Spontaneous symmetry breaking in fermion systems with functional RG: Purely fermionic approach

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Why purely fermionic RG?

Introduction → Walter Metzner's talk

- Partially bosonized RG:
  - Relatively simple description of collective fluctuations
  - Ward identities can be implemented easily
  - More biased for simple truncations → Dynamical (re-) bosonization
- Purely fermionic RG:
  - Unbiased even for simple truncations
  - Accurate results for superfluid gap in ground state of attractive Hubbard model
  - Fermionic two-particle vertex at strong coupling contains contributions that cannot be described as boson exchange

Outline

1. Fermionic fRG for symmetry breaking
   - Katanin scheme
   - Example: Reduced BCS model
2. Application: Attractive Hubbard model
3. More recent developments
   - Other symmetry broken states
   - fRG+MF in 1PI formalism
4. Summary

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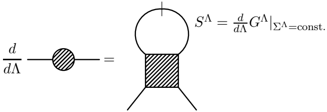
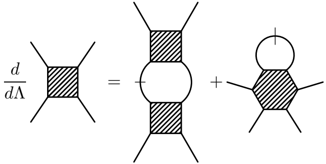
Reminder: Fermionic flow equations

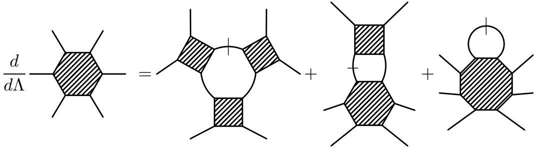
Σ^Λ:



Γ^Λ:



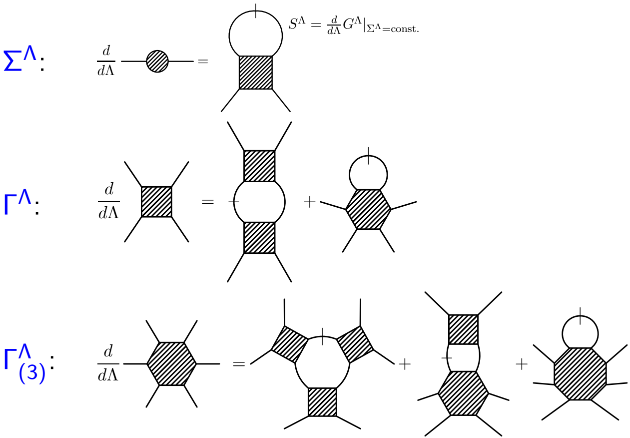
$\Gamma^\Lambda_{(3)}$:



and infinite hierarchy of higher-order equations

Simplest truncation for description of symmetry breaking?

Reminder: Fermionic flow equations



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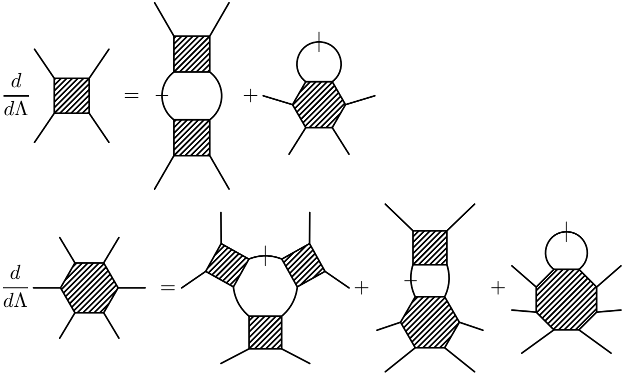
Simplest truncation for description of symmetry breaking?

