Phase structure, Thermodynamics and Fluctuations in QCD

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Agenda

- Phase transitions and QCD
- QCD-like model studies

  ➜ chiral and deconfinement aspects

- Significance of Fluctuations

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Experiments: Heavy-Ion Collision

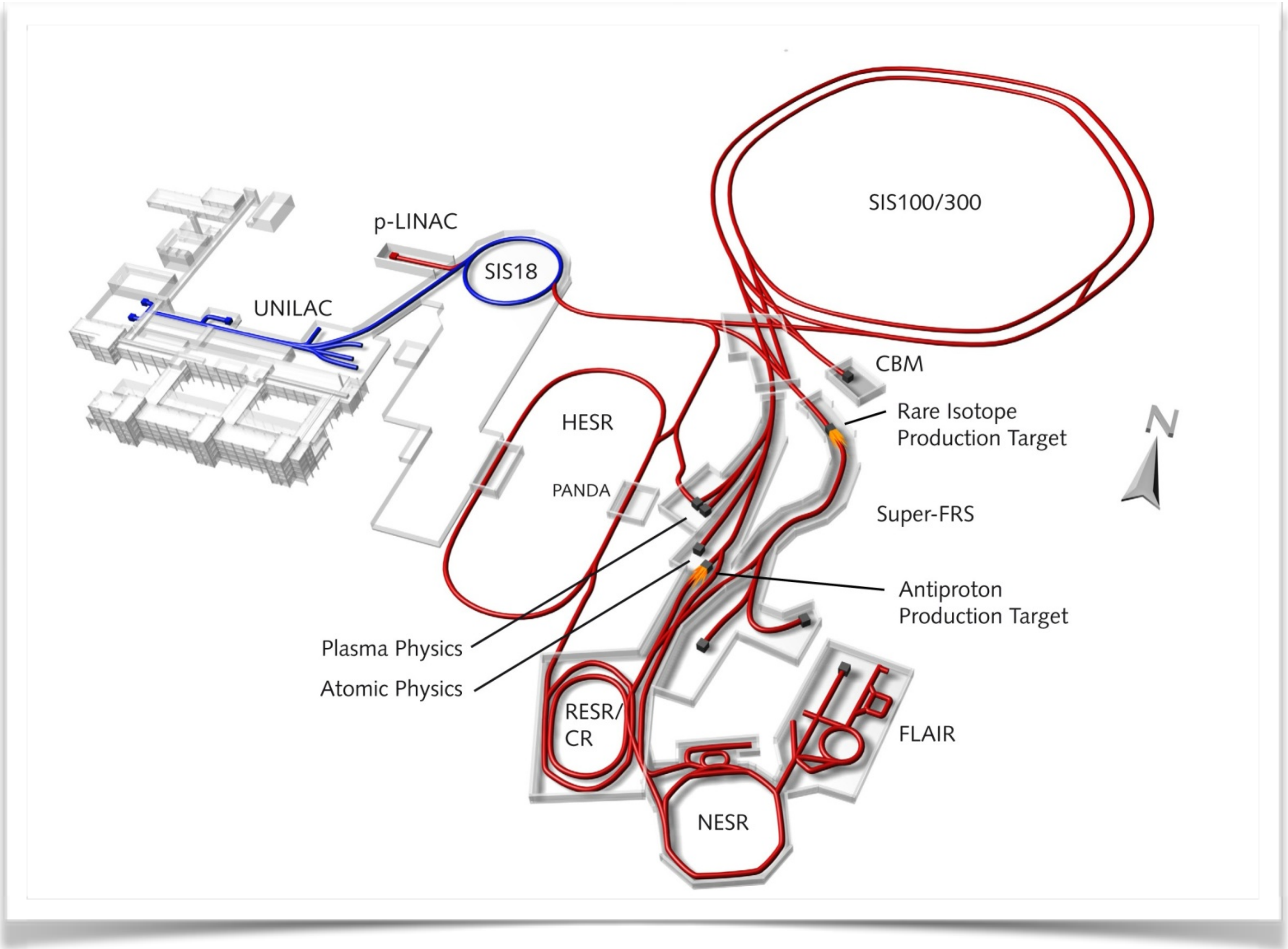
aim: create hot and dense QCD matter ➜ understanding strongly correlated systems

QCD under extreme conditions: very active field ➜ see e.g. FAIR construction (2014)

- Goals of HIC Experiments: learn QCD matter Equation of State

Understanding fundamental phenomena:

- color confinement
- nature of chiral & deconfinement transition
- early Universe history
- nuclear matter
- properties of stars
- ...



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FAIR construction start 2012 *Aug.2014*

Quantum Chromodynamics

Strongly-interacting matter: non-Abelian $SU(3)_c$ gauge theory

Lagrangian (without gauge fixing):

quark masses (input Electroweak) — chemical potentials

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}(i \not{D}_a T_a - m - \mu_f \gamma_0)\psi - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu,a}$$

quark fields — gauge fields

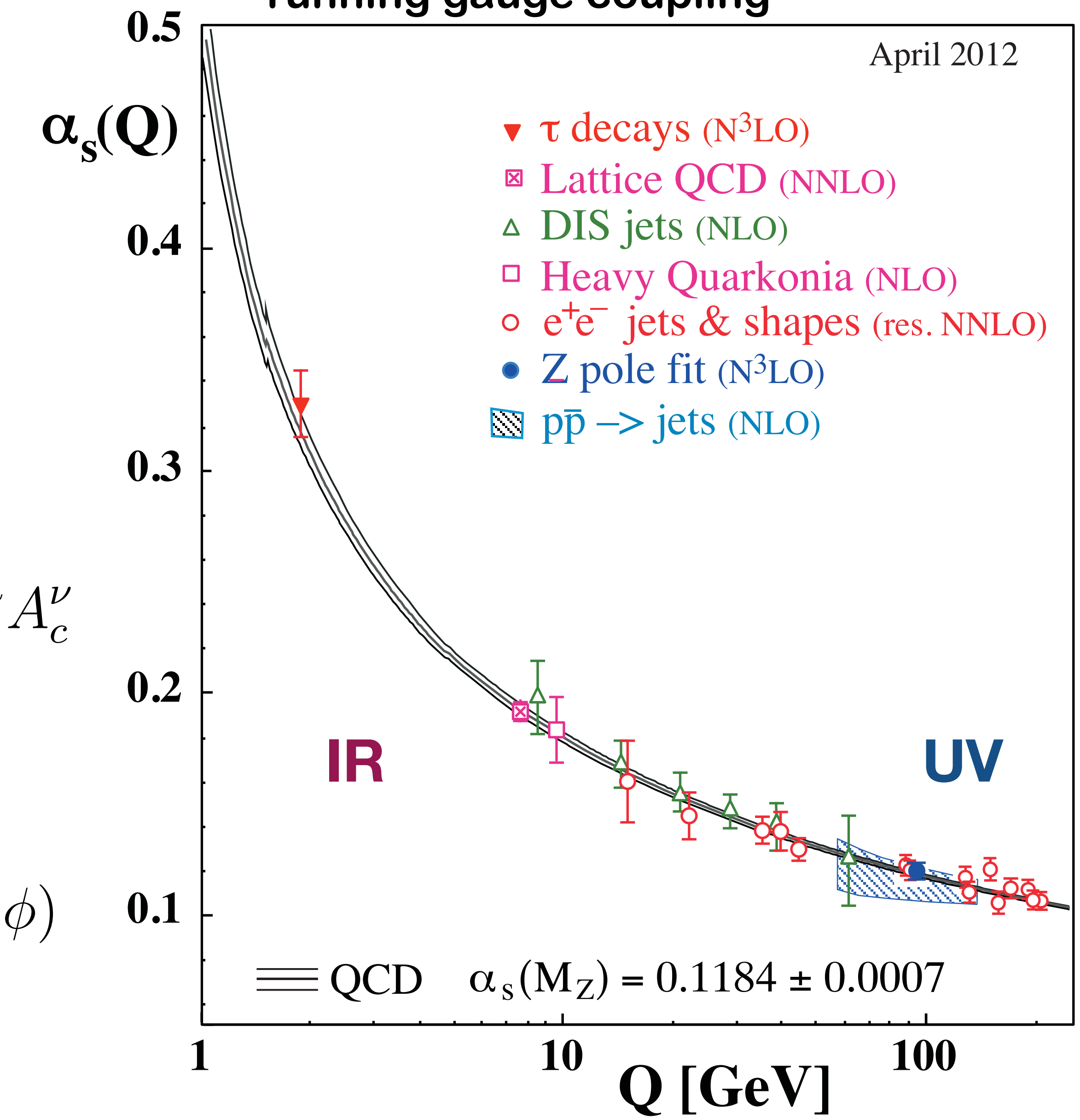
covariant derivative: $D_a^\mu = \partial^\mu + i g A_a^\mu$

gauge field tensor: $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$

Partition function:

$$\mathcal{Z}(T, \mu_f) = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\phi \, e^{-\int_0^\beta d\tau d^3x \, \mathcal{L}_{\text{QCD}}(\bar{\psi}, \psi, \phi)}$$

running gauge coupling



[S. Bethke, 2012]

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Quantum Chromodynamics

QCD at finite temperatures and densities

➜ "transitions" partial deconfinement & partial chiral symmetry restoration

For physical quark masses: smooth phase transitions ➜ deconfinement: analytic change of d.o.f.

➜ associated global QCD symmetries only exact in two mass limits:

1.) infinite quark masses ➜ center symmetry: Order parameter: VEV of traced Polyakov loop

(alternatives: dual observables, e.g. dressed Polyakov loop)
[Gattringer et al. 06/07]

$$\Phi = \langle l(\vec{x}) \rangle = \exp(-\beta F_q) \quad ; \quad \bar{\Phi} = \langle l^\dagger(\vec{x}) \rangle = \exp(-\beta F_{\bar{q}})$$

Free energy F_q of a static quark (anti-quark) in hot gluonic medium

<u>confined (disordered) phase</u>

- free energy diverges
- Polyakov loop vanishes
- correlations vanishes

<u>deconfined (ordered) phase</u>

- free energy finite
- Polyakov loop non-vanishing
- correlations finite

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2.) massless quarks ➜ chiral symmetry: **Order parameter: chiral condensate**

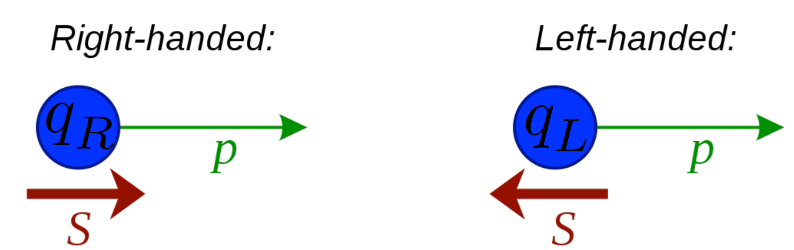
$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle$$

$$\underbrace{SU(N_f)_L \times SU(N_f)_R}_{} \times U(1)_B \times U(1)_A$$

$$\downarrow$$

$$SU(N_f)_{L+R\equiv V} \times U(1)_B$$

➜ $N_f^2 - 1$ massless Nambu-Goldstone bosons



$U(1)_A$ broken explicitly to Z_{2N_f} by quantum effects

<u>broken (ordered) phase</u>

- condensate $\langle \bar{q}q \rangle \neq 0$

<u>symmetric (disordered) phase</u>

- condensate $\langle \bar{q}q \rangle = 0$

Quantum Chromodynamics

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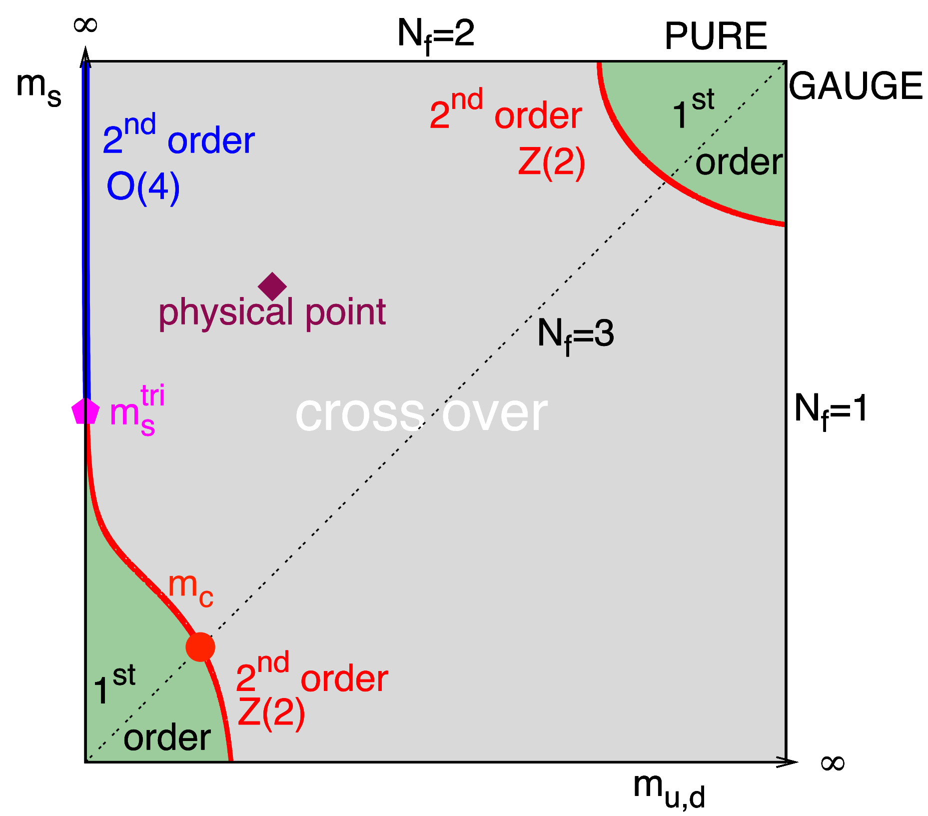
for finite quark masses: both symmetries explicitly broken



Quantum Chromodynamics

QCD at finite temperatures and densities

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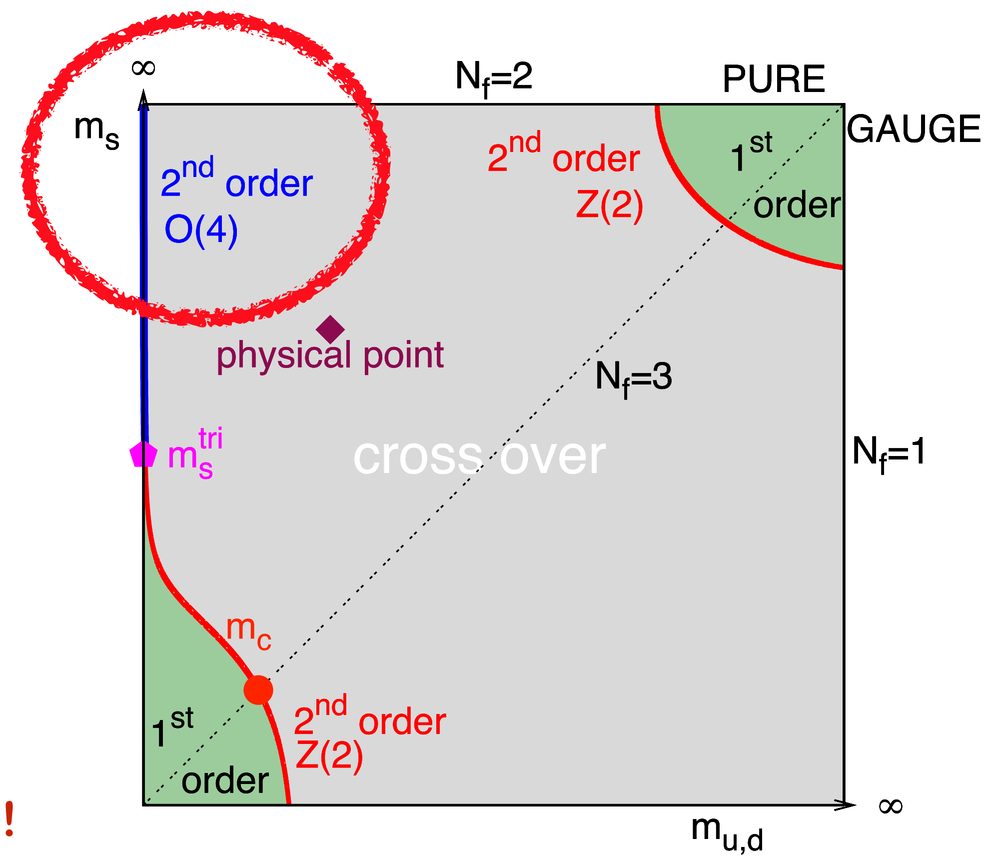


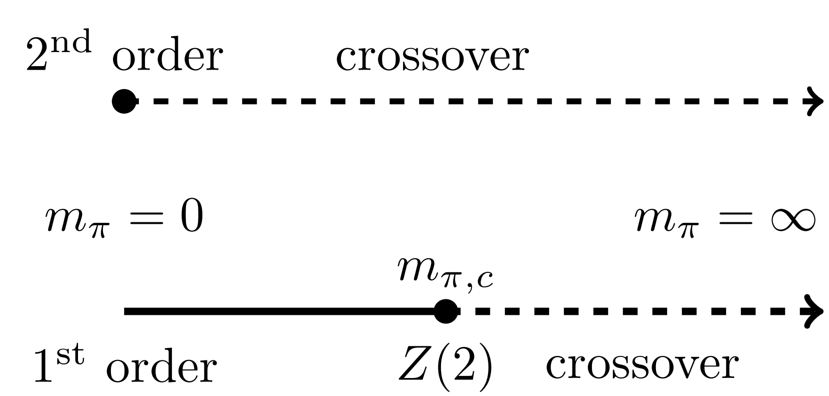
open issue: N_f=2: O(4)?

