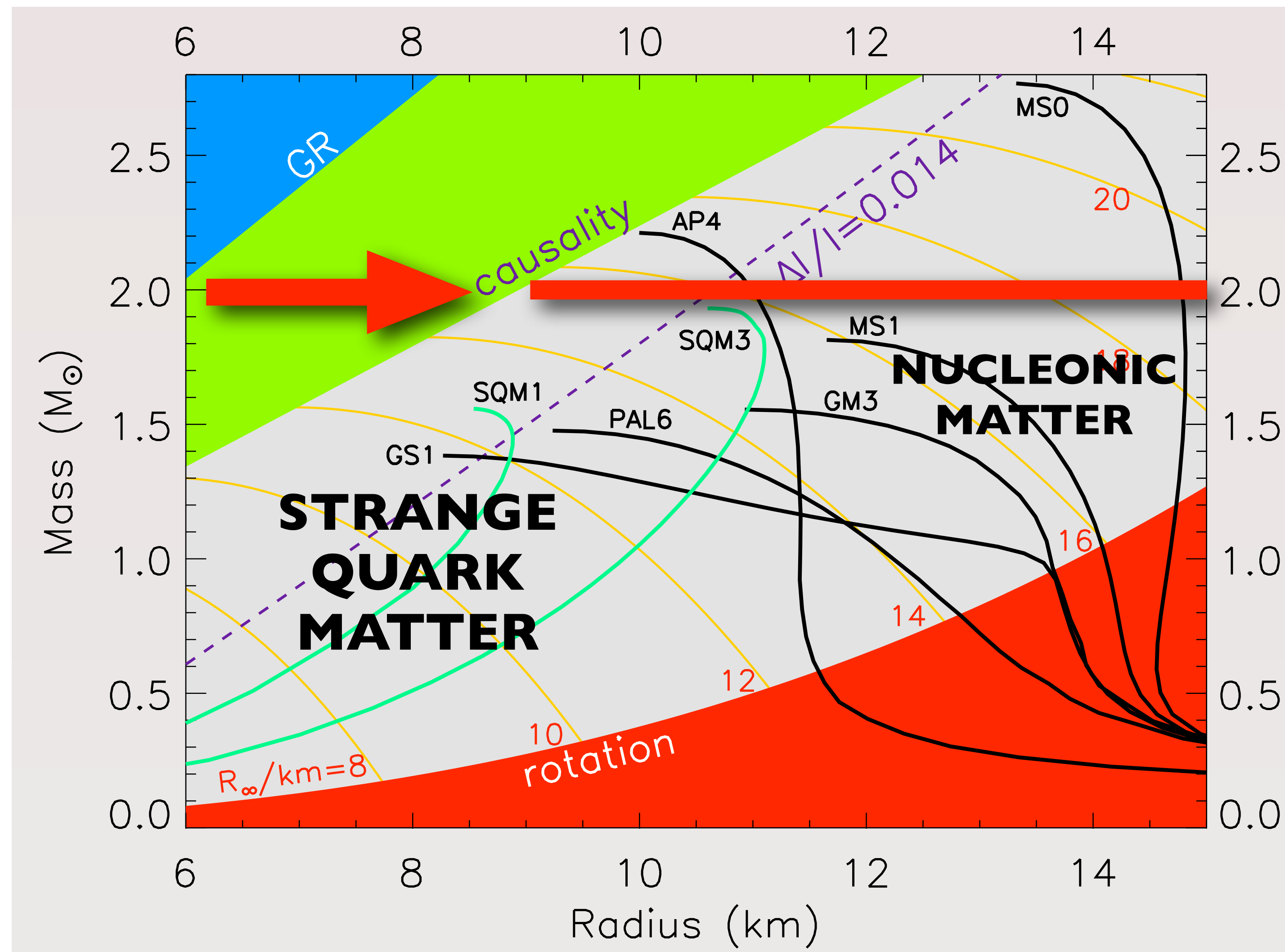
Neutron Star Scenarios

Tolman-**O**ppenheimer-**V**olkov Equations

$$\frac{dP}{dr} = -\frac{G}{c^2}\frac{(M + 4\pi P r^3)(\mathcal{E} + P)}{r(r - GM/c^2)}$$

