Spectral Functions from the Functional RG

7th International Conference
on the Exact Renormalization Group

ERG 2014

22/09/14 Lefkada, Greece

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2-Point Functions

…and their momentum dependencies

Euclidean momenta

- towards quantitative precision
- mass definitions
- mismatches in fluctuation scales

Minkowski momenta

- real time observables: require analytical continuation
- key observables: spectral functions
- here: real-time calculations embedded in Euclidean framework
- application: transport coefficients,…

Euclidean Iteration

- Helmboldt, Pawlowski, NSt arXiv:1409.8414

Why momentum dependence?

Quantitative precision

- QCD perspective on low-energy effective models:
  - ✓ **UV parameters fixed** by QCD flows
    - ➢ Talks by L. Fister, M. Mitter, J. Pawlowski, F. Rennecke
  - ✓ Increase in predictive power

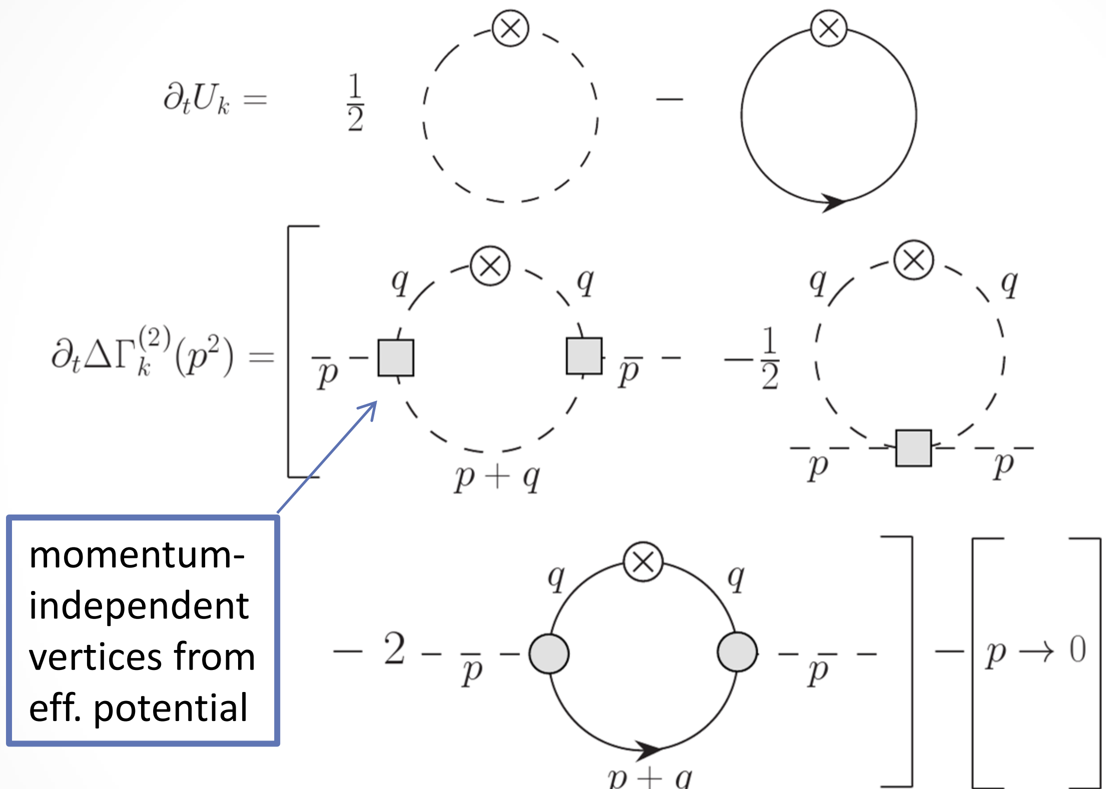
- Models have to be treated **quantitatively**
  - ✓ Full effective potential (grid or fixed Taylor expansion)
  - ✓ Higher order quark-meson scattering ➢ Pawlowski, Rennecke arXiv:1403.1179
  - ✓ **Momentum dependence**

- **Benchmark** of popular **truncation schemes** (LPA and LPA')

- Momentum dependence crucial for critical physics

Euclidean Iteration I

Momentum dependence of 2-point functions in an **iterative procedure**
Example: mesonic propagators in a quark meson model



Iteration procedure

$$\underbrace{\Gamma^{(2)}_{\mathrm{cl},k} \longrightarrow U_k}_{\#0} \longrightarrow \underbrace{\Gamma^{(2)}_{k} \longrightarrow U_k}_{\#1} \longrightarrow \underbrace{\Gamma^{(2)}_{k} \longrightarrow \ldots}_{\#2} \; \cdots$$

Euclidean Iteration II

$$\underbrace{\Gamma^{(2)}_{\mathrm{cl},k} \longrightarrow U_k}_{\#0} \longrightarrow \underbrace{\Gamma^{(2)}_{k} \longrightarrow U_k}_{\#1} \longrightarrow \underbrace{\Gamma^{(2)}_{k} \longrightarrow \dots}_{\#2}$$

- **Numerically inexpensive upgrade** for existing Euclidean calculations

- Here: **Quark-meson model at finite T; fixed ren. Yukawa coupling**
- **4d exponential** regulator function

- Convergence properties:

step	m_{cur} [MeV]	m_{pol} [MeV]	σ_{min} [MeV]
0	412.8	412.8	16.8
1	144.8	142 ± 2	83.5
2	136.4	135 ± 2	91.8
3	135.1	134 ± 2	93.1
4	134.9	133 ± 2	93.2
5	134.9	133 ± 2	93.2

Mass Definitions

Renormalized 2-point function: $\bar{\Gamma}^{(2)}(p_0, \vec{p}^2) = \Gamma^{(2)}(p_0, \vec{p}^2)/\bar{Z}$

Pole mass: $\bar{\Gamma}^{(2)}(\mathrm{i}m_{\mathrm{pol}}, 0) = 0$

Temporal screening: $T \sum_{p_0} \Gamma^{(2)}(p_0, 0)^{-1} e^{\mathrm{i}p_0 t} \sim e^{-m_{\mathrm{pol}}|t|}$

Screening mass: $\bar{\Gamma}^{(2)}(0, -m_{\mathrm{scr}}^2) = 0$

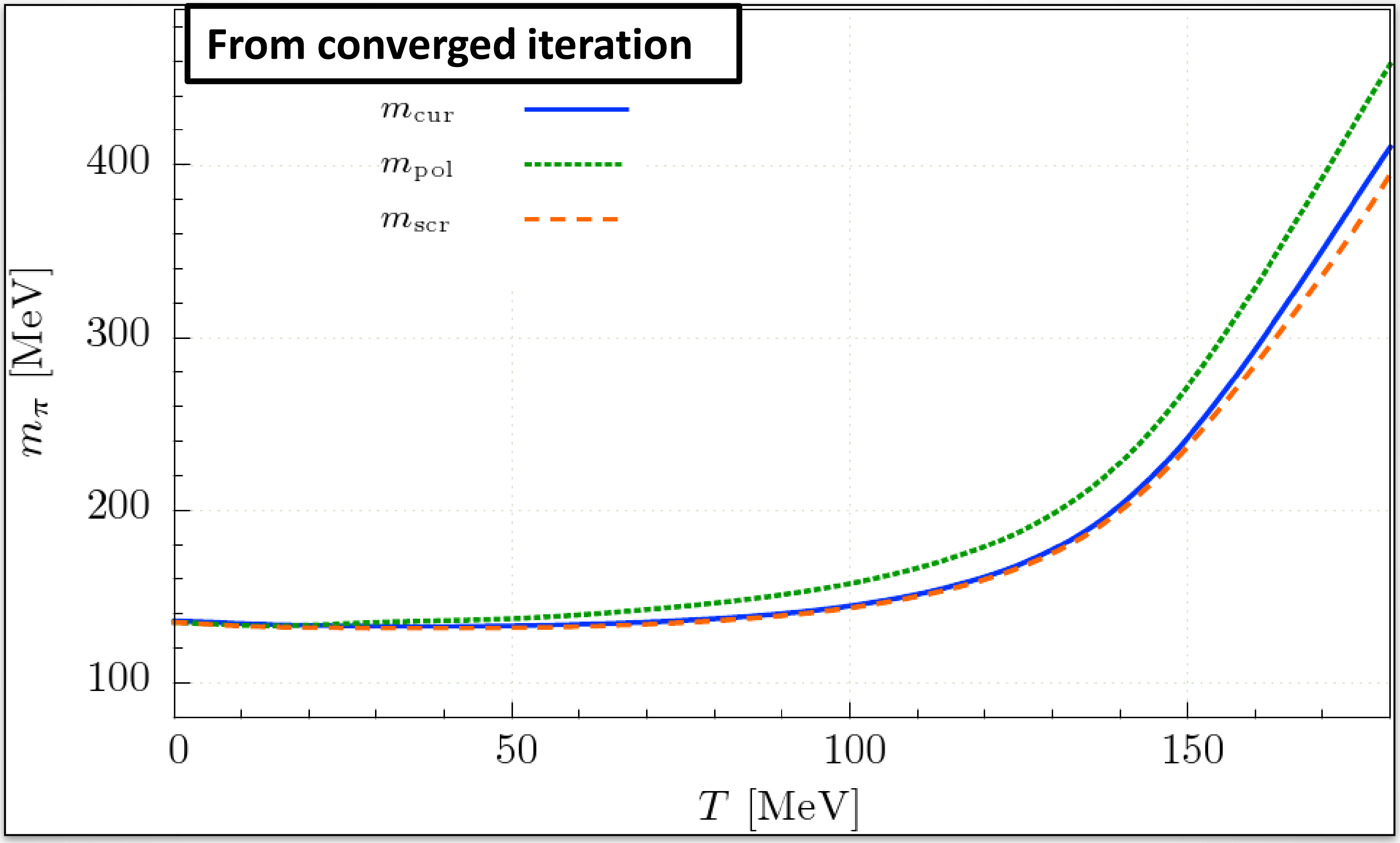
Spatial screening: $\int \mathrm{d}^3 p\, \Gamma^{(2)}(0, \vec{p}^2)^{-1} e^{\mathrm{i}\vec{p}\vec{x}} \sim e^{-m_{\mathrm{scr}}|x|}$

