Functional renormalization group in Floquet space and its application to periodically driven quantum dots

Katharina Eissing, Dante M. Kennes, Stefan Göttel, Volker Meden

7th International Conference on the Exact Renormalization Group

24th of September 2014

RWTH AACHEN UNIVERSITY
DFG – FOR 723
JARA Jülich Aachen Research Alliance

Motivation

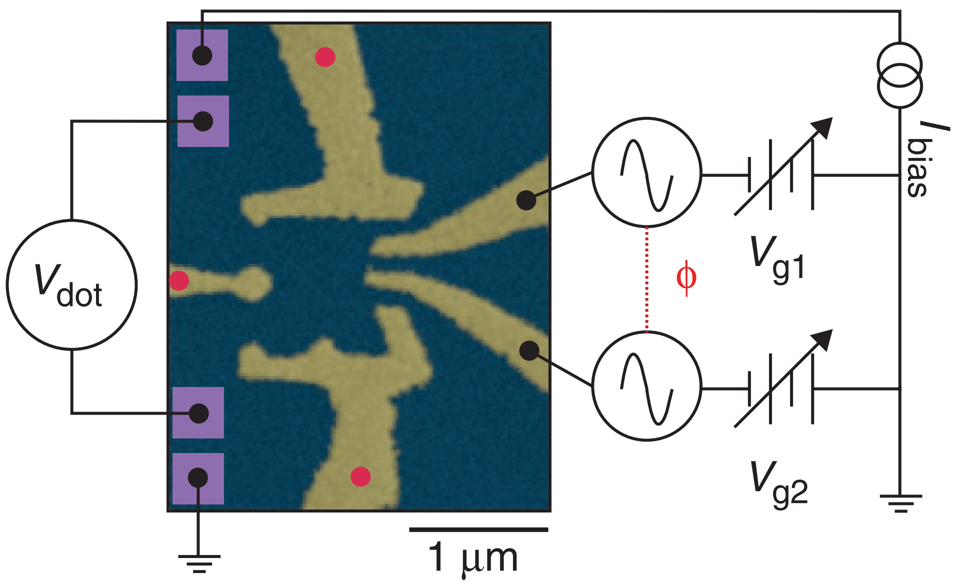
Interest : Long time behavior of periodically driven system

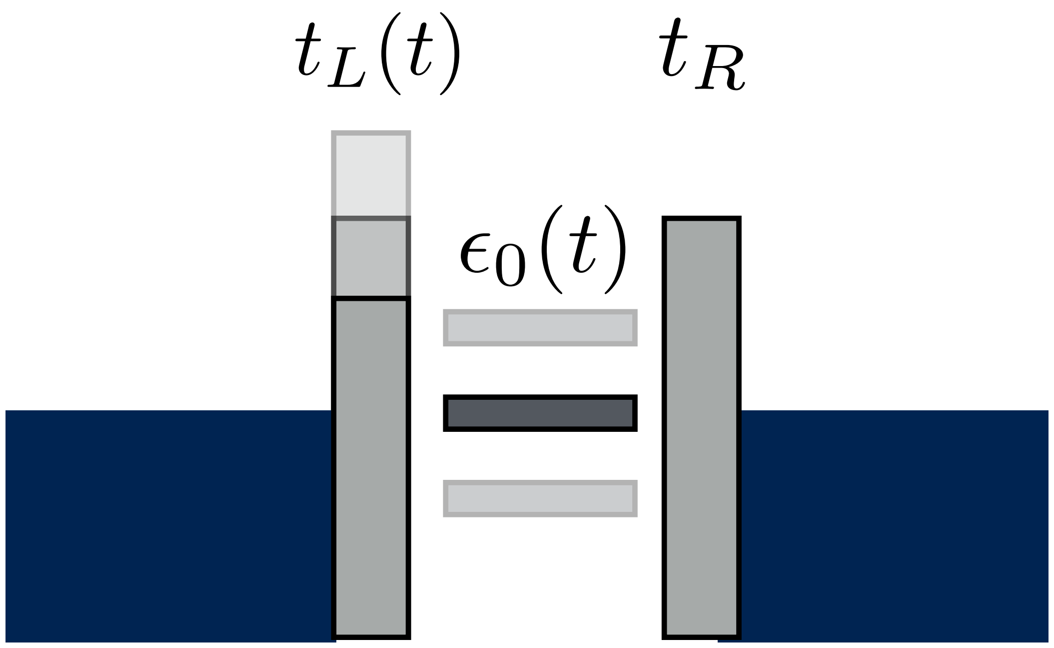
Electron pump: varying two independent parameter with phase shift

