Phases of Unitary Imbalanced Fermi Gases

Dietrich Roscher

[I. Boettcher, J. Braun, T.K. Herbst, J.M. Pawlowski, D. Roscher, C. Wetterich, arXiv:1409.5070]

[D. Roscher, J. Braun, J.E. Drut, arXiv:1410.xxxx in preparation]

ERG 2014

Institut für Kernphysik / Technische Universität Darmstadt

September 24, 2014

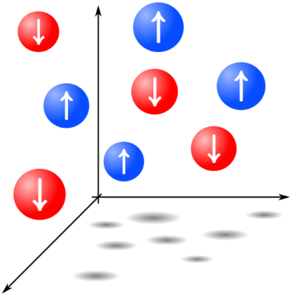
Deutsche Forschungsgemeinschaft BR 4005/2-1 DFG

HIC for FAIR Helmholtz International Center

Ultracold Fermi Gases:

$$S = \int_0^{\frac{1}{T}} \int \mathrm{d}^3x \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \psi_\sigma$$

- Spatial dimensions: 3
- Two components: $\sigma = \uparrow, \downarrow$



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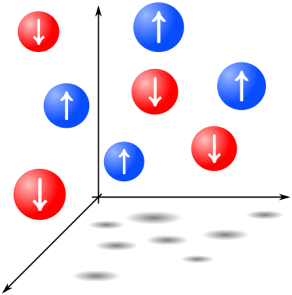
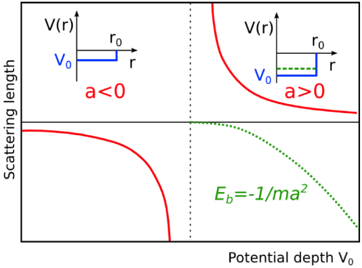
- Spatial dimensions: 3
- Two components: $\sigma = \uparrow, \downarrow$
- System temperature $T \sim T_F = \frac{1}{2m} \left(6\pi^2 n\right)^{\frac{1}{3}} \sim \mathcal{O}(\mu K)$



Unitary Fermi Gases:

$$S = \int_0^{\frac{1}{T}} \int \mathrm{d}^3x \left[\sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \psi_\sigma + \frac{\bar{g}}{2} \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow \right]$$

- Contact interaction: effective range $r_e = 0$
- Scattering length a tunable due to Feshbach resonance in experiment
- Here: *unitary* limit $\frac{1}{a} \to 0$ (strong coupling)



[M. Randeria, E. Taylor '13]

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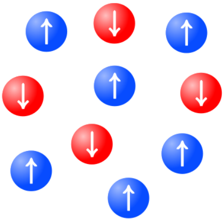

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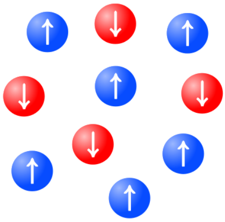
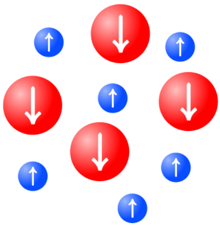
- Population/Spin imbalance: $h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$
- Mass imbalance $\bar{m} = \frac{m_\downarrow - m_\uparrow}{m_\uparrow + m_\downarrow}$
- Set reduced mass $\frac{4 m_\uparrow m_\downarrow}{m_\uparrow + m_\downarrow} = 1$ and average chemical potential $\frac{\mu_\uparrow + \mu_\downarrow}{2} = 1$ for convenience





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Physical relevance:

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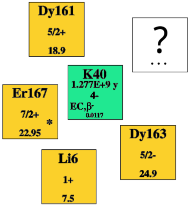

[Isotopes explored by *Grimm* group, Innsbruck]

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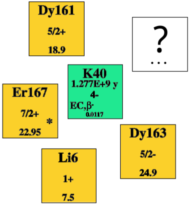
Physical relevance:

- Upcoming cold atoms experiments
- General insight: symmetry breaking patterns of fermionic field theories [*e.g.* V. Schoen, M. Thies '00]
- „Simulation" of more complicated systems (heavy ion collisions, color superconductivity) [*e.g.* A. Adams, L.D. Carr, T. Schaefer, P. Steinberg, J. Thomas '12]