Curvature mass: $\bar{\Gamma}^{(2)}(0, 0) = m_{\mathrm{cur}}^2$

No physical observable; dependent on renormalization procedure, parameterization of the propagator

Onset mass: Silver Blaze property links mass to critical chemical potential; coincides with pole mass

Physics Results



T=0:

$m_{\rm pol} = m_{\rm scr}$ by O(4) invariance

$$m_{\rm pol} \approx m_{\rm cur} \, : \; m_{\rm cur}^2 = \frac{Z_\parallel(p_0{=}\mathrm{i}m_{\rm pol}, \vec{p}^2{=}0)}{\bar{Z}} m_{\rm pol}^2$$

T>0:

$$\frac{m_{\rm pol}^2}{m_{\rm scr}^2} = \frac{Z_\perp(p_0{=}0, \vec{p}^2{=}-m_{\rm scr}^2)}{Z_\parallel(\mathrm{i}m_{\rm pol}, \vec{p}^2{=}0)}$$

LPA: Mismatches of Fluctuation Scales

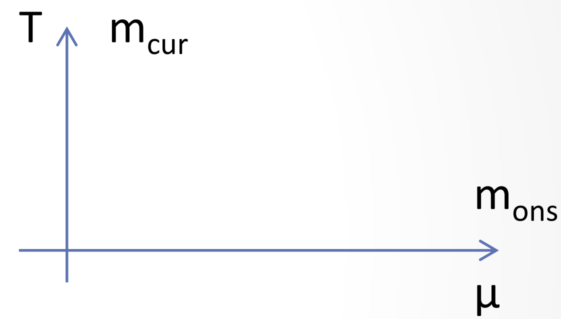
More than an academic exercise…

$$m_{\rm cur} \approx m_{\rm pole} = m_{\rm ons}$$

quantum fluctuations (→ $m_{\rm cur}$)

density fluctuations (→ $m_{\rm ons}$)

in the full calculation
but **large deviations in LPA**



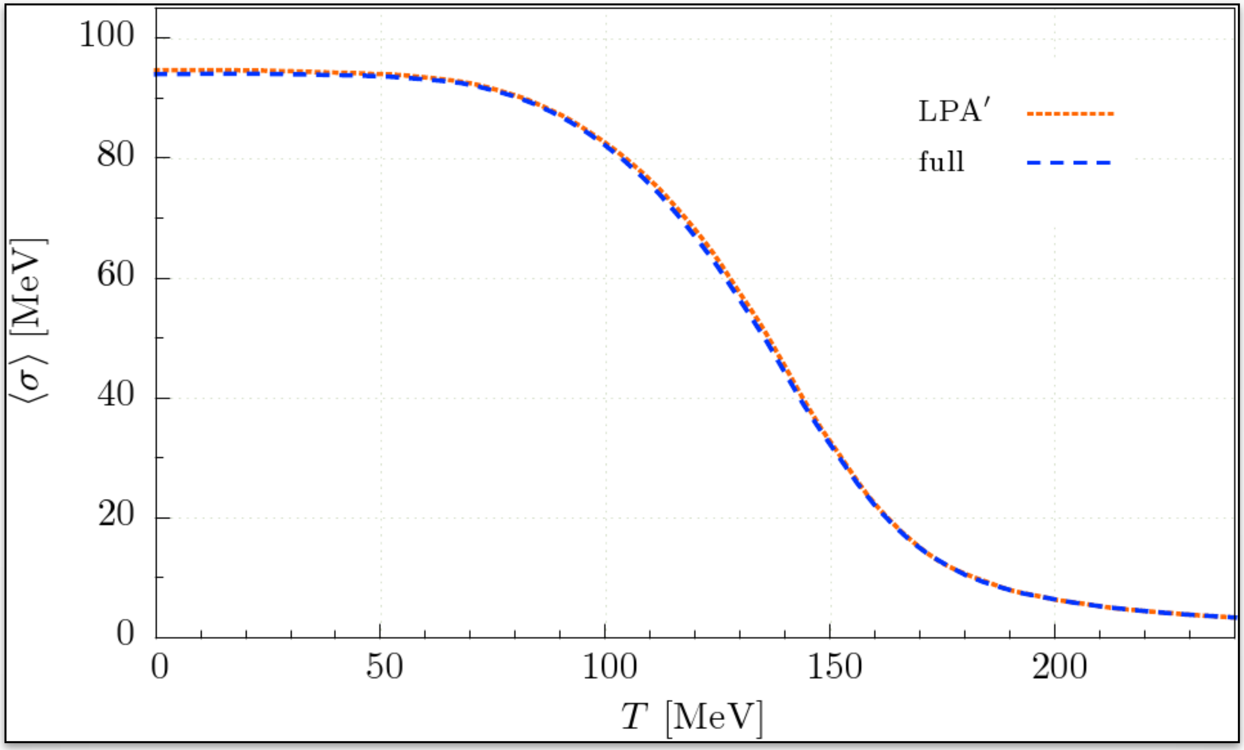
Rough estimate:

$$\left[\frac{\mu_c}{T_c}\right]_{\rm full} \Big/ \left[\frac{\mu_c}{T_c}\right]_{\rm LPA} \approx \left[\frac{m_{\rm cur}}{m_{\rm ons}}\right]_{\rm LPA} \approx 1.4$$

- **mismatch of fluctuation scales**
  => **large systematic errors at finite μ (curvature, CEP)**
- resolved by including momentum dependence

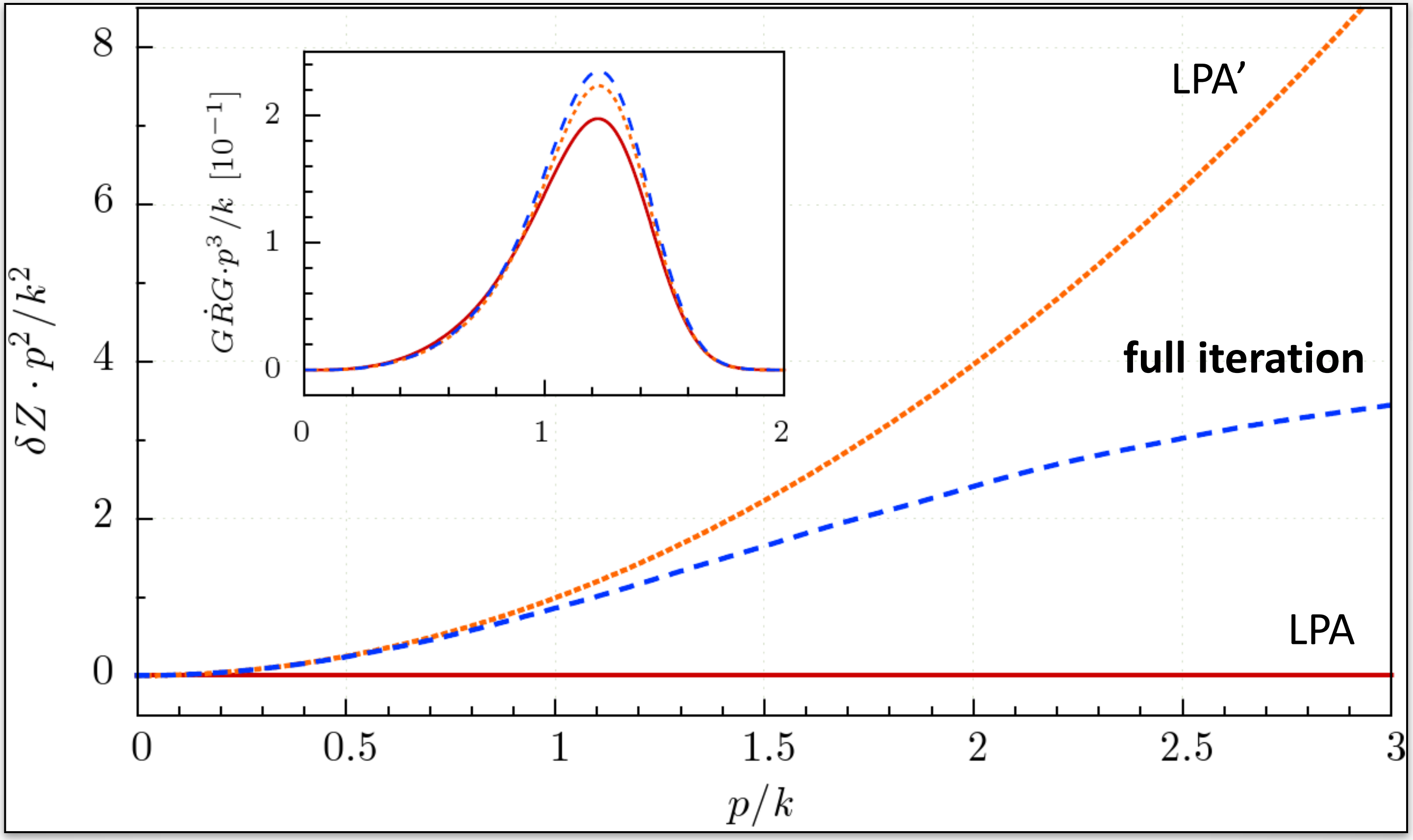
Comparison: Fixed UV

QCD perspective



- LPA with these initial conditions => no χSB
- Full calculation and LPA' in quantitative agreement