→ time-dependent onsite energy and hopping to left reservoir

$$\epsilon_0(t) = \bar{\epsilon}_0 + \Delta\epsilon\cos(\Omega t)$$

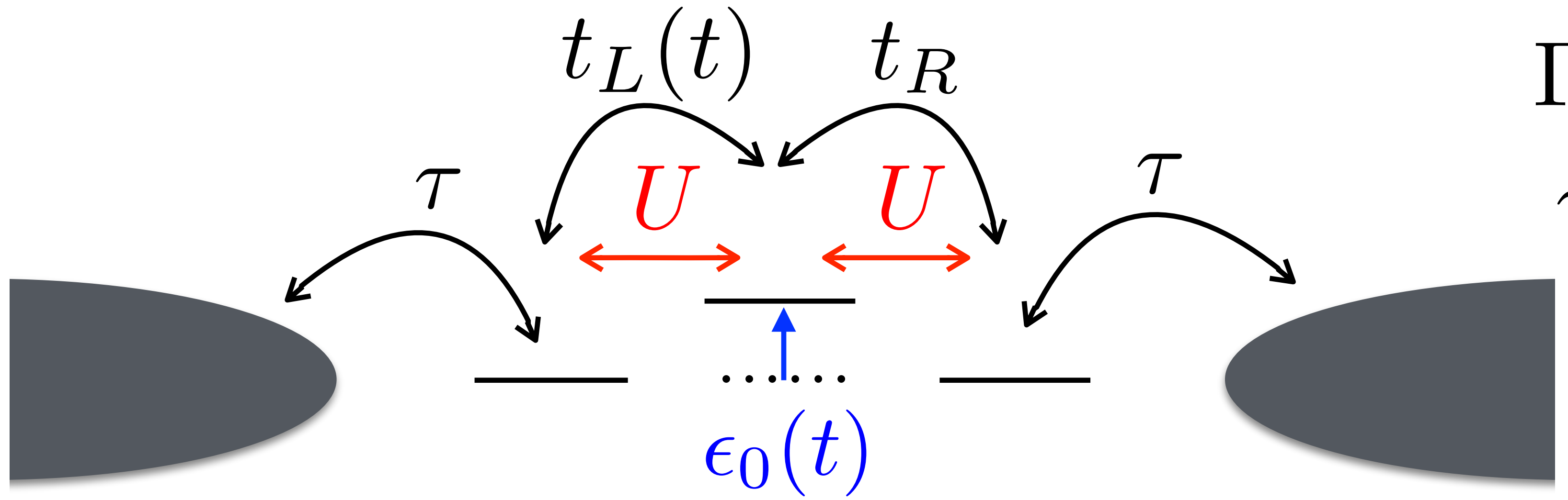
$$t_L(t) = \bar{t}_L + \Delta t\sin(\Omega t)$$





Interacting resonant level model

$\mu = T = 0$



$\Gamma = \pi \tau^2 \rho_{\text{res}}$

$\tau \gg t_{L/R}$

$$H = H_{\text{dot}} + \sum_{\alpha} H_{\text{coup},\alpha} + H_{\text{lead},\alpha}$$

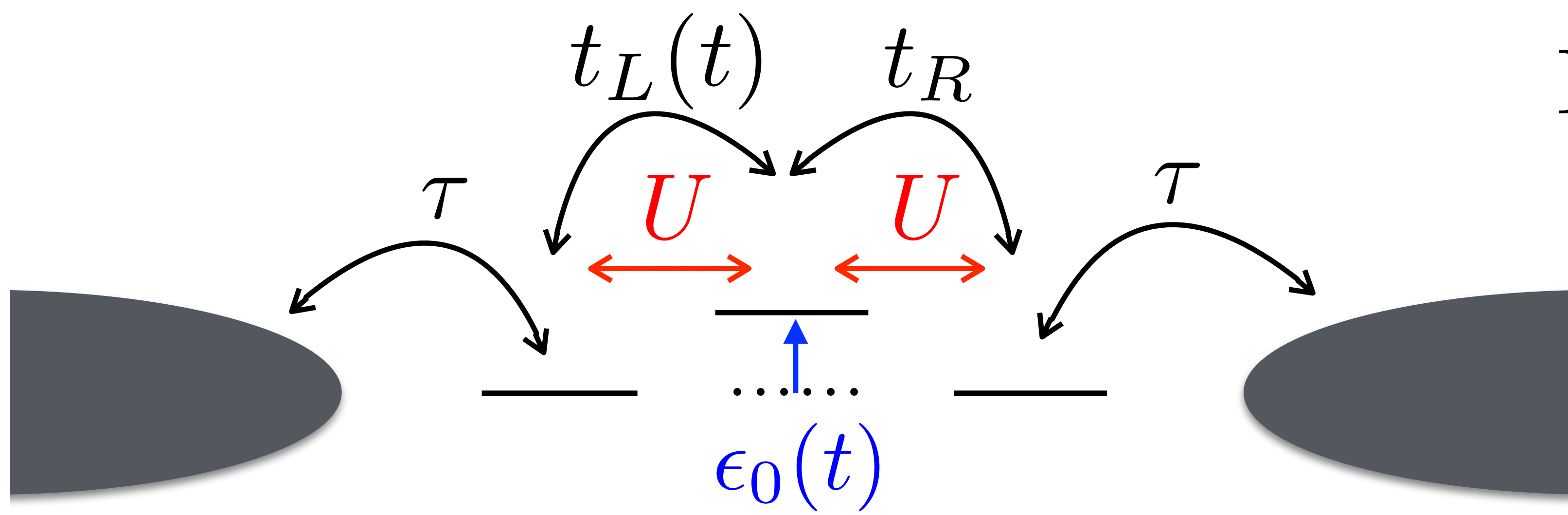
$$H_{\text{dot}} = \epsilon_0(t) n_2 + (t_L(t)\, d_1^\dagger d_2 + t_R\, d_2^\dagger d_3 + \text{H.c.}) + U\left[(n_1 - \tfrac{1}{2})(n_2 - \tfrac{1}{2}) + (n_2 - \tfrac{1}{2})(n_3 - \tfrac{1}{2})\right]$$

$$H_{\text{coup},\alpha} = \sum_{k_\alpha, i} \tau_{\alpha,i}\, d_i^\dagger c_{k_\alpha} + \text{H.c.}$$

$$H_{\text{lead}} = \sum_{k_\alpha} (\epsilon_{k_\alpha} - \mu_\alpha) c_{k_\alpha}^\dagger c_{k_\alpha}$$

