Nonperturbative dynamics of scalar fields in de Sitter space

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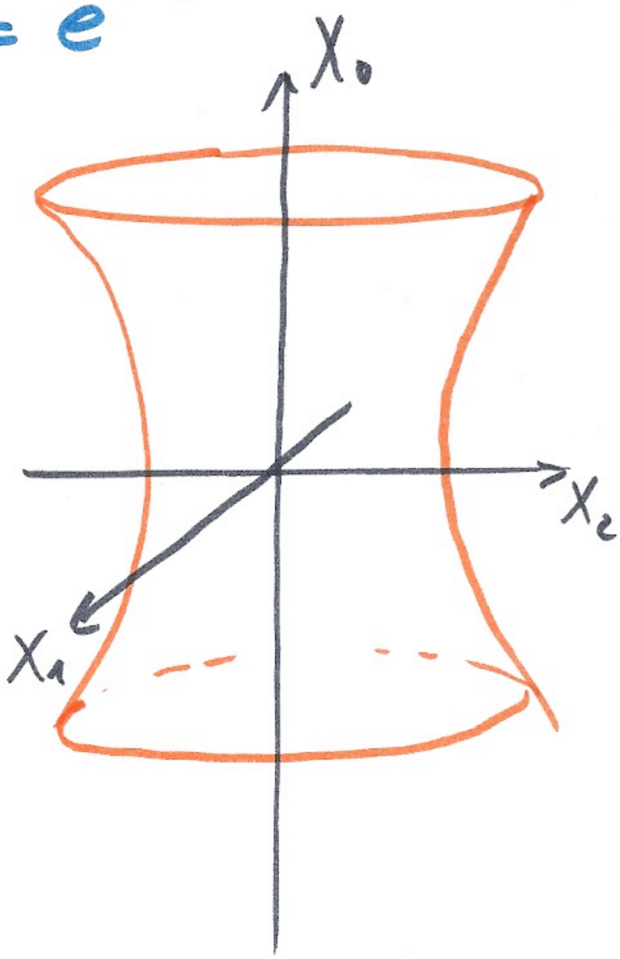
Motivations

- Radiative corrections to inflationary dynamics
- (Analog) black hole radiation
- Curvature-induced phase transitions
- Foundations of QFT in curved space-times
- ⋮

Scalar fields in de Sitter space (I)

$$ds^2 = -dt^2 + \bar{a}^2(t)\, d\vec{X}^2$$

$$\bar{a}(t) = e^{Ht}$$



$$\boxed{d\eta = dt/\bar{a}(t)}$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\vec{X}^2\right)$$

spatially homogeneous
but nonstationary

$$\boxed{\vec{x} = a(t)\vec{X}}$$

$$ds^2 = -(1-x^2)dt^2 - 2\vec{x}.d\vec{x}\,dt + d\vec{x}^2$$

stationary but inhomogeneous

Scalar fields in dS space (II)

$$S = \int d^D x \sqrt{-g(x)} \left(\frac{1}{2} \phi \Box \phi - \frac{m^2}{2} \phi^2 \right)$$

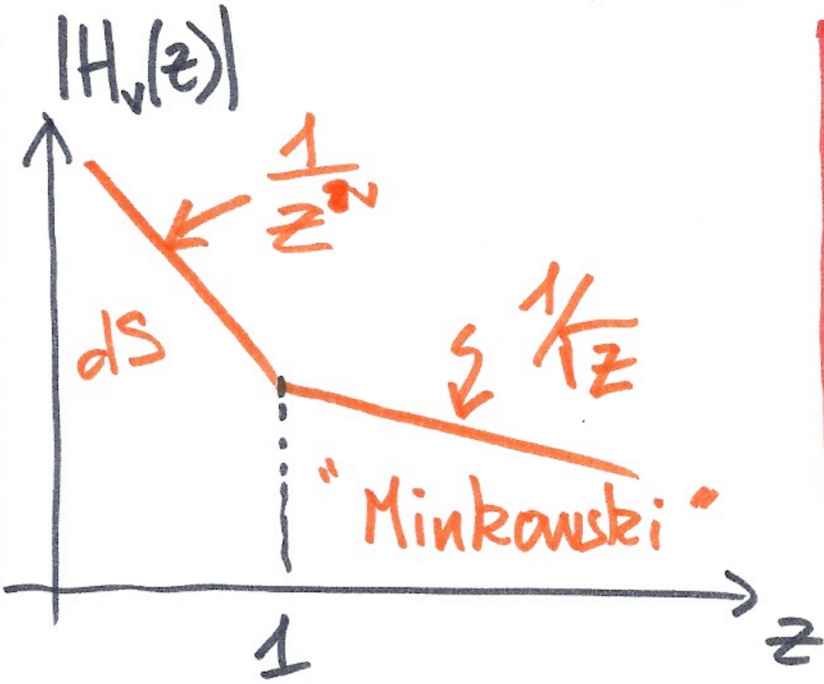
$$D = d + 1 \; ; \; \Box = \frac{1}{a^2(\eta)} \left(-\partial_\eta^2 + \frac{d-1}{\eta} \partial_\eta + \vec{\nabla}_x^2 \right)$$

