Superfluid matter beyond BCS, hyperthermal matter beyond KPZ

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Discussions:
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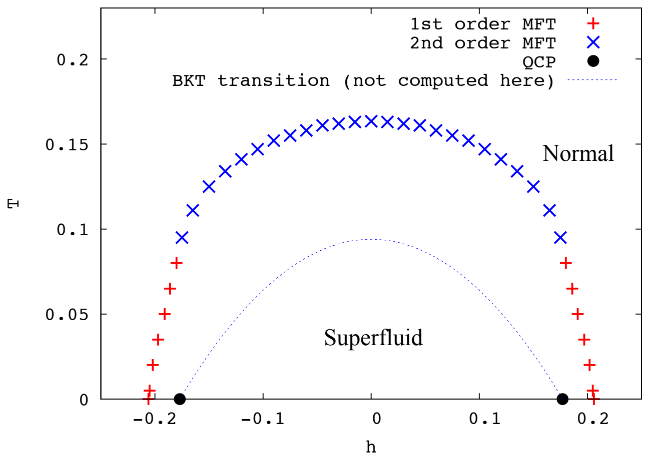
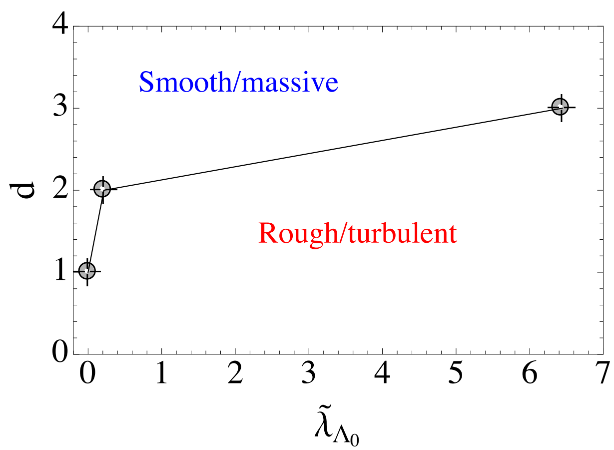
Road map for today



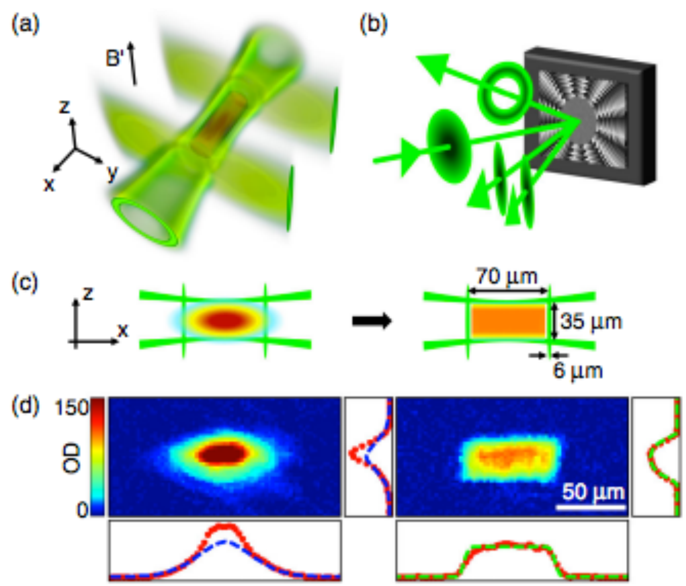
- Imbalanced Fermi gases in 2d
- Breakdown of homogeneous superfluidity
- Effective potential flow with fermionic mean-field as initial condition
- Potential quantum criticality toward Sarma-Liu-Wilczek phase
- Outlook on Larkin-Ovchinikov transitions

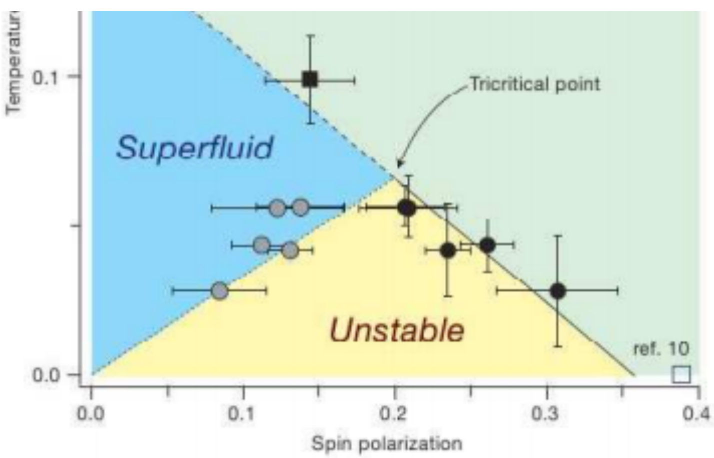


- KPZ interfaces dual to attractive Lieb-Liniger bosons in 1d
- Break Galilean invariance/integrability
- 1-loop flow with frequency cutoff technique
- Hyperthermal, self-organized phase
- Outlook on equilibration after quench

Interacting Fermi systems in two dimensions with ultracold atoms

- Prepare and inform correlated electron problems (high-T_c's, quantum Hall)
- Designer Hamiltonians
- Isolated systems (no phonons)
- Preparation and dynamics of (many-body) states
- Advances in cooling
  - Quantum degeneracy regime $T/T_F \sim$ 1-5% in reach
- Advances in homogeneous trapping
- Electrically neutral particles, sometimes good (no Coulomb, gauge fields)
- Atoms are still heavy and slow
- Coupling to optics/many-body photonics
- Explore beyond solid-state Hilbert space (SU(N) magnets, **spin imbalance**)





See also

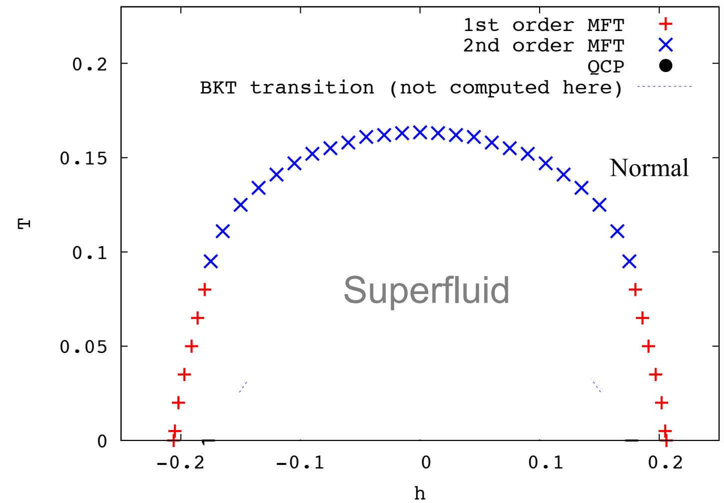
- **Today, Session VI A**:
  - 18:10 – 18:30 Herbst: Sarma phase in relativistic and non-relativistic systems
  - 18:50 – 19:10 Roscher: Phases of unitary imbalanced Fermi gases