Fermionic flow equations: Katanin scheme

- Scheme proposed by Katanin in PRB **70**, 115109 (2004)
- Aim: Derive truncation that captures ladder approximation *exactly*



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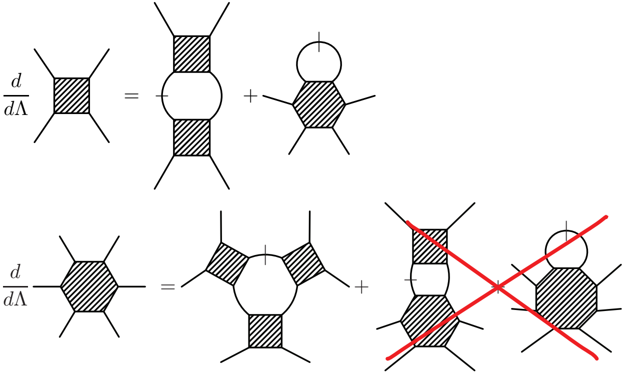

Neglect terms of $\mathcal{O}(\Gamma^4)$ or higher

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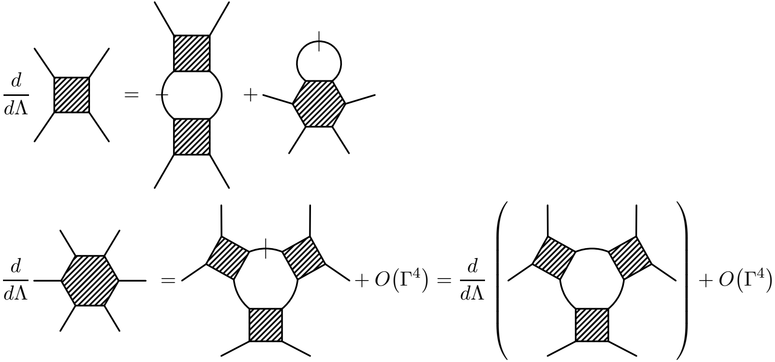

Right hand side can be rewritten as total derivative

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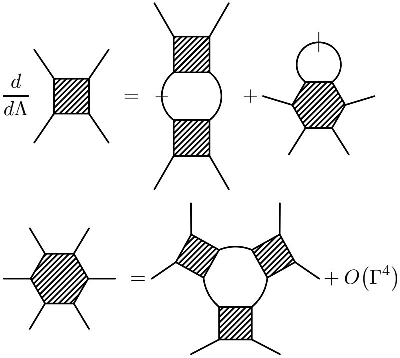

Simple approximation for $\Gamma_{(3)}$ $\rightarrow$ Insert into equation for Γ

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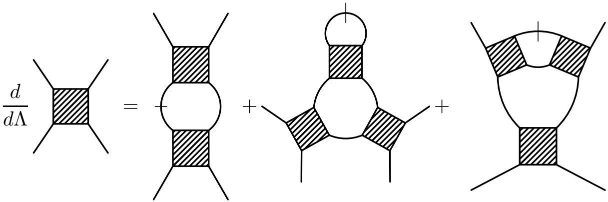
Two-loop flow equation for Γ:



Next step: Neglect terms with overlapping loops

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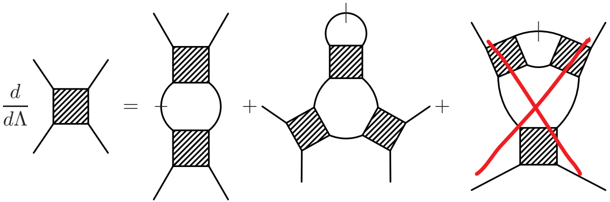

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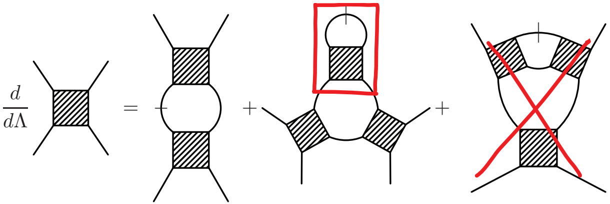

Second term: $\dot{\Sigma}$ insertions for one-loop contribution
Exploit $\dot{G}^\Lambda = S^\Lambda + G^\Lambda \dot{\Sigma}^\Lambda G^\Lambda$