$$\boxed{(-\Box + m^2)\phi = 0}$$

$$\phi(\eta, \vec{x}) \sim \int \frac{d^d k}{(2\pi)^d} \left(e^{i \vec{k} \cdot \vec{x}} H_\nu\left(\frac{k}{a(\eta)} \right) a_k + h.c. \right)$$

$$\nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}$$

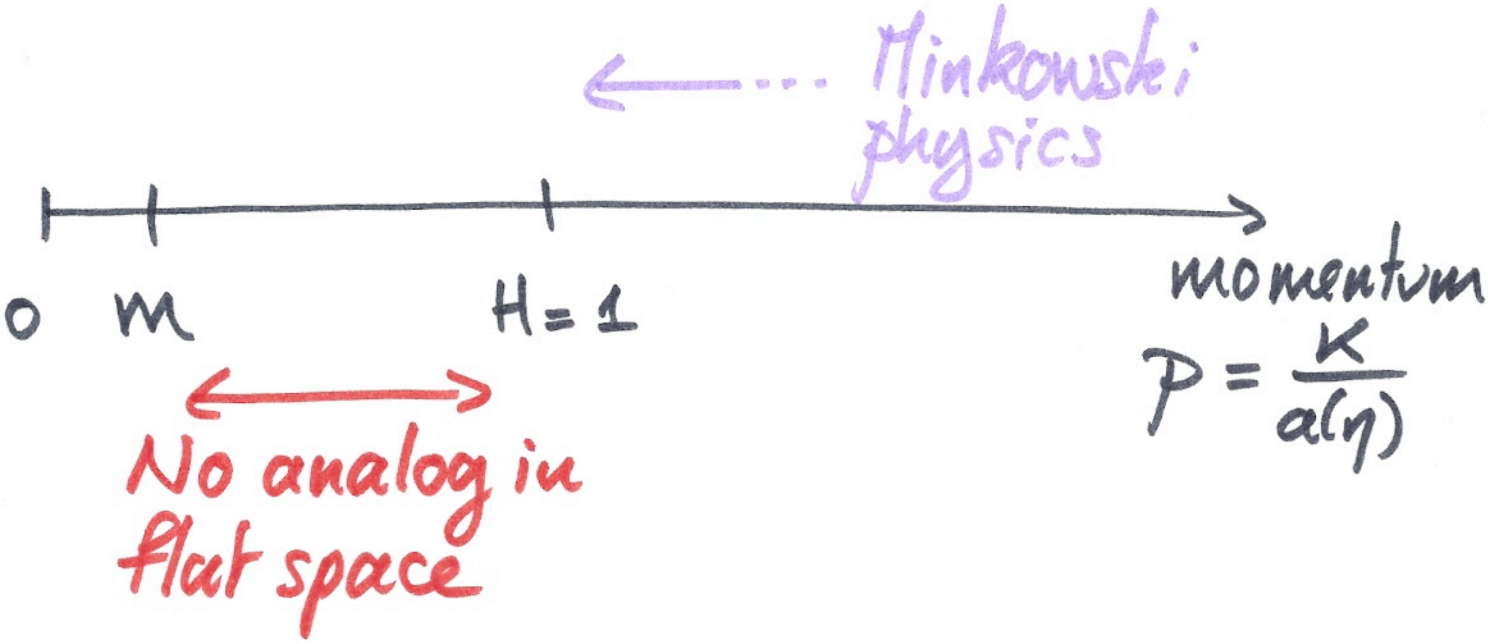
Redshift.