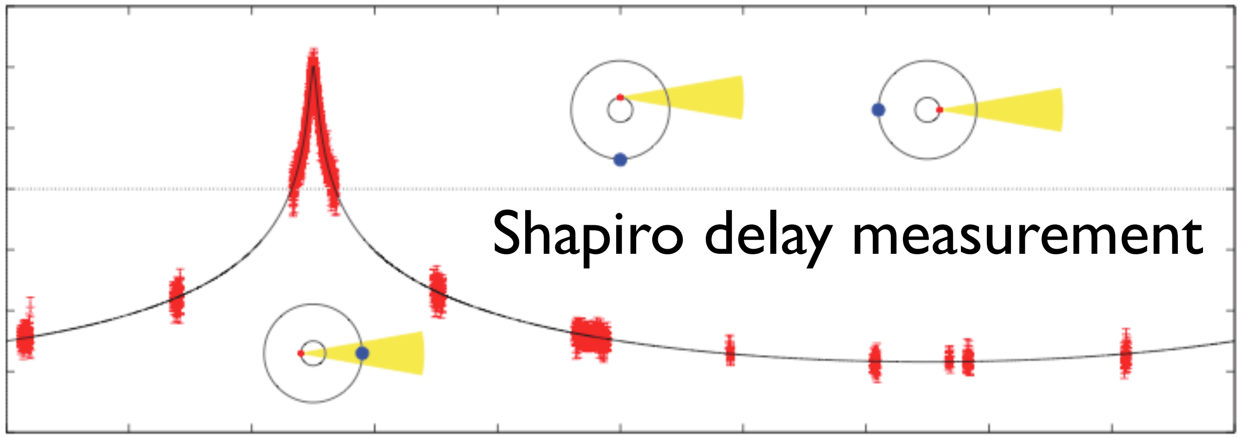
$$\frac{dM}{dr} = 4\pi r^2 \frac{\mathcal{E}}{c^2}$$