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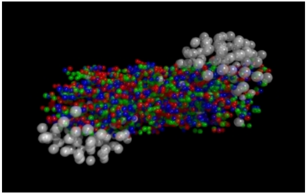
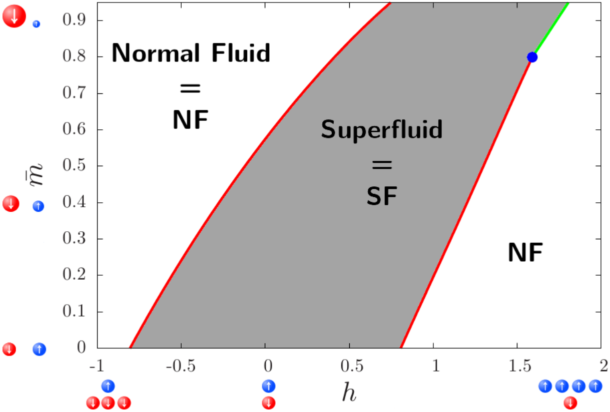


First Impressions: Mean Field at $T = 0$

$$U_{\mathrm{MF}}(\Delta_0) \sim \int \mathrm{d}q q^2 \left\{ \frac{|\Delta_0|^2}{2q^2} - T \sum_{\sigma=\pm 1} \ln\left[1 + e^{-\frac{1}{T}\left(q^2 \bar{m} - h + \sigma\sqrt{(q^2-1)^2 + |\Delta_0|^2}\right)}\right]\right\}$$



[M.M. Parish, F.M. Marchetti, A. Lamacraft, D.B. Simons '07]

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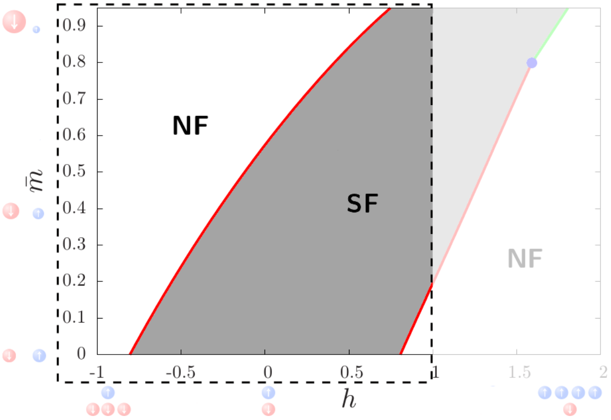
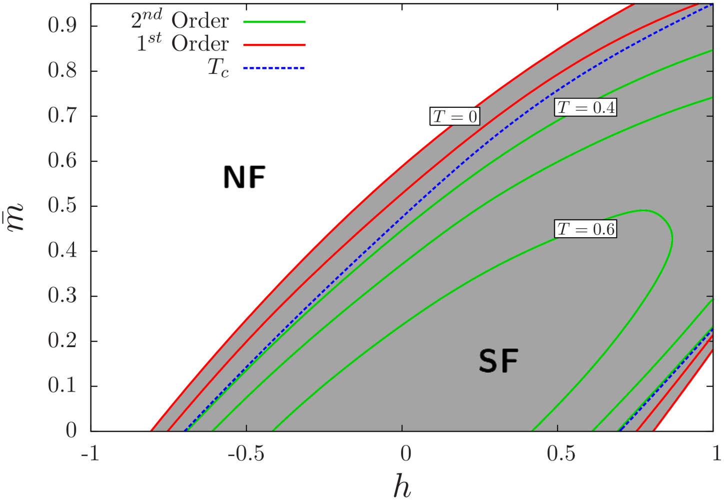


[M.M. Parish, F.M. Marchetti, A. Lamacraft, D.B. Simons '07]

Mean Field: Finite Temperature



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Examining Δ_0 More Closely

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- Implicit assumption so far: $\Delta_0 \neq \Delta_0(\vec{x})$
- Analytical calculations for general $\Delta_0(\vec{x})$ non-trivial even at mean field [see e.g. M. Thies *et al* '06 ; G. Basar, G.V. Dunne '08]