Controversy: Sarma-Liu-Wilczek superfluids unstable at mean-field

- Designer Hamiltonian

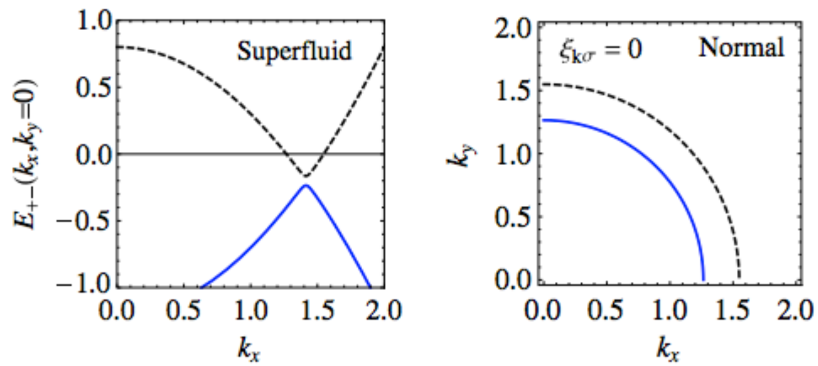
$$H = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}\sigma} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{g}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^\dagger_{\mathbf{k}+\mathbf{q}/2,\uparrow} c^\dagger_{-\mathbf{k}+\mathbf{q}/2,\downarrow} c_{\mathbf{k}'+\mathbf{q}/2,\downarrow} c_{-\mathbf{k}'+\mathbf{q}/2,\uparrow}$$

$$\xi_{\mathbf{k}\sigma} = \frac{\mathbf{k}^2}{2m_\sigma} - \mu_\sigma \qquad g < 0 \qquad \sigma = \uparrow, \downarrow$$



- Pairing gap opens away from both Fermi surfaces

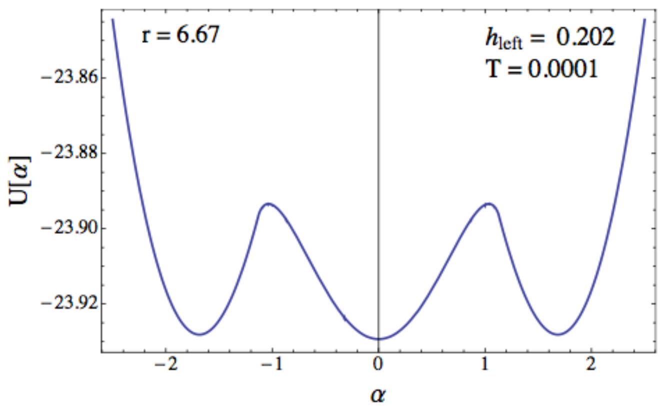
$$E_\pm = \frac{\xi_{\mathbf{k}\uparrow} - \xi_{-\mathbf{k}\downarrow}}{2} \pm \sqrt{\frac{\alpha^2}{2} + \left(\frac{\xi_{-\mathbf{k}\downarrow} + \xi_{\mathbf{k}\uparrow}}{2}\right)^2}$$



- Generically first order at mean-field when gap ~ imbalance (as are many magnetic metals)
- In 2d, mean-field *qualitatively incorrect*

Order parameter fluctuations *qualitatively crucial* in 2d: capture with full potential flow, link to fermionic initial conditions

- Example mean-field potential at first order transition at small T



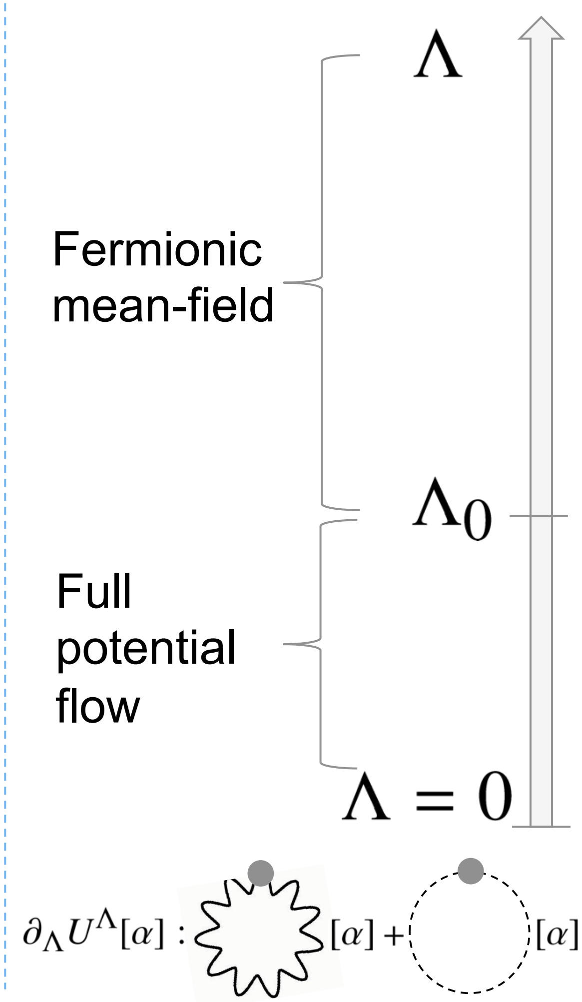
$$U^{\Lambda=\Lambda_0}(\alpha) = \frac{-1}{2g}\alpha^2 - T\sum_{k_0,\mathbf{k}} \ln\left[\frac{\left(-ik_0+\xi_{\mathbf{k}\uparrow}\right)\left(ik_0+\xi_{-\mathbf{k}\downarrow}\right)+\alpha^2/2}{\left(-ik_0+\xi_{\mathbf{k}\uparrow}\right)\left(ik_0+\xi_{-\mathbf{k}\downarrow}\right)}\right]$$

- Goldstone and amplitude fluctuations drive flow:

$$\frac{d}{d\Lambda}\Gamma^{\Lambda}[\sigma,\pi] = \frac{1}{2}\mathrm{Tr}\left\{\dot{R}^{\Lambda}\left[\Gamma^{(2)\Lambda}[\sigma,\pi]+\mathbf{R}^{\Lambda}\right]^{-1}\right\}$$

- Combined frequency and momentum cutoff

$$R_{\Lambda}(q_\tau,\mathbf{q}) = Z_{\mathbf{q}}\left(\Lambda^2 - q^2 - \frac{Z_\Omega}{Z_{\mathbf{q}}}q_\tau^2\right)\theta\left(\Lambda^2 - q^2 - \frac{Z_\Omega}{Z_{\mathbf{q}}}q_\tau^2\right)$$



