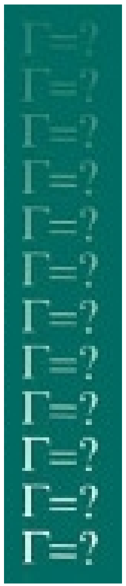
Super-renormalizable and Finite gravitational theories



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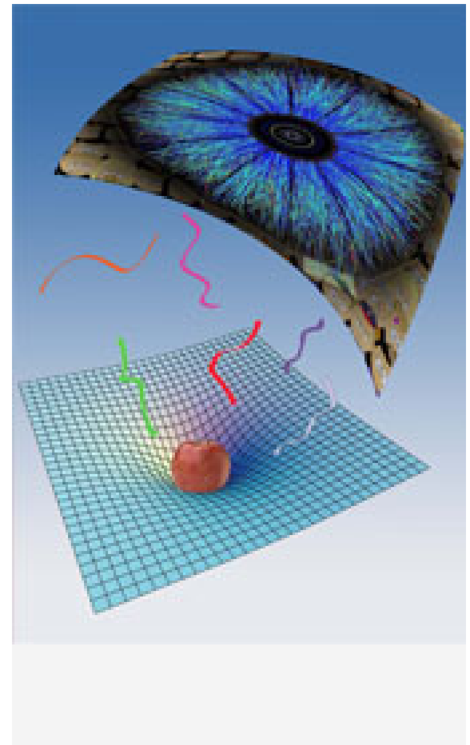

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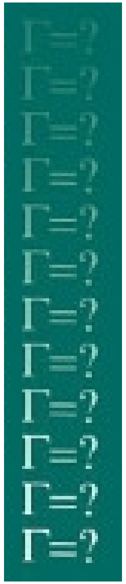

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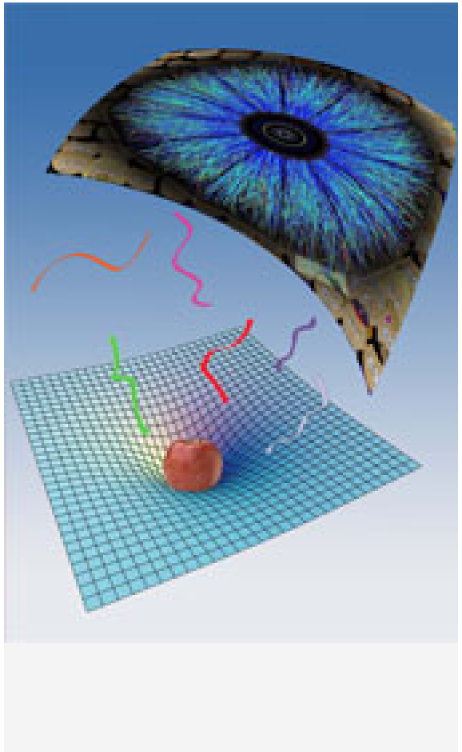

[based on arXiv: hep-th/1407.8036]

In collaboration with
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Finite Quantum Gravity

One of the simplest Lagrangian for finite QG theory in *d=4*

$$L=\lambda+\kappa^{-2}R+\kappa^{-2}G_{\mu\nu}\frac{e^{H(z)}-1}{\Box}R^{\mu\nu}+s_1R^2\Box R^2+s_2R_{\mu\nu}R^{\mu\nu}\Box R_{\rho\sigma}R^{\rho\sigma}+$$

$$+\sum_i c_i^{(3)}(R^3)_i+\sum_i c_i^{(4)}(R^4)_i+\sum_i c_i^{(5)}(R^5)_i \qquad z=\frac{\Box}{\Lambda^2}$$

with $$H(z)=\frac{1}{2}\Gamma(0,p^2(z))+\frac{1}{2}\gamma_E+\frac{1}{2}\log p^2(z) \quad p(z)=z^4 \quad \gamma=3$$

Lagrangian in UV

$$L_{\mathrm{UV}}=\lambda+\kappa^{-2}R+\omega R_{\mu\nu}\Box^3R^{\mu\nu}-\frac{\omega}{2}R\Box^3R+s_1R^2\Box R^2+s_2R_{\mu\nu}^2\Box R_{\rho\sigma}^2+$$

$$+\sum_i c_i^{(3)}(R^3)_i+\sum_i c_i^{(4)}(R^4)_i+\sum_i c_i^{(5)}(R^5)_i \qquad \omega=\frac{e^{\gamma_E/2}}{\Lambda^8\kappa^2}$$