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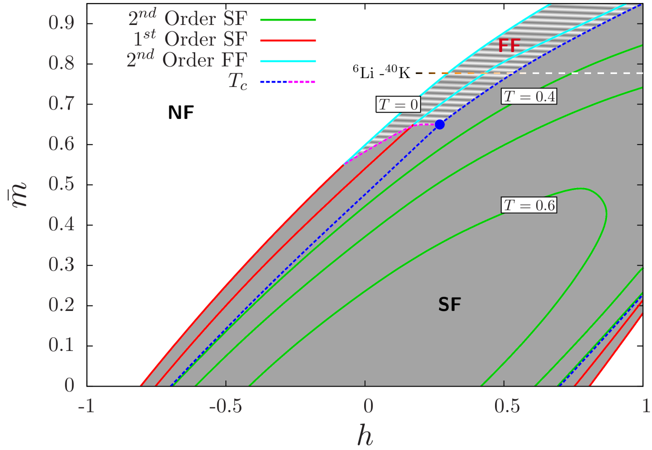
- Analytical calculations for general $\Delta_0(\vec{x})$ non-trivial even at mean field [see e.g. M. Thies *et al* '06 ; G. Basar, G.V. Dunne '08]
- FF-Ansatz: $\Delta_0(\vec{x}) \equiv \Delta_0 e^{i2\vec{Q}\cdot\vec{x}}$ [P. Fulde, R.A. Ferrell '64; A.I. Larkin, Y.N. Ovchinnikov '64]

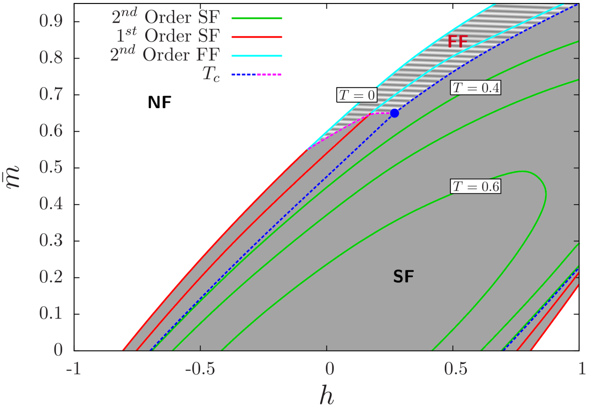
Mean Field Phase Diagram Including FF Region



[For ^{6}Li-^{40}K: J.E. Baarsma, H.T.C. Stoof '13]

Spontaneous Breaking of Galilean Invariance!

...and beyond Mean Field?



- What happens to the critical T upon inclusion of fluctuations?

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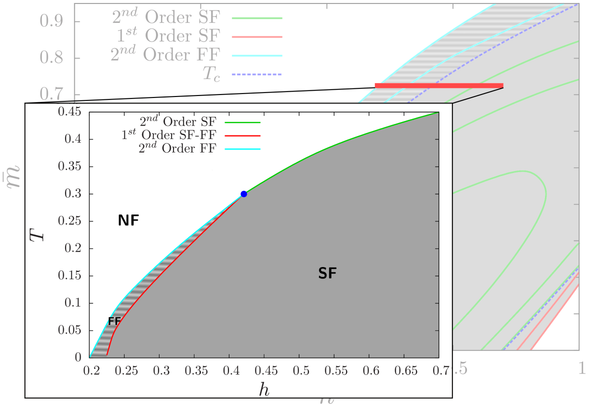

- What happens to the critical T upon inclusion of fluctuations?
- How stable is the inhomogeneous phase beyond mean field?

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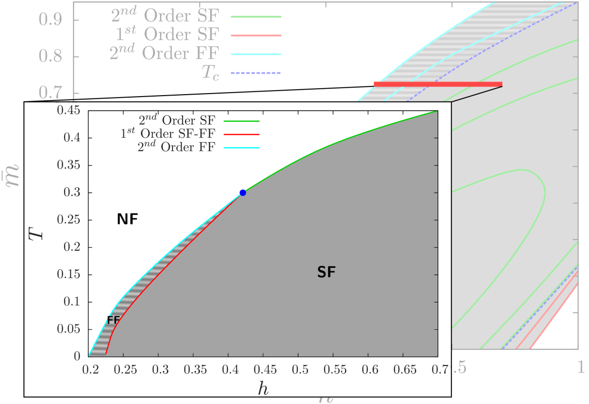

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- How stable is the inhomogeneous phase beyond mean field?

⇒ FRG Analysis

Based on framework by: [S. Diehl *et al* '07, '10; S. Floerchinger *et al* '08; ...]