Stationary gravitational redshift leads to strong infrared (IR) fluctuations

Scalar fields in dS (III)

The case of light fields $m \ll H = 1$



Loop corrections :

 $\sim \frac{H^4}{m^2}$: IR divergencies

 $\sim H^2 \ln\left(P/H\right)$: large logs (secular divergencies)

$\Rightarrow$ **NEED FOR RESUMMATION**

Resummation methods in dS

difficulty : Nonequilibrium system

- Stochastic approach
  [Starobinsky, Yokoyama, ('94)]
- Dynamical R.G.
  [Burgess et al ('10)]
- Euclidean dS
  [Rajaraman ('10), Beneke ('12)]
- Wigner-Weisskopf method
  [Boyanovsky ('12)]
- Large-N
  [Riotto, Sloth ('08), Serreau ('11)]
- Schwinger-Dyson eqns.
  [Gautier, Serreau ('13)]

static quantities so far

The p-representation (I)

[Parentani, Serreau ('13); Adamek, Busch, Parentani ('13)]

In principle $G(x,x') = \langle \phi(x)\phi(x') \rangle \equiv \hat{G}(z)$

$z \equiv$ dS-invariant distance

BUT : difficult to implement in practice
[see however : Youssef, Kreimer ('13)]

Momentum representations

- $ds^2 = a^2(\eta)(-d\eta^2 + d\vec{X}^2)$

$$G(x,x') \sim \int \frac{d^dK}{(2\pi)^d} e^{i\vec{K}.\vec{X}} \boxed{G_c(\eta,\eta',K)}$$

- $ds^2 = -(1-x^2)dt^2 - 2\vec{x}.d\vec{x}\,dt + d\vec{x}^2$

$$G(x,x') \sim \int \frac{d^dp}{(2\pi)^d} \frac{d^dp'}{(2\pi)^d} e^{i\vec{p}.\vec{x} + i\vec{p}'.\vec{x}'} \boxed{G_p(t-t',p,p')}$$

Scaling law : $G_c(\eta,\eta',K) = \frac{1}{K}\hat{G}(p,p')$

$p = K/a(\eta)$

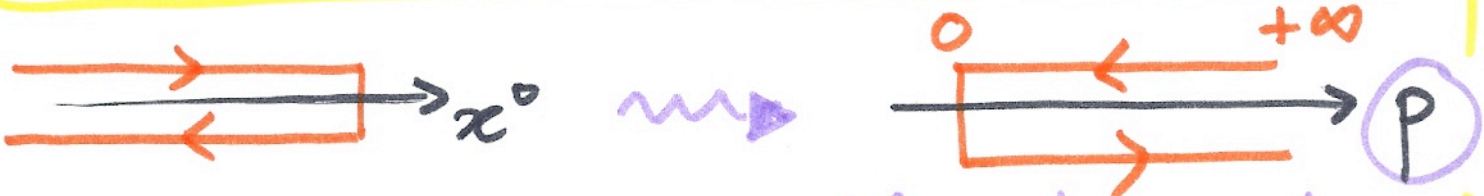
The p-representation (II)

example : Schwinger-Dyson equation

$$\left(-\Box + m^2\right) G(x,x') = -i\,\delta^{(D)}(x,x') - i\int_z \Sigma(x,z)\,G(z,x')$$

$$\int_z \equiv \int d^D z \sqrt{-g(z)}$$

+ standard closed-time-path techniques



Momentum contour

$$\boxed{\hat{G}(p,p') = F(p,p') - \frac{i}{2}\,\mathrm{sign}_c(p-p')\,\rho(p,p')}$$

$$\left[\partial_p^2 + 1 + \frac{\nu^2 - 1/4}{p^2}\right] F(p,p') = -\int_p^{+\infty} dp''\, \Sigma_\rho(p,p'')\,F(p'',p') + \int_{p'}^{+\infty} dp''\, \Sigma_F(p,p'')\,\rho(p'',p')$$

+ similar eq. for $\rho(p,p')$

The p-representation (III)

$$G_c(\eta,\eta';K) = \frac{1}{K}\hat{G}(p,p') \quad , \quad p = \frac{K}{a(\eta)}$$

✓ Gravitational redshift accounted for

✓ Reduces to a dynamical 0+1 d pb.