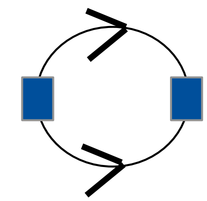
Obtain initial values of propagators from fermionic contractions

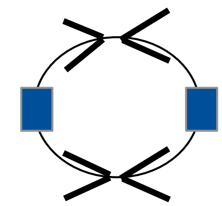
- X, and Z factors evaluated at potential minimum

$$\Gamma^{\Lambda}_{(2)}(q;\alpha) = \begin{pmatrix} \Gamma_{\sigma\sigma}(q;\alpha) & \Gamma_{\sigma\pi}(q;\alpha) \\ \Gamma_{\pi\sigma}(q,\alpha) & \Gamma_{\pi\pi})(q,\alpha) \end{pmatrix}\Bigg|_{\alpha}$$
$$= \begin{pmatrix} Z_{\Omega}[\alpha_0]q_0^2 + Z_{\mathbf{q}}[\alpha_0]\mathbf{q}^2 + U'[\alpha] + \alpha^2 U''[\alpha] & -X[\alpha_0]q_0 \\ X[\alpha_0]q_0 & Z_{\Omega}[\alpha_0]q_0^2 + Z_{\mathbf{q}}[\alpha_0]\mathbf{q}^2 + U'[\alpha] \end{pmatrix}$$

- Initial values from fermionic normal and anomalous particle-particle ladder

$$\mathcal{S}[\sigma,\pi,\alpha] = \int_q \frac{\sigma_{-q}\sigma_q}{2}\Big(Q_{\sigma\sigma}(q;\alpha) + U''[\alpha]\Big) + \frac{(2\pi)^2}{T}U[\alpha]$$
$$+ \int_q \frac{\pi_{-q}\pi_q}{2}\left(Q_{\pi\pi}(q;\alpha) + \frac{1}{\alpha}U'[\alpha]\right)$$
$$+ \int_q \left(\frac{\sigma_{-q}\pi_q}{2}Q_{\sigma\pi}(q;\alpha) + \frac{\pi_{-q}\sigma_q}{2}Q_{\sigma\pi}(-q;\alpha)\right),$$

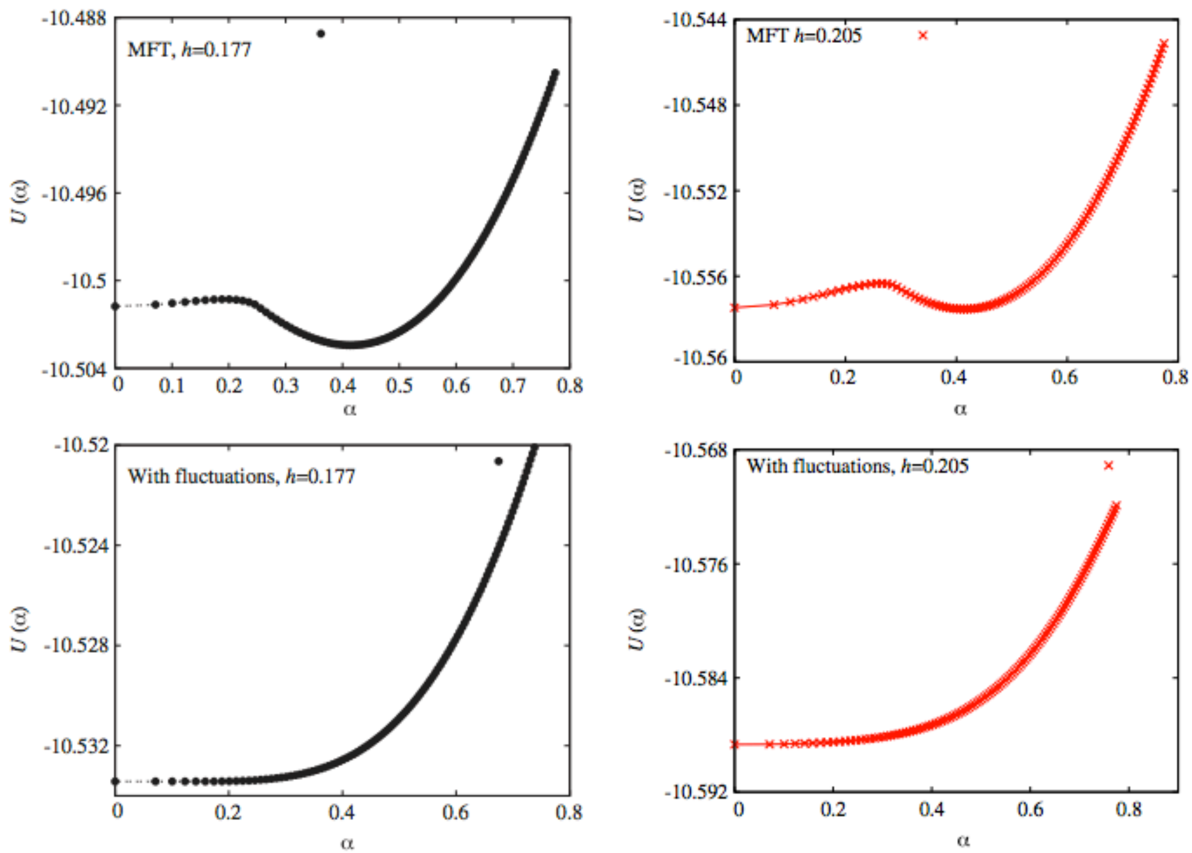
K: 

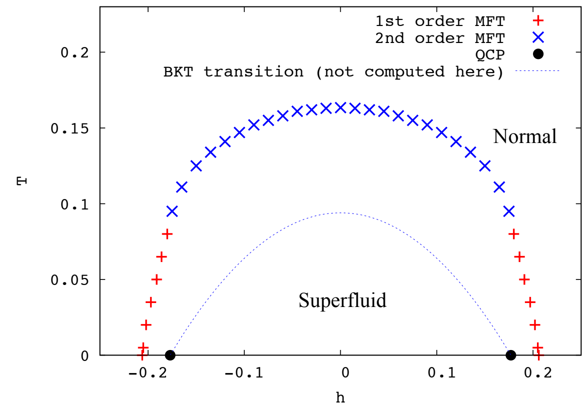
L: 

$$Q_{\sigma\sigma}(q) = K(q) + L(q) - (K(0) + L(0))$$
$$Q_{\pi\pi}(q) = K(q) - L(q) - (K(0) - L(0))$$
$$Q_{\sigma\pi}(q) = iK^{\text{odd}}(q)$$