FRG Implementation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \cdot (\partial_t R_k)\right]$$ [Wetterich '93]

Ansatz for Γ_k:

$$\Gamma_k = \int_0^{\frac{1}{T}} \int \mathrm{d}^3x \left[\sum_{\sigma=\uparrow\downarrow} \psi_\sigma^* \left(\partial_\tau - \Delta(1+\sigma\bar{m}) - \mu(1+\sigma h)\right)\psi_\sigma + \varphi^*\left(\frac{Z_\varphi}{A_\varphi}\partial_\tau - \frac{1-\bar{m}^2}{2}\Delta\right)\varphi \right.$$
$$\left. + U_k(\rho = \varphi^*\varphi) - \frac{h_\varphi}{2}\left(\varphi^*\psi_\uparrow\psi_\downarrow - \varphi\psi_\uparrow^*\psi_\downarrow^*\right)\right]$$

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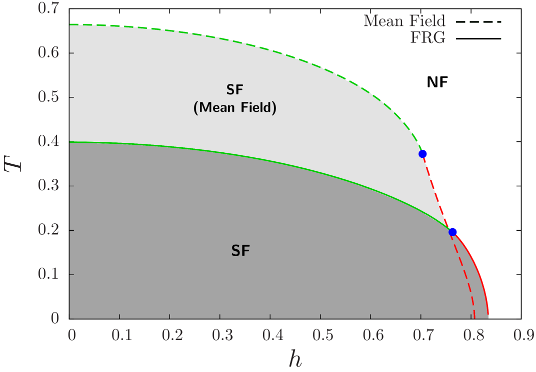
$$\left. + U_k(\rho = \varphi^* \varphi) - \frac{h_\varphi}{2} \left(\varphi^* \psi_\uparrow \psi_\downarrow - \varphi \psi_\uparrow^* \psi_\downarrow^* \right) \right]$$

- Flow of the effective potential:

$$k \partial_k U_k = \eta_{A_\varphi} \rho U_k' + [k \partial_k U_k]^\psi + [k \partial_k U_k]^\varphi$$

- Order parameter of $U(1)$ symmetry breaking: $\Delta_{0,k} = h_\varphi \sqrt{\rho_{0,k}}$, $U_k(\rho_{0,k})$ minimum
- Numerical implementation: discretization of $U_k(\rho)$ an a grid

First Results & Benchmarking: $\bar{m} = 0$ case

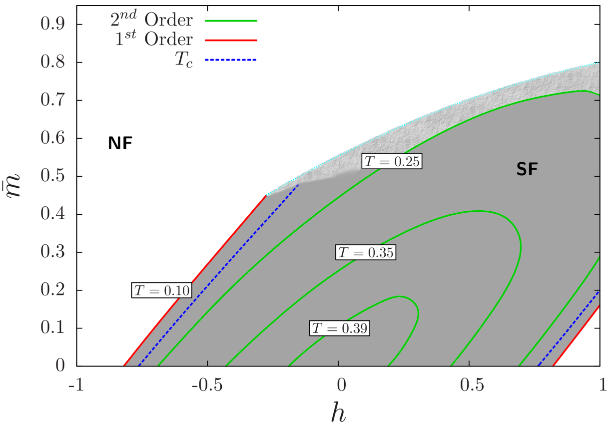


[I. Boettcher, J. Braun, T.K. Herbst, J.M. Pawlowski, D. Roscher, C. Wetterich '14]

See also: talk by Tina Herbst

- For small h: $T_{c,\mathrm{FRG}} < T_{c,\mathrm{MF}}$
- Endpoint of the transition line at $T = 0$: $h_{c,\mathrm{FRG}} > h_{c,\mathrm{MF}}$
- Quantitative agreement for two different regularization schemes

FRG Phase Diagram with Mass Imbalance



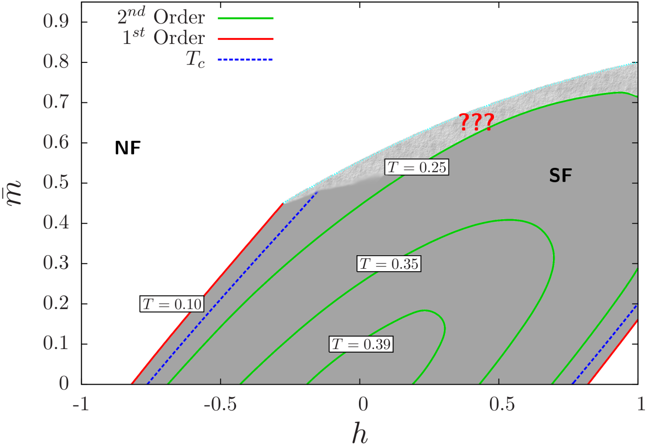
[D. Roscher, J. Braun, J.E. Drut *in prep.*]

Similar as for the $\bar{m} = 0$ case:

- For small h and $\bar{m}$: $T_{c,\mathrm{FRG}} < T_{c,\mathrm{MF}}$
- Superfluid region slightly broadened for low T

Full Phase Diagram with Mass Imbalance

Convergence Problems?