✓ SD eqns $\Leftrightarrow$ momentum-flow-like equations

⇒ **Efficient formulation of resummation / nonpert. techniques**

- Analytical solutions of SD eqs. with $\Sigma^{\text{two-loop}}$ or Σ^{NLO} in $1/N$ [Serreau, Parentani ('13); Gautier, Serreau ('15)]
- Nonperturbative renormalization group [Serreau ('14); Guilleux, Serreau (?)]

Phase structure of O(N) theories in dS space

Strong IR fluctuations restore spontaneously broken symmetries <u>in any spacetime dim.</u>

✓ O(2) [Ratra ('85)]

✓ O(∞) [Serreau ('11)]

What about N=1 ; 2<N<∞ ?

- perturbation theory
- Hartree approx.
- Wigner-Weisskopf

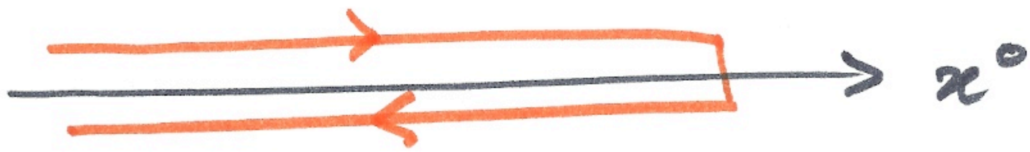
⇒ first order phase transition with H + massive "Goldstone modes"

Non perturbative R.G. in dS space (I)

[Kaya ('13), Serreau ('14)]

Flow equation on the closed time-contour:

[see also: Gasenzer, Pawlowski ('08)
Berges, Mesterhazy ('12)]



$$S \longrightarrow S + \Delta S_\kappa$$

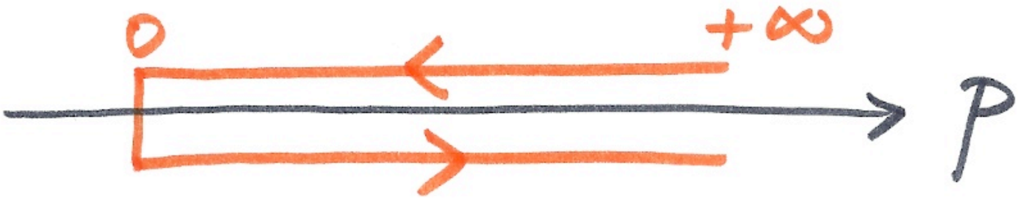
$$\Delta S_\kappa = \frac{1}{2}\int_{x,y} \phi(x) R_\kappa(x,y) \phi(y)$$

$$\dot{\Gamma}_\kappa[\phi] = \frac{1}{2} \mathrm{Tr}\left\{ \dot{R}_\kappa \cdot G_\kappa[\phi] \right\}$$

$$G_\kappa[\phi] = i \left(\Gamma_\kappa^{(2)}[\phi] + R_\kappa \right)^{-1}$$

NPRG in dS space (II)

p-representation : contour in momentum



$$\boxed{\hat{R}_k(p,p') = \frac{\delta_c(p-p')}{p^2} R_k(p)}$$

ex: flow of the effective potential

$$\boxed{\dot{V}_k(\phi) = \frac{1}{2}\int \frac{d^dp}{(2\pi)^d}\, \dot{R}_k(p)\, \frac{G_k(p,p)}{p}}$$

Local potential approximation (I)

$$\Gamma_\kappa[\phi] = \int d^D x \sqrt{-g(x)} \left(\frac{1}{2}\phi \Box \phi - V_\kappa(\phi) \right)$$

$$R_\kappa(p) = (\kappa^2 - p^2)\,\theta(\kappa^2 - p^2)$$

$$\hat{G}_\kappa(p,p') = \mathrm{Re}\left\{ u_\kappa(p)\, u_\kappa^*(p') \right\}$$