$U(2)_L \times U(2)_R / U(2)_V$?

➜ crit. exp. similar

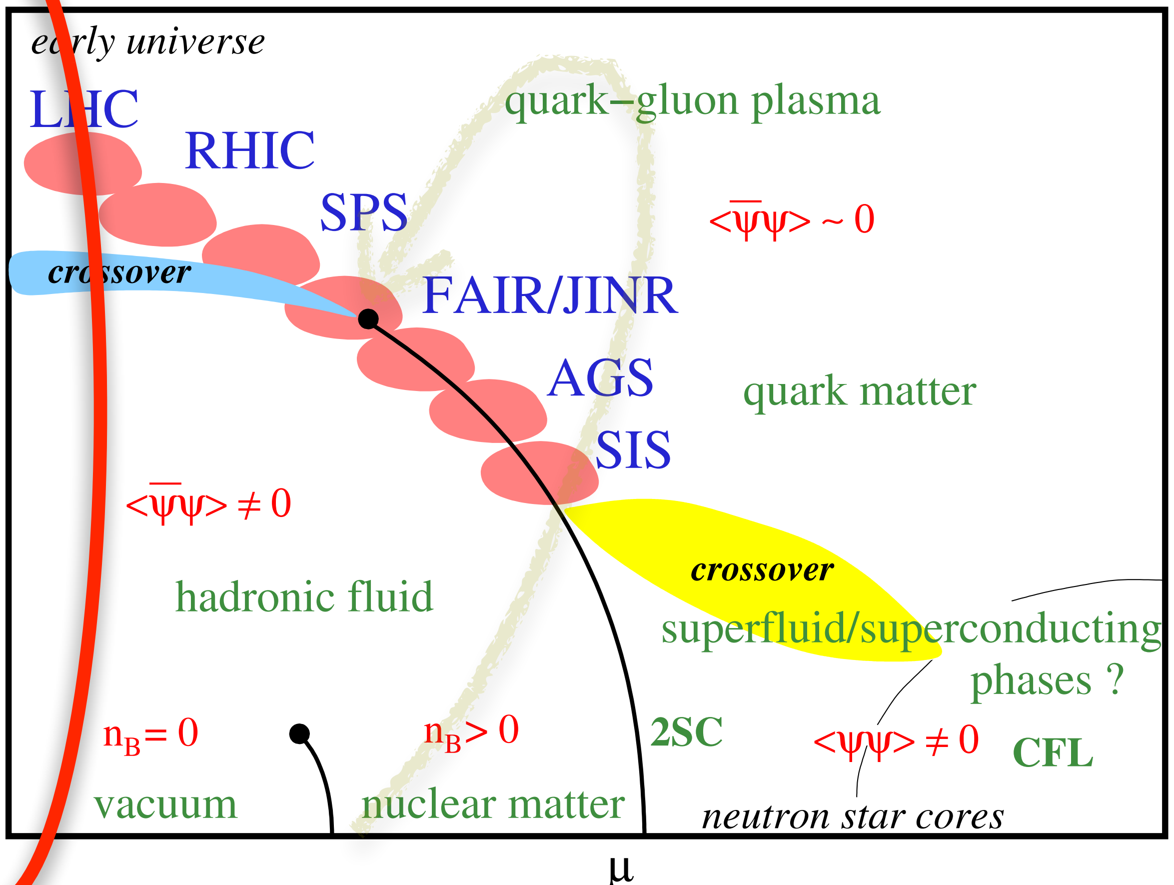
or even 1^{st} order?

dep. on strength of axial anomaly!



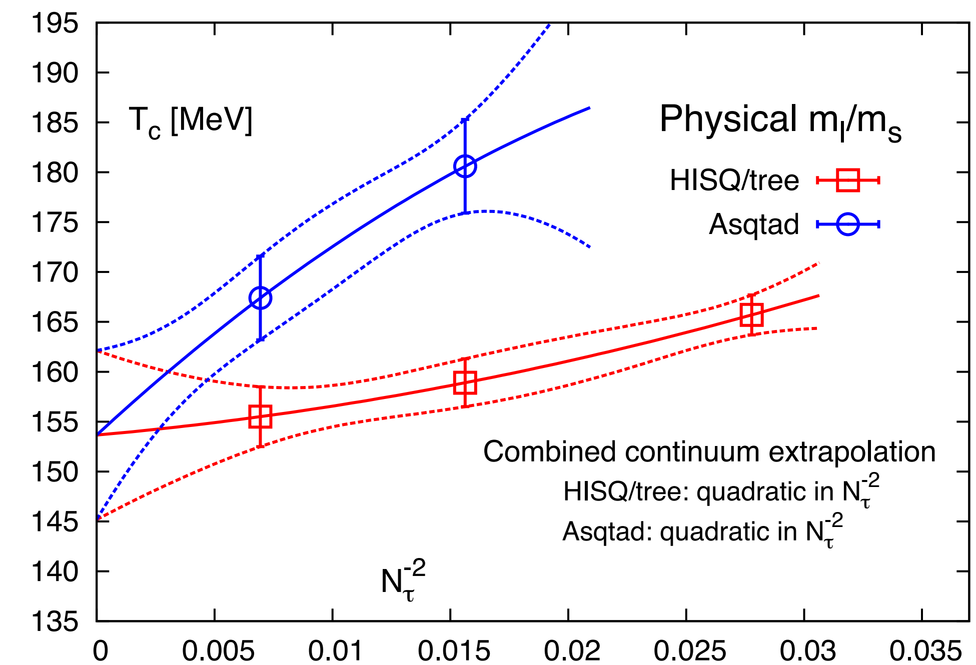
still conflicting lattice results!

Conjectured QCD phase diagram



Lattice simulations

QCD lattice simulations: no final answer



courtesy of F. Karsch

➜ can one improve the model calculations?

➜ remove model ambiguities

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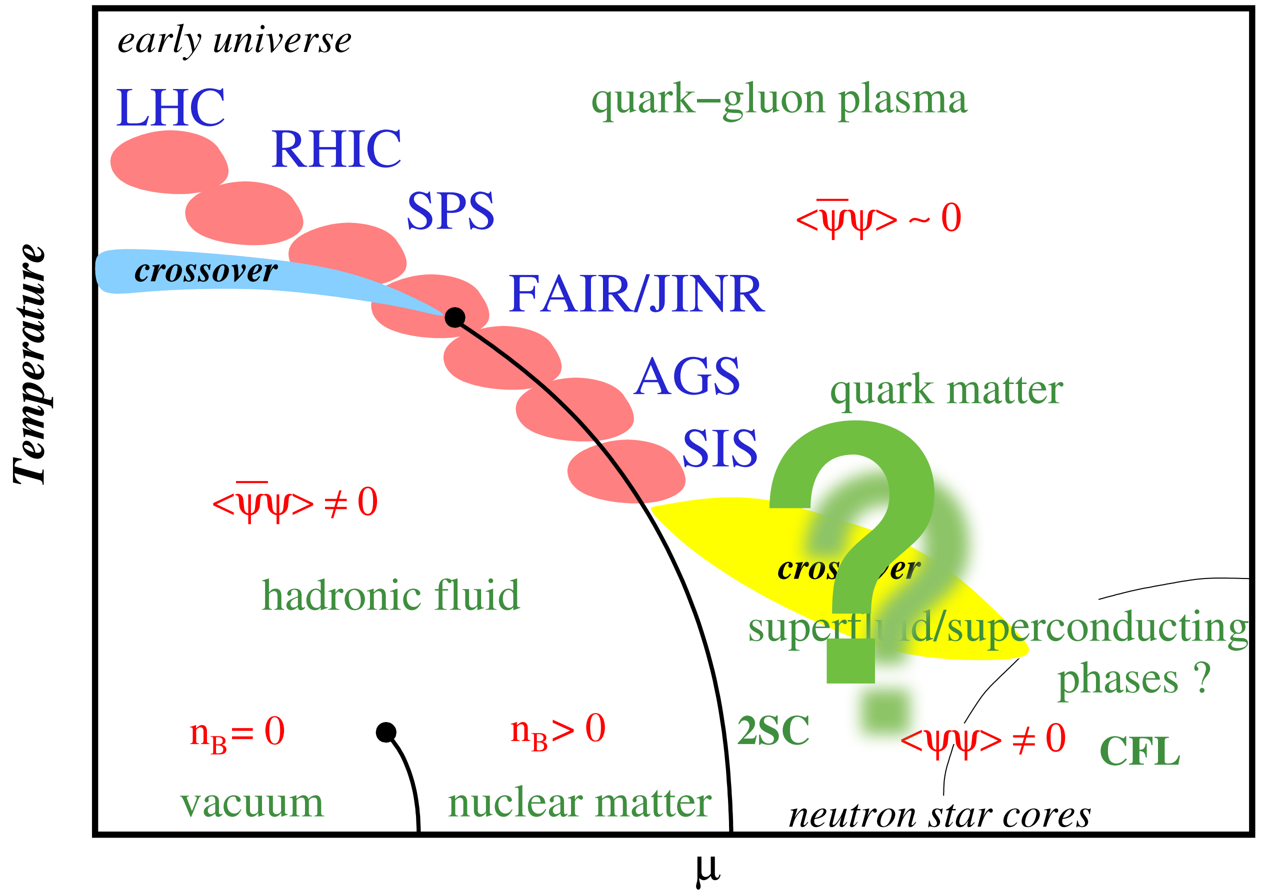
Conjectured QC$_3$D phase diagram



➜ can one improve the model calculations?

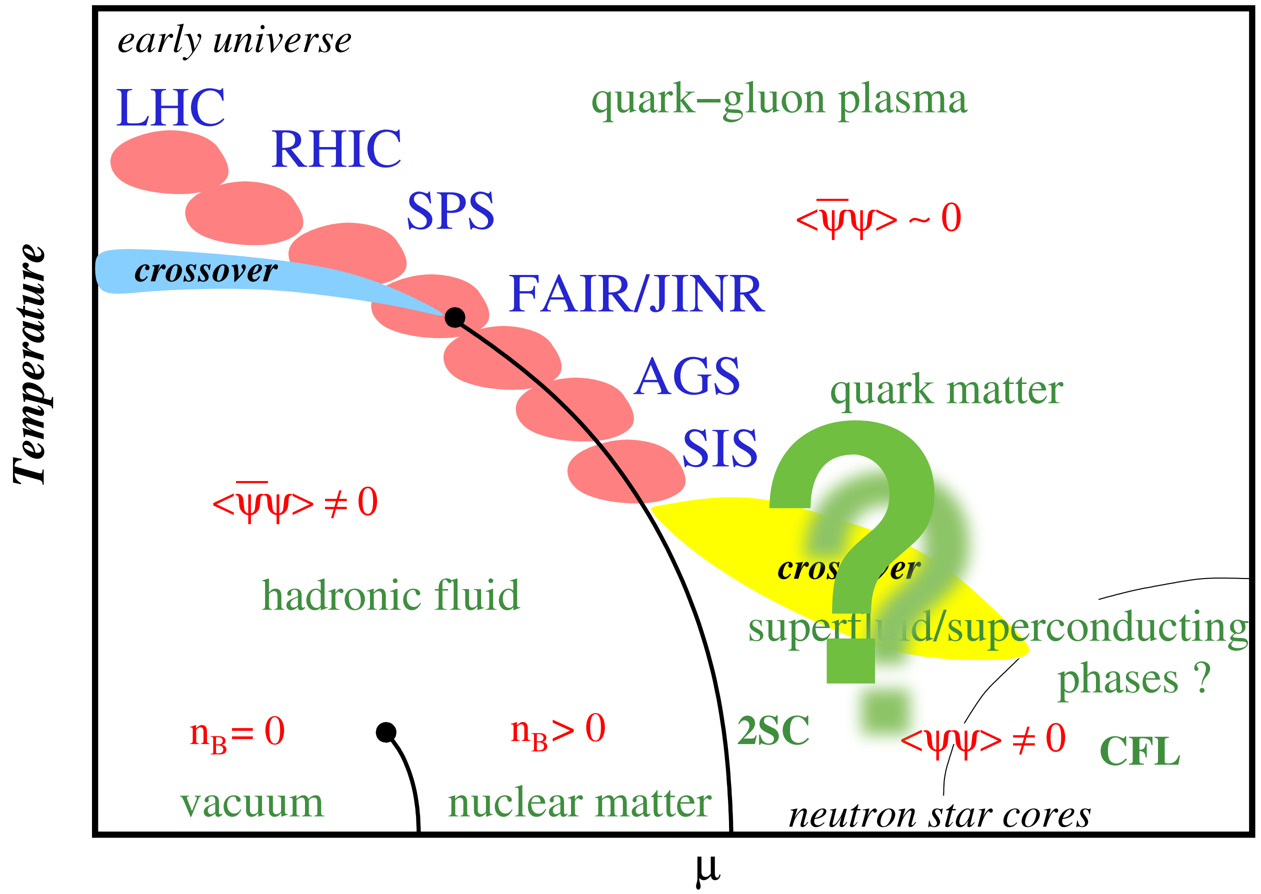
➜ remove model ambiguities

Theoretical questions: **chiral & deconfinement transition**

- **CEP:** existence/location/number
- **Quarkyonic phase:** coincidence of both transitions at $\mu = 0$ & $\mu > 0$?
- relation between chiral & deconfinement?
  chiral CEP/deconfinement CEP?
  [Braun, Janot, Herbst 12/14]
- **finite volume effects?** ➜ lattice comparison
- **inhomogeneous phases?** ➜ more favored?
- **role of fluctuations?** so far mostly mean-field results
  ➜ **effects of fluctuations are important**
  e.g. size of critical region around CEP
- **axial anomaly restoration** around chiral transition**?**
- **good experimental signatures?**

➜ higher moments more sensitive to criticality
deviation from HRG model

Conjectured QC₃D phase diagram



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deviation from HRG model

➜ can one improve the model calculations?