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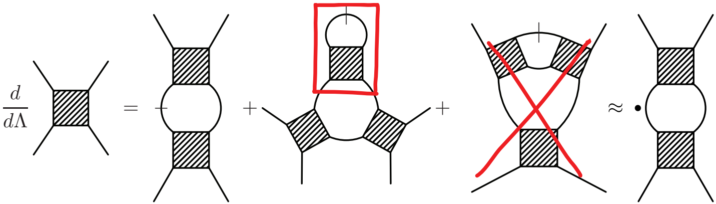

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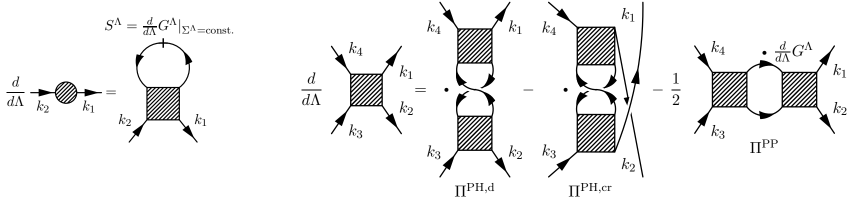
Fermionic flow equations: Katanin scheme

Result: Simple one-loop scheme (Katanin, PRB **70**, 115109 (2004))



- Standard truncation for instability analyses (See Metzner *et al.*, RMP **84**, 299 (2012))
- Solves mean-field models exactly (Salmhofer *et al.*, PTP **112**, 943 (2004))
- Yields reasonable results for SC gap in Hubbard model

  (Gersch *et al.*, NJP **10**, 045003 (2008); AE and Metzner, PRB **87**, 174523 (2013), PRB **89**, 035126 (2014))

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Reduced BCS model

- Microscopic action:

$$S[\bar\psi,\psi] = \sum_{k,\sigma} \bar\psi_{k\sigma}(-G_{0,k})^{-1}\psi_{k\sigma} - \Delta_0 \sum_k (\bar\psi_{k\uparrow}\bar\psi_{-k\downarrow} + \psi_{-k\downarrow}\psi_{k\uparrow}) + g \sum_{k,k'} \bar\psi_{k\uparrow}\bar\psi_{-k\downarrow}\psi_{-k'\downarrow}\psi_{k'\uparrow}$$

- Model exactly solvable in thermodynamic limit
  Solution: MF + RPA
- Exactly solvable with fRG using Katanin scheme

  (Salmhofer *et al.*, PTP **112**, 943 (2004))

How to obtain exact solution with fRG?

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How to obtain exact solution with fRG?

Reduced BCS model

- Ansatz for flowing action:

$$S^\Lambda[\bar\psi,\psi] = \sum_{k,\sigma} \bar\psi_{k\sigma}(-G_{0,k}^\Lambda)^{-1}\psi_{k\sigma} - \Delta_\Lambda \sum_k (\bar\psi_{k\uparrow}\bar\psi_{-k\downarrow} + \psi_{-k\downarrow}\psi_{k\uparrow})$$
$$+ v_\Lambda \sum_{k,k'} \bar\psi_{k\uparrow}\bar\psi_{-k\downarrow}\psi_{-k'\downarrow}\psi_{k'\uparrow}$$
$$- \frac{w_\Lambda}{2} \sum_{k,k'} (\bar\psi_{k\uparrow}\bar\psi_{-k\downarrow}\bar\psi_{-k'\downarrow}\bar\psi_{k'\uparrow} + \text{conj.})$$

- w_Λ important for symmetry breaking and fulfillment of Ward identity

- Flow of vertex: $\frac{d}{d\Lambda}\Gamma^\Lambda = \Pi^\Lambda_{\mathrm{PH,d}} + \Pi^\Lambda_{\mathrm{PH,cr}} + \Pi^\Lambda_{\mathrm{PP}}$

- Solution: $\Gamma^\Lambda = \Gamma^{\Lambda_0} + \Gamma^{\Lambda_0} L^\Lambda \Gamma^\Lambda$