➤ Goldstone's theorem respected and not broken during flow

Quantum fluctuations smoothen effective potential



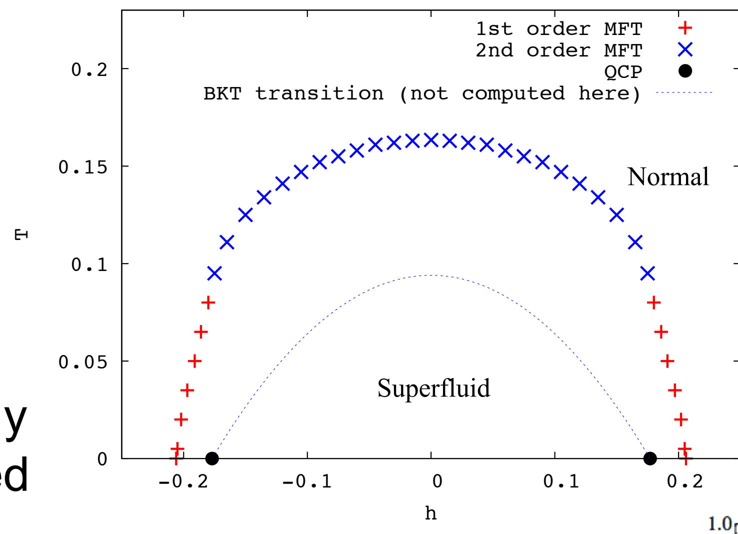


- BCS regime, weak attraction
- Renormalization strongest for regions with curvature

- Extension to KT phase/finite temperature desirable
- Coupling to fermions including their self-energies in flow

Fluctuation-corrected phase diagram of imbalanced fermions in 2d

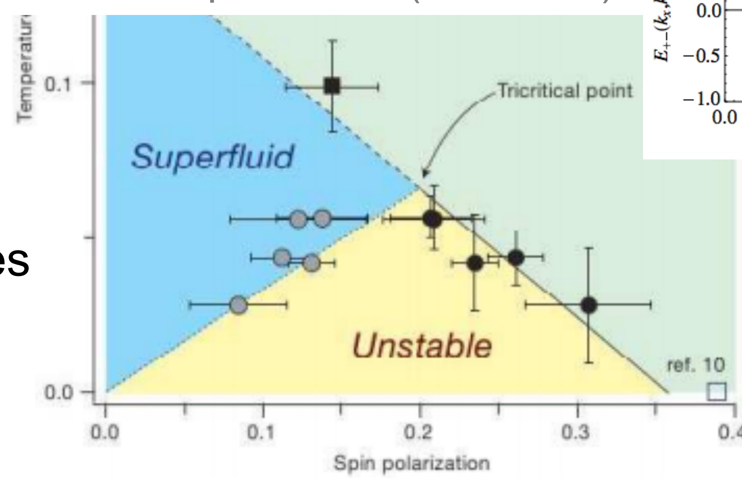
- Mean-field tri-critical points renormalized to T=0, h_c
- New quantum critical points to Sarma-Liu-Wilczek phase?
- Goldstone phase fluctuations and BKT transition at finite T
- Second "Lifshitz"« transition to fully gapped state at smaller h expected

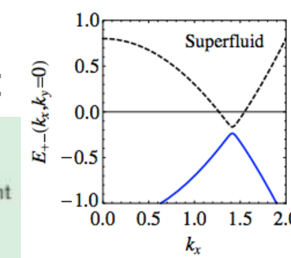


<u>Predictions for future 2d experiment:</u>

- at least substantial suppression of tri-critical point
- potentially anomalous thermodynamic/transport signatures at finite T in quantum critical fan
- Interplay with KT vortices?

3d experiments (MIT 2006):

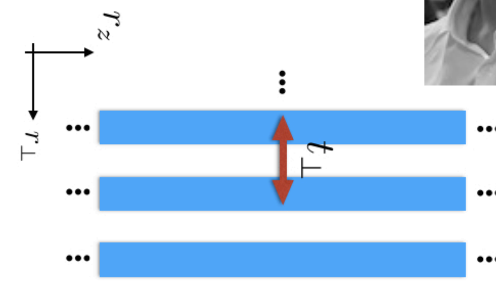




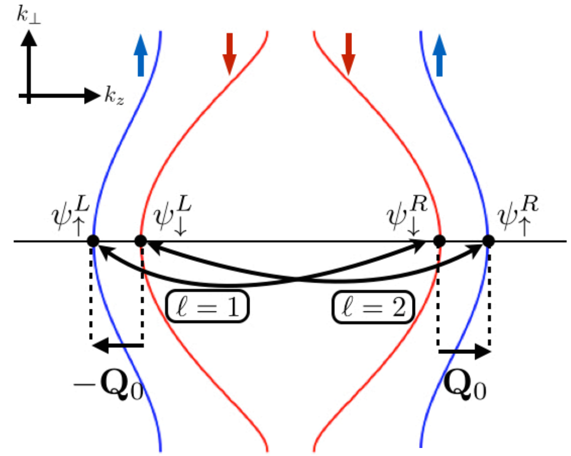
Non-Fermi liquid criticality at onset of Larkin-Ovchinikov pairing

Piazza, Zwerger, Strack,
to appear (2014)

- Coupled wires: $\xi_\sigma(\mathbf{k}) = \frac{k_z^2}{2m} - 2t_\perp \cos(dk_\perp) - \mu - \sigma h$

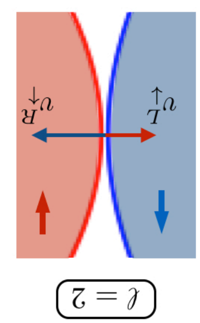


- Cooper pairing susceptibility maximal at Q_0
- Amplitude-modulated pairing field: $\Delta(\mathbf{x}) \propto \cos(\mathbf{Q}_0 \cdot \mathbf{x})$
- Transition continuous on mean-field level



- Low-energy Lagrangian around two hot spots:

$$\mathcal{L}_{LE} = \frac{2}{g}\Delta_{LO}^2 + \sum_{\substack{\sigma=\uparrow,\downarrow \\ j=R,L}} \bar{\psi}_\sigma^j \left(\partial_\tau - i v_\sigma^j \partial_z + \frac{\partial_\perp^2}{2m_\perp}\right)\psi_\sigma^j$$
$$- \Delta_{LO}\left(\psi_\downarrow^R \psi_\uparrow^L + \psi_\downarrow^L \psi_\uparrow^R + \text{h.c}\right)$$