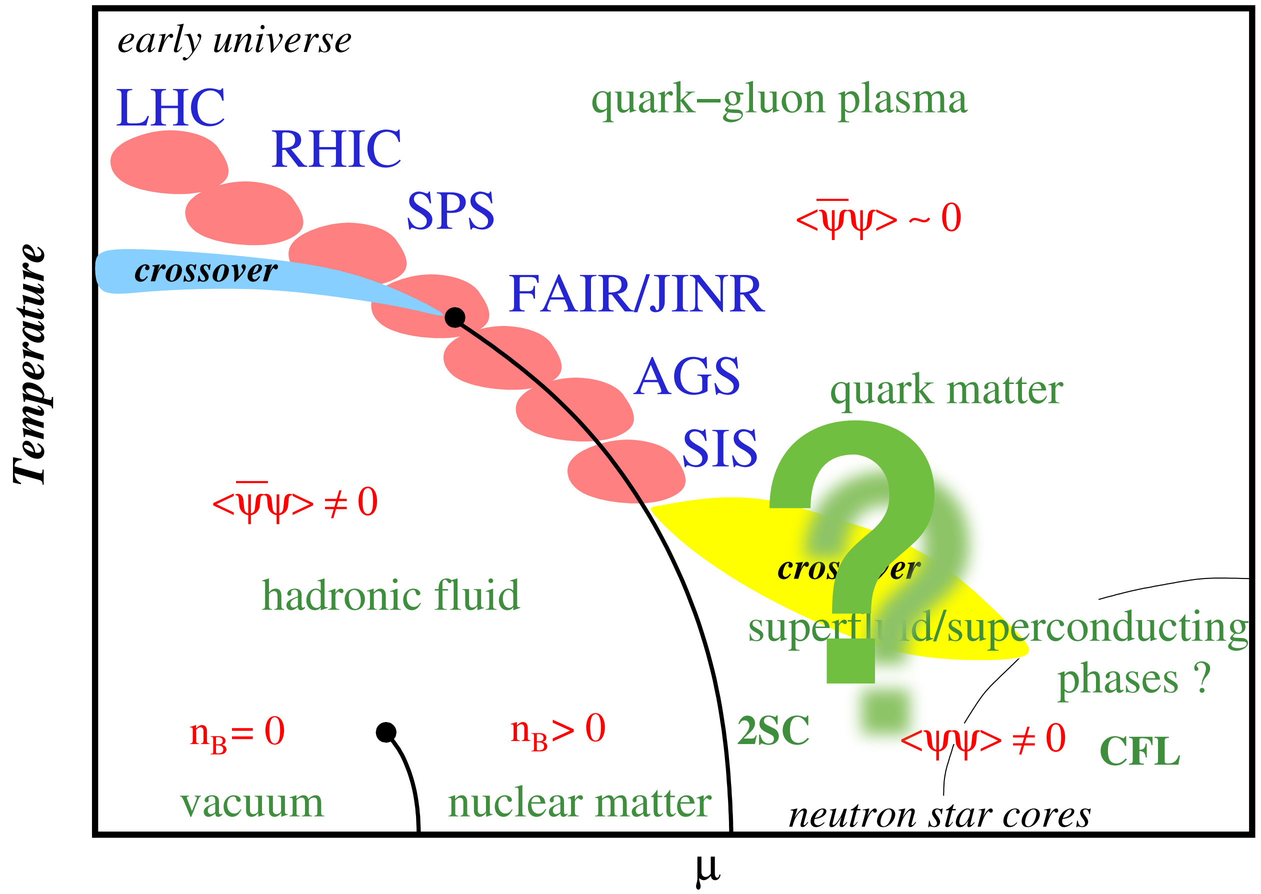
➜ **remove model ambiguities**

<u>non-perturbative continuum functional methods (DSE, FRG, nPI)</u>

➜ complementary to lattice

⇒ no sign problem μ>0 ⇒ chiral symmetry/fermions/small masses/chiral limit

Conjectured QC$_3$D phase diagram



Method of choice:
Functional Renormalization Group

e.g. (Polyakov)-quark-meson model truncation

- good description for chiral sector
- implementation of gauge dynamics
  (deconfinement sector)

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  deviation from HRG model

Chiral transition

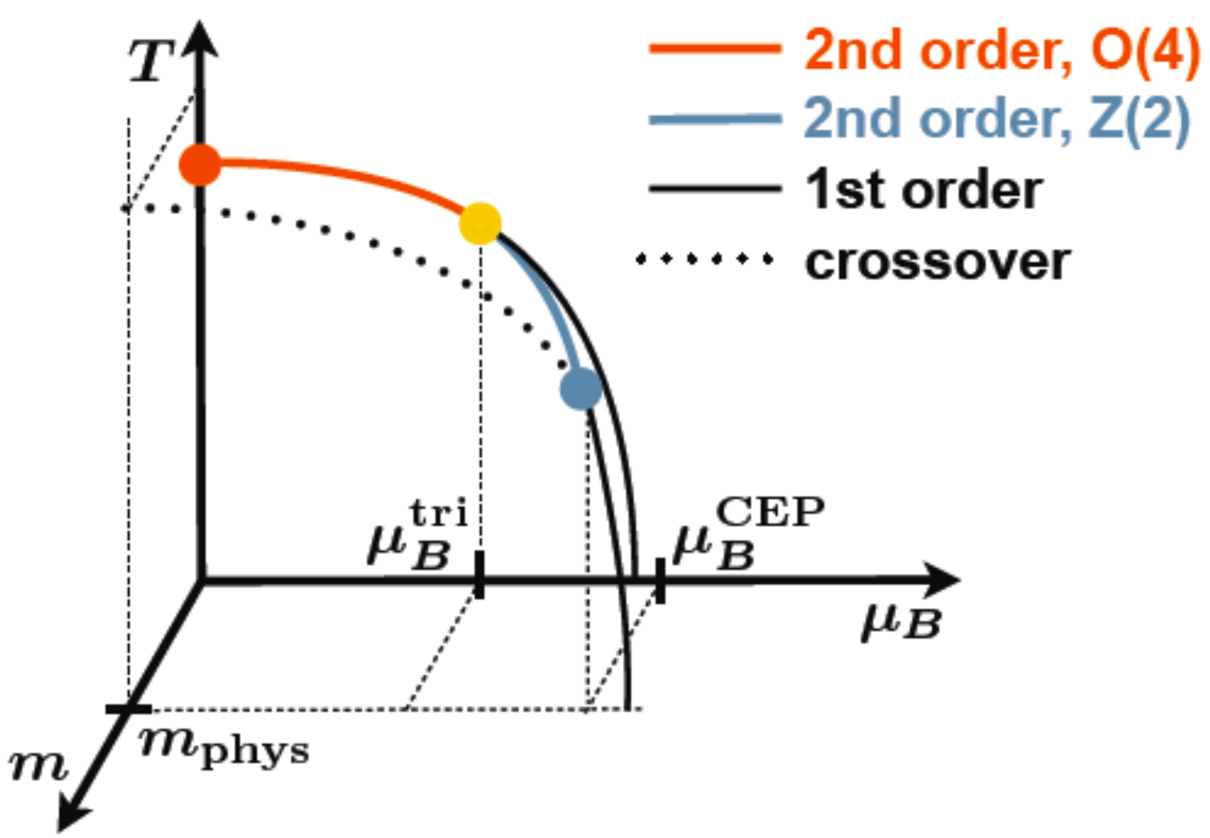
Fluctuations of oder parameter $\to \infty$ at 2nd order transition

critical fluctuations ➜ phase boundary?

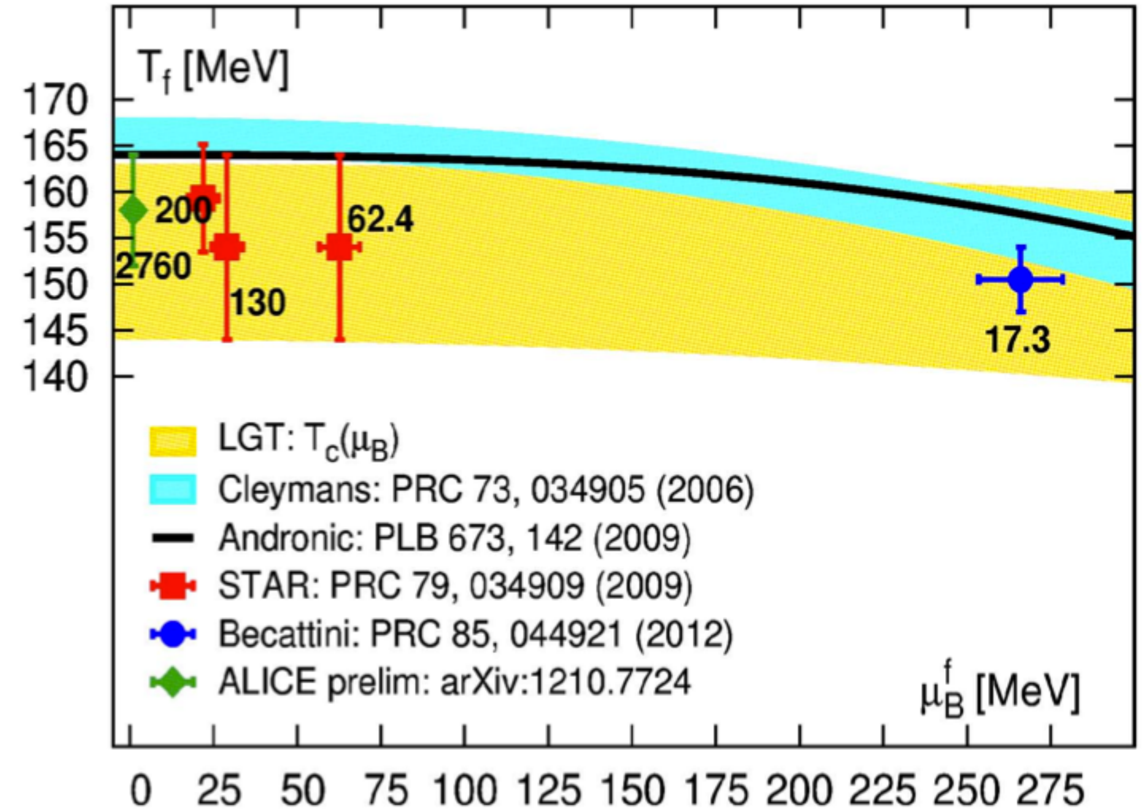
How can we probe a transition?

- singular behaviour in $\frac{\partial^n p(X)}{\partial X^n}$ with $X = T, \mu, \ldots$
- higher order cumulants $c_n \equiv \frac{\partial^n p(T,\mu)}{\partial(\mu/T)^n}$

. . . more sensitive to criticality



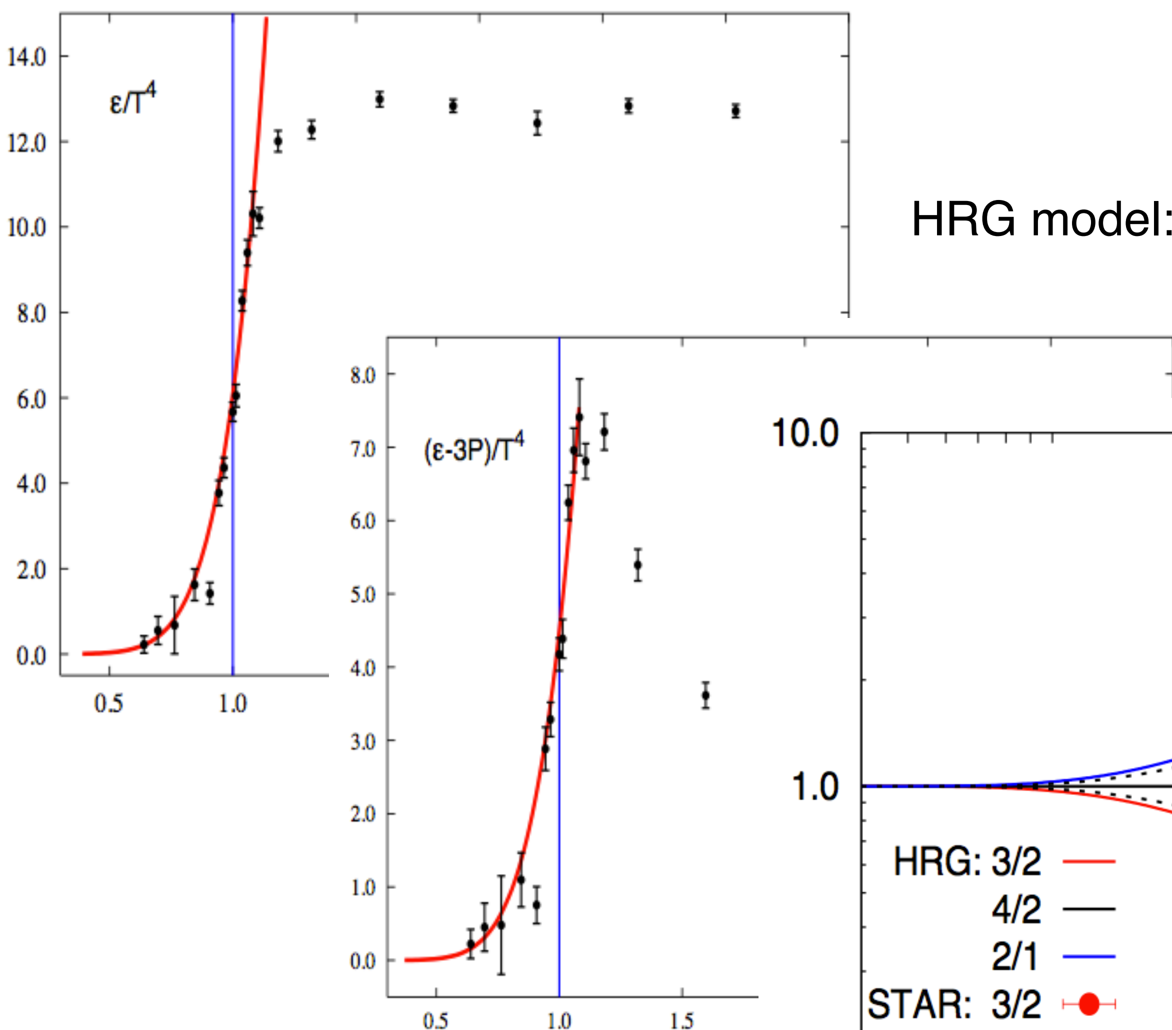
freeze-out close to chiral crossover line



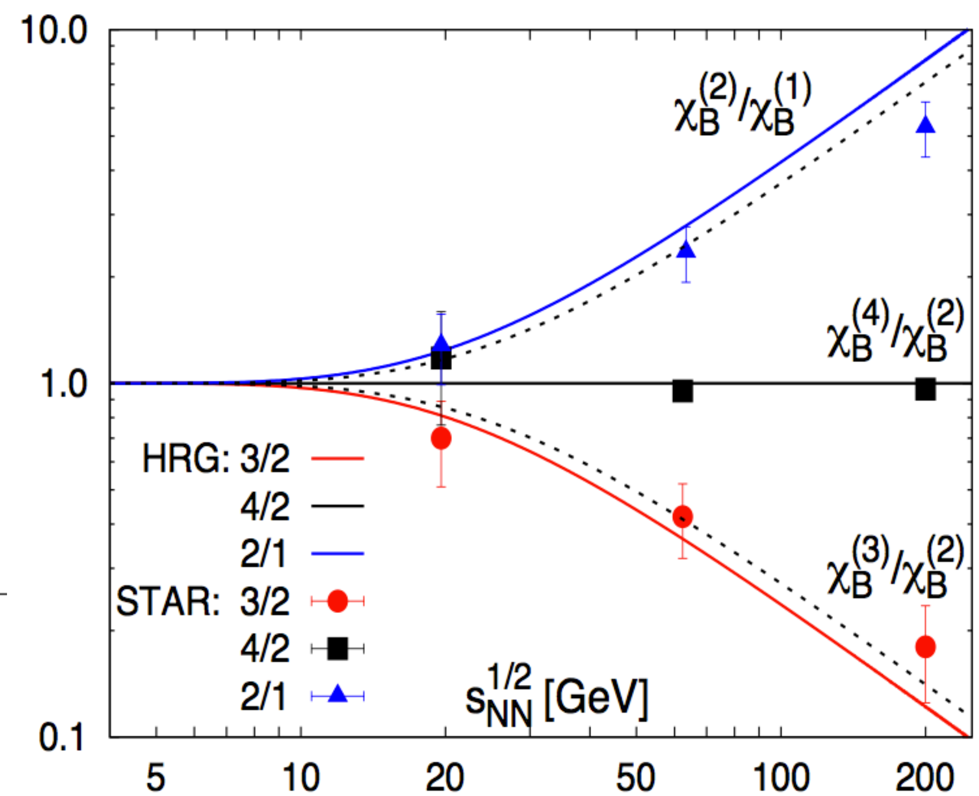
[HotQCD, QM 2012]