Interacting resonant level model

$\mu = T = 0$



$\Gamma = \pi \tau^2 \rho_{\mathrm{res}}$

$\tau \gg t_{L/R}$

IRLM in perturbation theory: $\Sigma \sim U \ln(\frac{t_{L/R}}{\Gamma})$

$\longrightarrow$ RG-based resummation

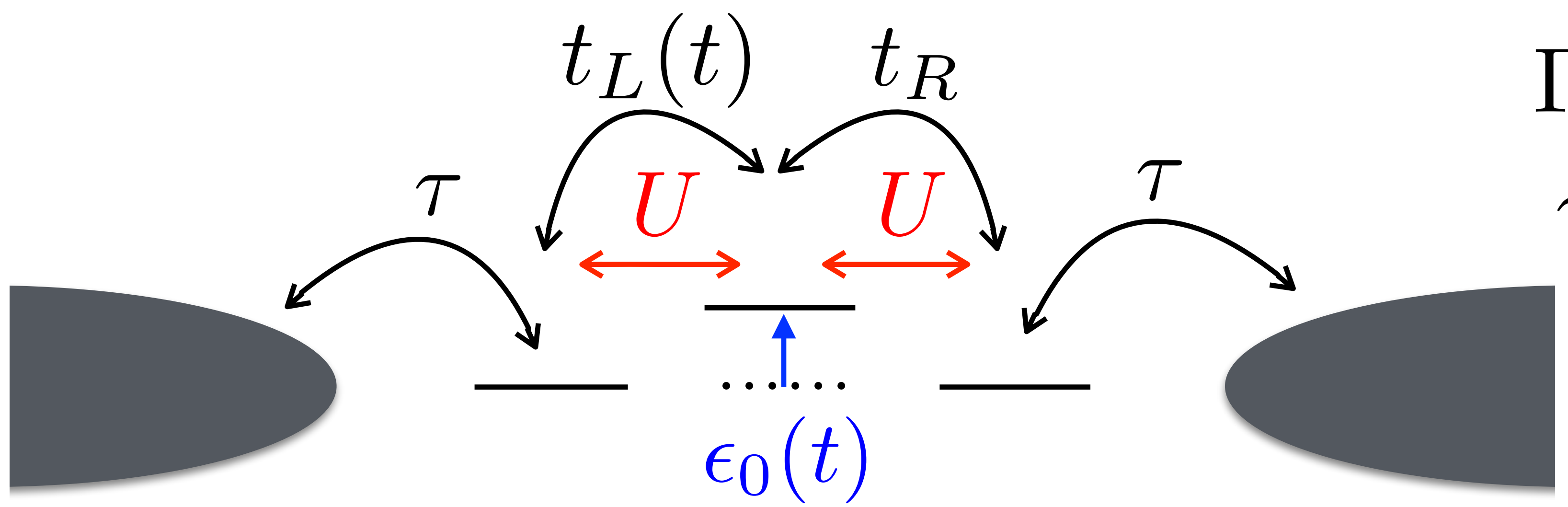
(Schlottmann '80 - 82,
Borda et al. '07,
Karrasch '10 …)

$$T_K = -\frac{2}{\chi\,\pi} = -\frac{2}{\frac{dn_2}{d\epsilon_0}\big|_{\epsilon=0}\,\pi}$$

$$\frac{T_K}{\Gamma} \sim \Big(\frac{t_L/t_R}{\Gamma}\Big)^{2-2\frac{2U/\pi\Gamma}{1+2U/\pi\Gamma}}$$

Interacting resonant level model

$$\mu = T = 0$$



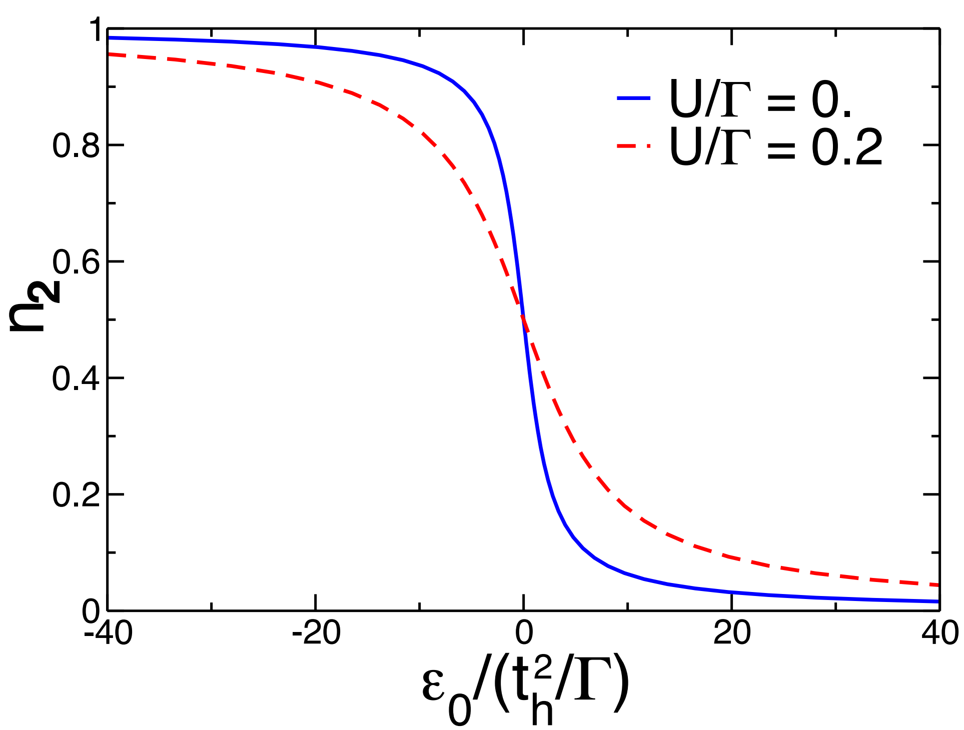
$$\Gamma = \pi\tau^2\rho_{\mathrm{res}}$$

$$\tau \gg t_{L/R}$$

IRLM in perturbation theory: $\Sigma \sim U \ln(\frac{t_{L/R}}{\Gamma})$

$\longrightarrow$ RG-based resummation

(Schlottmann ’80 - 82,
Borda et al. ’07,
Karrasch ’10 …)