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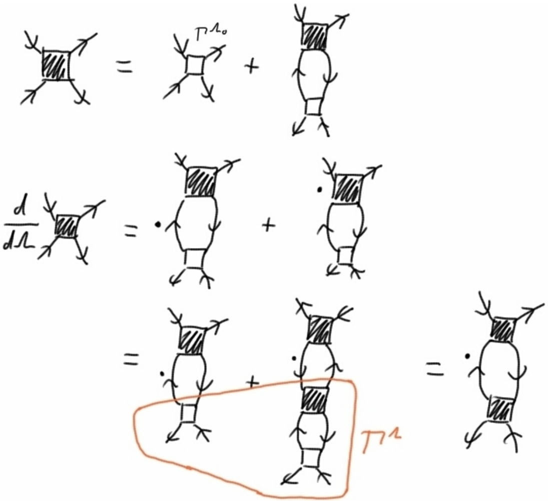
Reduced BCS model

Solution for vertex: “Proof”



Reduced BCS model

Explicit solution for vertex

- Flow equations for coupling constants:

$$\dot{v}_\Lambda = (v_\Lambda^2 + w_\Lambda^2)\dot{B}^\Lambda + 2 v_\Lambda w_\Lambda \dot{A}^\Lambda$$
$$\dot{w}_\Lambda = (v_\Lambda^2 + w_\Lambda^2)\dot{A}^\Lambda + 2 v_\Lambda w_\Lambda \dot{B}^\Lambda$$

where $B^\Lambda = -\sum_p G^\Lambda(p) G^\Lambda(-p)$ and $A^\Lambda = \sum_p (F^\Lambda(p))^2$

- Introduction of amplitude and phase mode of superconducting gap:

$$a_\Lambda = v_\Lambda + w_\Lambda \qquad \varphi^\Lambda = v_\Lambda - w_\Lambda$$
$$\dot{a}_\Lambda = a_\Lambda^2 \frac{d}{d\Lambda}(B^\Lambda + A^\Lambda) \qquad \dot{\varphi}_\Lambda = \varphi_\Lambda^2 \frac{d}{d\Lambda}(B^\Lambda - A^\Lambda)$$

Simple solution:

$$a_\Lambda = \frac{g}{1 - g(B^\Lambda + A^\Lambda)} \qquad \varphi_\Lambda = \frac{g}{1 - g(B^\Lambda - A^\Lambda)}$$

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Reduced BCS model

Solution for self-energy

- Flow of self-energy: $\dot{\Delta}_\Lambda = -a_\Lambda \sum_k S_F^\Lambda(k)$
- Solution: $\Delta_\Lambda = -g \sum_k F(k)$

  BCS gap equation! Can be seen by using solution for Γ^Λ and $\dot{G}^\Lambda = S^\Lambda + G^\Lambda \dot{\Sigma}^\Lambda G^\Lambda$

- Ward identity for global $U(1)$ charge symmetry:

$$\Delta_\Lambda = \Delta_0 + \Delta_0 \varphi_\Lambda (B^\Lambda - A^\Lambda) \quad \leftrightarrow \quad \varphi_\Lambda = g \frac{\Delta_\Lambda}{\Delta_0} \sim \Delta_0^{-1}$$

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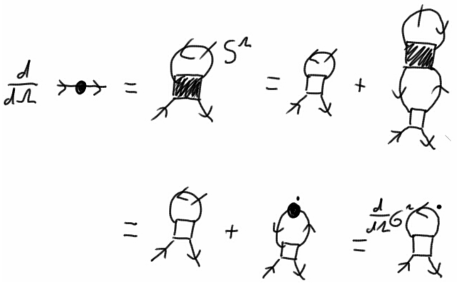