Non-local form-factors

$$L=\lambda+\kappa^{-2}R+R\,F_1(\Box)R+R_{\mu\nu}F_2(\Box)R^{\mu\nu}$$

- Extension of the quadratic in curvature terms Tomboulis, Krasnikov

- The most general theory describing gravitons' propagation around flat spacetime

- Intrinsically non-local due to non-polynomial functions F_1 and F_2

- Example with one form-factor (multiplicative modification of the graviton propagator)

$$L=\lambda+\kappa^{-2}R+\kappa^{-2}G_{\mu\nu}\frac{e^{H(\Box/\Lambda^2)}-1}{\Box}R^{\mu\nu}$$

Non-local form-factors

Requirements:
- Propagator has only first single poles with real masses (no tachyons) and with positive residues (no ghosts)
- In the spectrum only physical massless transverse graviton (spin 2)

Demands on a form-factor $e^{H(z)}$: $z = \frac{\Box}{\Lambda^2}$
- is real and positive on the real axis and has no zeros on the complex plane, is analytic on the whole complex plane
- has proper asymptotics for large *z* (in UV) along and around real axis (nonpolynomial or polynomial with degree ≥1)

- Example:

$$H(z) = \frac{1}{2}\Gamma(0, p^2(z)) + \frac{1}{2}\gamma_E + \frac{1}{2}\log p^2(z)$$

Non-local form-factors

$$H(z) = \frac{1}{2}\Gamma(0, p^2(z)) + \frac{1}{2}\gamma_E + \frac{1}{2}\log p^2(z)$$

- If *p(z)* is a polynomial then UV behavior is asymptotically polynomial, so asymptotically in UV HDQG
- But in *H(z)* there are no poles of *p(z)* due to analytic properties of *H(z)* !

- Unitarity of the theory secured at the perturbative level

- If degree of *p(z)* greater than zero, then theory is automatically multiplicatively renormalizable in *d=4*
- Define

$$\deg p(z) = \gamma + 1$$

Super-renormalizability

In $d=4$ divergences are present:

- formally for $\gamma=-1$ at any loop order and Δ grows with growing $L \Rightarrow$ non-renormalizability of EH gravity
- for $\gamma=0$ at any loop order and $\Delta \leq 4 \Rightarrow$ renormalizability of R^2 gravity
- for $\gamma=1$ at loop order 1,2,3 $\Rightarrow$ 3-loop super-renormalizability
- for $\gamma=2$ at loop order 1,2 $\Rightarrow$ 2-loop super-renormalizability
- for $\gamma=3$ at loop order 1 $\Rightarrow$ 1-loop super-renormalizability

- Divergences remain only at 1-loop order for $\gamma \geq 3$

We achieved 1-loop super-renormalizability!

Finiteness

- No divergences at the quantum level
- Divergent part of the effective action ~ beta functions of the theory

$$\beta_i = 0$$

related to scale (conformal) invariance and FP of RG flow

- In 1-loop superrenormalizable theory perturbative contributions only at one loop only to four couplings $\lambda \quad \kappa^{-2} \quad \omega_0^1 \quad \omega_0^2$

$$L_{\text{div}} = \lambda + \kappa^{-2} R + \omega_0^1 R^2 + \omega_0^2 R_{\mu\nu}^2$$

- Contributions only from generally covariant terms, with $2\gamma+4$ to 2γ (partial) derivatives on the metric

Finiteness

- Contributions to beta functions
- For cosmological constant
- For Planck constant
- For quadratic in curvature terms

$$\beta_\lambda \sim \frac{\omega_{\gamma-2}}{\omega_\gamma}, \left(\frac{\omega_{\gamma-1}}{\omega_\gamma}\right)^2$$

$$\beta_{\kappa^{-2}} \sim \frac{\omega_{\gamma-1}}{\omega_\gamma}, O\left(\mathrm{Riem}^3\right)$$

$$\beta_{\omega_0^{1,2}} \sim \frac{\omega_\gamma^1}{\omega_\gamma^2}, O\left(\mathrm{Riem}^3\right), O\left(\mathrm{Riem}^4\right)$$