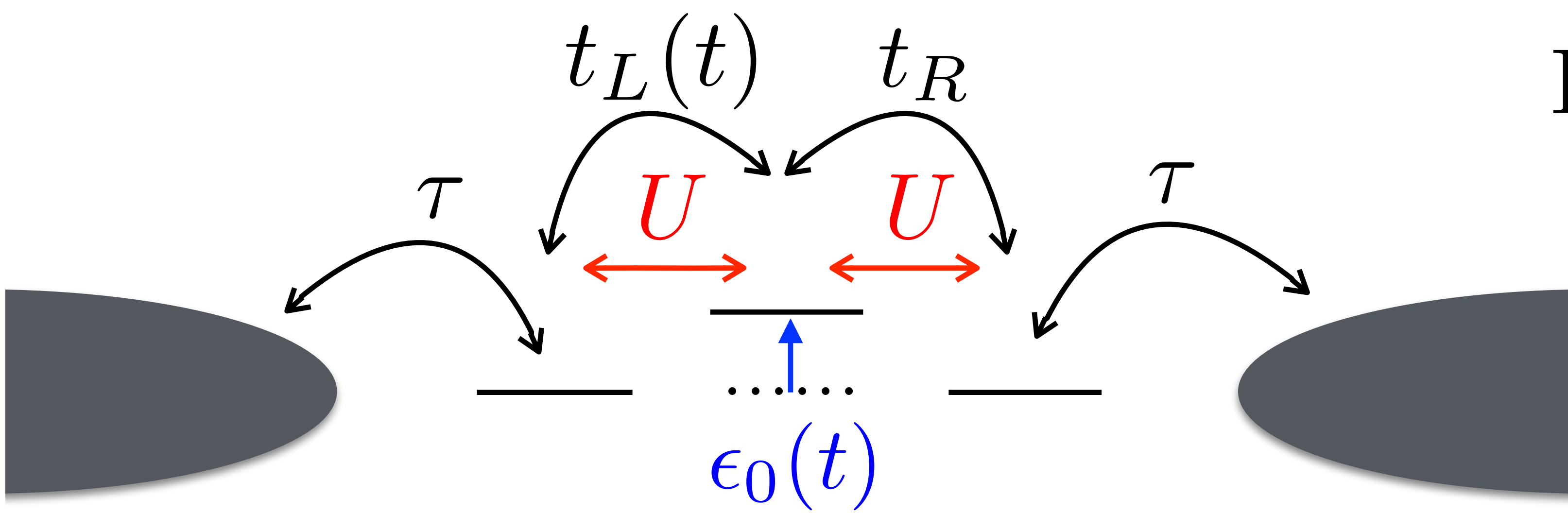


$$T_K = -\frac{2}{\chi\,\pi} = -\frac{2}{\frac{dn_2}{d\epsilon_0}\big|_{\epsilon=0}\,\pi}$$

$$\frac{T_K}{\Gamma} \sim \left(\frac{t_L/t_R}{\Gamma}\right)^{2-2\frac{2U/\pi\Gamma}{1+2U/\pi\Gamma}}$$

Interacting resonant level model

$\mu = T = 0$



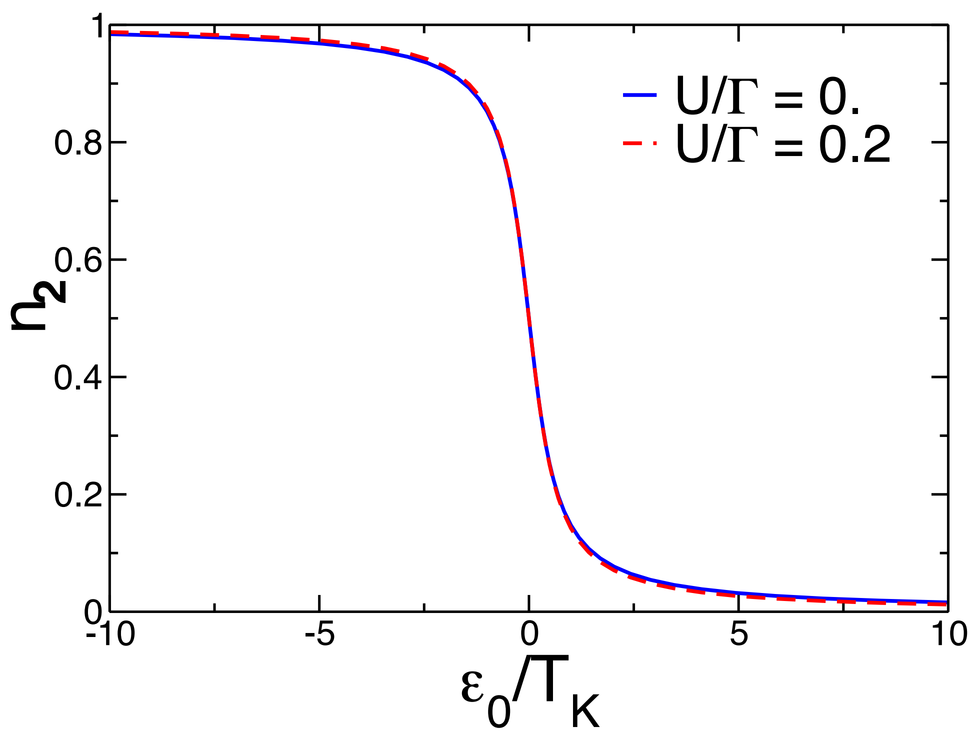
$\Gamma = \pi \tau^2 \rho_{\text{res}}$

$\tau \gg t_{L/R}$

IRLM in perturbation theory: $\Sigma \sim U \ln(\frac{t_{L/R}}{\Gamma})$

$\longrightarrow$ RG-based resummation

(Schlottmann '80 - 82, Borda et al. '07, Karrasch '10 ...)