Reduced BCS model

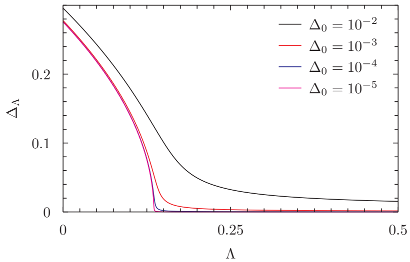
Numerical results

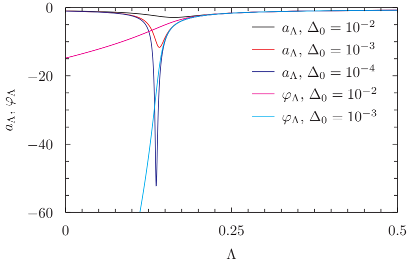
Constant density of states,

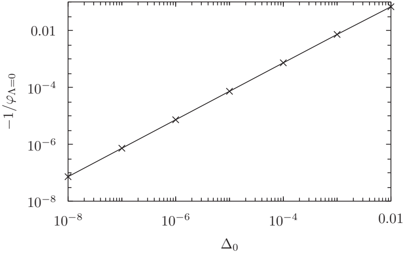
bandwidth $D = 2$,

momentum cutoff,

$g = -0.5$







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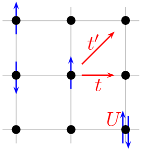
Attractive Hubbard model

- Simple model for attractively interacting lattice fermions
- Can be simulated with cold atoms in optical lattices
- Hamiltonian:

$$\mathcal{H} = \sum_{i,j,\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

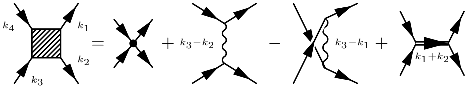


- $U < 0$, $d = 2$, $T = 0$
- Ground state: s-wave singlet superfluid;
  Studied with fRG by Gersch *et al.*, NJP, **10**, 045003 (2008); AE and Metzner, PRB **87**, 174523 (2013); Strack *et al.*, PRB **78**, 014522 (2008); Obert *et al.*, PRB **88**, 144508 (2013)

Results presented here from AE and Metzner, PRB **87**, 174523 (2013)

Approximation for effective interactions and self-energies

- Vertex: Decomposition in interaction channels



Description of effective interactions: Boson exchange

$$V_{kk'}(q) = \sum_{\alpha,\beta} V_{\alpha\beta}(q) h_\alpha(q,k) h_\beta(q,k') \overset{U<0}{\approx} V(q) g(k_0) g(k_0')$$

(where V represents A, Φ, C, M, ν, X, $\tilde{X}$)

Extension of ideas by Karrasch *et al.* JPCM **20**, 345205 (2008) and Husemann and Salmhofer PRB **79**, 195125 (2009)

for description of symmetry breaking in Cooper channel

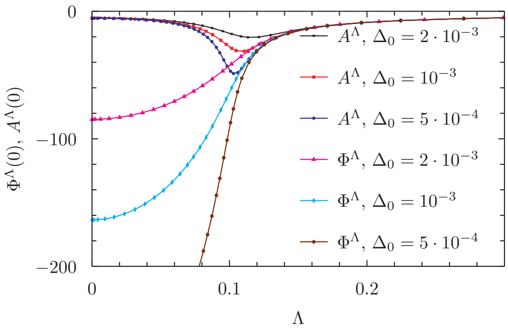
- Regulator and self-energy:

Regulator: $ik_0 + R(k) = i\,\mathrm{sgn}(k_0)\sqrt{\Lambda^2 + k_0^2}$

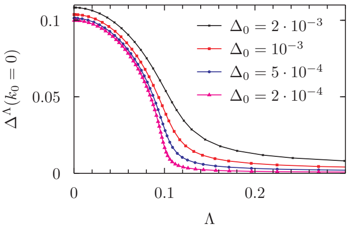
Self-energy: $\Sigma(k) \approx \Sigma(k_0), \quad \Delta(k) \approx \Delta(k_0)$