- Compute quasi-particle scattering rates
- Non-analyticities in pairing channel

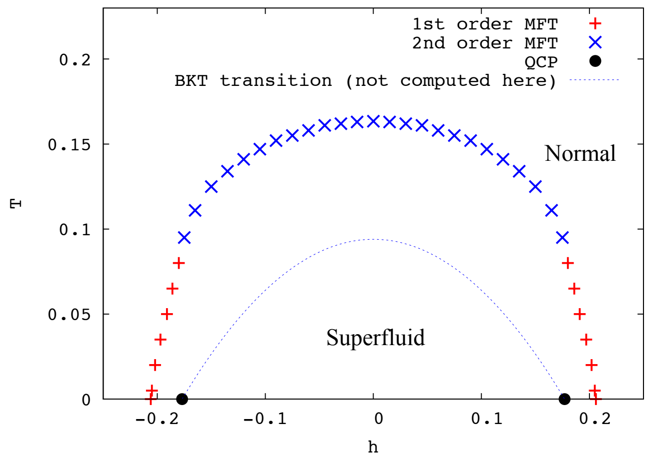
Established imbalanced superfluids in 2d as plain-vanilla[1] non-Fermi liquid 'metal' quantum phase transitions at finite fermion density

	Broken symmetry	*Collective momentum*	*Effective model*	*Bare collective dynamics*
Spin-density wave	SU(2) spin rotation	Commensurate Q=(π,π) particle-hole pair (Ferromagnet at Q=(0,0) also possible)	Hot spots on Fermi surface coupled to magnon	$\lvert\Omega\rvert\,,\ z=2$ 2 Goldstone modes
Nematic Fermi surface deformation	C_4 lattice orientation	Forward scattered Q=(0,0) particle-hole pair	One Fermi surface coupled to photon	$\lvert\Omega\rvert/\lvert\mathbf{q}\rvert\,,\ z=3$ No Goldstone mode
Imbalanced superfluids	U(1) number conservation	Homogeneous Q=(0,0) particle-particle pair (LOFF at finite Q also possible)	Two mismatched Fermi surfaces coupled to Cooperon	$i\Omega\,,\ z=2$ 1 Goldstone mode (Landau damping also possible)

[1]Excluding fractionalization/emergent gauge field scenarios and insulating states

- Larkin-Ovchinikov QCP candidate for "almost naked" QCP?
- Weak violation of cosmic censorship of metals?

Road map for today



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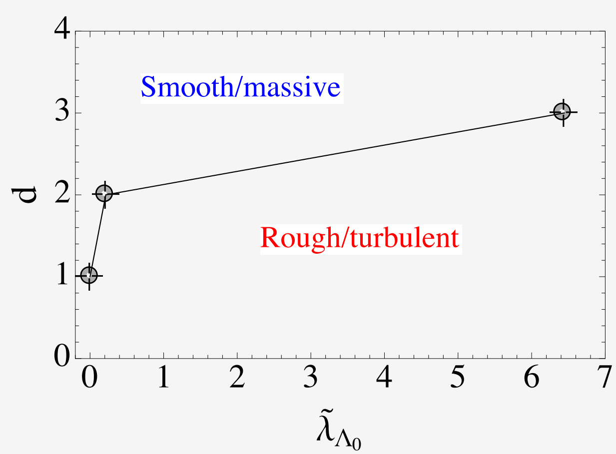

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Hyperthermal matter?

Hyperthermia

From Wikipedia, the free encyclopedia

Hyperthermia is elevated body temperature due to failed thermoregulation that occurs when a body produces or absorbs more heat than it dissipates. Extreme temperature elevation then becomes a medical emergency requiring immediate treatment to prevent disability or death.

Turbulence: Berges, Canet, next session
KPZ: Mathey, Kloss in parallel session VIA
Dynamic criticality: Diehl, next session

KPZ interfaces dual to ground state (T=0) of attractive Lieb-Liniger bosons

$$\frac{\partial h}{\partial t} = \nu_0 \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta$$

$$H = -\frac{1}{2}\sum_{\alpha=1}^{n}\frac{\partial^2}{\partial x^2} + \gamma \sum_{\alpha<\beta}\delta(x_\alpha - x_\beta)$$

$$\gamma < 0$$

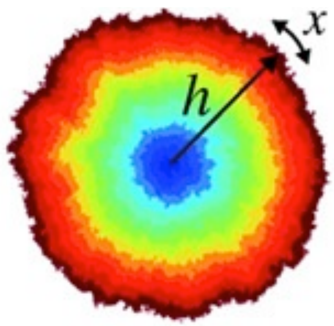
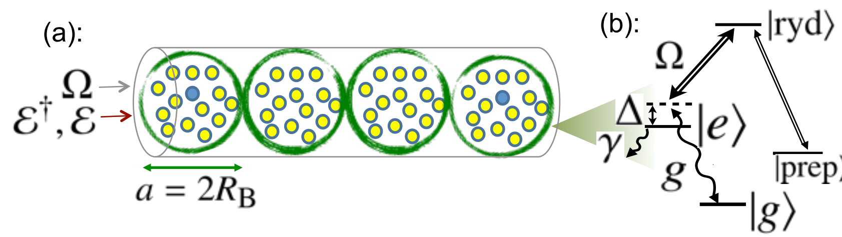


Image from K. A. Takeuchi et al. Sci. Rep. 1, 34 (2011)



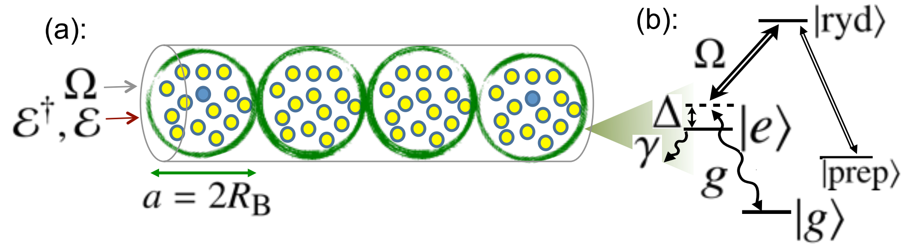
- Noisy height fluctuations in space and time around growing base
- Archetype of dynamic criticality away from equilibrium
- Symmetries: Galilean, height shift, … facilitate solutions in 1d

- Bosons in 1d optical lattice; photons in Rydberg quantum wires
- Archetype of interacting quantum many-body system
- Integrability, many conserved quantities facilitate solutions (Bethe Ansatz, CFT's) for ground state

Kardar, Nucl. Phys. B (1987); Brunet, Derrida, PRE (2000); Calabrese, LeDoussal, PRL (2011); Strack, Choi, Demler, Lukin, to appear (2014)

“Broken” KPZ equation as diffusion equation with multiplicative noise

- Physical reality:
  - Integrability broken
  - Less conserved quantities (typically 3,4, or so)