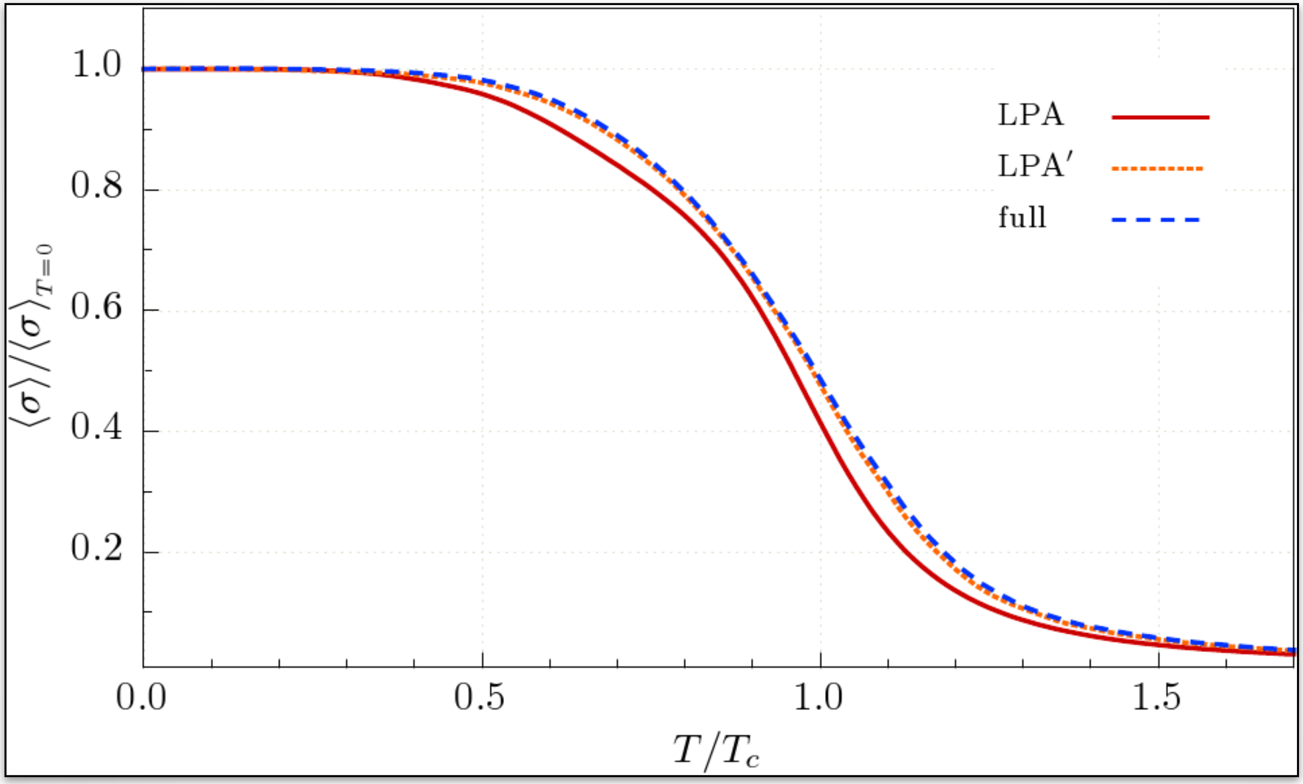
LPA’ Comparison



- **LPA’** includes only a **scale-dependent Z**
- Very good approximation to the full calculation **(deviation < 3 %)**
- Upgrade: calculate momentum dependence on LPA’ solution (1 step)

Comparison: Fixed IR

Model perspective



- Reasonably good agreement at μ=0 (in terms of relative scales)
- But in LPA still **large systematic error at finite μ**

Spectral Functions

- Kamikado, NSt, von Smekal, Wambach; Eur.Phys.J. **C74** (2014) 2806
- Tripolt, NSt, von Smekal, Wambach; Phys.Rev. **D89** (2014) 034010

Spectral Functions

Real-time observable from Euclidean framework

$$\Gamma^{(2)}_R(\omega,\vec{p}) = -\lim_{\epsilon\to 0}\Gamma^{(2)}_E(-\mathrm{i}(\omega+\mathrm{i}\epsilon),\vec{p})$$

$$\rho(\omega,\vec{p}) = \frac{\mathrm{Im}\,\Gamma^{(2)}_R(\omega,\vec{p})}{\mathrm{Im}\,\Gamma^{(2)}_R(\omega,\vec{p})^2+\mathrm{Re}\,\Gamma^{(2)}_R(\omega,\vec{p})^2}$$

requires analytical continuation from Euclidean to Minkowski signature
numerically hard or even ill-posed problem

Popular approaches (based on Euclidean data)

- Maximum Entropy Method (MEM)
- Padé Approximants

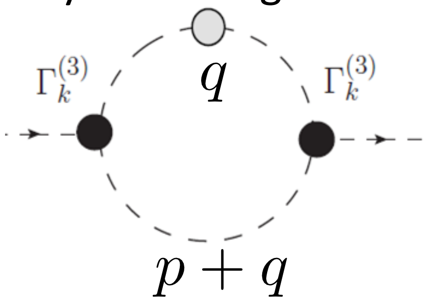
Alternative: analytic continuation on the level of the flow equation

- Kamikado, NSt, von Smekal, Wambach; Eur.Phys.J. **C74** (2014) 2806
- Floerchinger; JHEP 1205 (2012) 021

Analytical continuation

➢ Kamikado, NSt, von Smekal, Wambach; Eur.Phys.J. **C74** (2014) 2806

- Compute flow equation for Euclidean 2-point function
  perform analytically for 3d regulator function $R = \vec{p}^2 r(\vec{p}^2)$



- Perform analytical continuation in ext. momentum

$$p_0 \to -\mathrm{i}(\omega + \mathrm{i}\epsilon)$$

- Ensure correct continuation

$$n_{B/F}(E + \mathrm{i}p_0) \to n_{B/F}(E)$$

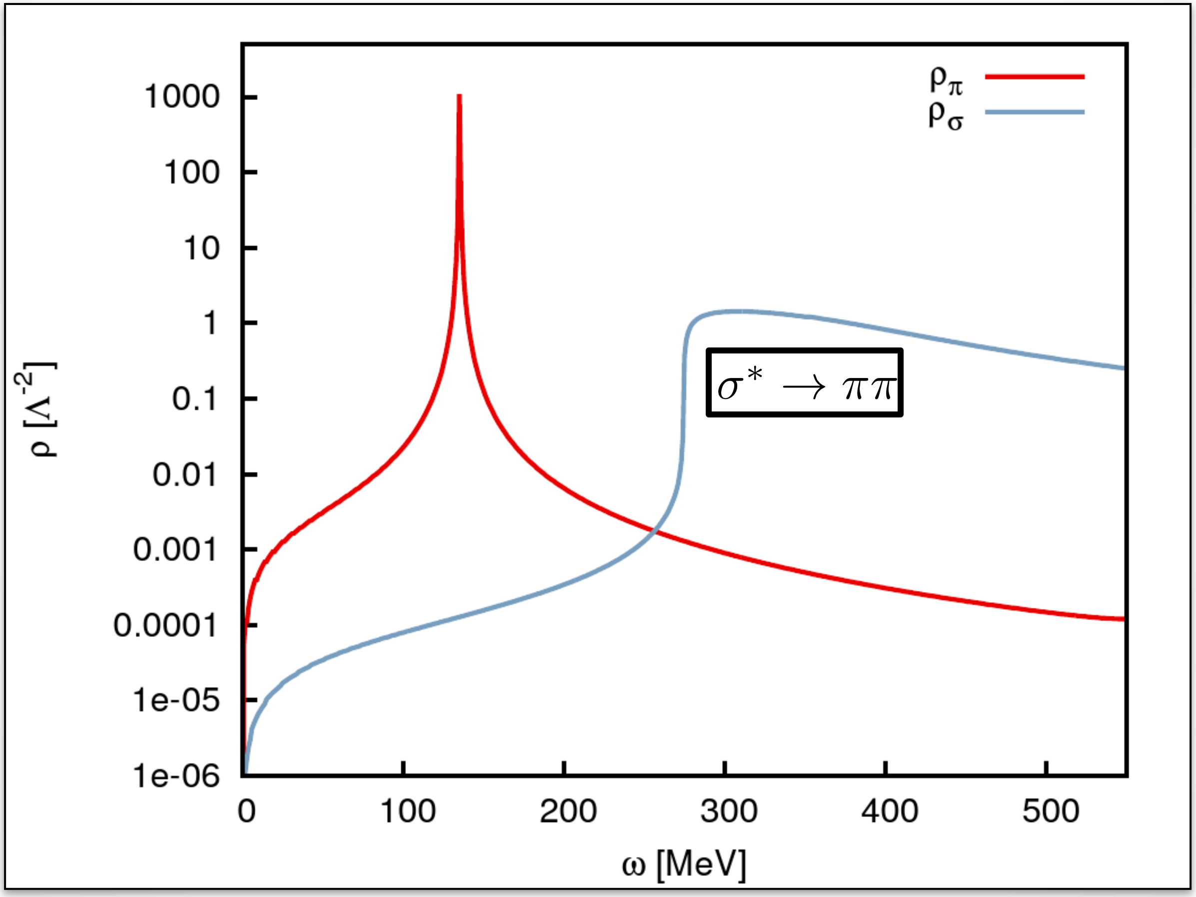
- For small but finite ε compute real and imaginary part of