Hadron Resonance Gas Model

HRG model: good lattice data description



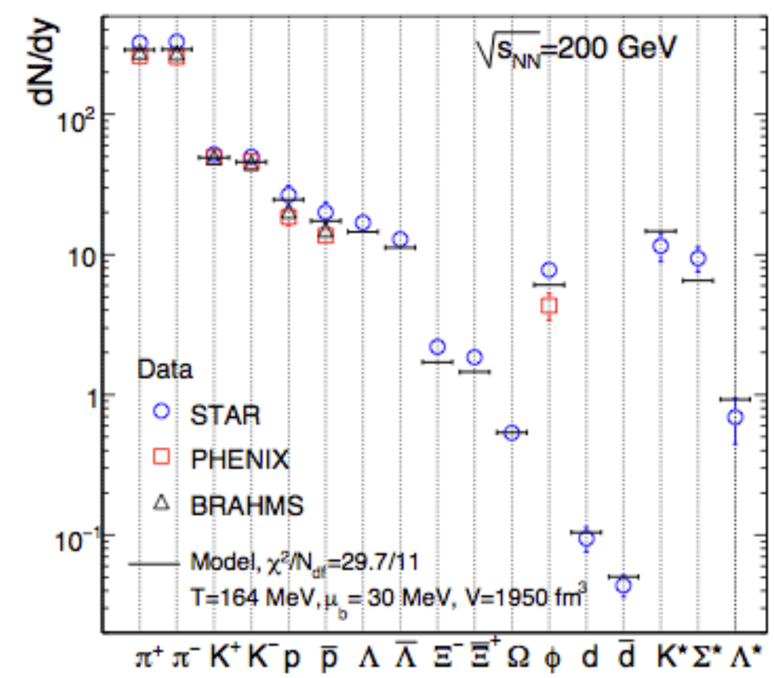
HRG model: no critical fluctuations



[Karsch, Redlich 2010]

HRG model versus experiment

[Andronic et al. 2011]



Agenda

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- **QCD-like model studies**

  ➜ **chiral and deconfinement aspects**

- **Significance of Fluctuations**

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Vacuum Fluctuations

Partition function:

$$\mathcal{Z} = \int \underbrace{\mathcal{D}\bar{\psi}\mathcal{D}\psi\cancel{\mathcal{D}\phi}}\, e^{-\int d^4x\, \mathcal{L}(\bar{\psi},\psi,\phi)}$$

replace with (const.) condensate σ

Grand potential in Mean-field approximation

$$\Omega(T,\mu;\sigma) = \Omega_{\text{vac}} + \Omega_{\text{T}} + V_{\text{MF}}(\sigma) \qquad (+\mathcal{U}_{\text{Poly}}(\Phi)\ )$$

vacuum term: regularize e.g. with sharp three-momentum cutoff

$$\Omega_{\text{vac}}(\Lambda) = -4\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^2 + m_q^2}$$

for each cutoff: adjust model parameters like f_π, m_σ, m_π

standard MFA: $\Lambda = 0$

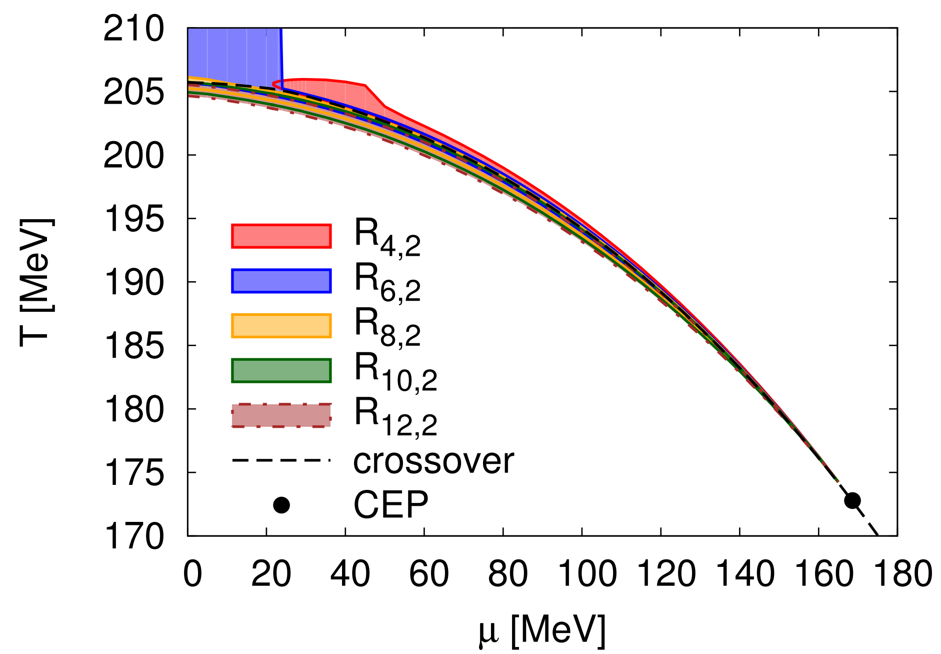
Role of fluctuations in (P)QM models

Fluctuations of higher moments exhibit **strong variation from HRG model**

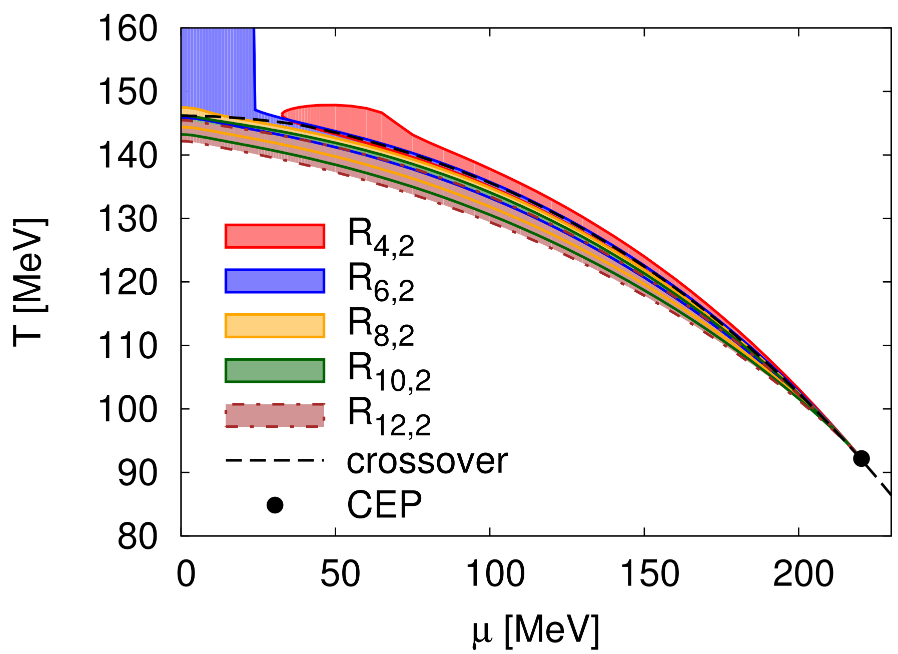
[Karsch, Redlich, Friman, Koch et al. 2011]

- $\rightarrow$ turn negative
- higher moments: $R^q_{n,m} = c_n / c_m$ c_n: Taylor expansion coefficients of pressure
- regions where $R_{n,2} < 0$ along crossover in the phase diagram

unquenched PQM MFA



QM MFA w/o vacuum



role of vacuum term in (P)QM models see

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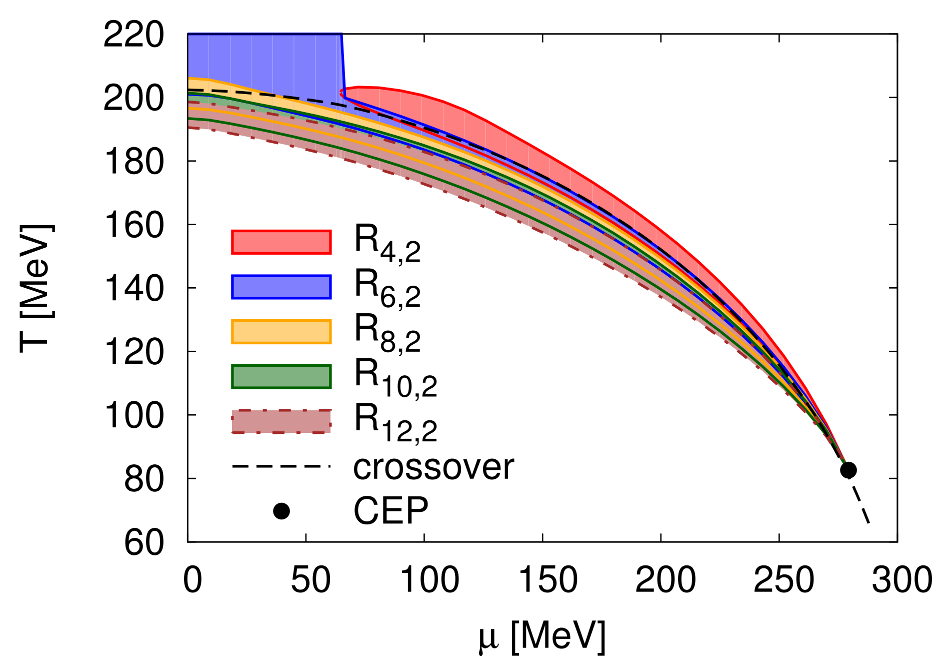
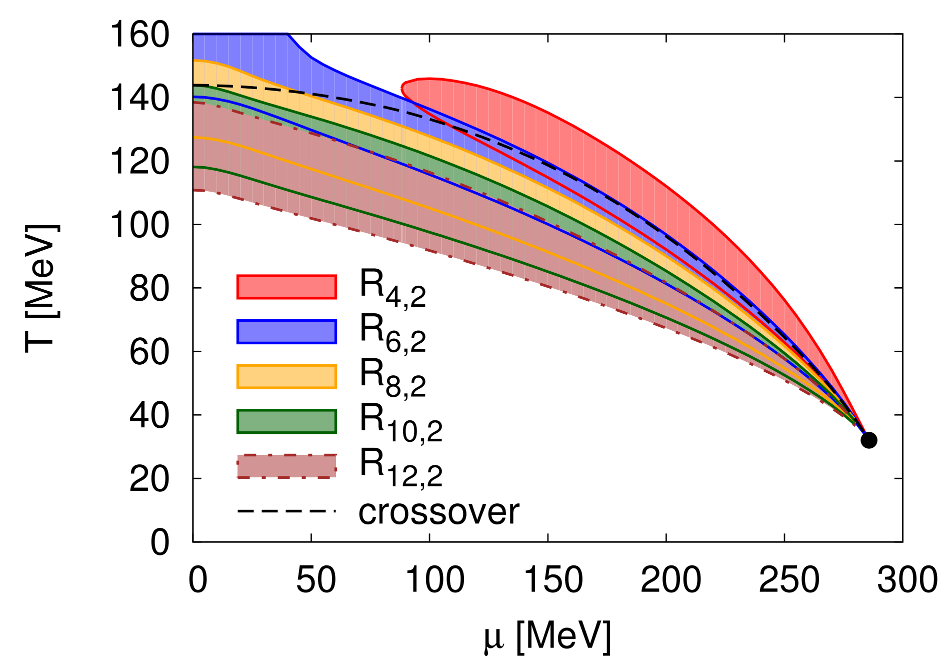
unquenched PQM MFA renormalized



QM MFA renormalized



role of vacuum term in (P)QM models see: **[BJS, Wagner 2011/12]**

Mean-Field PQM N_f=2+1

improvement of pure (YM) Polyakov-loop potential:

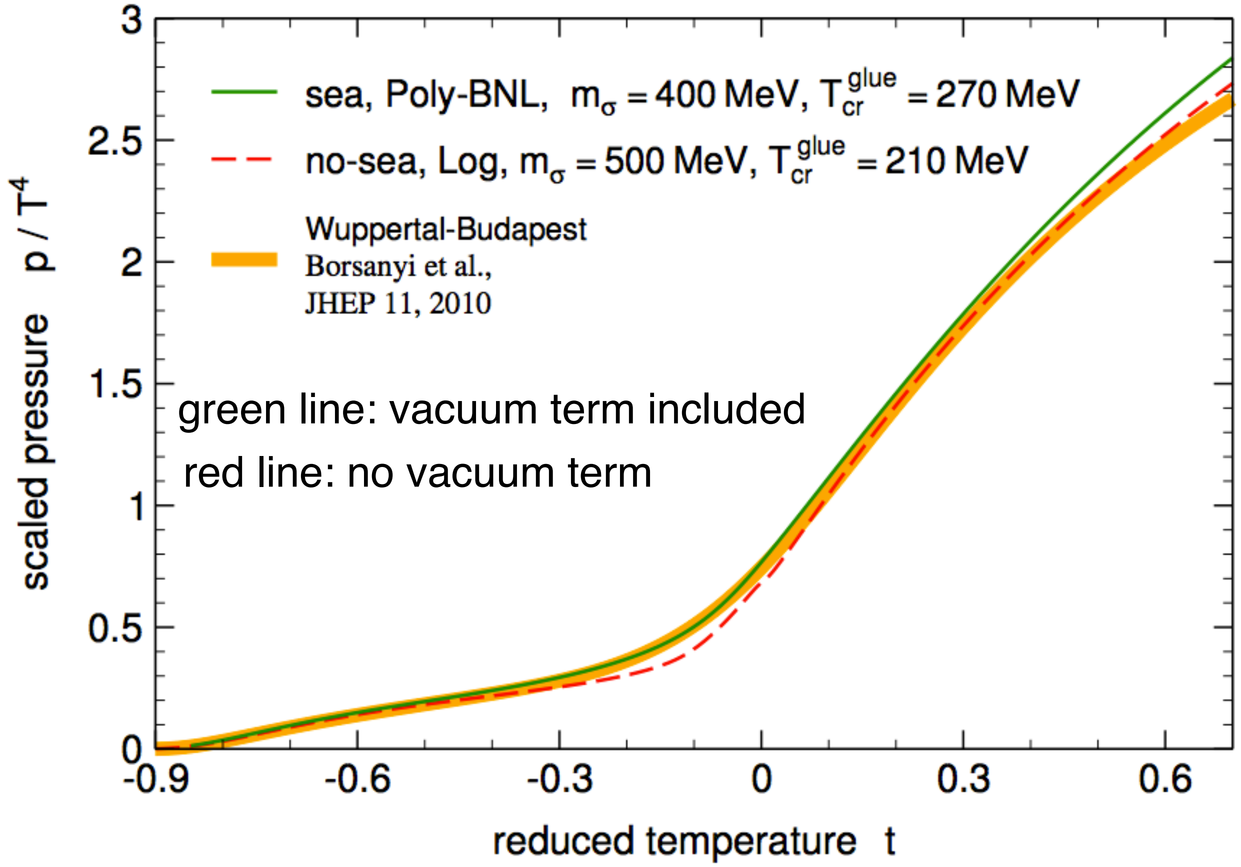
matter back-coupling on gluodynamics

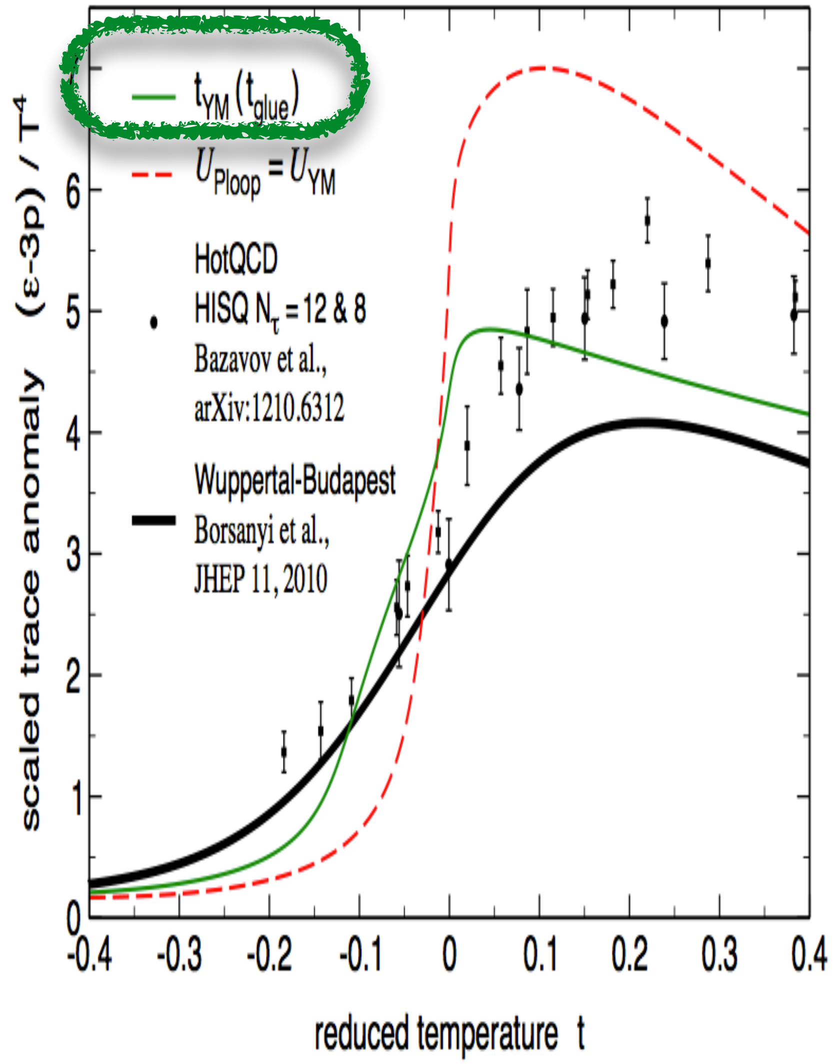
an effective unquenching

$$\mathcal{U}_{glue}(t_{glue}) = \mathcal{U}_{\text{YM}}(t_{\text{YM}})$$

with $t_{\text{YM}}(t_{glue}) = 0.57\, t_{\text{glue}}$

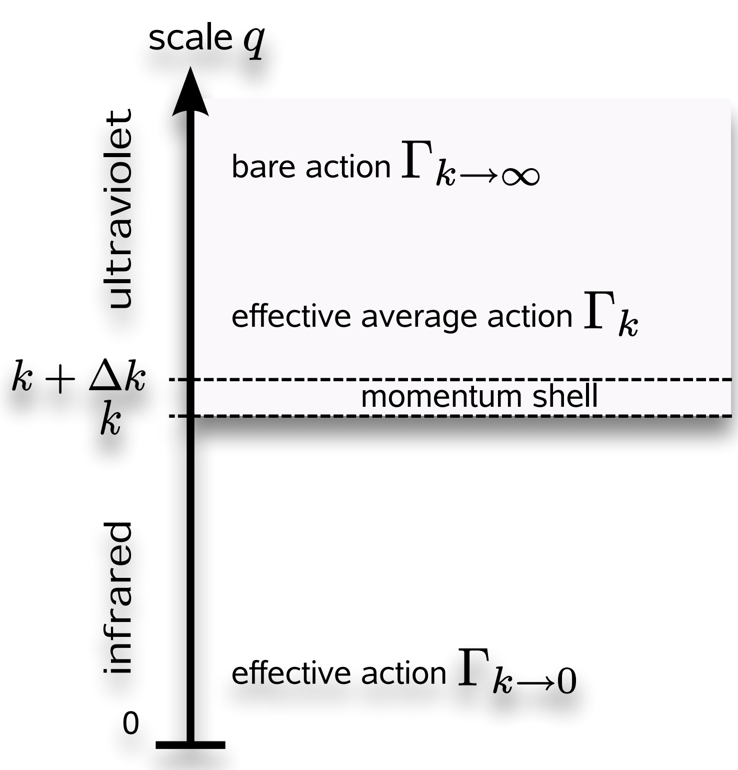
[Herbst, Mitter, Stiele, Pawlowski, BJS 2014]





Functional Renormalization Group

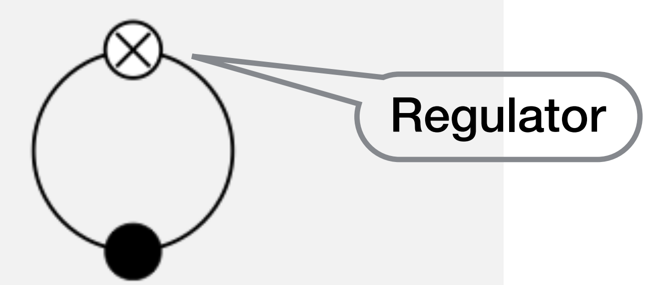
- $\Gamma_k[\phi]$ scale dependent effective action $\quad t = \ln(k/\Lambda) \quad R_k$ regulators $\quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$



FRG (average effective action)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$



[Wetterich 1993]

- Ansatz for Γ_k: **Leading order derivative expansion** **arbitrary potential**

$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu \partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V_k(\phi^2)$$

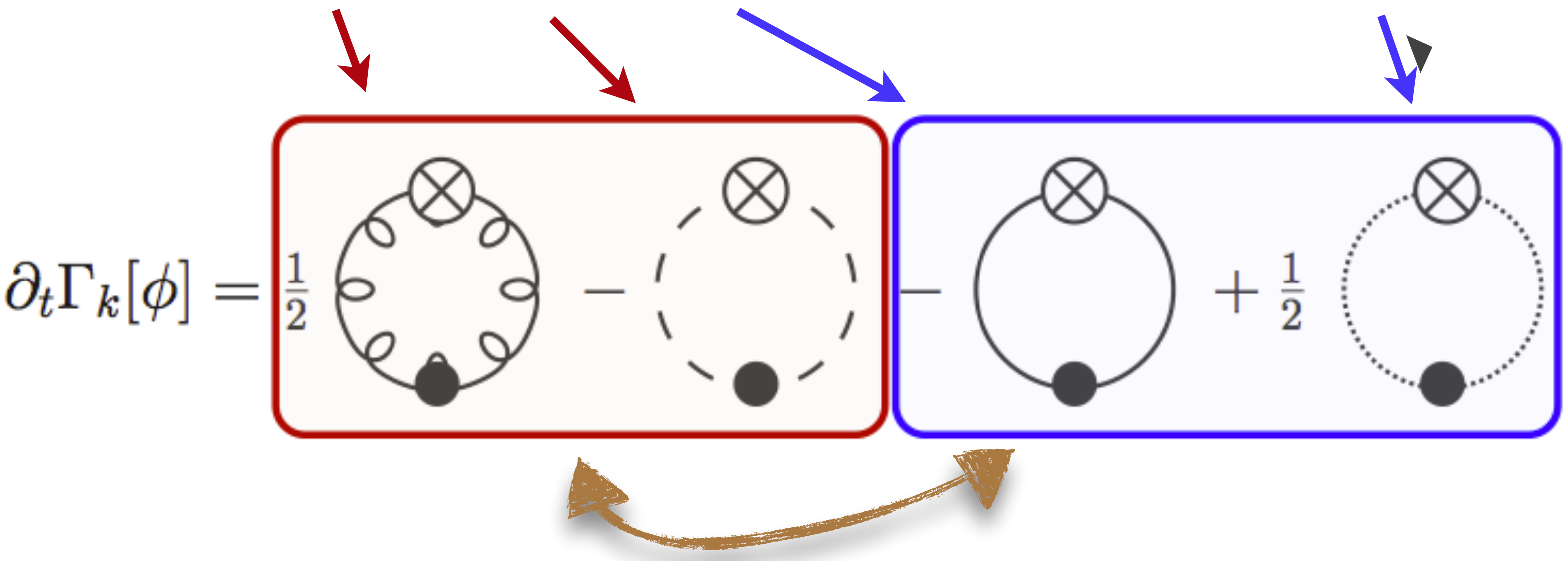
$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

solutions with grid/polynomial techniques

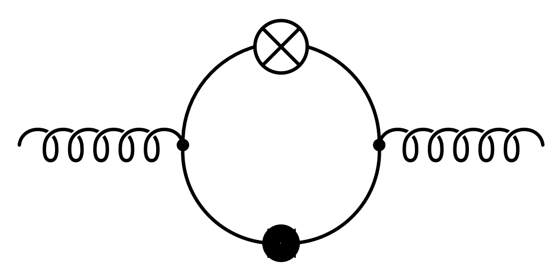
FRG and QCD

- **full dynamical QCD FRG flow:** [Braun, Haas, Pawlowski 2009/12]

fluctuations of **gluon**, **ghost**, **quark** and (via hadronization) **meson**



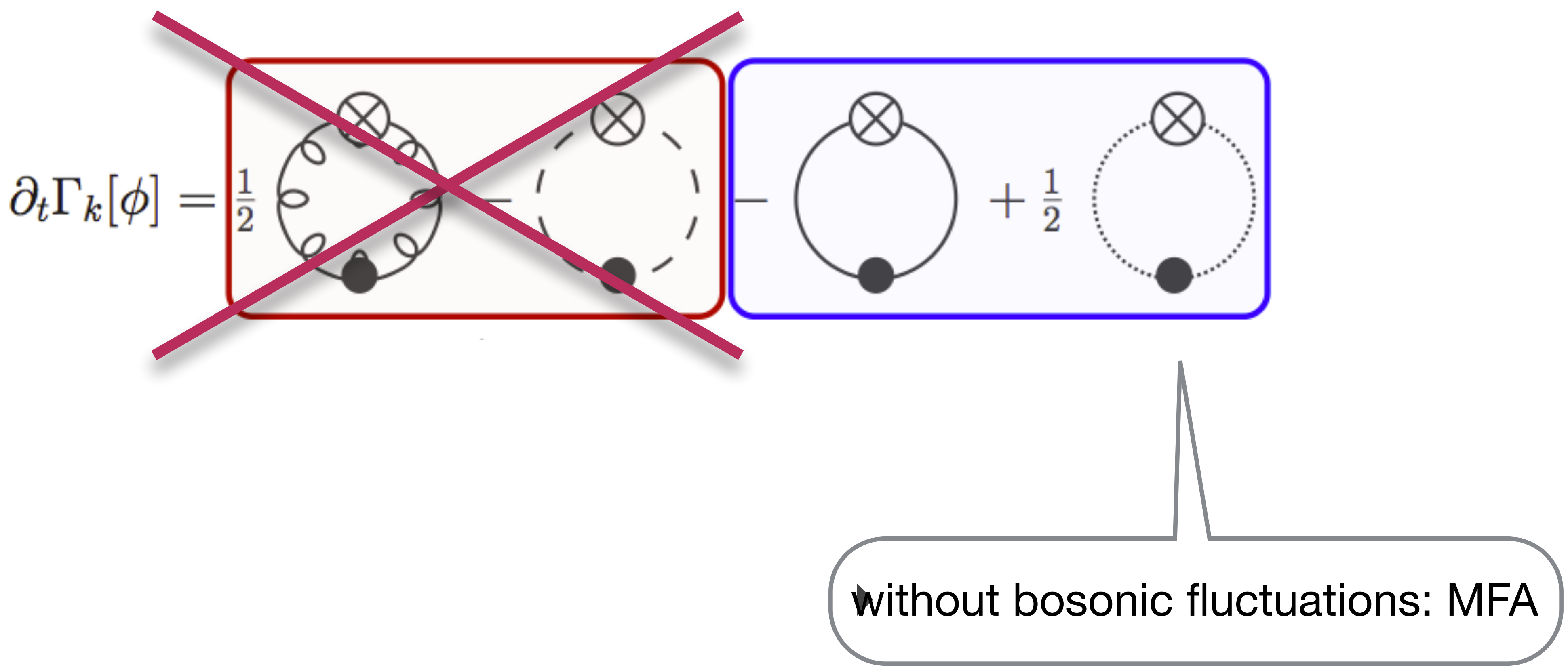
in presence of **dynamical quarks**:
gluon propagator is modified



pure Yang Mills flow + matter back-coupling

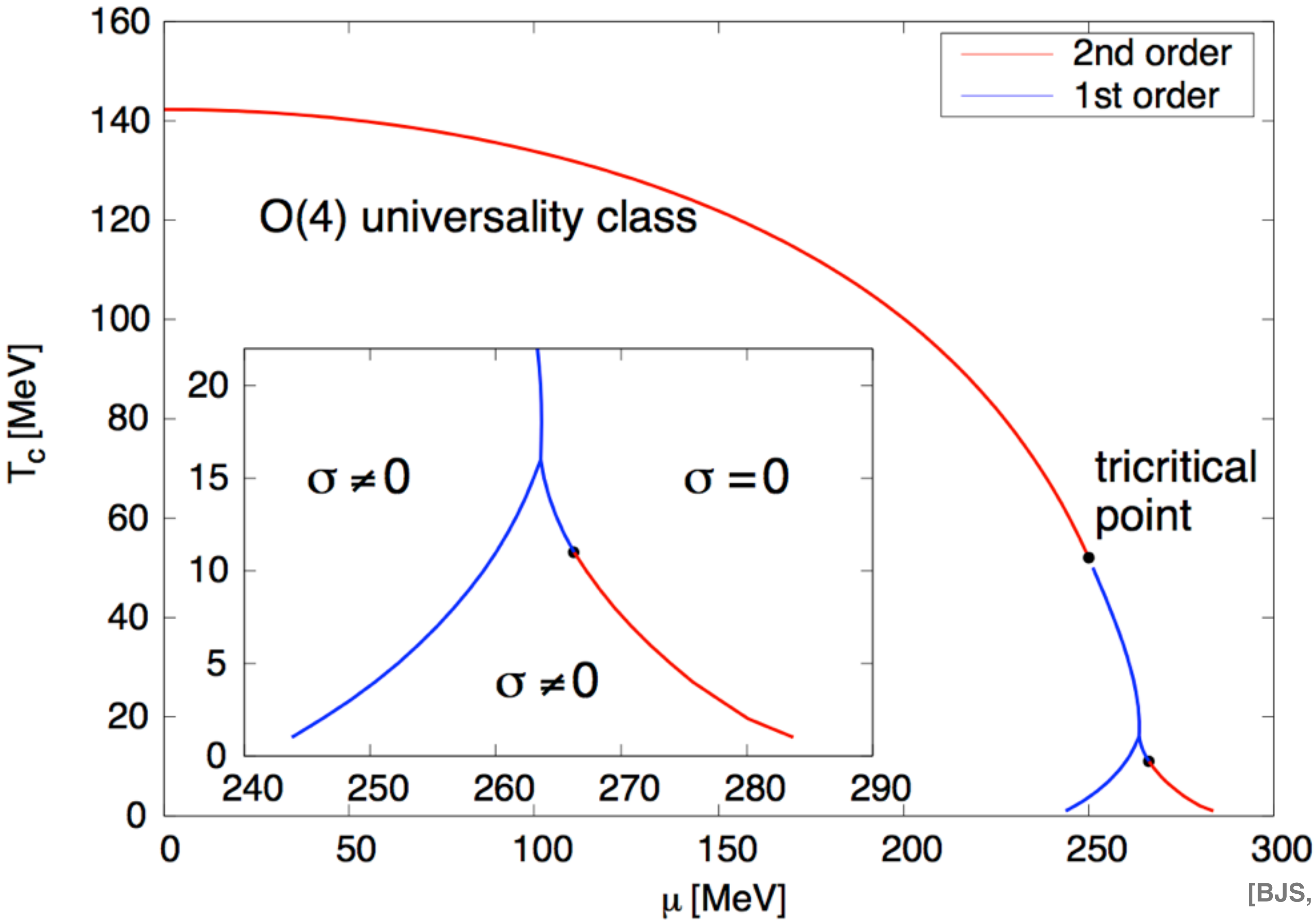
FRG: quark-meson truncation

First step: flow for **quark-meson** model truncation: neglect **YM contributions**