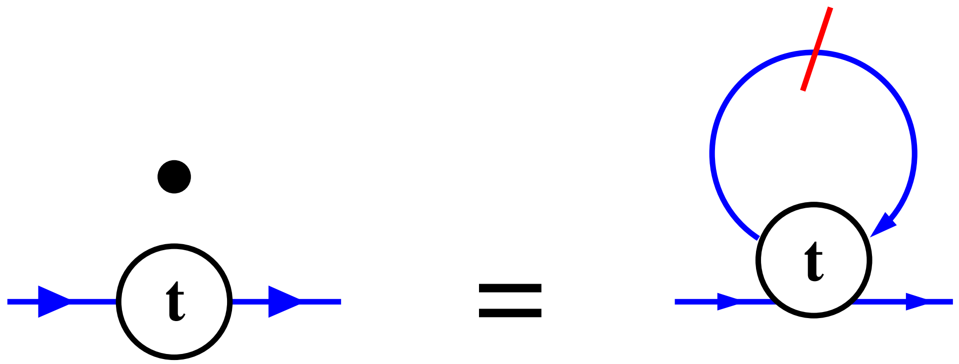


$$T_K = -\frac{2}{\chi \pi} = -\frac{2}{\frac{dn_2}{d\epsilon_0}\big|_{\epsilon=0} \pi}$$

$$\frac{T_K}{\Gamma} \sim \left(\frac{t_L/t_R}{\Gamma}\right)^{2-2\frac{2U/\pi\Gamma}{1+2U/\pi\Gamma}}$$

1PI scheme and vertex expansion:

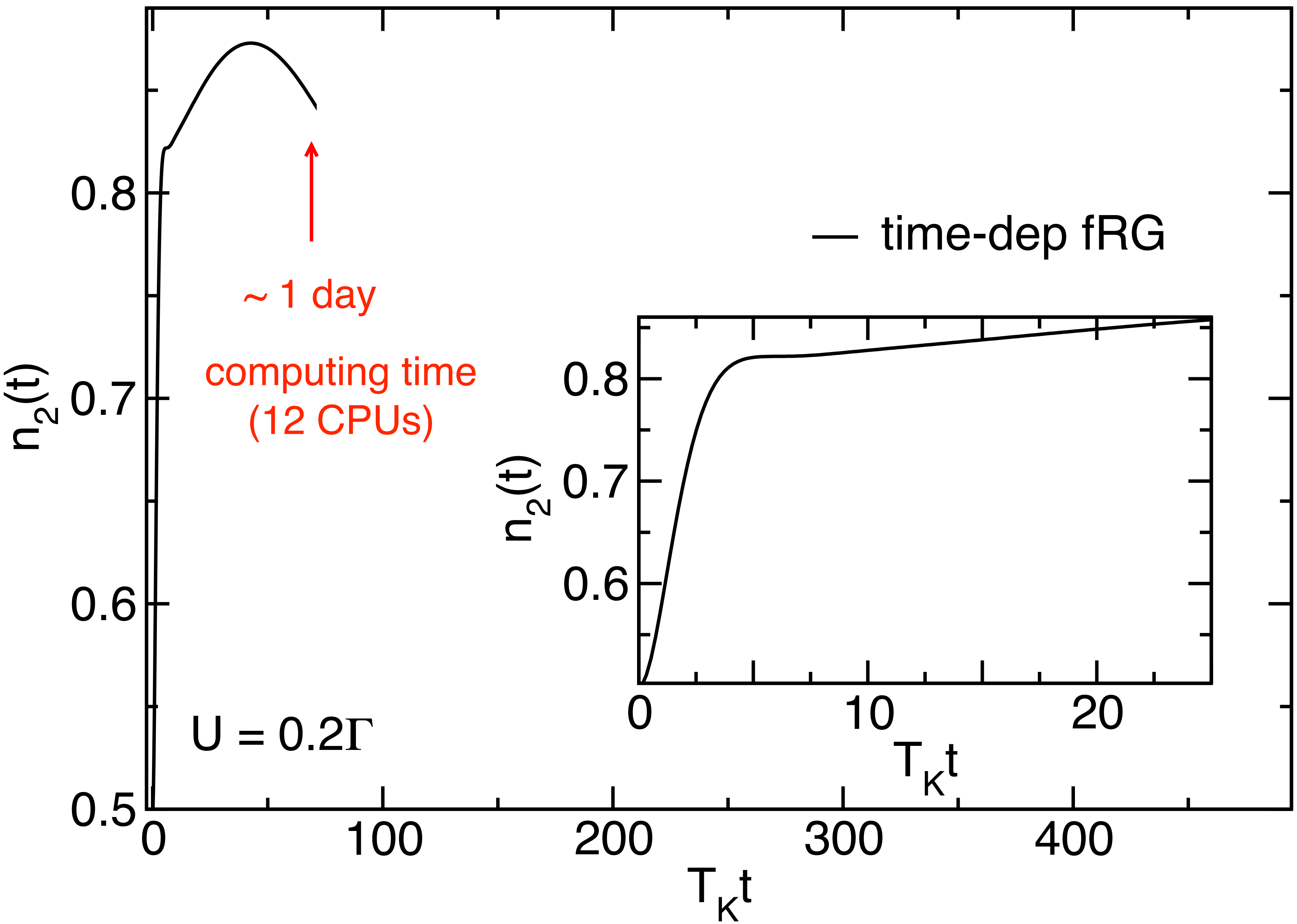
Flow equation:



- Keldysh formalism for nonequilibrium
- $G^0 \to G^{0,\Lambda}$
- auxiliary reservoirs to introduce cutoff