$$-\Gamma_E^{(2)}(-\mathrm{i}(\omega + \mathrm{i}\epsilon), \vec{p})$$

Test cases: simple bosonic/ Yukawa models

O(N) Model at T=0

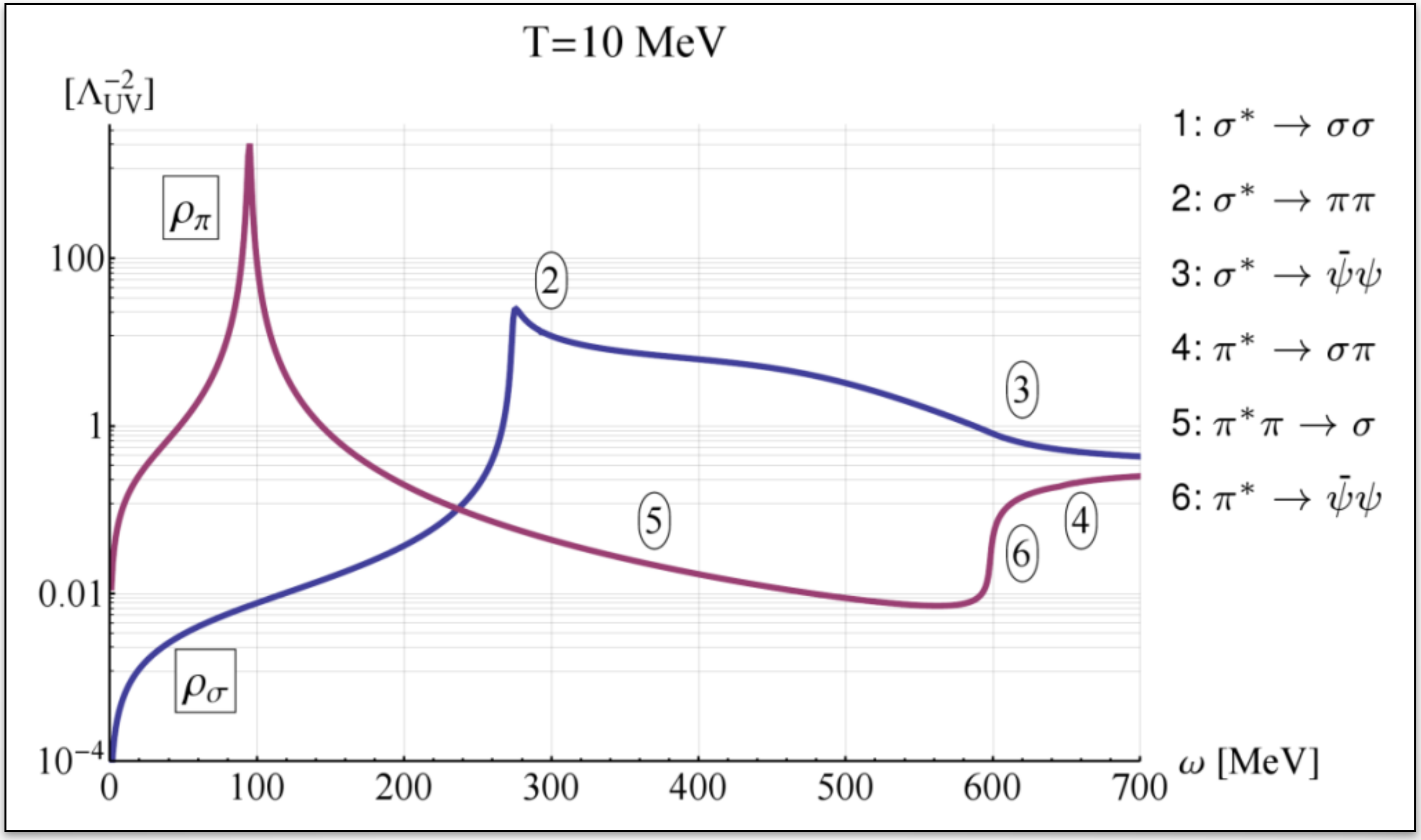
- Kamikado, NSt, von Smekal, Wambach; Eur.Phys.J. **C74** (2014) 2806



QM Model at T,μ > 0

- Tripolt, NSt, von Smekal, Wambach; Phys.Rev. **D89** (2014) 034010

Including quarks at finite temperature and density:
additional decay channels involving quarks and particles from heat bath