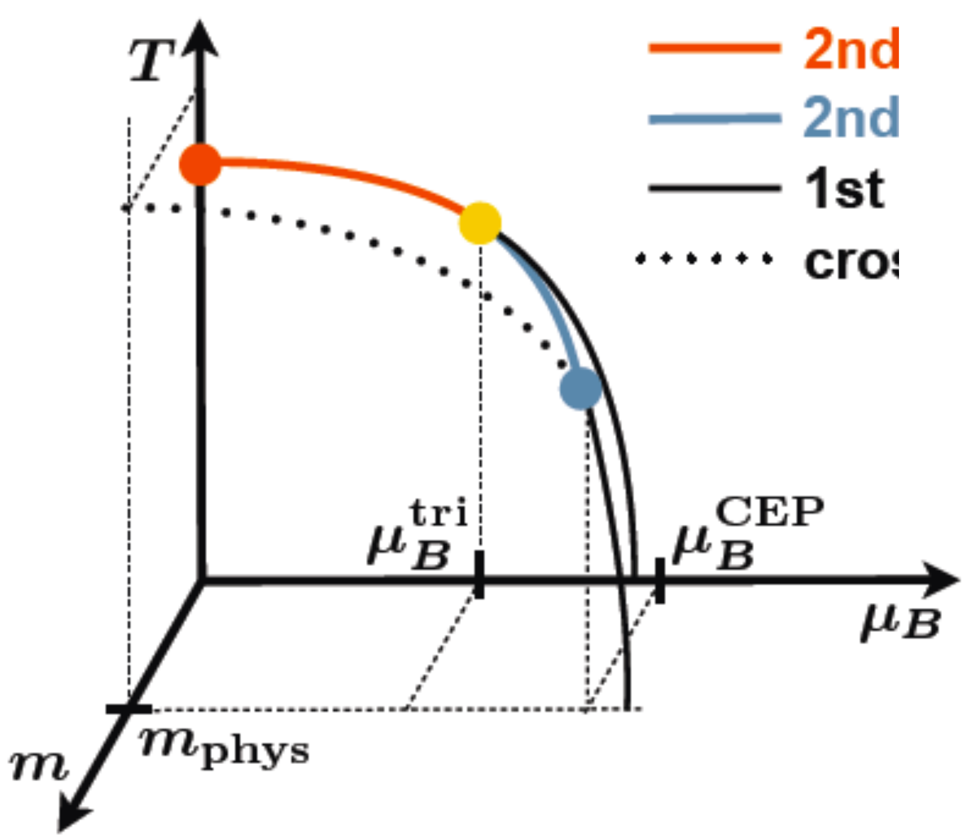
Phase diagram N_f=2 QM

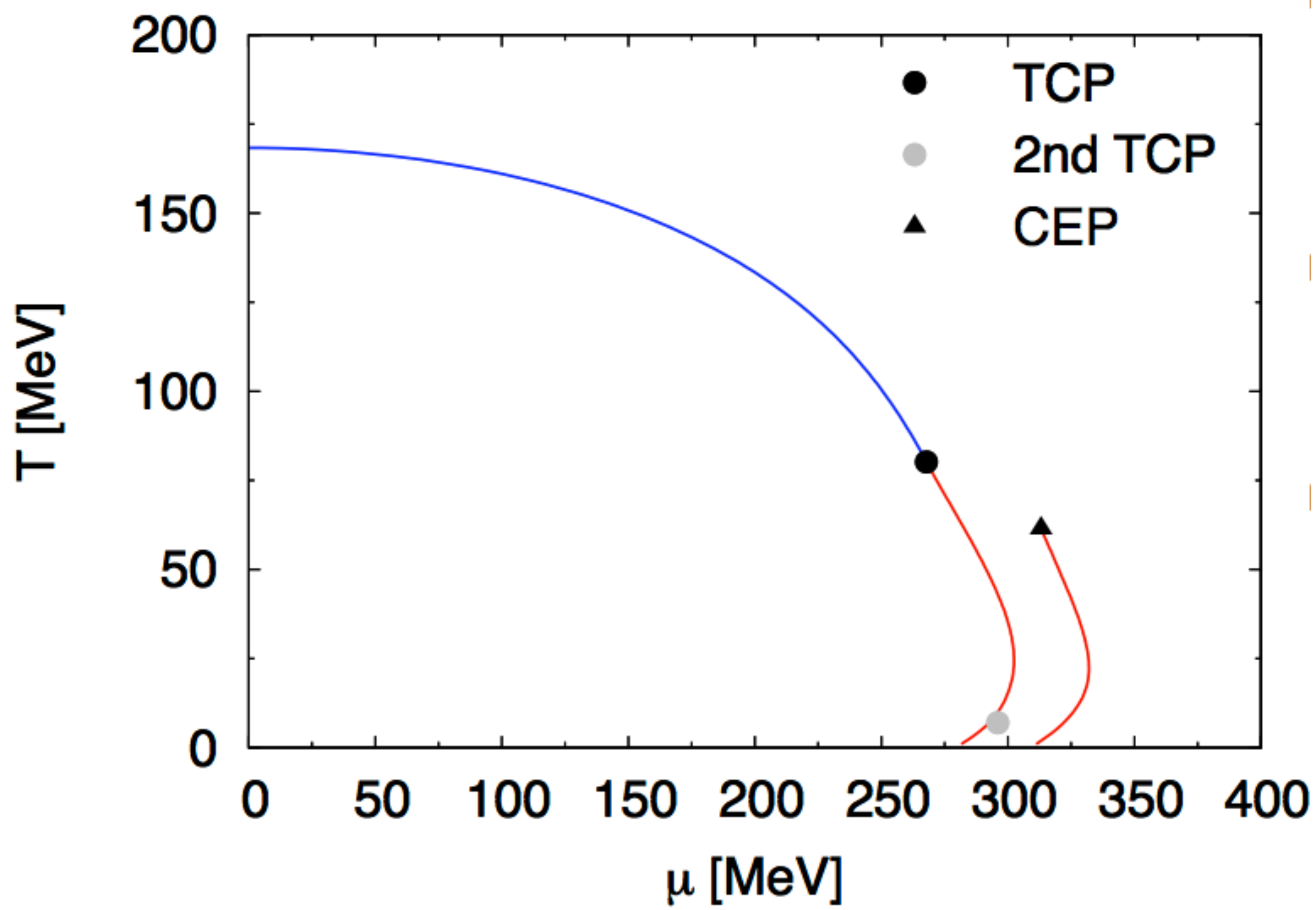
$O(4) \sim SU(2) \times SU(2)$ chiral limit no spinodal lines!



[BJS, J Wambach 2005]

Phase diagram N_f=2 QM





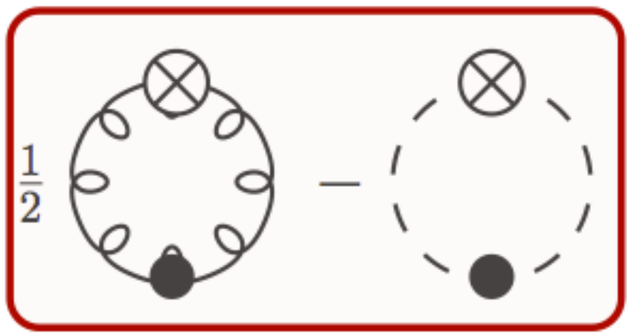
[BJS, J Wambach 2005]

FRG and QCD

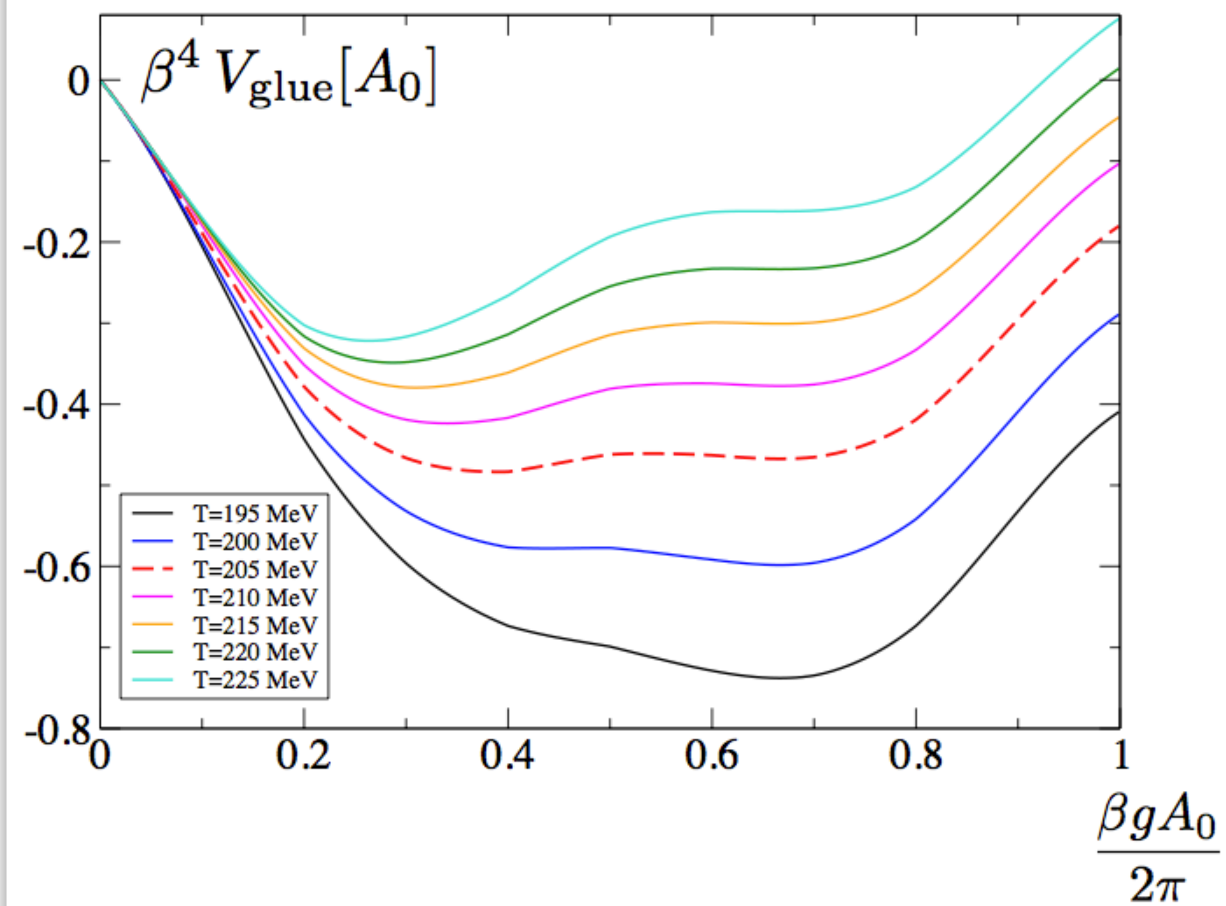
- **pure Yang Mills flow:**

  fluctuations of **gluon**, **ghost**

$$\partial_t \Gamma_k[\phi] =$$



Glue potential



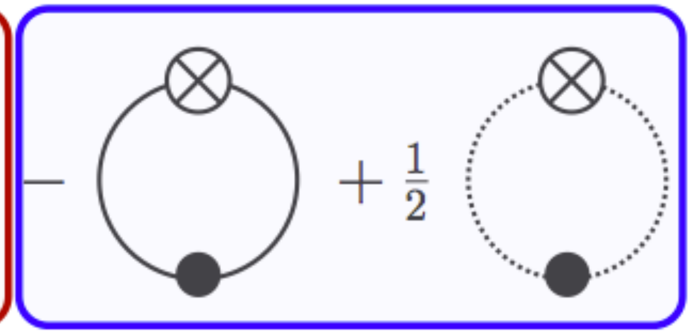
[Haas, Stiele, Braun, Pawlowski, Schaffner-Bielich, (2013)]

FRG and QCD

- **Polyakov-loop improved quark-meson flow:** [Herbst, Pawlowski, BJS 2007 2013]

  fluctuations of **Polyakov-loop**, **quark** and **meson**

$$\partial_t \Gamma_k[\phi] = \rightarrow \mathcal{U}_{\text{Pol}}(\Phi)$$

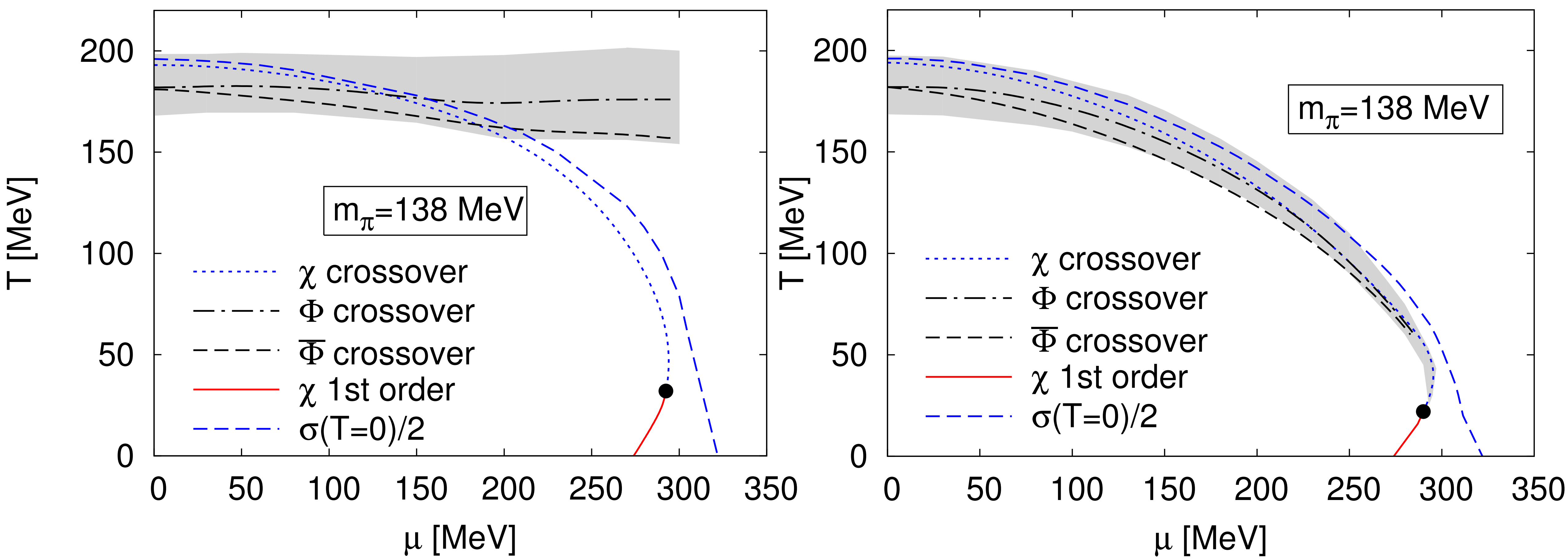


Yang-Mills flow replaced by
➜ **effective Polyakov-loop potential** $\rightarrow \mathcal{U}_{\text{Pol}}(\Phi)$

fitted to lattice Yang-Mills thermodynamics

FRG: Quark-Meson with Polyakov

w/o $T_0(\mu)$ | with $T_0(\mu)$



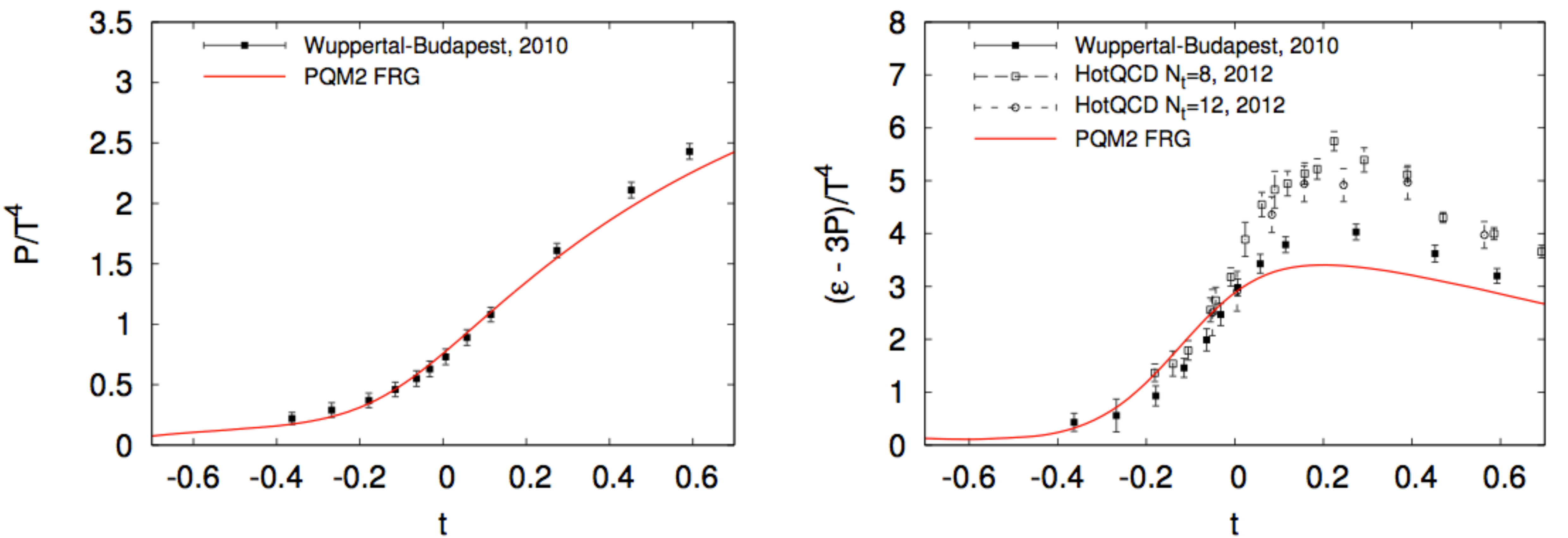
[Herbst, Pawlowski, BJS 2010,2013]