$$\Lambda_{\text{initial}} = \infty \qquad \Lambda_{\text{final}} = 0$$

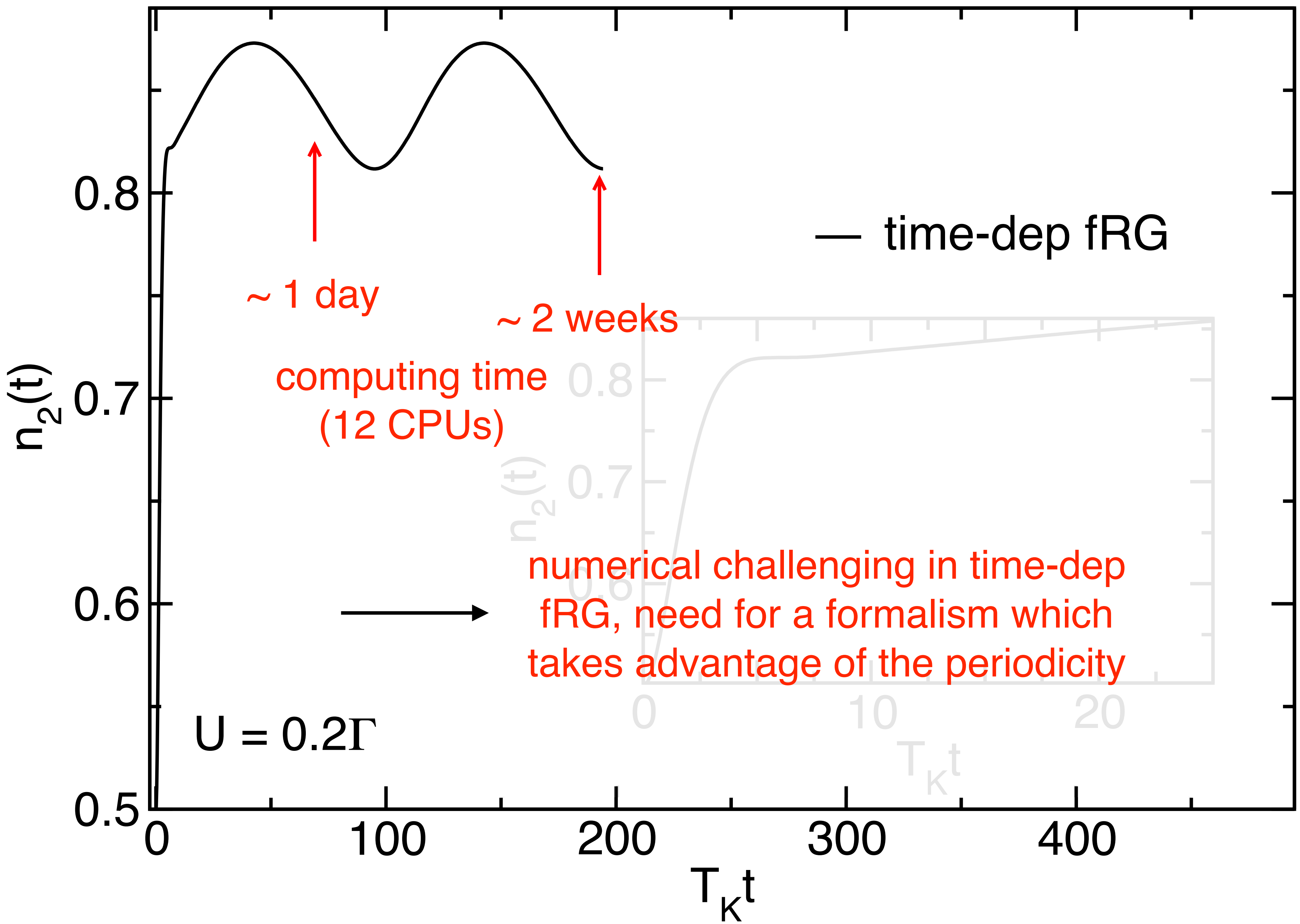
time-dep fRG



$\epsilon_0 = -T_K$

$\Delta t = 0.05 \quad \Delta\epsilon = \Delta t / t_h \, T_K$

time-dep fRG



$\epsilon_0 = -T_K$

$\Delta t = 0.05 \quad \Delta\epsilon = \Delta t / t_h \, T_K$

Floquet-Theorem

$$\left(i\frac{\partial}{\partial t} - \epsilon^{\text{eff}}(t)\right)G^R(t,t') = \delta(t,t')$$

Floquet theorem: $\dot{\phi}(t) = A(t)\phi(t)$ $\quad A(t+T) = A(t)$

Floquet state solution: $\phi(t) = e^{imt}u(t)$ $\quad u(t) = u(t+T)$

Floquet-Hamiltonian $\mathcal{H}(\mathbf{q},t) = H(\mathbf{q},t) - i\hbar\frac{\partial}{\partial t}$

Floquet space formalism

Transformation in the Floquet space : $\mathcal{R} \otimes \mathcal{T}$

$\mathcal{R}$: real space

$\mathcal{T}$: space of time-periodic functions

$$|i\rangle = c_i^\dagger \,|0\rangle$$

$$|k\rangle = e^{-ik\Omega t} \qquad k \in \mathbb{Z} \qquad \Omega = \frac{2\pi}{T}$$

$$\epsilon(t) = \sum_k \epsilon_k \, e^{ik\Omega t}$$

$$\langle k \,|\, \epsilon(t) \,|\, k' \rangle = \epsilon_{k'-k} = \epsilon_{k',k}$$