- Set to zero all $\omega_{\gamma-2}$ and $\omega_{\gamma-1}$ and terms cubic, quartic in curvature
- Add two killers of beta functions $\beta_{\omega_0^1}$ and $\beta_{\omega_0^2}$

Killers

- Quadratic in curvature ("kinetic") part of the Lagrangian

$$L=\omega^1_\gamma R\Box^\gamma R+\omega^2_\gamma R_{\mu\nu}\Box^\gamma R^{\mu\nu}$$

- One of the simplest choice

$$s_1 R^2\Box^{\gamma-2}R^2+s_2 R_{\mu\nu}R^{\mu\nu}\Box^{\gamma-2}R_{\rho\sigma}R^{\rho\sigma}$$

- Contribution to beta functions from killers $\beta_{\omega_0^{1,2}\,\mathrm{kill}}\sim\frac{s}{\omega_\gamma}$

Finiteness if $\beta_{\omega_0^{1,2}}+\beta_{\omega_0^{1,2}\,\mathrm{kill}}=0$

- Contribution of killers to be computed using Barvinsky-Vilkovisky technology for traces of covariant operators on any background and in Dimensional Regularization (d=4-ε)

Computation

- 1-loop Quantum Effective Action $\Gamma = \frac{i}{2} \mathrm{Tr} \ln \hat{H}$ $\quad \Gamma_{\mathrm{div}} = -\frac{1}{\varepsilon} \sum_i \beta_i X_i$
- Kinetic operator for quantum fluctuations on any curved background

$$H^{\mu\nu,\rho\sigma} = \frac{\delta^2 S}{\delta g_{\mu\nu} \delta g_{\rho\sigma}}$$

- Contribution from killers we need only to quadratic in curvature order
- In BV trace technology killers contribute only to *U* terms (with 2γ derivatives)

$$\mathrm{Tr} \ln \hat{H}_{K1} = s_1 \frac{i}{\varepsilon} \frac{24 R^2}{3 \omega_\gamma^1 + \omega_\gamma^2}$$

$$\mathrm{Tr} \ln \hat{H}_{K2} = s_2 \frac{i}{\varepsilon} \left(\frac{(-10 \omega_\gamma^1 + \omega_\gamma^2) R^2}{3 \omega_\gamma^2 (3 \omega_\gamma^1 + \omega_\gamma^2)} + \frac{2 (20 \omega_\gamma^1 + 7 \omega_\gamma^2) R_{\mu\nu}^2}{3 \omega_\gamma^2 (3 \omega_\gamma^1 + \omega_\gamma^2)} \right)$$

Finiteness

- Beta functions of quadratic in curvature couplings

$$\beta_{R^2} := a_1 s_1 + a_2 s_2 + c_1 \qquad \beta_{R^2_{\mu\nu}} := b_2 s_2 + c_2$$

- c_1 and c_2 are contributions from terms in "kinetic" part of Lagrangian

- Coefficients of killers required to kill beta functions

$$s_1 = \frac{-(3\omega_\gamma^1 + \omega_\gamma^2)(40 c_1 \omega_\gamma^1 + 10 c_2 \omega_\gamma^1 + 14 c_1 \omega_\gamma^2 - c_2 \omega_\gamma^2)}{24(20\omega_\gamma^1 + 7\omega_\gamma^2)}$$

$$s_2 = \frac{-3 c_2 \omega_\gamma^2 (3\omega_\gamma^1 + \omega_\gamma^2)}{20\omega_\gamma^1 + 7\omega_\gamma^2}.$$

Conclusions

- E-H QG is valid non-renormalizable EFT below Planck scale
- HDQG is renormalizable, can be made even 1-loop super-renormalizable, has massive ghosts
- Nonlocality in formfactors solves unitarity problems, HDQG revival !!
- Still possible polynomial behaviors for propagation asymptotically in UV
- Divergences only at one-loop order
- Perturbative finiteness obtained by adding killers
- Easy generalizations to higher dimensions and for higher curvature terms in the action

Conclusions

Finite Quantum Gravity Exists!!!

Thank you
for attention!