- Break Galilean invariance/symmetries by temporal correlations in noise
- Map to diffusion equation with multiplicative noise

$$\frac{\partial h}{\partial t} = \nu_0 \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta \qquad \Rightarrow \qquad \partial_t \phi = \nu_0 \nabla^2 \phi + \frac{\lambda}{2\nu_0} \phi \eta$$

$$\phi(t, \mathbf{x}) = \exp\left[(\lambda/2\nu_0) h(t, \mathbf{x})\right]$$

<u>Noise spectrum</u>
(scale-free inspired by turbulence[1])

$$\overline{\eta(\omega', \mathbf{x}')\eta(\omega, \mathbf{x})} = D_{1/f}(\omega)\delta(\omega + \omega')\delta^{(d)}(\mathbf{x}' - \mathbf{x})$$

$$D_{1/f}(\omega) = \frac{1}{|\omega|}$$

- Compute fluctuations around growing average
- Compare to known KPZ results

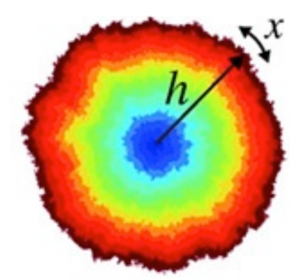


Image from K. A. Takeuchi et al. Sci. Rep. 1, 34 (2011)

[1]See e.g. Yakhot and Orszag, PRL (1986)

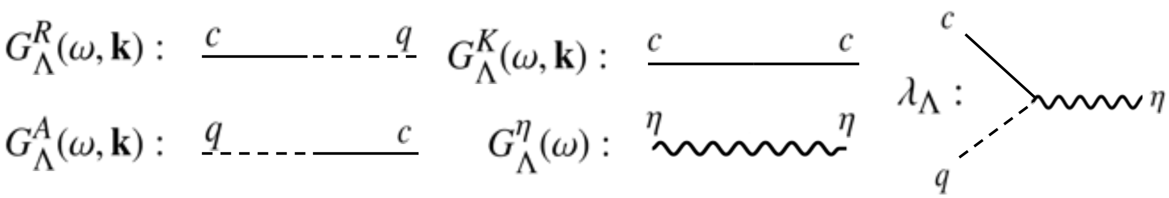
Unified noise and field integration in Keldysh path integral

$$\partial_t \phi = \nu_0 \nabla^2 \phi + \frac{\lambda}{2\nu_0}\phi\eta$$

- Random forces Gaussian: $W[\eta] \propto \exp\left\{-\int d^d x \int d\omega \frac{1}{2}\eta(\omega,\mathbf{x})|\omega|\eta(\omega,\mathbf{x})\right\}$
- Unified Keldysh generating functional

$$Z = \int \mathcal{D}\eta W[\eta] \mathcal{D}(\phi,\tilde{\phi}) e^{i(S_\phi[\phi,\tilde{\phi}] - \int_{t,\mathbf{x}} \eta\phi\tilde{\phi})} \equiv \int \mathcal{D}(\eta,\phi,\tilde{\phi}) e^{i(S_\phi[\phi,\tilde{\phi}] + S_\eta[\eta] + S_\lambda[\phi,\tilde{\phi},\eta])}$$

- Propagators (G^K next slide) $G^R(\omega,\mathbf{k}) = \frac{1}{i\gamma\omega - \nu_0 \mathbf{k}^2}$,
- Tri-linear noise vertex $S_\lambda[\phi,\tilde{\phi},\eta] = -\int_t dt \int d^d x \frac{\lambda}{2\nu_0}\eta(t,\mathbf{x})\tilde{\phi}(t,\mathbf{x})\phi(t,\mathbf{x})$



Compare with Frey, Täuber (1994) in terms of h-field: $\frac{\partial h}{\partial t} = \nu_0 \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta$

- Effective Keldysh noise spectrum appears second order in noise vertex (not first)
- Vertices not momentum-dependent
- Temporal color in noise generate propagator corrections perturbatively

Perform one-loop RG with frequency cutoff technique

- Wetterich equation on Keldysh contour

$$\partial_\Lambda \Gamma_\Lambda[\phi,\eta] = \frac{i}{2}\mathrm{Tr}\left[\frac{\dot{\mathcal{R}}}{\Gamma^{(2)}_\Lambda[\phi,\eta]+\mathcal{R}}\right] \qquad \mathcal{R} = \begin{pmatrix} 0 & R^A_\Lambda & 0 \\ R^R_\Lambda & 0 & 0 \\ 0 & 0 & R^\eta_\Lambda \end{pmatrix}$$

$$R^\eta_\Lambda(\omega) = \left(-|\omega| + \Lambda^2\right)\theta\left[\Lambda^2 - |\omega|\right]$$

$$\partial_\Lambda R^\eta_\Lambda(\omega) = 2\Lambda\theta\left(\Lambda^2 - |\omega|\right)$$

$$R^R_\Lambda(\omega) = \gamma\left(-i\omega + i\mathrm{sgn}(\omega)\Lambda^2\right)\theta\left[\Lambda^2 - |\omega|\right]$$

$$\dot{R}^R_\Lambda(\omega) \equiv \partial_\Lambda R^R_\Lambda(\omega) = 2\Lambda i\gamma\mathrm{sgn}(\omega)\theta\left[\Lambda^2 - |\omega|\right]$$

- Do *not* impose any fluctuation-dissipation relation on flow
- Derivative expansion plus mass term for broken Galilean invariance

$$G^K_\Lambda(\omega,\mathbf{k}) = \frac{-2id^K_\Lambda}{\left|i\gamma_\Lambda\omega - \left(A_\Lambda\mathbf{k}^2 + \Delta_\Lambda\right) + R^R_\Lambda(\omega)\right|^2} \qquad G^\eta_\Lambda(\omega) = \frac{-i}{|\omega| + R^\eta_\Lambda(\omega)} \qquad \Gamma^{(3)}_\Lambda = -\int_{t,\mathbf{x}} \lambda_\Lambda \eta\tilde{\phi}\phi$$

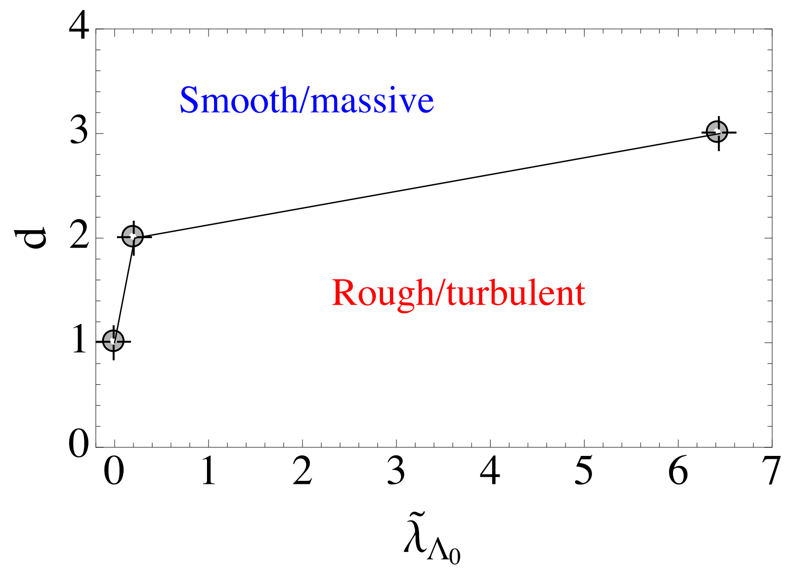
- Noise vertex relevant in d < 4, perturbative control only for ε = 4 – d small[1]
- fRG: crossover scales and flexibility to rescale frequencies/time