Amplitude and phase mode of superfluid gap

Scale dependence for $U = -2$, $t' = -0.1$, $n = 1/2$



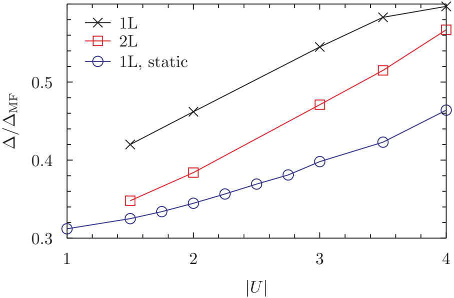
Flow of gap:



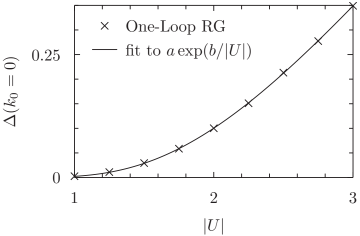
- Flow of couplings in Cooper channel similar to reduced BCS model

Reduction of superfluid gap by fluctuations

$n = 1/2,\ t' = -0.1$



(AE and Metzner, PRB **87**, 174523 (2013))

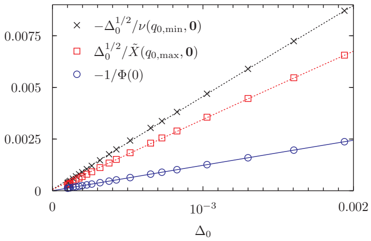


→ Well inside BCS regime

→ Good agreement with literature!

Δ/Δ_{MF}	Approximation, parameter
0.3	Perturbation theory, $n = 1/2$ or $n = 0.8$, $U \to 0$
0.5	One-loop fermionic RG, $n = 1/2$, $\|U\| = 1 - 3.5$ (Gersch *et. al.* (2008))
0.39	DMFT, $n = 1/2$, $\|U\| = 1.4$ (Bauer *et. al.* (2009))
0.76	One-loop mixed RG, $n = 1/2$, $\|U\| = 4$ (Obert *et al.* (2013))
0.5	QMC, $n = 0.2$, $\|U\| = 4$ (Singer *et. al.* (1996))

Singular behaviour of vertex for $\Delta_0 \to 0$ ($U = -3$, $n = 1/2$, $t' = -0.1$)



One-loop level:

$$\Phi(0, \mathbf{0}) \sim \Delta_0^{-1}$$

$$\nu(q_0 \sim \Delta_0^{1/2}, \mathbf{0}) \sim \Delta_0^{-1/2}$$

$$\tilde{X}(q_0 \sim \Delta_0^{1/2}, \mathbf{0}) \sim \Delta_0^{-1/2}$$

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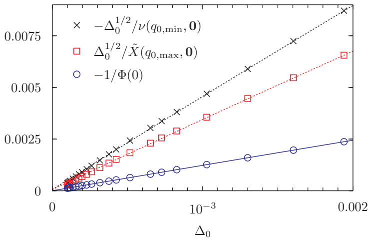

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(AE and Metzner, PRB **87**, 174523 (2013))

In addition at two-loop level:

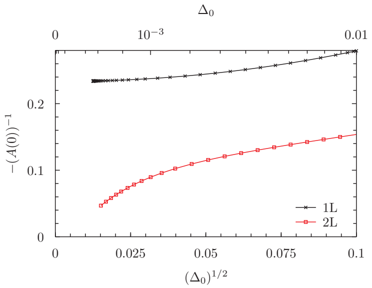
(AE, PRB **90**, 115125 (2014))

$$A(0) \sim \Delta_0^{-1/2}$$

$$\partial_\Lambda A^\Lambda(0) \sim \int \frac{d^3 p}{(2\pi)^3} \Phi^\Lambda(p) \partial_{\Lambda, V} \Phi^\Lambda(p)$$

Expected, see Strack *et al.*, PRB **78**, 014522 (2008)

No other singularities for $\Delta_0 \to 0$!