FRG: Quark-Meson with Polyakov

Pressure and interaction measure in comparison with lattice data (polynomial Polyakov-loop potential)

$N_f = 2$

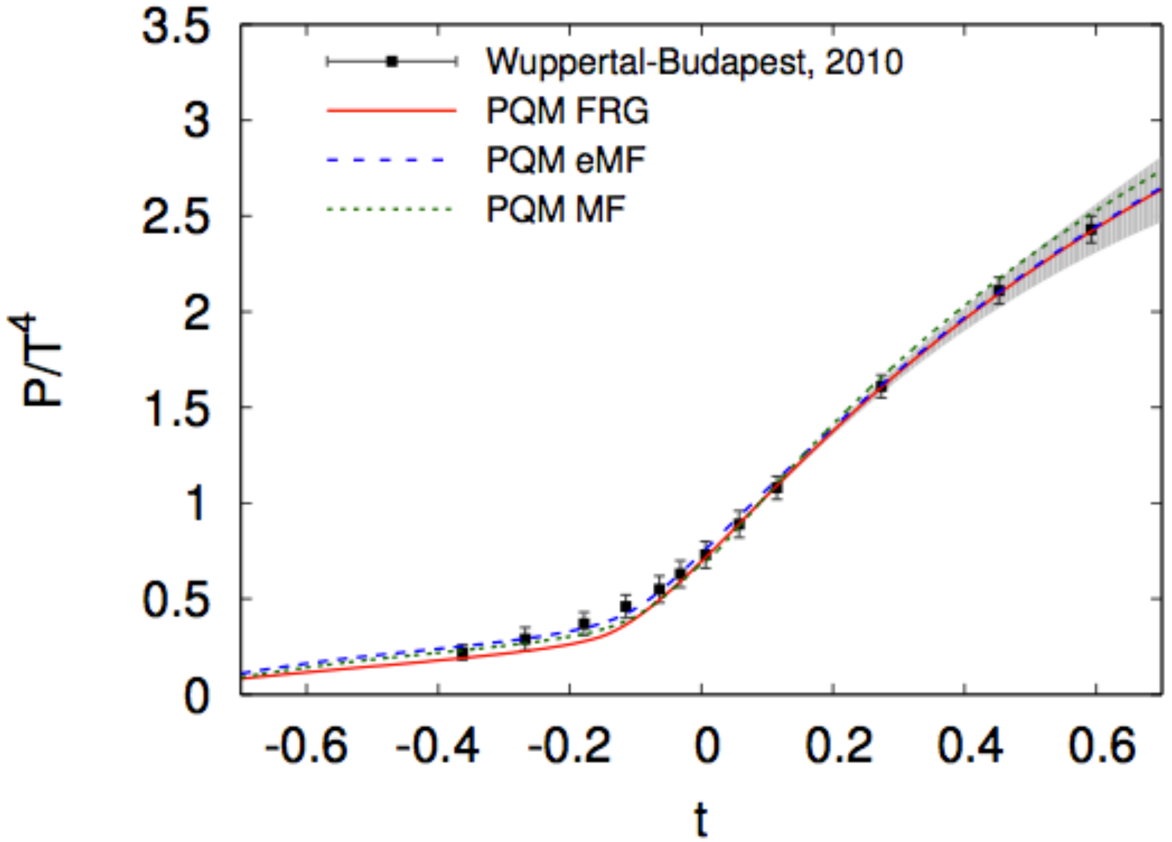
[Herbst, Mitter, Stiele, Pawlowski, BJS 2014]

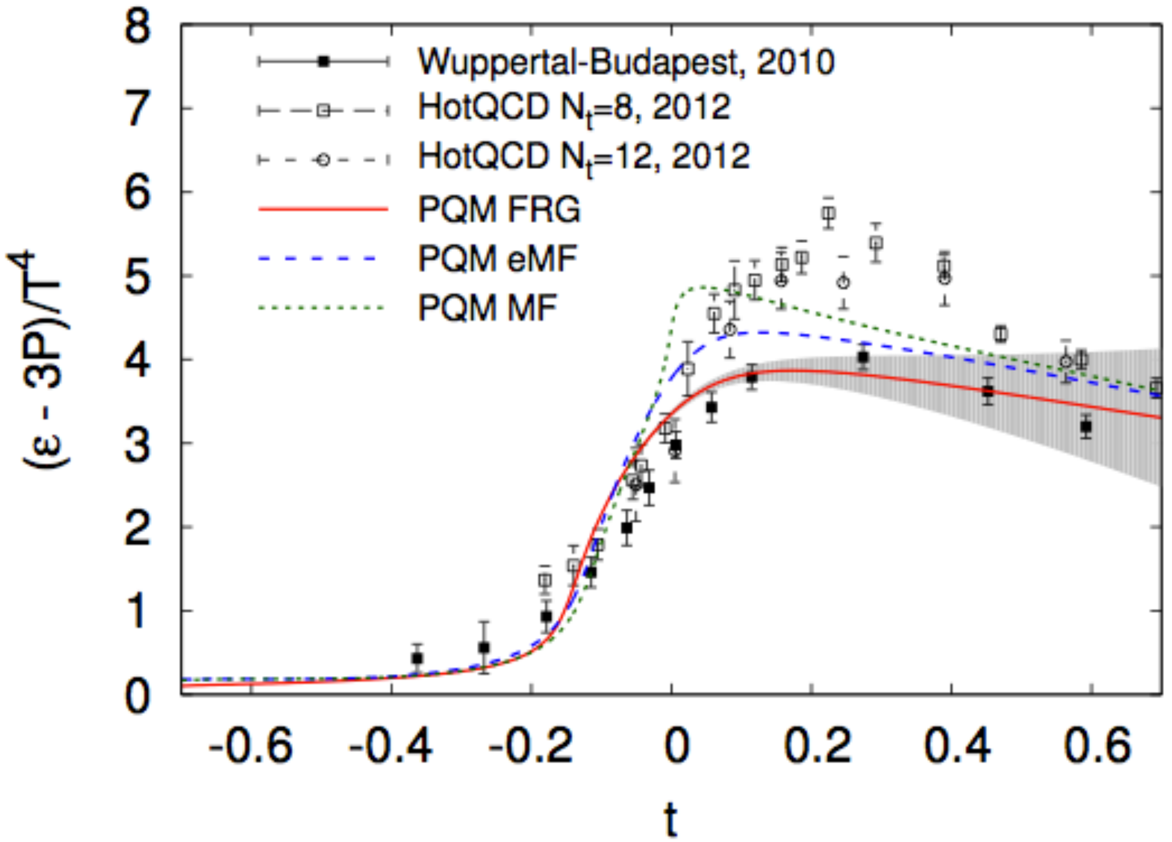


FRG: Quark-Meson with Polyakov

$N_f = 2+1$

[Herbst, Mitter, Stiele, Pawlowski, BJS 2014]





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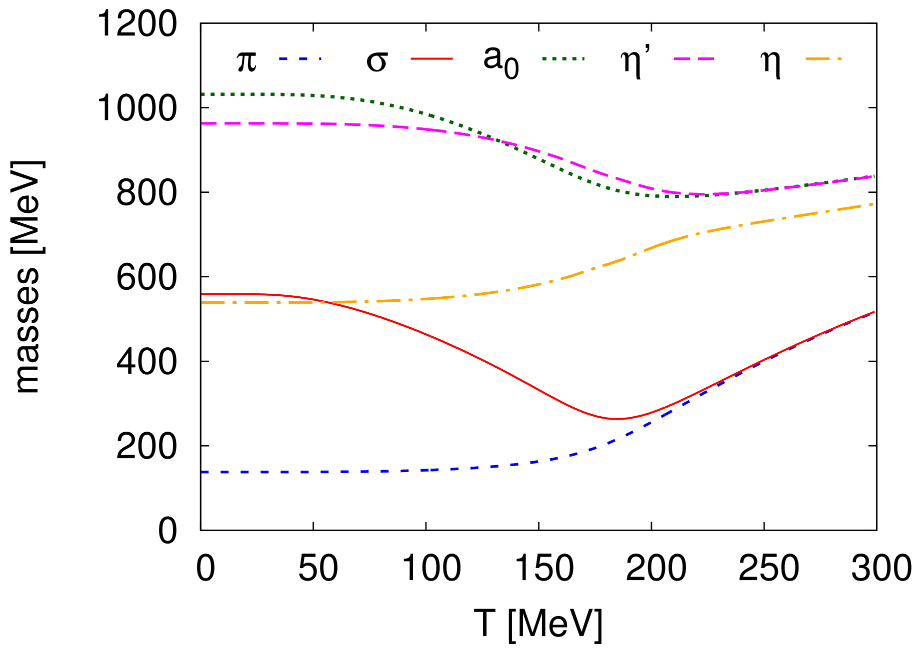
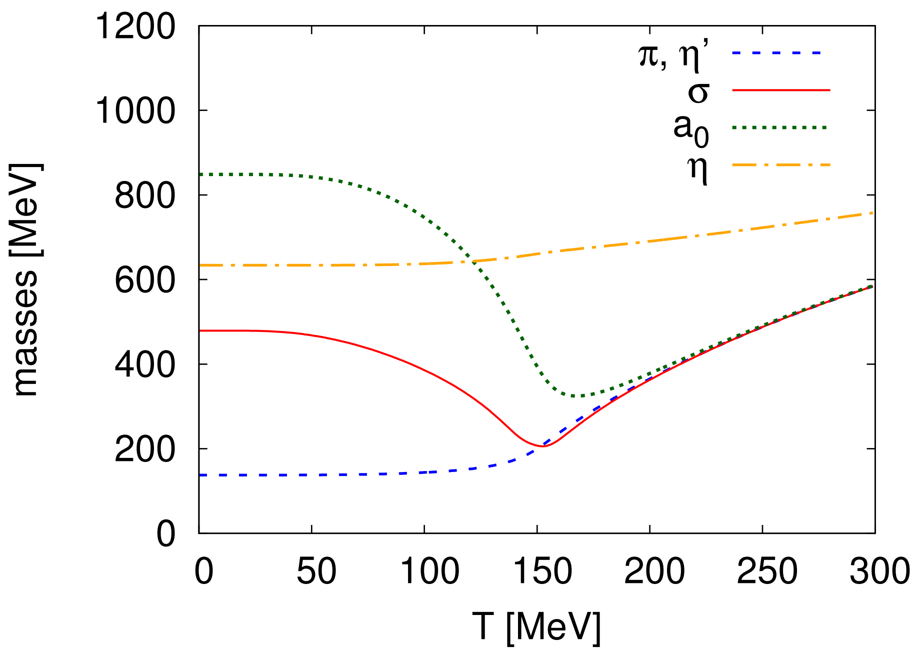
Influence axial anomaly