- $u_\kappa'' + \left(1 - \frac{\nu_\kappa^2 - 1/4}{p^2}\right) u_\kappa = 0 \,, \quad p \geq \kappa$
- $u_\kappa'' + \frac{\kappa^2 - \nu_\kappa^2 - 1/4}{p^2}\, u_\kappa = 0 \,, \quad p \leq \kappa$

with $\nu_\kappa = \sqrt{\frac{d^2}{4} - V_\kappa''(\phi)}$

$$\boxed{\dot{V}_\kappa(\phi) = \frac{\Omega_d}{(2\pi)^d}\, \kappa^2 \int_0^\kappa dp\, p^{d-1} \, |u_\kappa(p)|^2}$$

Local potential approx (II)

- $u_k(p) = \frac{\sqrt{\pi p}}{2} e^{i\varphi_k} H_{\nu_k}(p)$, $\underline{\underline{p \geq k}}$

- $u_k(p) = \frac{\sqrt{\pi p}}{2} e^{i\varphi_k} \left\{ c_k^- \left(\frac{k}{p}\right)^{\bar{\nu}_k} + c_k^+ \left(\frac{p}{k}\right)^{\bar{\nu}_k} \right\}$

$\underline{\underline{p \leq k}}$

$\varphi_k = \frac{\pi}{2}\left(\nu_k + \frac{1}{2}\right)$; $\bar{\nu}_k = \sqrt{\nu_k^2 - k^2}$

- Continuity of u_k and u_k' at $p = k$

$\Rightarrow$ $c_k^{\pm} = \frac{1}{2}\left(H_{\nu_k}(k) \pm \frac{k}{\bar{\nu}_k} H_{\nu_k}'(k)\right)$

RG flow in the IR

$\boxed{\kappa \ll 1}$ $(H=1)$

$$H_\nu(\kappa) \simeq \frac{2^\nu \Gamma(\nu)}{i\pi\kappa^\nu}$$

$(\nu \in \mathbb{R})$

IR enhancement

$$\boxed{\dot{V}_\kappa(\phi) \simeq \frac{\Omega_d}{(2\pi)^d} F_{\bar{\nu}_\kappa} \frac{\kappa^{d+2-2\bar{\nu}_\kappa}}{d-2\bar{\nu}_\kappa}}$$

$$\bar{\nu}_\kappa = \sqrt{\nu_\kappa^2 - \kappa^2}$$

$$F_\nu = \frac{[2^\nu \Gamma(\nu)]^2}{4\pi}$$

$$\nu_\kappa = \sqrt{\frac{d^2}{4} - V_\kappa''(\phi)}$$

Dimensional reduction

$$\rho = \frac{\kappa^2}{2}\phi^2 \quad ; \quad U_\kappa(\rho) = V_\kappa(\phi)$$

$$\Rightarrow \quad V_\kappa''(\phi) = \kappa^2\left[U_\kappa'(\rho) + 2\rho U_\kappa''(\rho)\right]$$

+ assume $\quad d - 2\nu_\kappa \sim O(\kappa^2)$

$$\boxed{\dot{U}_\kappa = -2\rho U_\kappa' + \frac{2A_d}{1 + U_\kappa' + 2\rho U_\kappa''}}$$

$$\boxed{A_d = \frac{d\,\Gamma(d/2)}{8\pi^{d/2+1}}}$$

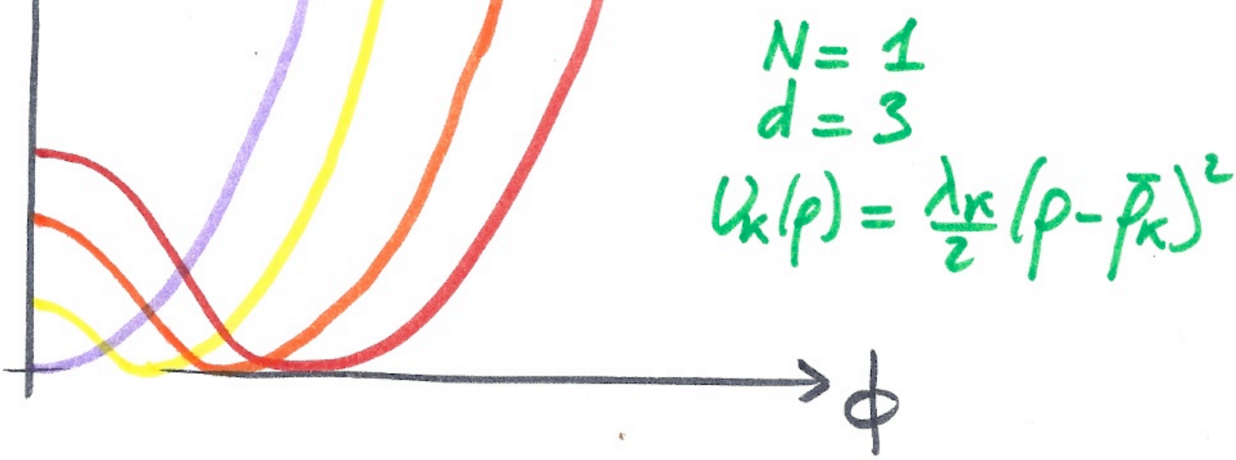
$\Rightarrow$ **Similar to Euclidean $\mathbb{R}^D$ flow with $D = 0$!!**