Outline

1. Fermionic fRG for symmetry breaking
   - Katanin scheme
   - Example: Reduced BCS model
2. Application: Attractive Hubbard model
3. More recent developments
   - Other symmetry broken states
   - fRG+MF in 1PI formalism
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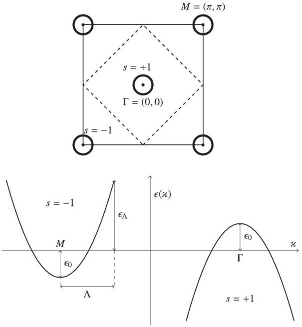
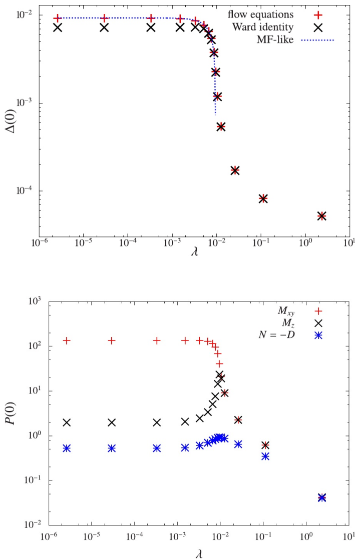
Commensurate antiferromagnetism

- Simple two-patch model with perfect nesting





- Translational and spin-rotational symmetries broken

(Maier and Honerkamp, PRB **86**, 134404 (2012),

Maier *et al.*, PRB **90**, 035140 (2014))

d-wave superconductivity

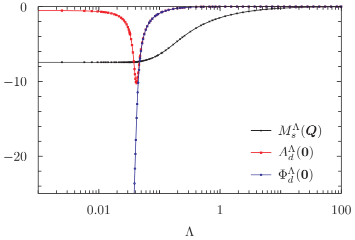
(AE and Metzner, PRB **89**, 035126 (2014))

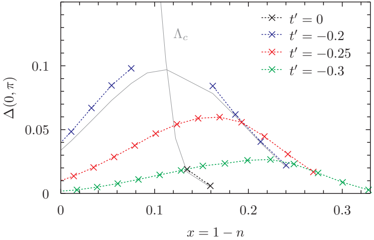
Hubbard model:

$$\mathcal{H} = \sum_{i,j,\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

- Simple model for electronic structure of CuO_2 planes
- $U > 0$, $d = 2$, $T = 0$
- Ground state: AF, dSC, ...

dSC: $\Delta(\boldsymbol{k}) \propto \cos k_x - \cos k_y$





Extends instability analyses by Zanchi and Schulz (2000), Halboth and Metzner (2000), Honerkamp *et al.* (2001), Husemann and Salmhofer (2009), Husemann *et al.* (2012), Giering and Salmhofer (2012), ...

See also mixed fermion-boson RG study by Friederich *et al.*, PRB **83**, 155125 (2011)

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fRG+MF in 1PI formalism

Simple description of competing order? Combine fRG and MF theory!

fRG+MF in Wick-ordered formalism: Reiss *et al.*, PRB **75**, 075110 (2007)

Idea in 1PI formalism: (Wang, AE, Metzner, PRB **89**, 121116 (2014))

- Solve full fRG flow including fluctuations for $\Lambda > \Lambda_{\text{MF}}$

$$\partial_\Lambda \Gamma^\Lambda = \Pi^\Lambda_{\text{PH,d}} - \Pi^\Lambda_{\text{PH,cr}} - \frac{1}{2}\Pi^\Lambda_{\text{PP}} \qquad \partial_\Lambda \Sigma^\Lambda = \Gamma^\Lambda S^\Lambda$$

- Continue fRG flow for $\Lambda < \Lambda_{\text{MF}}$ and in symmetry broken phase below Λ_c for reduced model

$$\partial_\Lambda \Gamma^\Lambda = \Pi^\Lambda_{\text{PH,d}} = \Gamma^\Lambda \dot{L}^\Lambda \Gamma^\Lambda \qquad \partial_\Lambda \Sigma^\Lambda = \Gamma^\Lambda S^\Lambda$$