[1] See also more sophisticated truncations with fRG Kloss, Canet, Delamotte, Wschebor

Truly far from equilibrium "rough phase" at high noise levels

- Rough phase: violation of thermal fluctuation dissipation relation

	$d=1$	d=2	d=3
ζ_{d^K}	15.68	8.30	5.56
ζ_γ	−6	−2	−2/3
ζ_{hyper}	21.68	10.30	6.23
z	8	4	2.66



- Response and statistical Keldysh component can scale differently

$$\mathcal{R}(\omega,\mathbf{k}) = -2\mathrm{Im}\overline{\langle\phi(-\omega,-\mathbf{k})\phi(\omega,\mathbf{k})\rangle}_R \quad\Rightarrow\quad \mathcal{R}(s^z\omega, s\mathbf{k}) \propto \frac{1}{s^{2-\zeta_\gamma}}\mathcal{R}$$

$$\zeta_{\text{hyper}} = \zeta_{d^K} - \zeta_\gamma$$

$$C(\omega,\mathbf{k}) = i\,\overline{\langle\phi(-\omega,-\mathbf{k})\phi(\omega,\mathbf{k})\rangle}_K \quad\Rightarrow\quad C(s^z\omega, s\mathbf{k}) \propto \frac{1}{s^{4-2\zeta_\gamma+\zeta_{d^K}}}C$$

- KPZ with Galilean invariance, exact exponent identity $\zeta_{d^K} = \zeta_\gamma$ $\zeta_{\text{hyper}} = 0$

Roughening transition fulfills fluctuation-dissipation relation

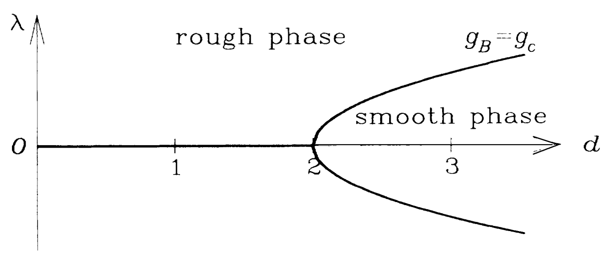
- d=2, 3, fine tuned flows at roughening transition

$$\zeta_\gamma^{\rm rt} = \zeta_{dK}^{\rm rt} = \frac{2d}{8+d} \quad \Rightarrow \quad z^{\rm rt} = 2 - \zeta_\gamma^{\rm rt} = \frac{16}{8+d}$$

- Approaching the rough phase, FDT is violated

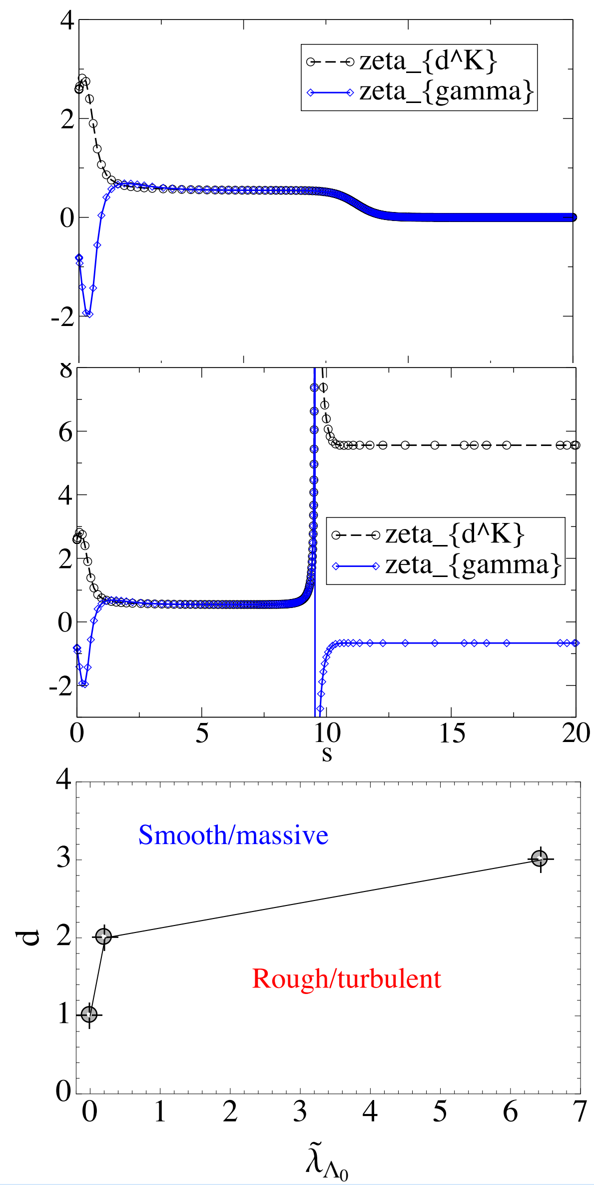
$$\zeta_\gamma = \frac{2(d-4)}{d}$$

- In d=1, interface always rough
- KPZ with Galilean invariance (Nattermann, PRA 1992; Frey, Täuber, PRB 1994):

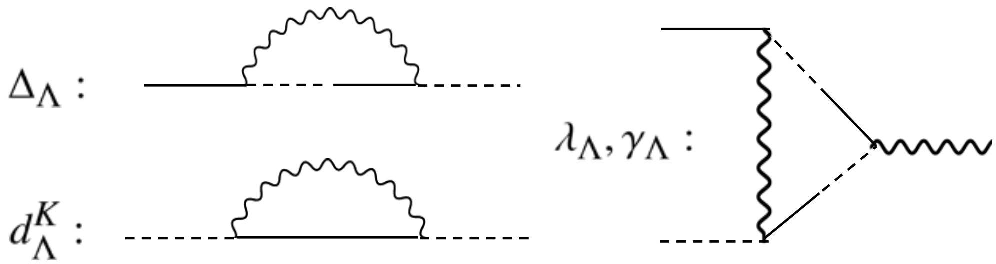


$$\chi + z = 2$$

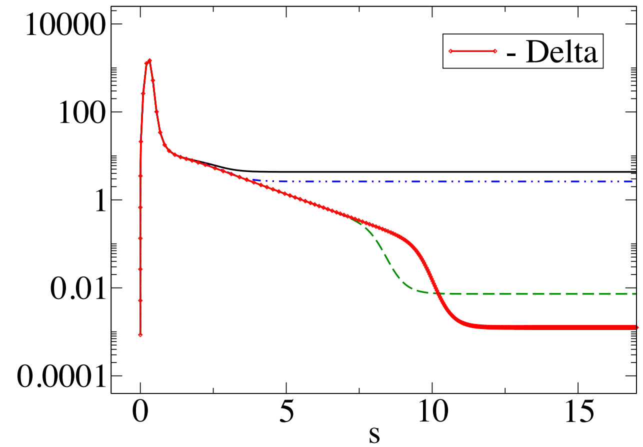
$$C(\mathbf{x},t) \equiv \langle [h(\mathbf{x}_0+\mathbf{x},t_0+t) - h(\mathbf{x}_0,t_0)]^2 \rangle$$
$$= x^{2\chi} F(t/x^z)$$