QM Model at T>0

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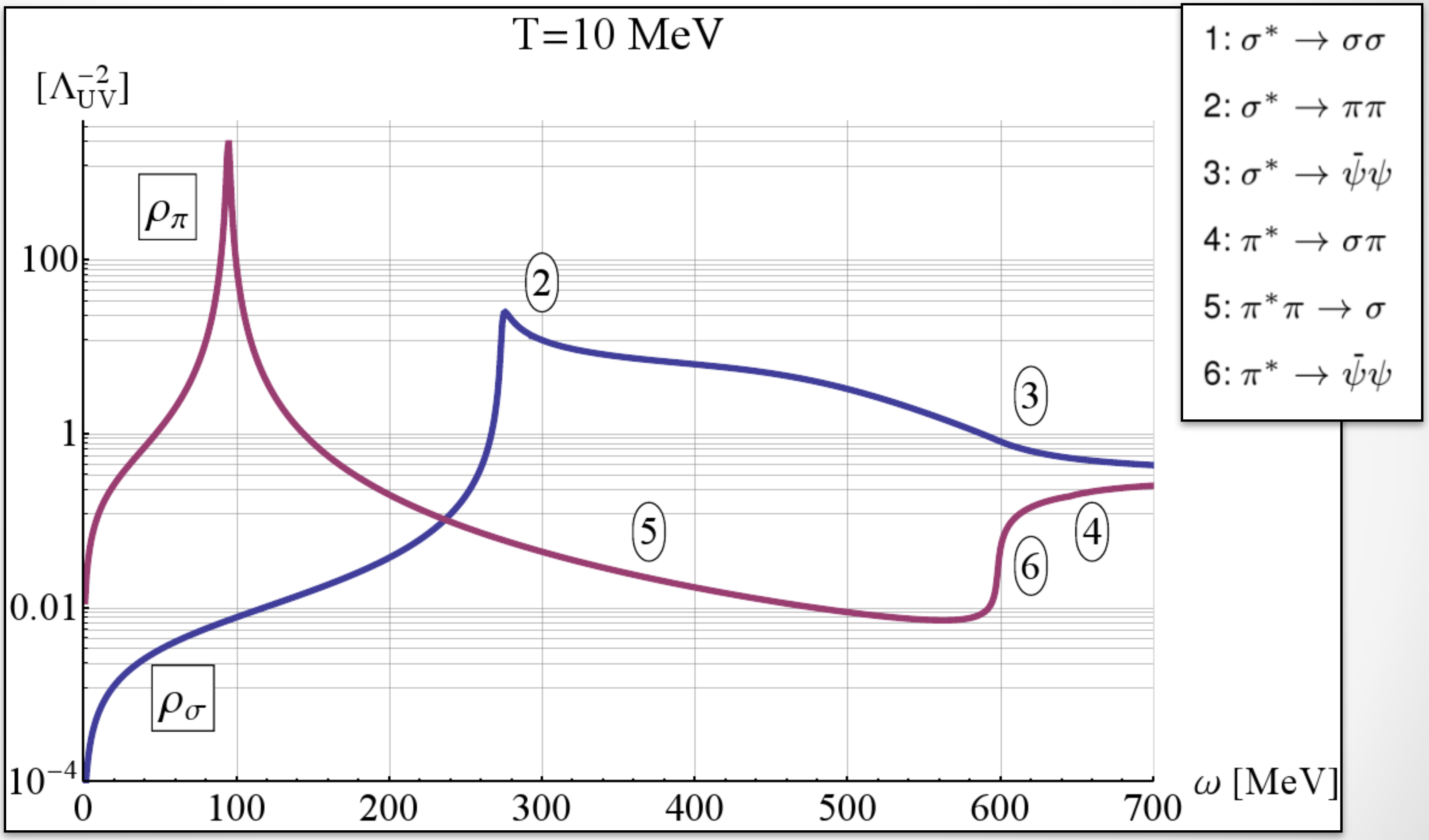
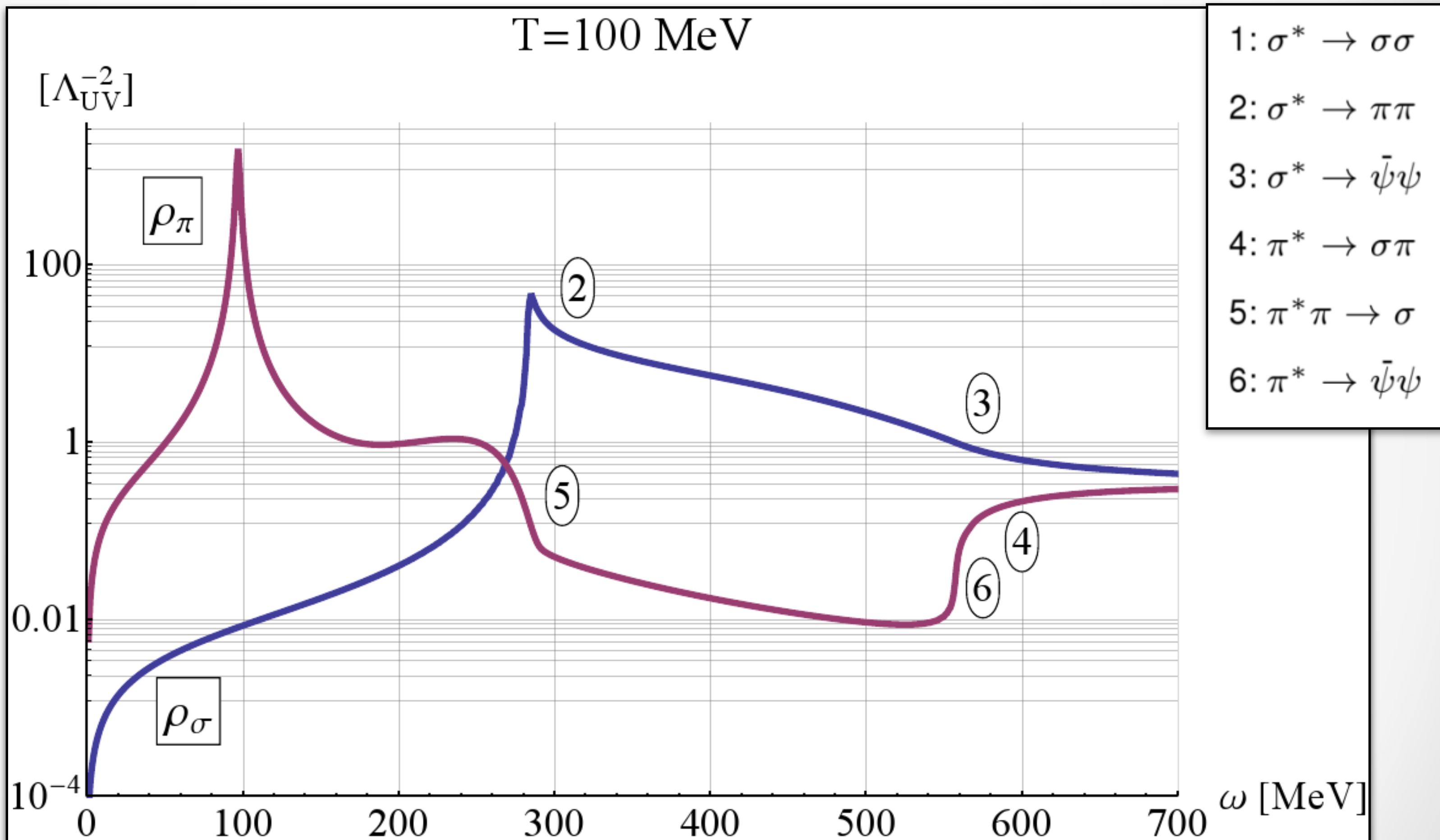
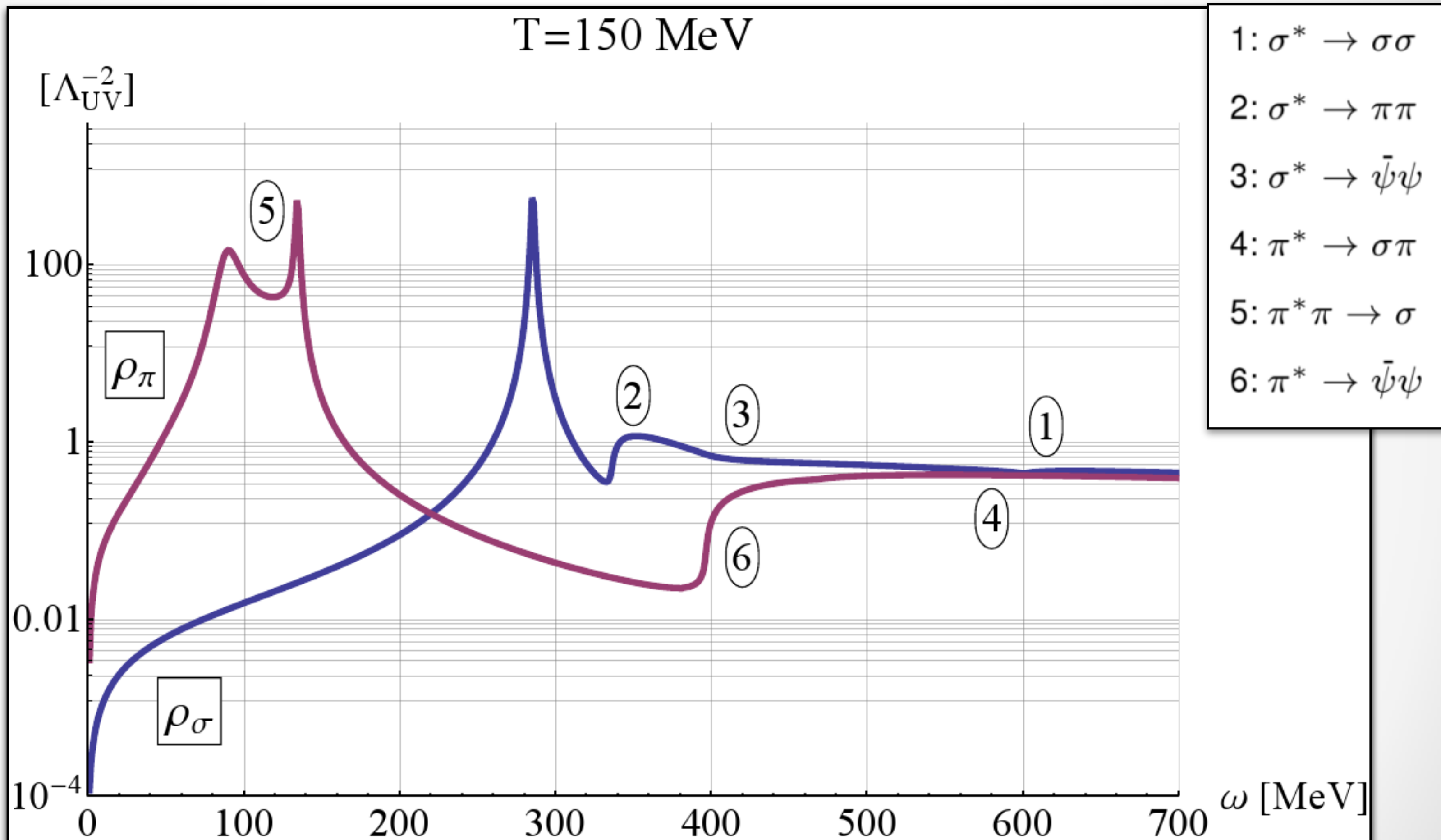