Example: periodically oscillating onsite energy of two-site model

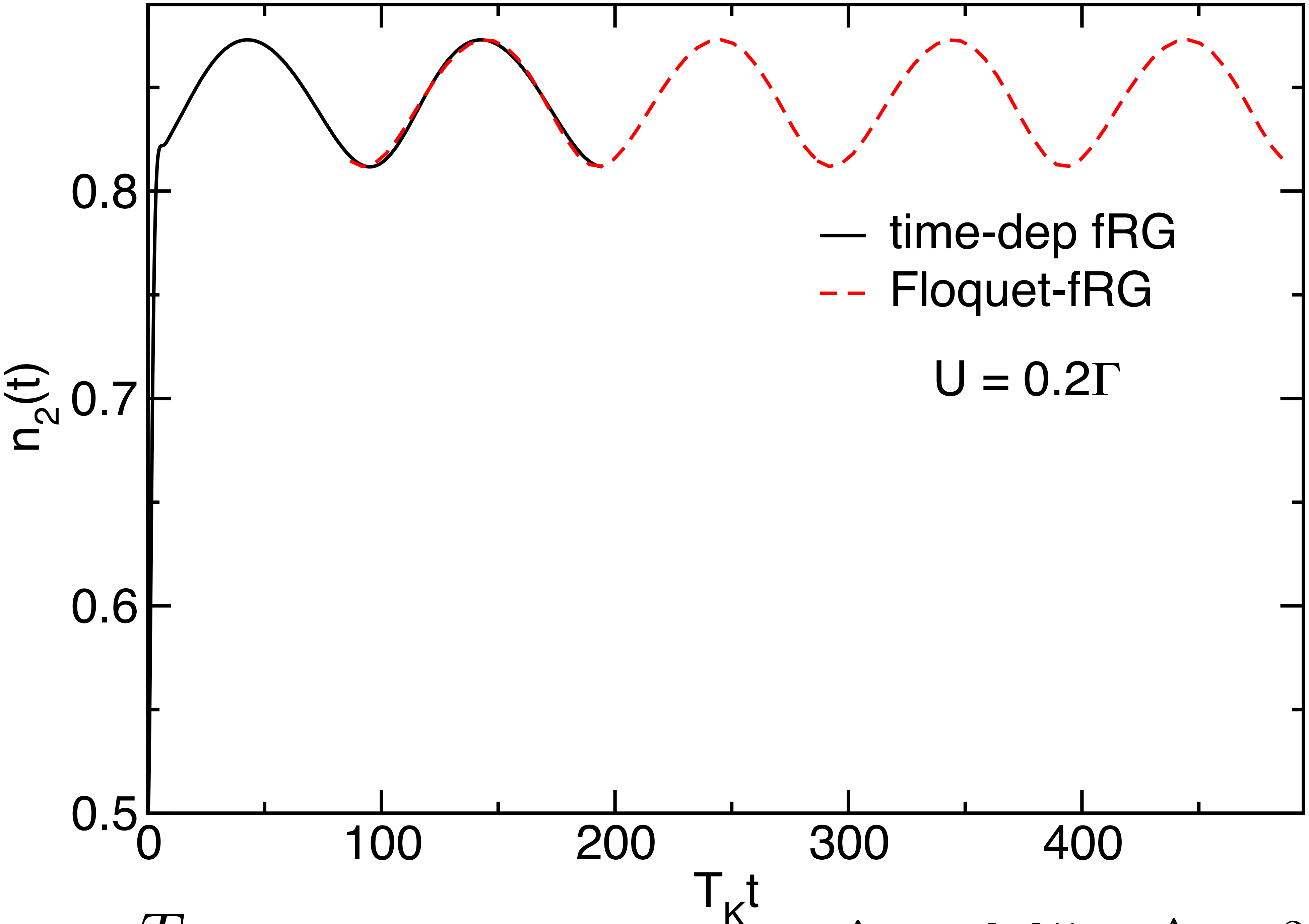
$$H(t) = \epsilon(t) d_1^\dagger d_1 + t_h (d_1^\dagger d_2 + \mathrm{H.c.})$$

$$\langle i,k|\mathcal{H}|j,k'\rangle = (\epsilon_0 - k\Omega)\delta_{i,j}\delta_{k,k'} + \epsilon_{k'-k}\delta_{i,j} + t_h \delta_{i,j\pm 1}\delta_{k,k'}$$