Self-organized criticality in rough phase



Smooth/massive phase:



- Rough phase: explicit cancellation in flow equations à la QED:

$$\Lambda\partial_\Lambda\tilde{\Delta} = \left(-2+\zeta_\gamma\right)\tilde{\Delta} + \tilde{\lambda}^2 D_{\lambda^2}[\tilde{\Delta}]$$

$$\Lambda\partial_\Lambda\tilde{\lambda} = \left(\frac{d-4}{2} + \frac{d}{4}\zeta_A + (1-\frac{d}{4})\zeta_\gamma - \zeta_\lambda\right)\tilde{\lambda}$$

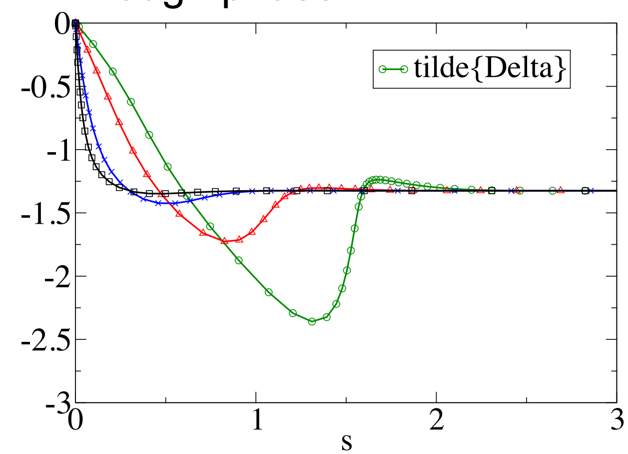
$$\zeta_A = 0$$

$$\zeta_{d^K} = \tilde{\lambda}^2 S_{\lambda^2}[\tilde{\Delta}]$$

$$\zeta_\gamma = \tilde{\lambda}^2 G_{\lambda^3}[\tilde{\Delta}] = \zeta_\lambda$$

$$\Lambda\partial_\Lambda\tilde{\lambda} = \left(\frac{d-4}{2} - \frac{d}{4}\zeta_\gamma\right)\tilde{\lambda} \qquad \zeta_\gamma = \frac{2(d-4)}{d}$$

Rough phase:



Line of fixed points, UV-robust vanishing of mass

KPZ-Lieb-Liniger duality beyond integrability/different symmetries?

Interaction quench in a Lieb-Liniger model and the KPZ equation with flat initial conditions

Pasquale Calabrese

] 6 Feb 2014

Glimmers of a Quantum KAM Theorem: Insights from Quantum Quenches in One Dimensional Bose Gases

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Real-time dynamics in a quantum many-body system are inherently complicated and hence difficult to predict. There are, however, a special set of systems where these dynamics are theoretically tractable: integrable models. Such models possess non-trivial conserved quantities beyond energy and momentum. These quantities are believed to control dynamics and thermalization in low dimensional atomic gases as well as in quantum spin chains. But what happens when the special symmetries leading to the existence of the extra conserved quantities are broken? Is there any memory of the quantities if the breaking is weak? Here, in the presence of weak integrability breaking, we show that it is possible to construct residual quasi-conserved quantities, so providing a quantum analog to the KAM theorem and its attendant Nekhoreshev estimates. We demonstrate this construction explicitly in the context of quantum quenches in one-dimensional Bose gases and argue that these quasi-conserved quantities can be probed experimentally.

5 Jul 2014

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Equilibration of a Tonks-Girardeau Gas Following a Trap Release

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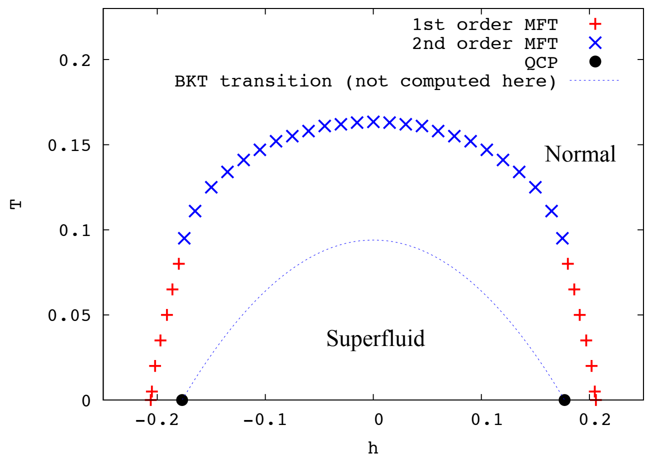
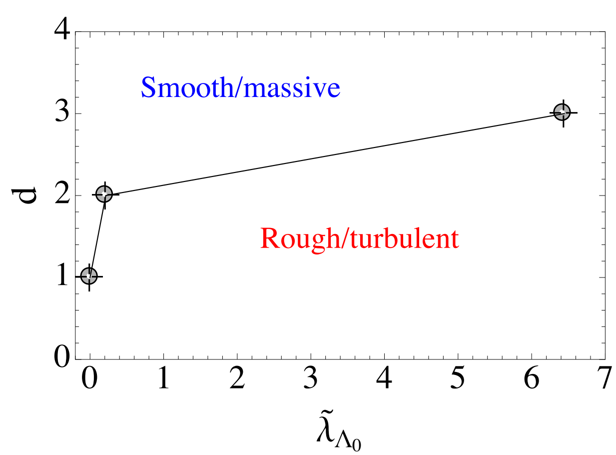
Summary



- Imbalanced Fermi gases in 2d
- Breakdown of homogeneous superfluidity
- Effective potential flow with fermionic mean-field as initial condition
- Potential quantum criticality toward Sarma-Liu-Wilczek phase
- Outlook on Larkin-Ovchinikov transitions



- KPZ interfaces dual to attractive Lieb-Liniger bosons in 1d
- Break Galilean invariance/integrability
- 1-loop flow with frequency cutoff technique
- Hyperthermal, self-organized phase
- Outlook on equilibration after quench

Further info: http://users.physics.harvard.edu/~pstrack/