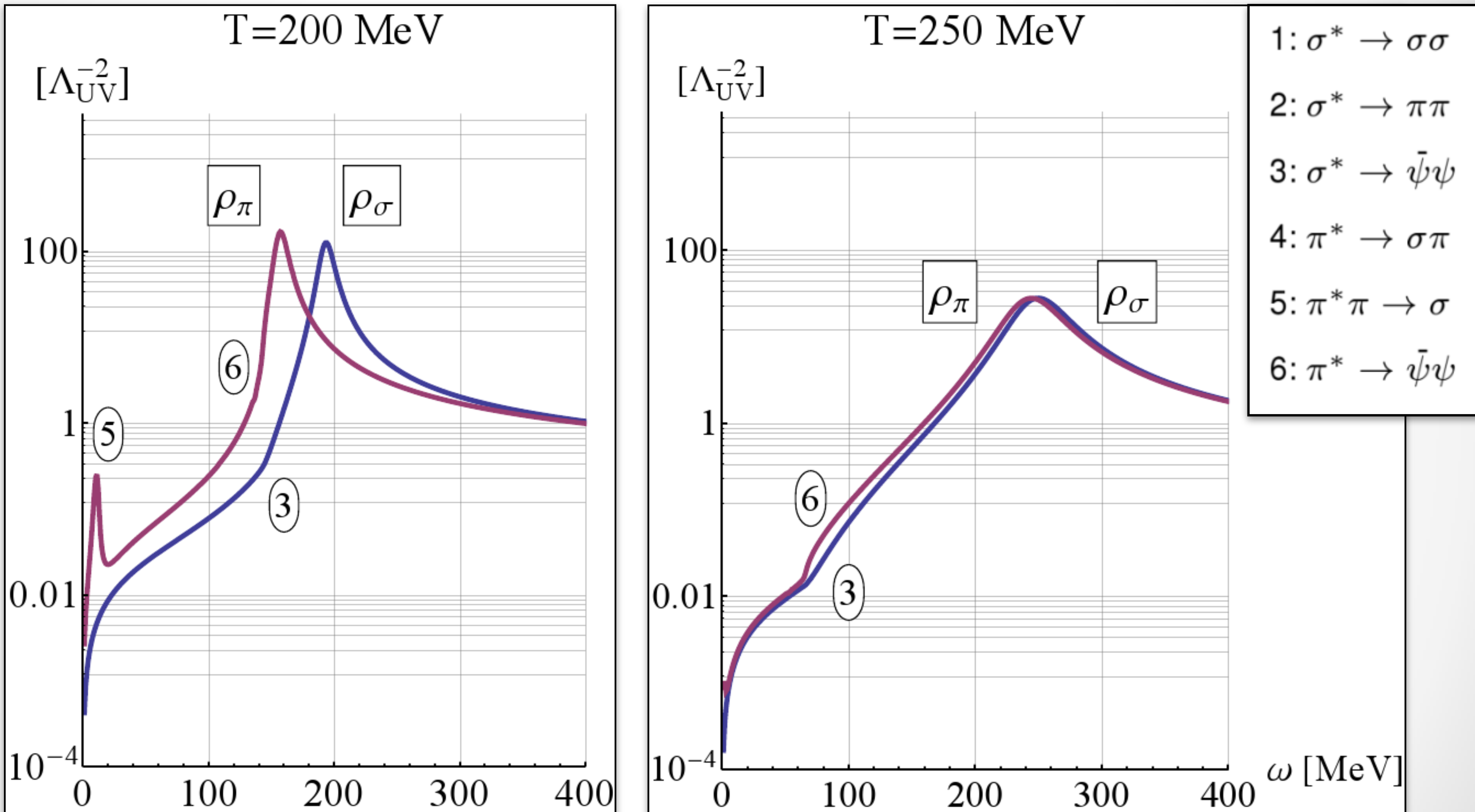
QM Model at T>0



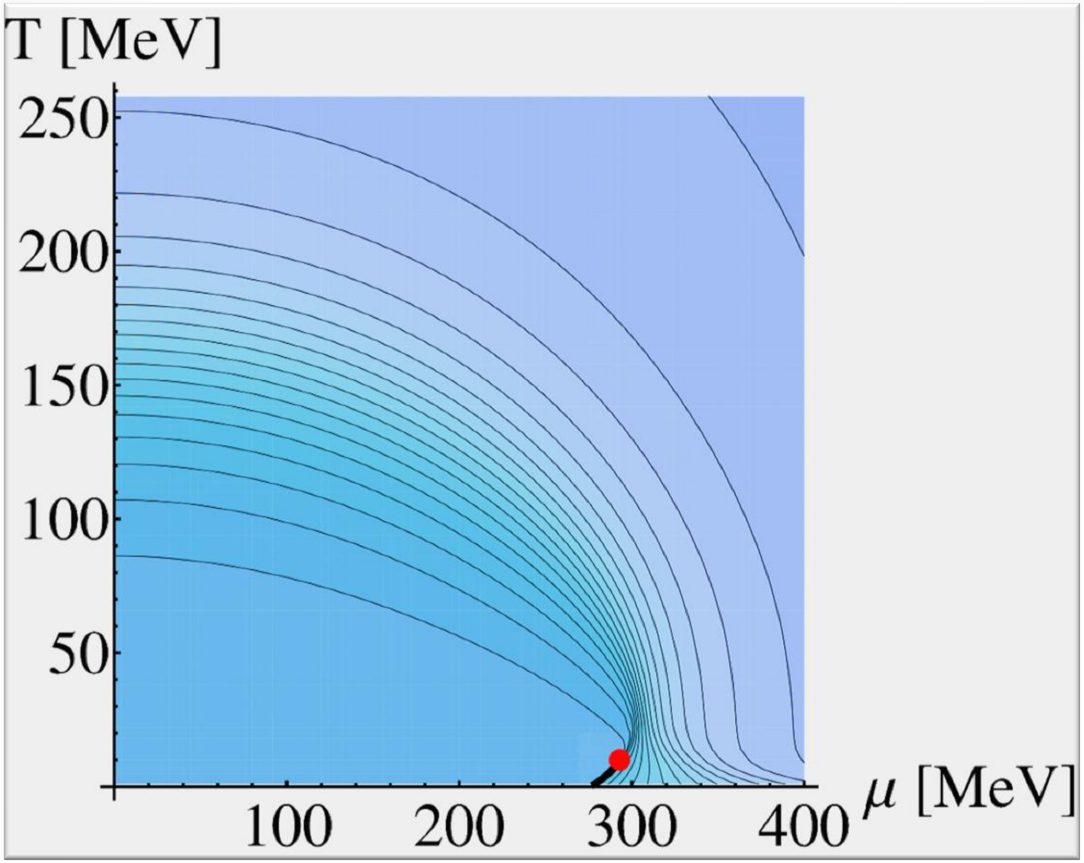
QM Model at T>0



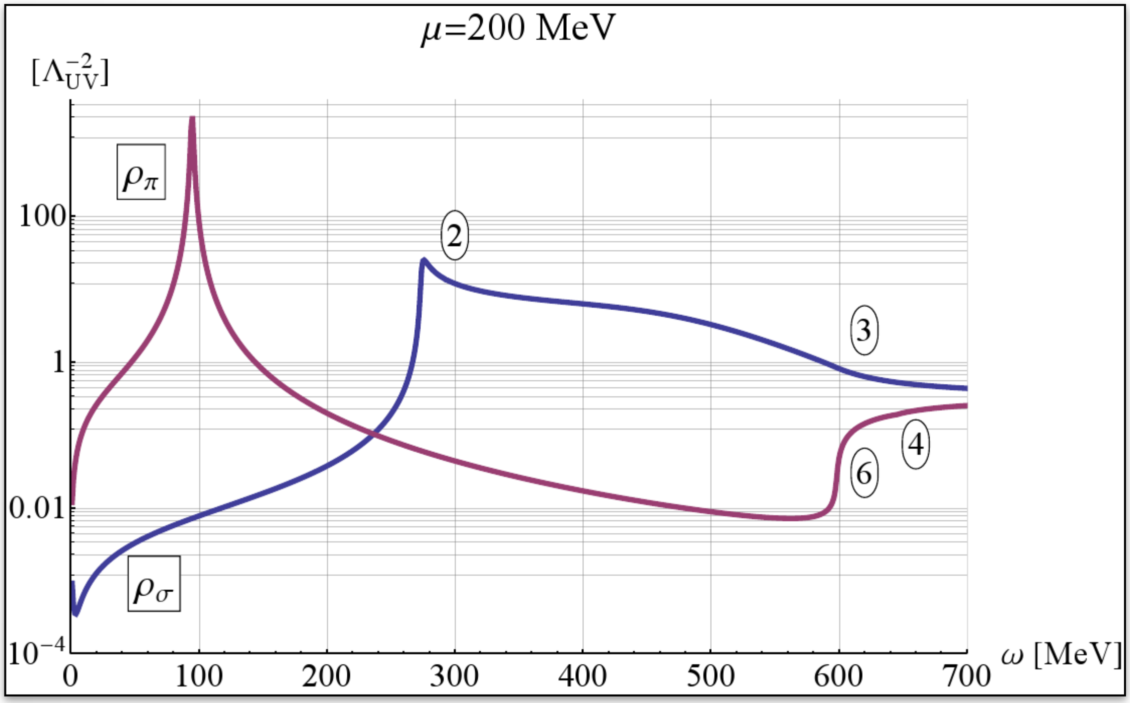
QM Model at T>0



QM Model at μ > 0



QM Model at μ > 0



1: $\sigma^* \to \sigma\sigma$

2: $\sigma^* \to \pi\pi$

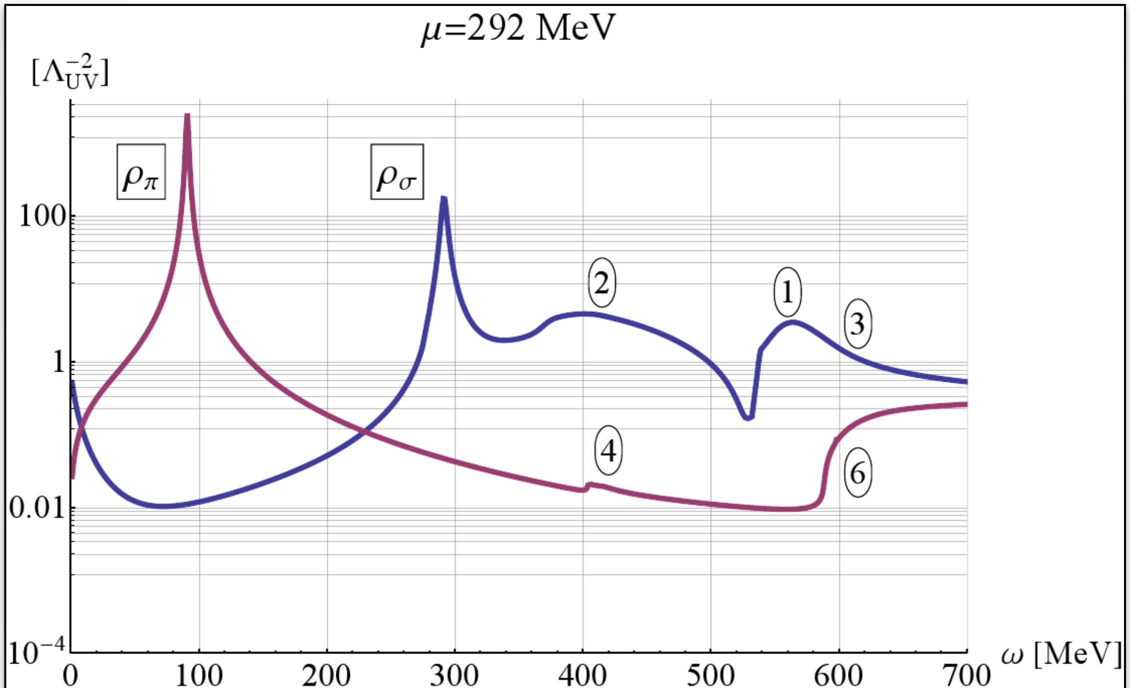
3: $\sigma^* \to \bar{\psi}\psi$

4: $\pi^* \to \sigma\pi$

5: $\pi^*\pi \to \sigma$

6: $\pi^* \to \bar{\psi}\psi$

QM Model at μ > 0



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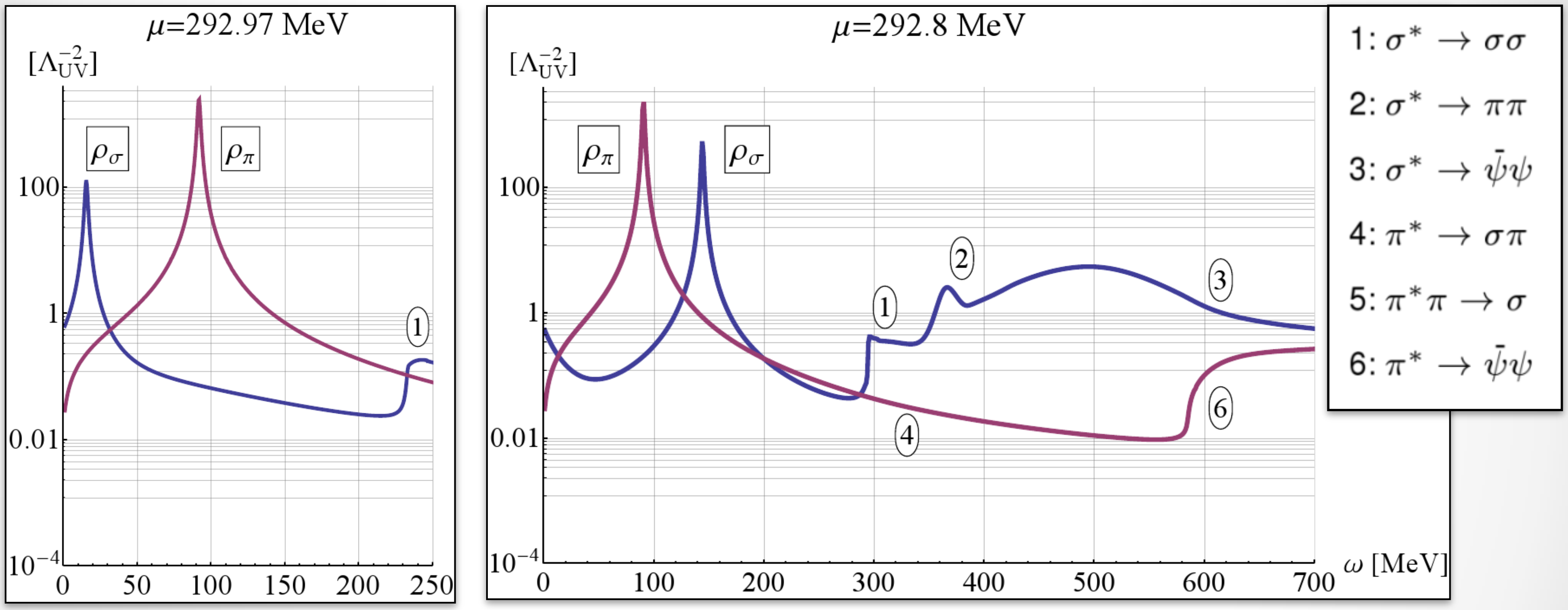
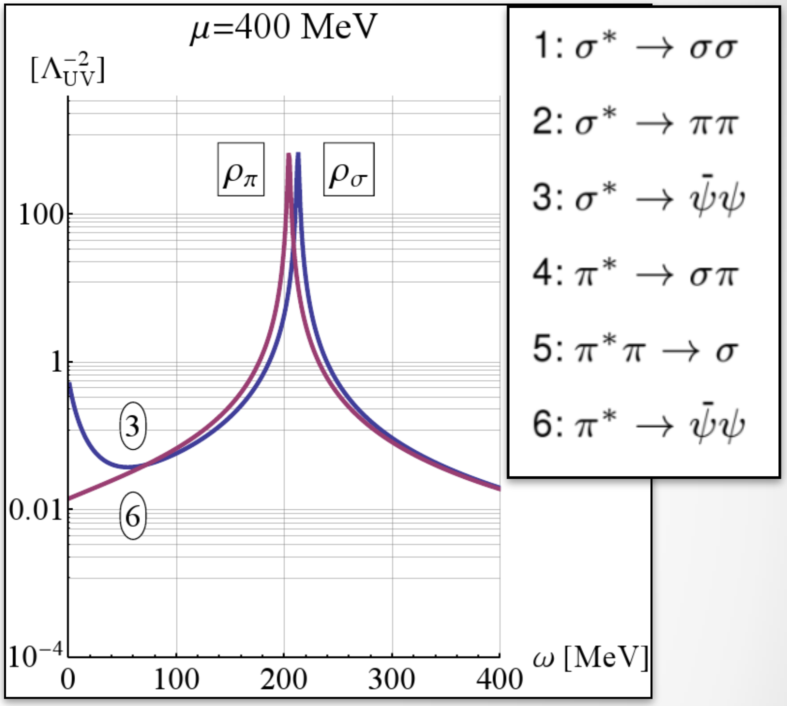