Symmetry Restoration

- IR enhancement leads to an effective dimensional reduction

- The dS flow in the deep IR is similar to that of the flat Euclidean theory in $D = 0$

$\Rightarrow$ $\boxed{\text{No broken phase } \forall d, \forall N}$



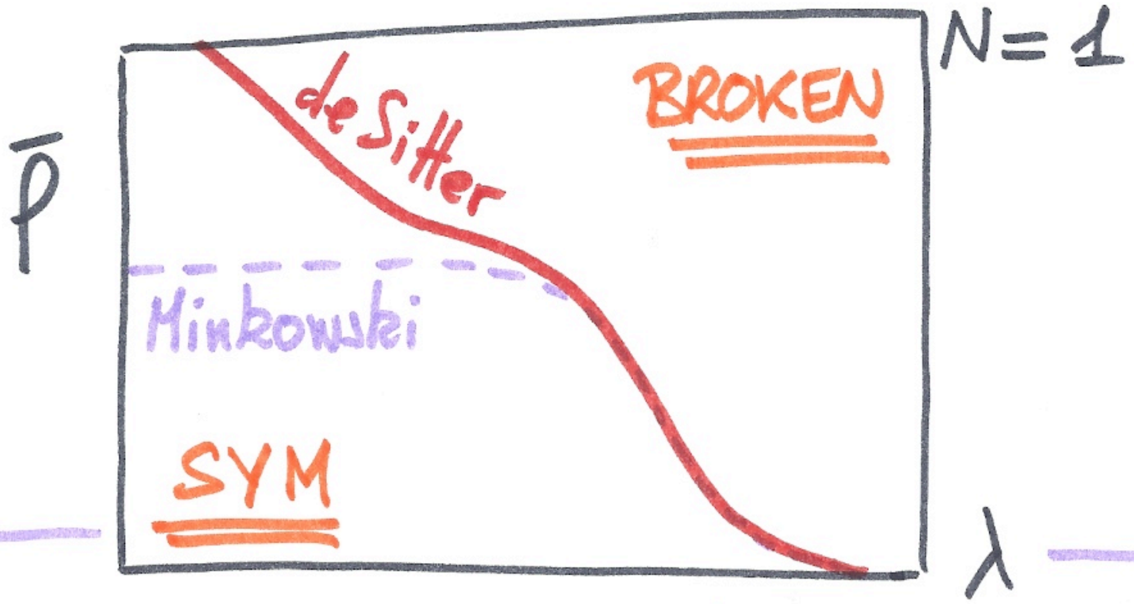
LPA : solving the full flow equation

See poster by Maxime Guilleux

- $N \geq 2$: Strong IR effects in the Goldstone sector

  $\Rightarrow$ Sym. restoration $\forall d$

- $N = 1$: A broken phase exists when the flow freezes in the UV (Mink.) regime



CONCLUSIONS

- IR physics in dS is nontrivial and interesting

- p-representation allows one to formulate powerful resummation/nonperturbative tools in dS space

- Strong IR fluctuations lead to symmetry restoration for $N \geqslant 2$ $\forall d$ and to strong modification of the phase structure for $N = 1$

PROSPECTS

- Implement a fully dS-invariant regulator

- Analysis of the phase structure of $N = 1$

- Include anomalous dimension

- BMW method in the p-representation

  $\hookrightarrow \hat{G}(p,p')$