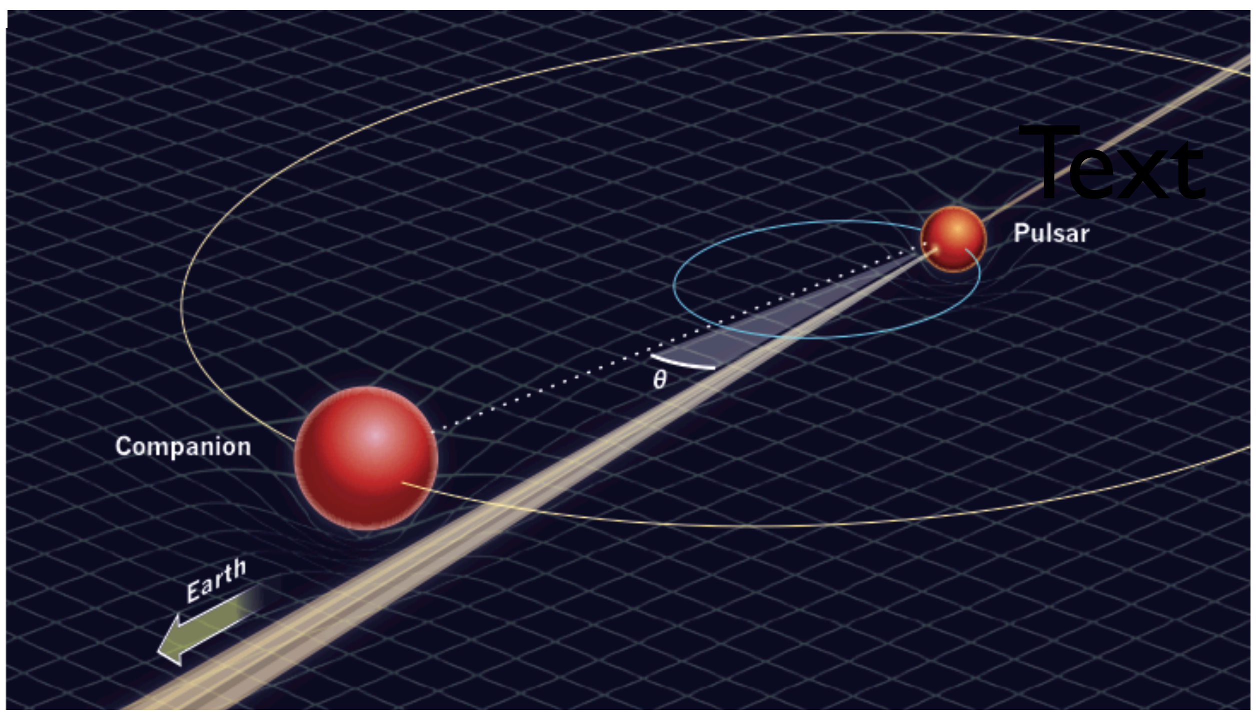
Mass-Radius Relation



New constraints from NEUTRON STARS

P.B. Demorest et al.
Nature 467 (2010) 1081

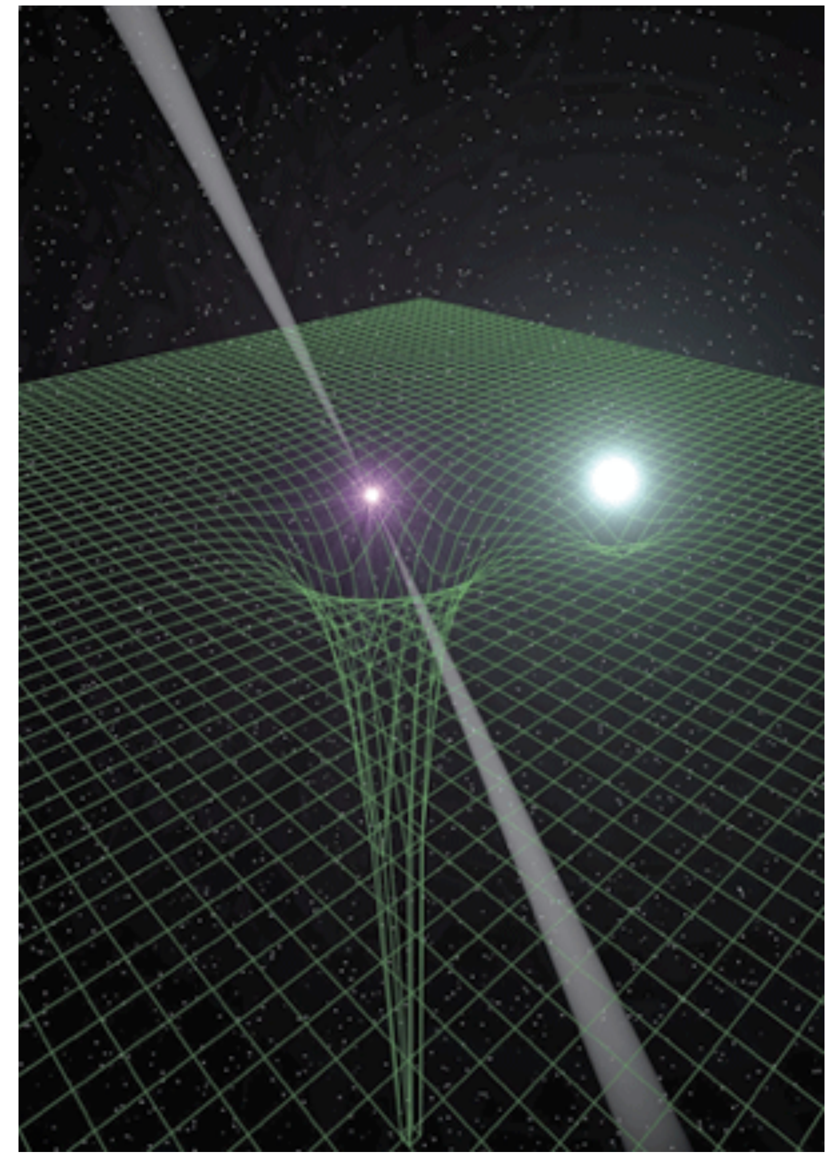




PSR J1614+2230

$$\mathrm{M} = 1.97 \pm 0.04\ \mathrm{M}_\odot$$

J. Antoniadis et al.
Science 340 (2013) 6131

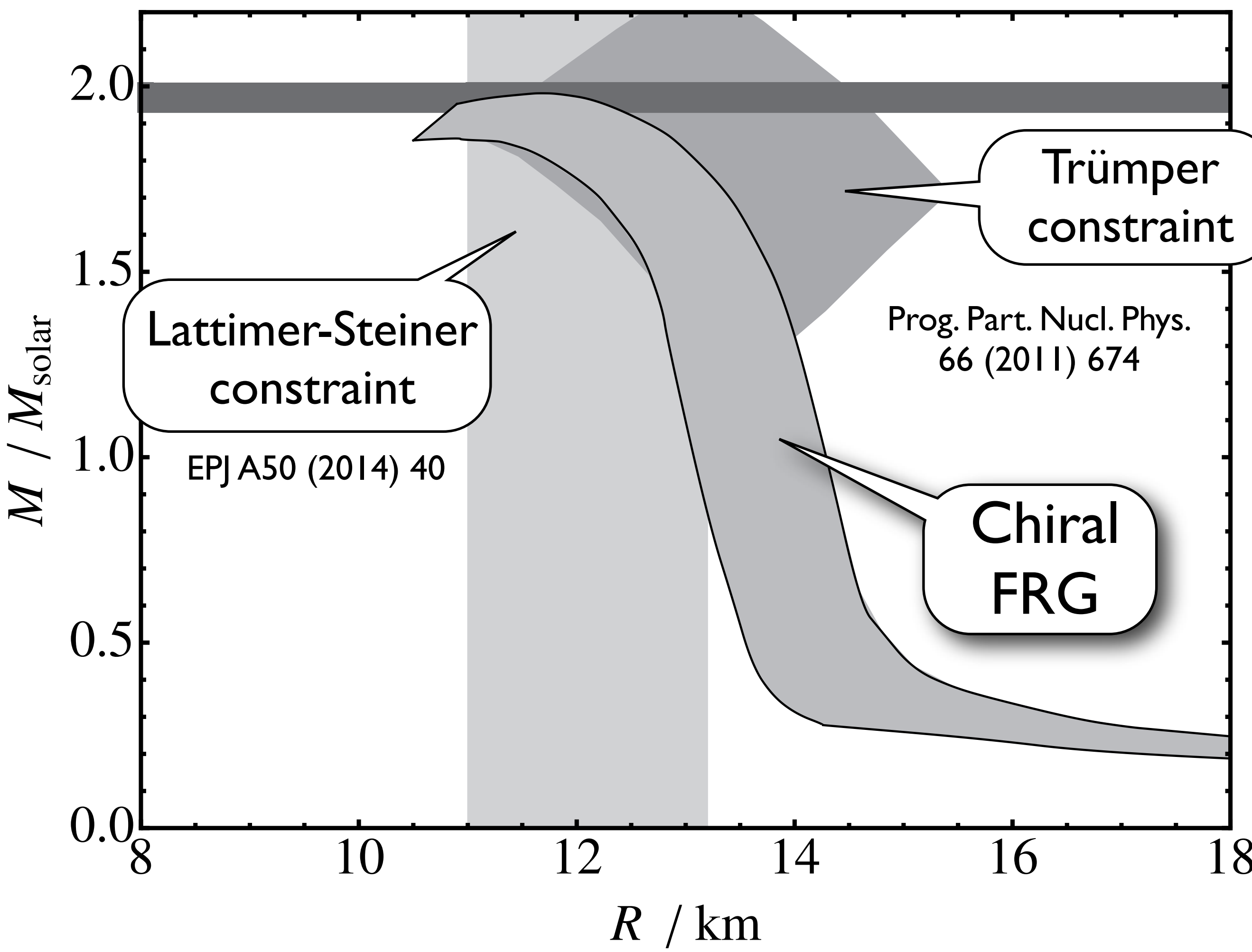


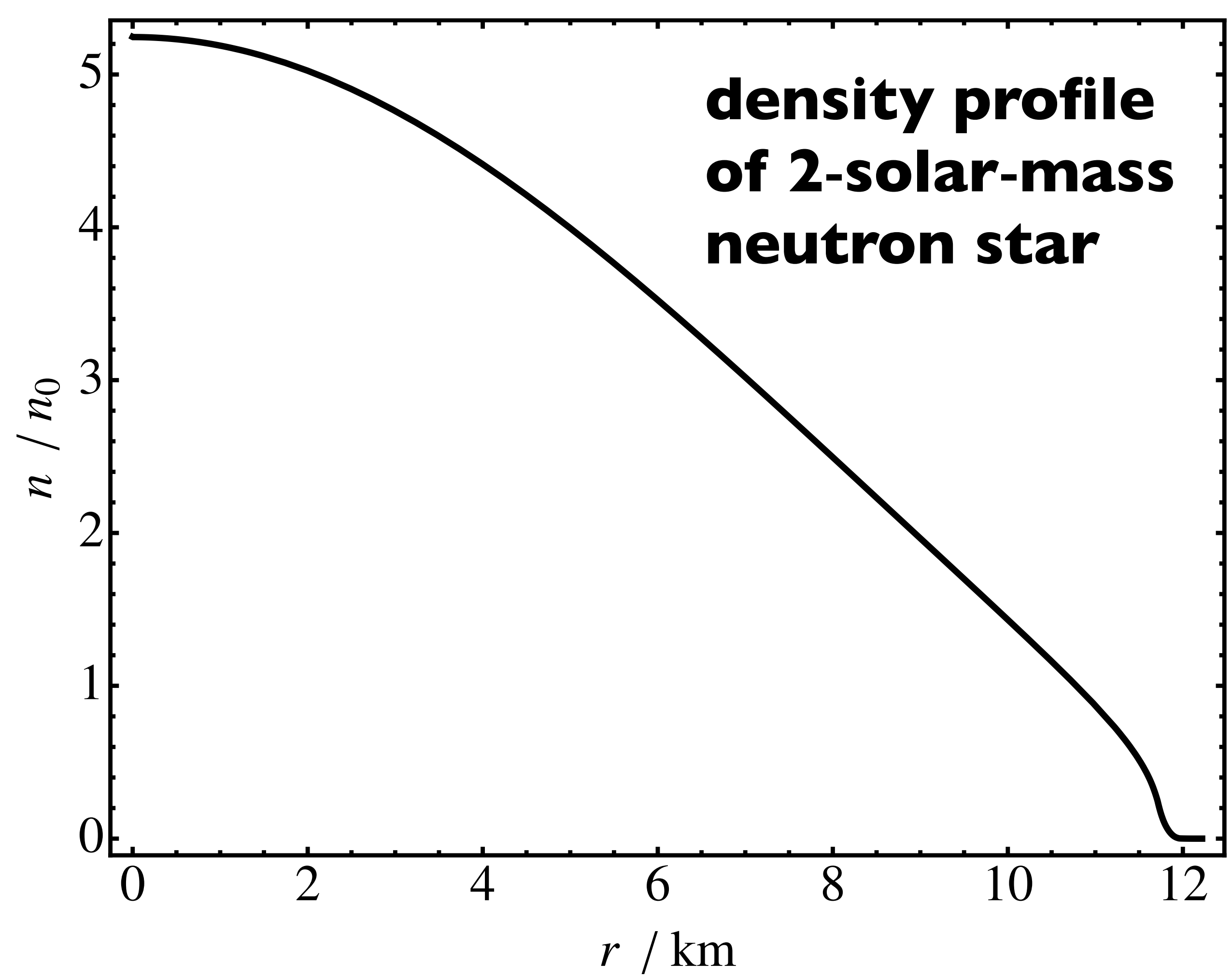
PSR J0348+0432

$$\mathrm{M} = 2.01 \pm 0.04\ \mathrm{M}_\odot$$

NEUTRON STAR MATTER
from
Chiral Nucleon-Meson Model
and **FRG**

M. Drews, W.W. (2014)





- Neutron matter plus proton admixture (beta equilibrium)
- Symmetry energies 30 - 37 MeV
- **No** ultrahigh densities in the core of the neutron star

CONCLUSIONS

- **FRG** provides non-perturbative approach to

  Nuclear Chiral Thermodynamics

  from symmetric to asymmetric nuclear matter and neutron matter

  - fluctuations beyond mean field include important two-pion exchange mechanisms and low-energy nucleonic particle-hole excitations
  - 1st order phase transition: Fermi liquid $\leftrightarrow$ interacting Fermi gas

- **No** indication of **first-order chiral phase transition**

  - fluctuations work against early restoration of chiral symmetry

- New constraints from **neutron stars** for the **equation-of-state** of **dense & cold baryonic matter**:

  - Mass - radius relation: **stiff equation of state** required !
    No ultrahigh densities $(\varrho_{\mathbf{max}} \sim 5\,\varrho_0)$
  - **Conventional** (nucleon-meson, "non-exotic") **EoS** fulfills constraints