$$G^{\mathrm{ret}}(\omega) = \frac{1}{\omega - \mathcal{H} + i0^+}$$

Occupancy on an oscillating, interacting dot

Transient and long time behavior

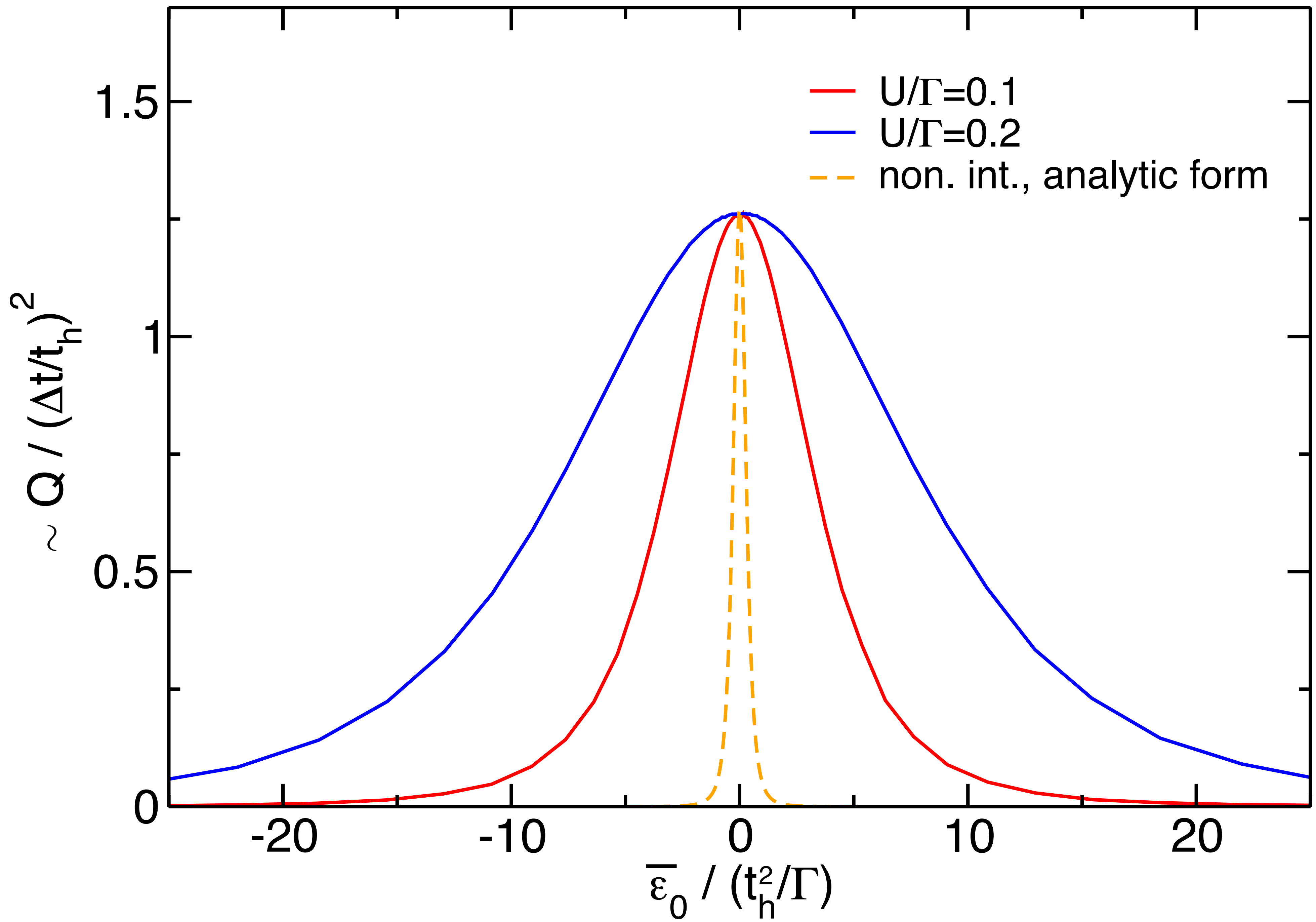


$\epsilon_0 = -T_K$

$\Delta t = 0.05$ $\Delta\epsilon = 2\Delta t/t_h\, T_K$

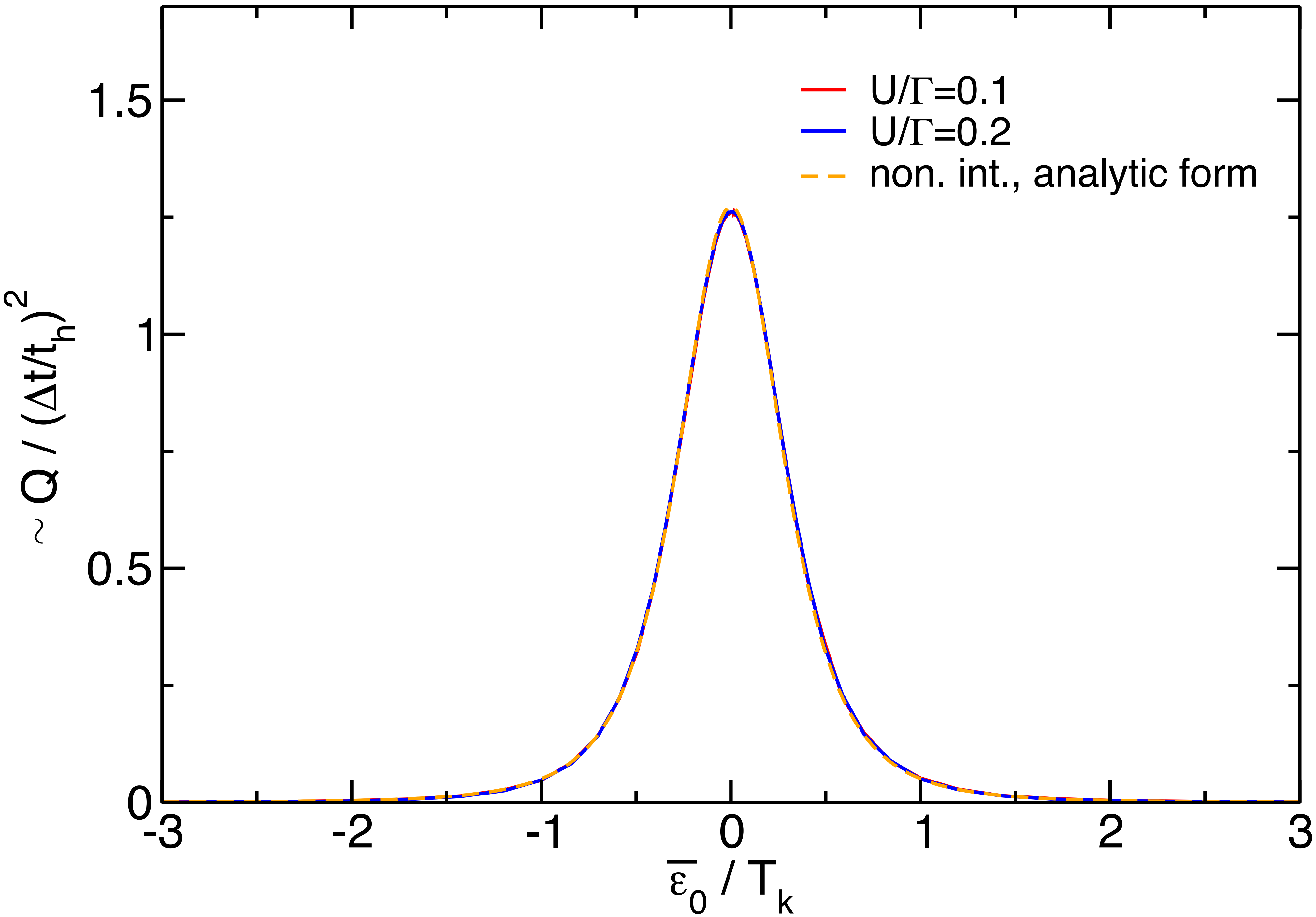
Pumped charge

$$Q = \int_0^T J_L(t)dt \qquad \Delta t = 0.005 \qquad \Omega = 2\pi T_K/100$$



Pumped charge

$$Q = \int_0^T J_L(t)dt \qquad \Delta t = 0.005 \qquad \Omega = 2\pi T_K/100$$



Conclusion and Outlook

- Floquet space natural choice to treat time-periodic systems
- fRG in Floquet space resembles stationary form
- wide range of possible system: non-adiabatic systems, different charge pumping situations, periodically varying bias voltage, time dependent interactions, arbitrary periodic variations