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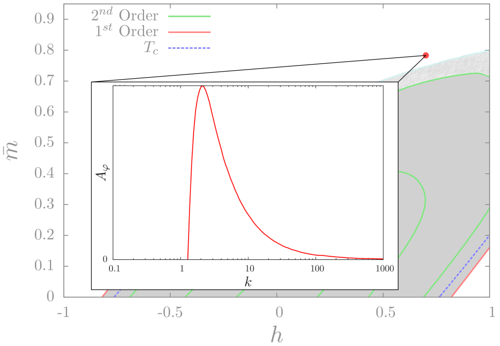
Behaviour of the Boson Propagator

$$P_\varphi^{-1}(q_0,\vec{q}) = iZ_\varphi q_0 + \frac{1-\bar{m}^2}{2} A_\varphi \vec{p}^2$$



[D. Roscher, J. Braun, J.E. Drut *in prep.*]

- Sign change of A_φ invalidates lowest order deriv. expansion for P_φ^{-1}
- Interpretation: formation of bosons with momentum $|\vec{Q}| > 0$ favored

[H.C. Krahl, S. Friederich, C. Wetterich '09]

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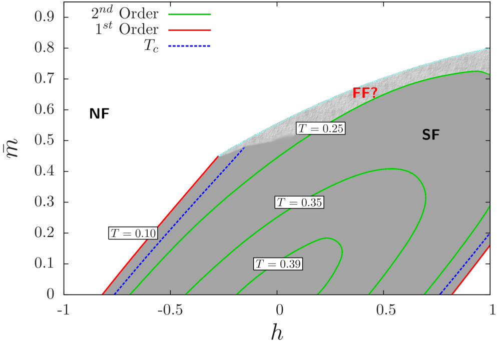

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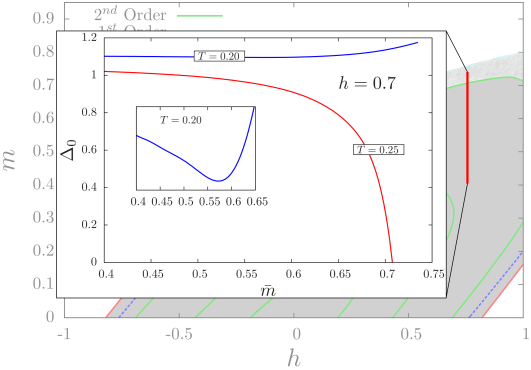
  [H.C. Krahl, S. Friederich, C. Wetterich '09]
- Rough coincidence with FF region at mean field

Further Precursors of Inhomogeneous Pairing



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Inside the problematic region:

- Unphysical growing of Δ_0 towards transition to normal phase

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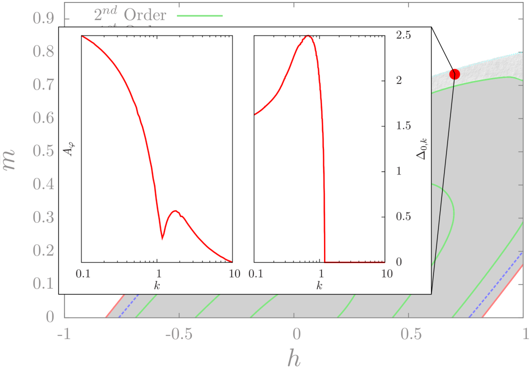

[D. Roscher, J. Braun, J.E. Drut *in prep.*]

Inside the problematic region:

- Unphysical growing of Δ_0 towards transition to normal phase
- Early occurrence of $\Delta_{0,k} > 0$ inhibits sign change of A_φ
- Conclusion: $\Delta_0 \neq \Delta_0(\vec{x})$ probably not the true ground state

Summary

- Application of FRG to compute the phase diagram of a unitary Fermi gas with spin and mass imbalance beyond mean field
- Observations: T_c mostly lowered, superfluid phase slightly extended
- Precursors of inhomogeneous phases found, qualitative agreement with mean field predictions

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Outlook

- More elaborate derivative expansion/explicit inhomogenenous condensate $\rightarrow$ w.i.p. with J. Braun, S. Rechenberger
- Full BEC-BCS crossover: finite a, dimensions $d \neq 3$ $\rightarrow$ planned with I. Boettcher, T. Herbst
- Complementary MC simulations $\rightarrow$ w.i.p. with J. Braun, J. Drut

[D. Roscher, J. Braun, J.-W. Chen, J. Drut '13; J. Braun, J. Drut, D. Roscher '14]