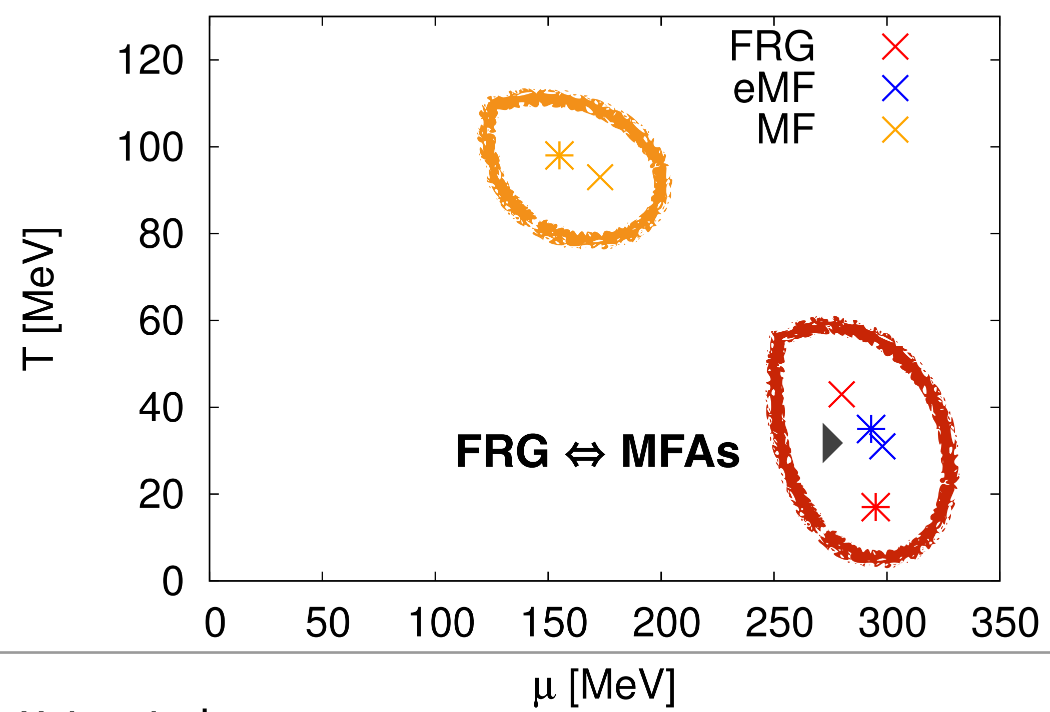
with $U_A(1)$ breaking term



without $U_A(1)$ breaking term

location of CEP

fluctuations push CEP down



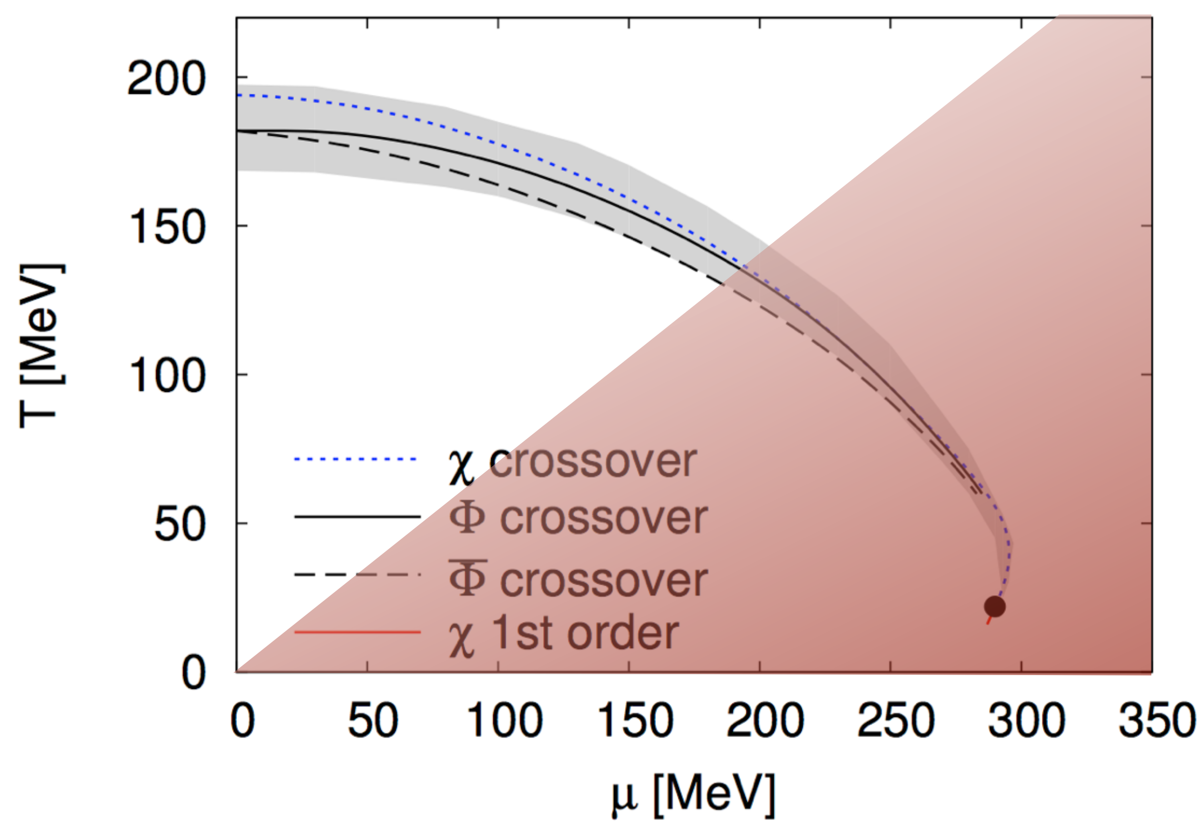
star: with $U_A(1)$ breaking term

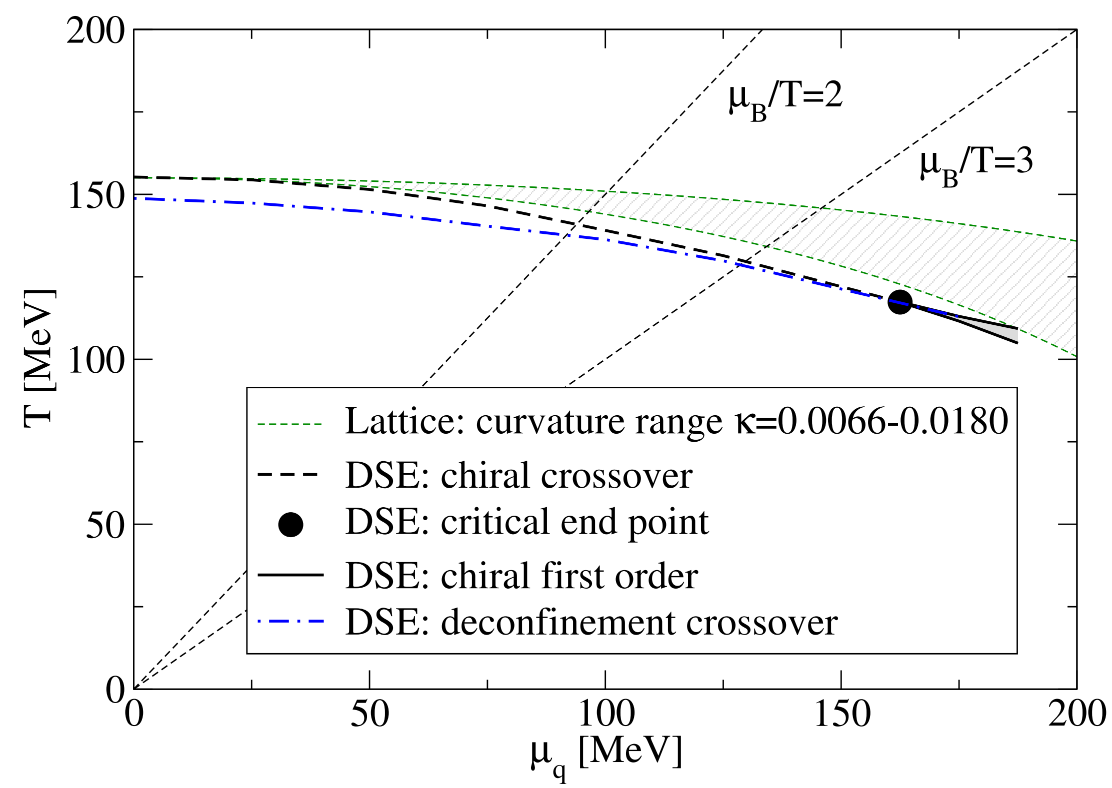
cross: w/o $U_A(1)$ breaking term

[M. Mitter, BJS 2014]

Critical Endpoint

Location of CEP **not** accessible with lattice, FRG & DSE





so far:

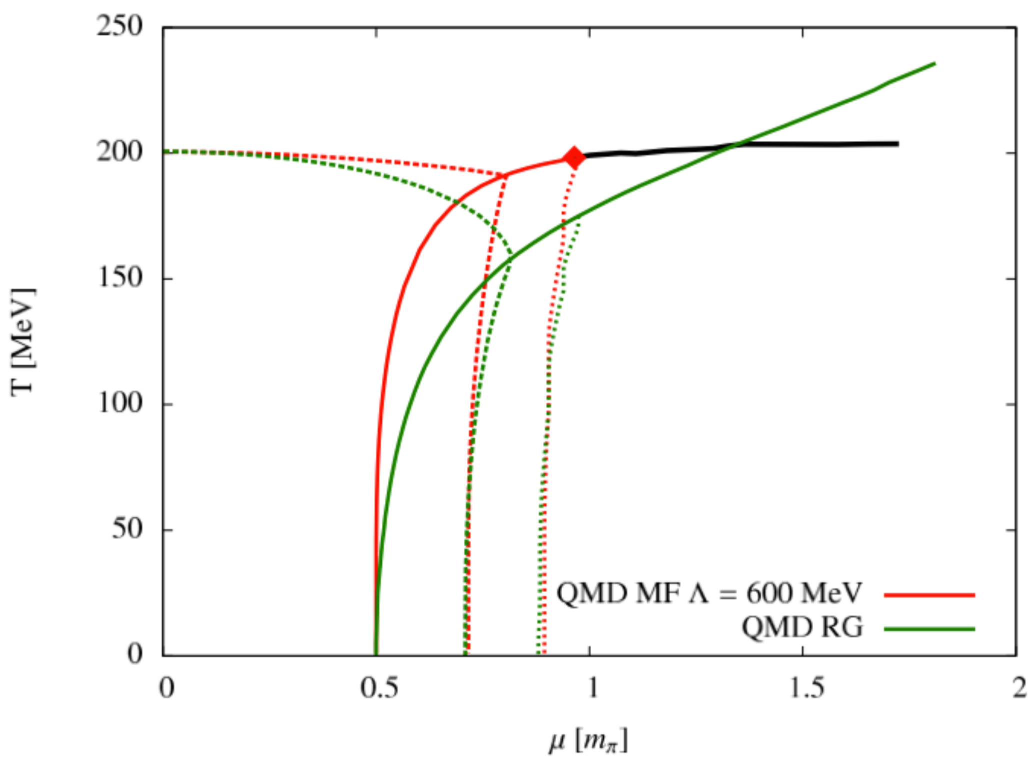
we can exclude CEP for small densities

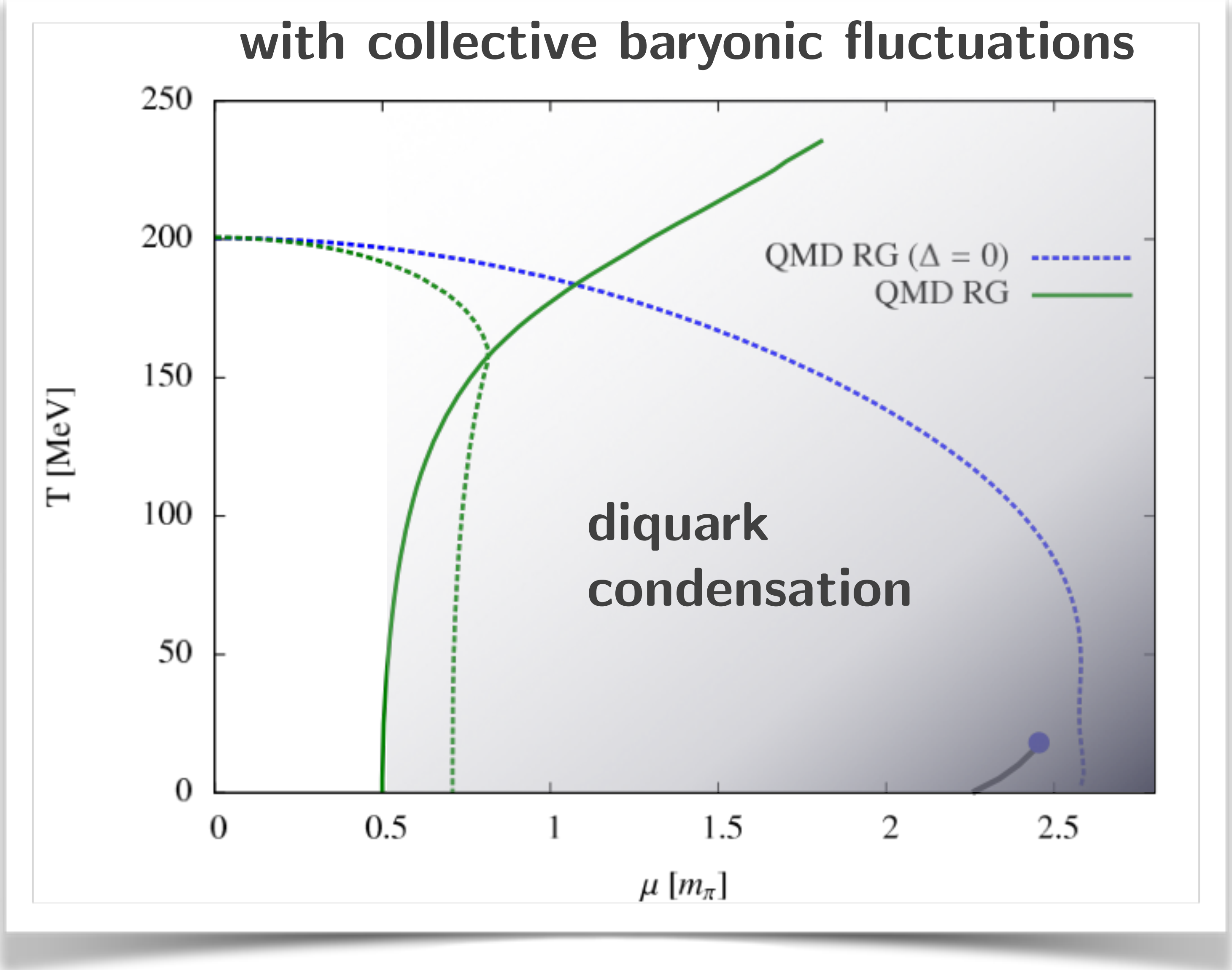
[C. Fischer, J. Lücker, C. Welzbacher 2014]

but no baryons!

N_c=2 : diquark condensation

[N. Strodthoff, BJS, L. von Smekal 12]





- no low-T $1^{\rm st}$ order transition, no CEP at $\mu \sim 2.5\ m_\pi$!

Outlook: Inhomogeneities

inhomogeneous chiral symmetry breaking:

phases characterized by spatially varying chiral condensate σ(x) which breaks translational variance

allowing for inhomogeneous phases ➜ cooper pairs with non-vanishing total momentum near Fermi surface

only one- and two-dimensional condensates (here, in this context, first work beyond mean-field approximation)

quark-meson model (renormalizable):

include vacuum term

in grand potential

(Dirac-sea contribution)

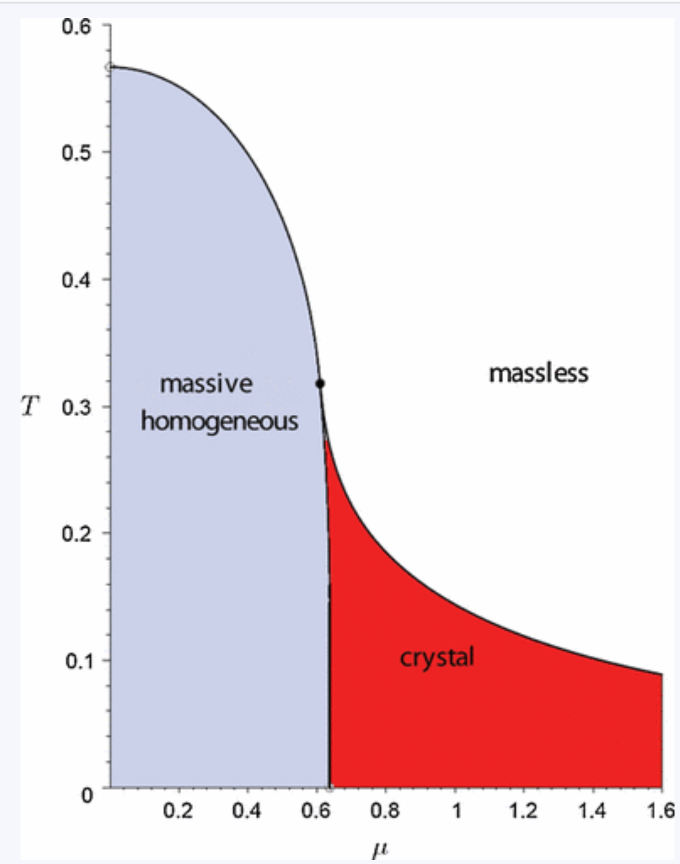
example:

Gross-Neveu 1+1

➜ chiral spirals

favored solution

for μ>0

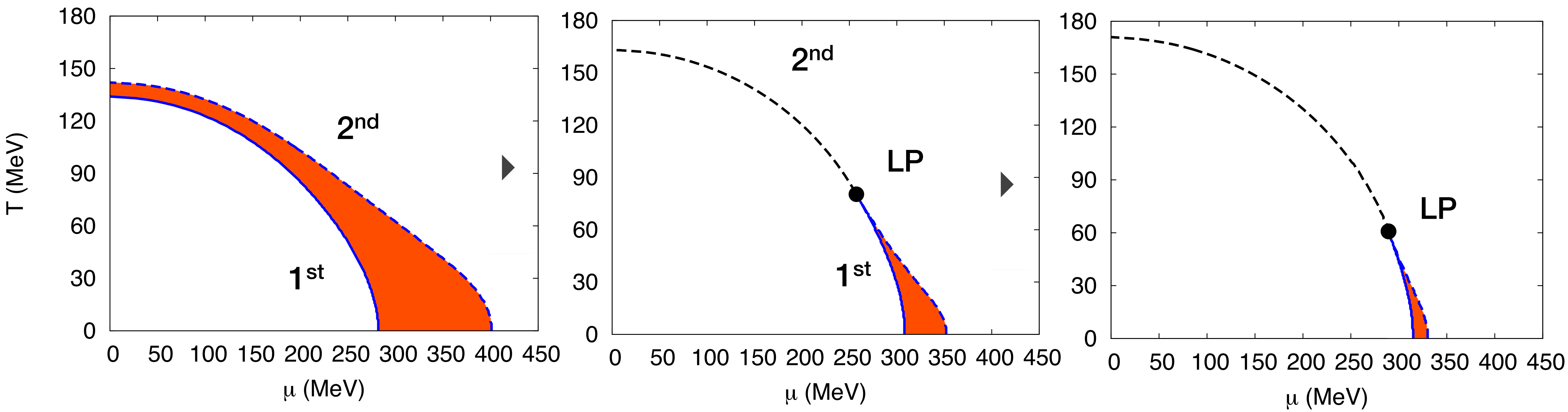


Outlook: Inhomogeneities

QM model: Phase diagram (two flavor, extended MFA)

Influence Dirac sea (left: Λ=0 middle: Λ=600 MeV right: Λ=5 GeV)

[S. Carignano, M. Buballa, BJS 2014]



LP: Lifshitz point (two homogeneous phases meet one inhomogeneous phase)

CP: Critical point (endpoint of 1st order transition)

For $m_\sigma = 2M_q$ LP=CP

outlook: full FRG treatment….

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Summary & Conclusions

- QCD-like model studies for two and three flavors
- effects of quantum and thermal fluctuations
  on QCD phase structure
- existence of critical points in phase diagram

▸ functional approaches (e.g. FRG) are suitable and controllable tools to investigate the QCD phase diagram and its boundaries