QM Model at μ > 0



QM Model at μ > 0



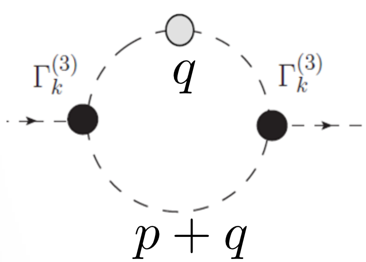
Going beyond...

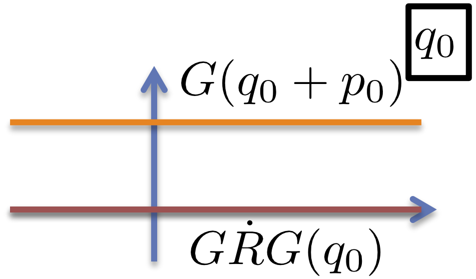
So far: 3d regulator function $R = \vec{p}^2 r(\vec{p}^2)$

Generalization:

- Either regulators which allow to perform Matsubara sums analytically
  - ➢ Floerchinger; JHEP 1205 (2012) 021

- Or **fully numerical procedure**- perform Matsubara sum numerically
  - Require: analytical regulator function for complex momenta
  - for free: finite chemical potential

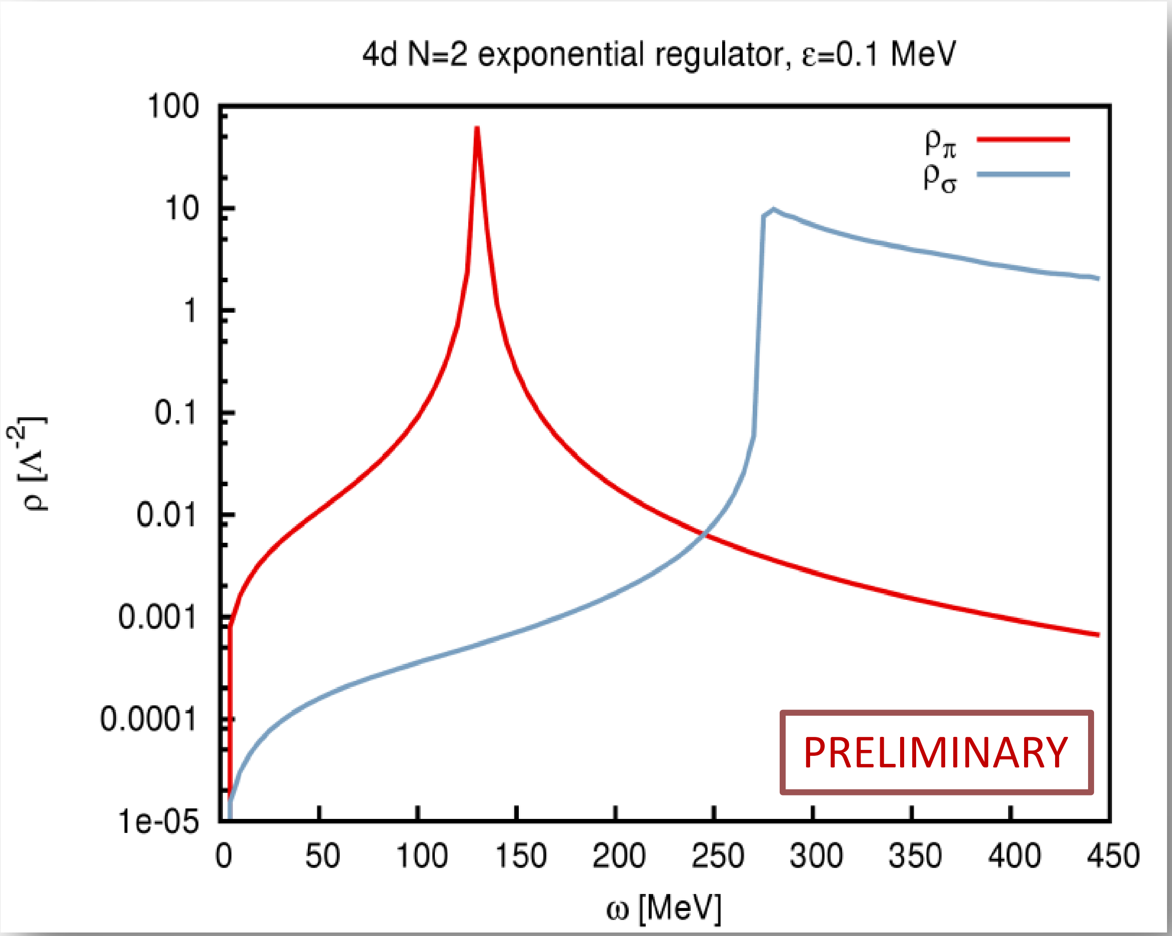




Shopping list:

- Proper regulator for complex external momenta
- Suitable for numerical applications
- Analytical functions
- Analytical structure of regularized propagator: as few poles as possible
  - Require pole procedures to obtain the correct real-time result

4d Spectral Functions



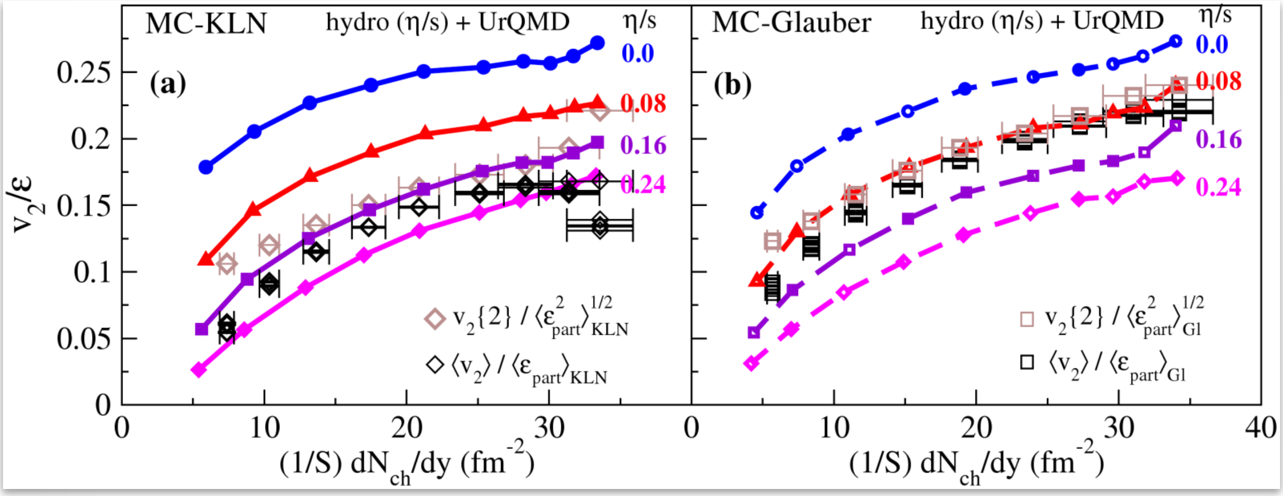
- Pawlowski, NSt [in prep.]

Transport Coefficients in YM

- Haas, Fister, Pawlowski arXiv:1308.4960
- Christiansen, Haas, Pawlowski, NSt; in prep.

Transport Coefficients

- Assume: given spectral functions of elementary fields
- Compute **transport coefficients via Kubo formulae** (ultracold atoms, QGP)



➢ Bass, Heinz, Hirano, Shen Phys.Rev.Lett. **106** (2011) 192301

$$\eta = \lim_{\omega\to 0} \frac{1}{20} \frac{\rho_{\pi\pi}(\omega,\vec{0})}{\omega}$$

Require $\rho_{\pi\pi}(\omega,\vec{p}) = \int_x e^{-\mathrm{i}\omega x_0 + \mathrm{i}\vec{p}\vec{x}} \langle[\pi_{ij}(x), \pi_{ij}(0)]\rangle$

Computing EM Correlators

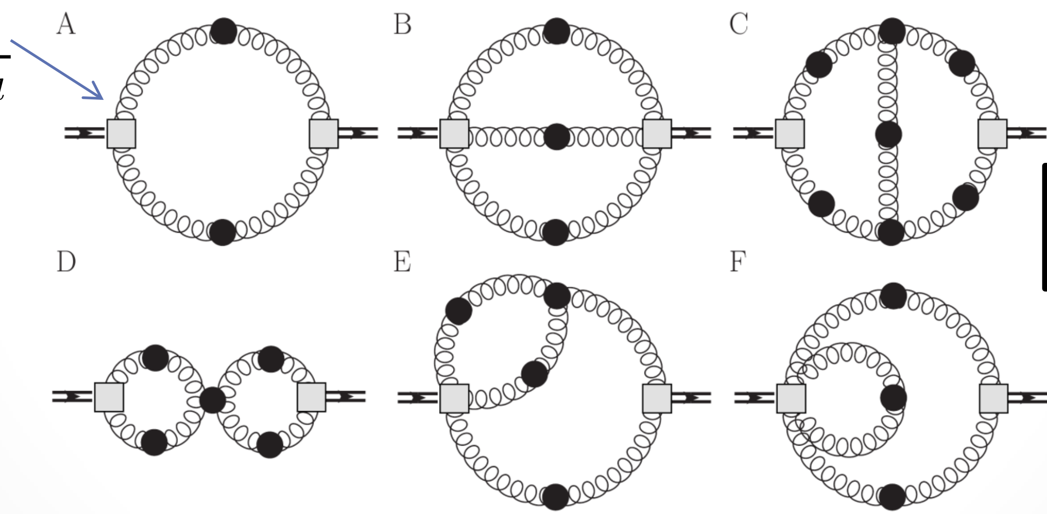
DSE-like expansion formula

- Pawlowski **Annals Phys. 322 (2007) 2831-2915**

$$\langle \pi_{ij}[\hat{A}]\pi_{ij}[\hat{A}]\rangle = \pi_{ij}[G_{A\phi_k}\frac{\delta}{\delta\phi_k} + A]\pi_{ij}[G_{A\phi_k}\frac{\delta}{\delta\phi_k} + A]$$

Finite number of diagrams involving **full** propagators/vertices

$$\frac{\delta^2\pi_{ij}}{\delta A_k \delta A_l}$$



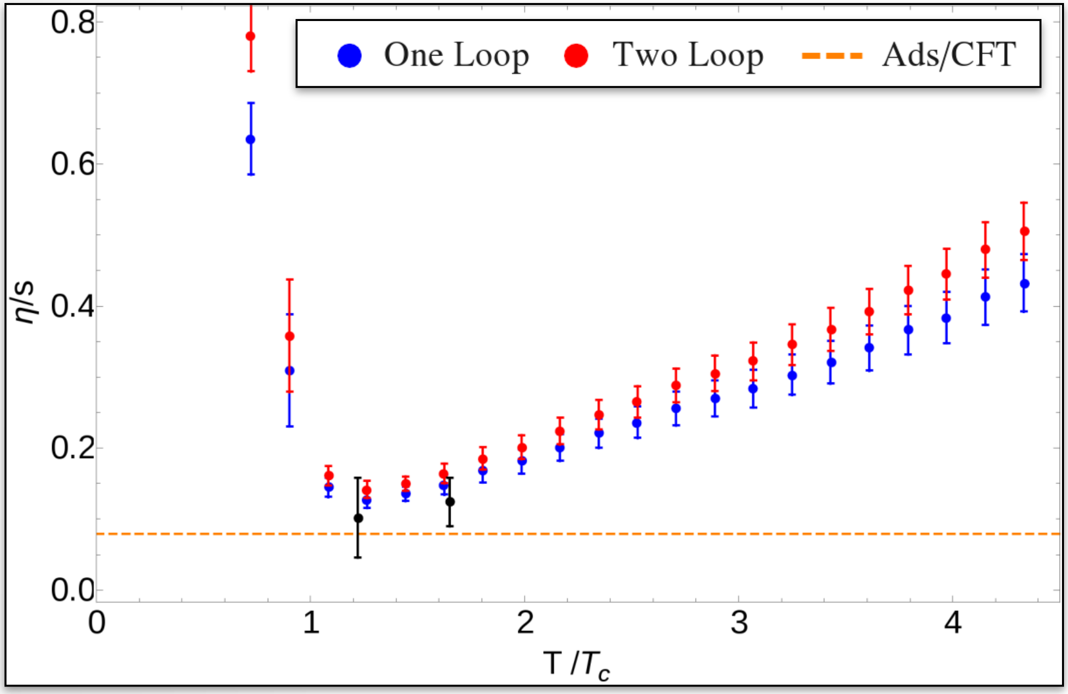
All diagrams to 2-loop order

2-Loop Result

Required input: distribution functions and **spectral functions of elementary fields**

Here: gluon spectral function from Euclidean FRG data using MEM

- Haas, Fister, Pawlowski arXiv:1308.4960



1-loop

- Haas, Fister, Pawlowski arXiv:1308.4960

2-loop

- Christiansen, Haas, Pawlowski, NSt; in prep.

- Only small deviations between 1- and 2-loop near T_c
- High temperature behaviour consistent with HTL

Summary

- **Euclidean Iteration**

  to take into account momentum dependence of the 2-pt function
  - ✓ mass definitions and implications for LPA fluctuation scales
  - ✓ quantitative reliability of LPA'

- **Spectral Functions**

  by analytical continuation on the level of the flow equation
  - ✓ tested in simple models (O(N), QM model)
  - ✓ generalization towards 4d regulator functions on the way

- **Transport Coefficients**

  from loop expansion involving full propagators and vertices
  - ✓ quantitative predictions for η/s in YM theory

Thank you for your attention!