- Solution:

$$\Gamma^\Lambda = \tilde{\Gamma}^{\Lambda_{\text{MF}}} + \tilde{\Gamma}^{\Lambda_{\text{MF}}} L^\Lambda \Gamma^\Lambda \qquad \Sigma^\Lambda = \Sigma^{\Lambda_{\text{MF}}} + \tilde{\Gamma}^{\Lambda_{\text{MF}}}(G^\Lambda - G^{\Lambda_{\text{MF}}})$$

$\tilde{\Gamma}^{\Lambda_{\text{MF}}}$: irreducible vertex

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fRG+MF in 1PI formalism

Recipe: (Wang, AE, Metzner, PRB **89**, 121116 (2014))

- Solve fRG flow for $\Lambda > \Lambda_{\mathrm{MF}}$ where $\Lambda_{\mathrm{MF}} \gtrsim \Lambda_c$

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$$\Sigma^{\Lambda=0} = \Sigma^{\Lambda_{\mathrm{MF}}} + \tilde{\Gamma}^{\Lambda_{\mathrm{MF}}} (G^{\Lambda=0} - G^{\Lambda_{\mathrm{MF}}})$$

Remarks:

- No anomalous vertices and propagators appear in fRG flow
- Full fRG flow below Λ_c mean-field-like at least for SC at $T = 0$

  (AE and Metzner, PRB **87**, 174523 (2013), PRB **89**, 035126 (2014); Obert *et al.*, PRB **88**, 144508 (2013))

- Scheme is exact for reduced models

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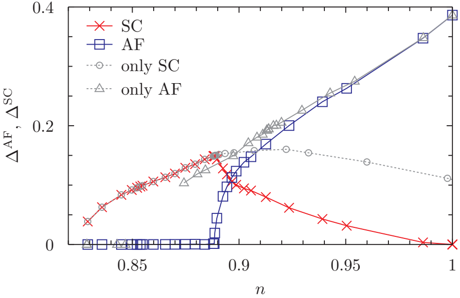
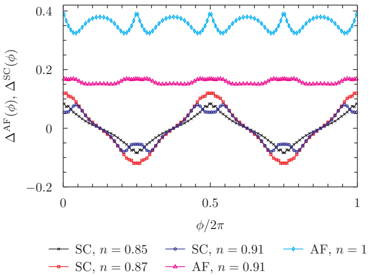
Competition and coexistence of AF and dSC

Hubbard model, $U = 3$, $t' = -0.15$

So far: Only dSC and commensurate AF (Wang, AE, and WM, PRB 89, 121116(R) (2014))





- AF and dSC compete and coexist in sizeable doping range, differently from Friederich *et al.*, PRB **83**, 155125 (2011)
- Quantum critical point between dSC and dSC+AF
- Note: Incommensurate AF important for these parameters
  (AE and Metzner, PRB **89**, 035126 (2014))

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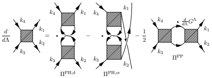
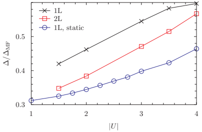
Summary

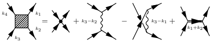
- Katanin scheme is simplest truncation for symmetry breaking within purely fermionic fRG



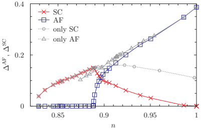
- Yields reasonable results for gaps in various models



- Decomposition of vertex in interaction channels allows to capture its singular dependences on momentum and frequency



- Competing order? Cumbersome with flow to symmetry broken phase
  $\rightarrow$ fRG+MF in 1PI formalism as simple alternative



The End

Thank you for your attention!

Summary

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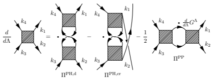

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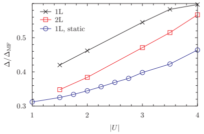

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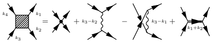

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