Fate of the Higgs mode in the vicinity of a quantum critical point

Adam Rançon

ENS-Lyon

Nicolas Dupuis

LPTMC (Paris)

ENS DE LYON

ERG 2014

JAMES FRANCK INSTITUTE

THE UNIVERSITY OF CHICAGO

Fate of the ~~Higgs~~ amplitude mode in the vicinity of a quantum critical point

Adam Rançon

ENS-Lyon

Nicolas Dupuis

LPTMC (Paris)

ENS DE LYON

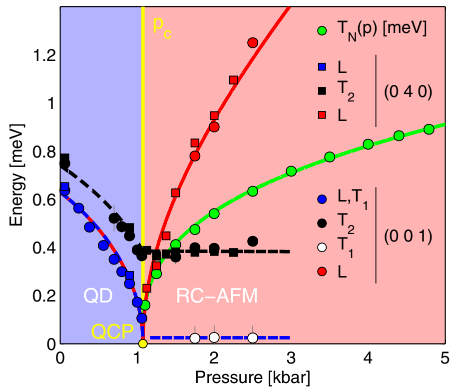
ERG 2014

JAMES FRANCK INSTITUTE

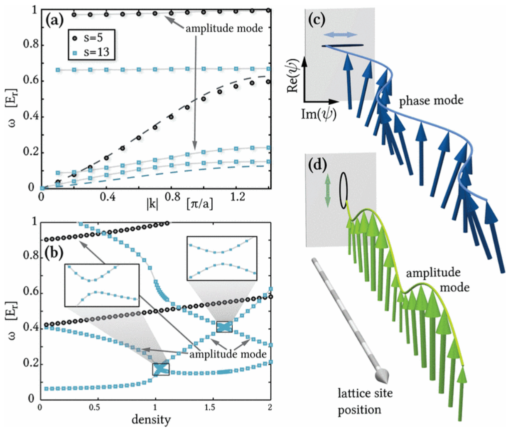
THE UNIVERSITY OF CHICAGO

Experimental observations

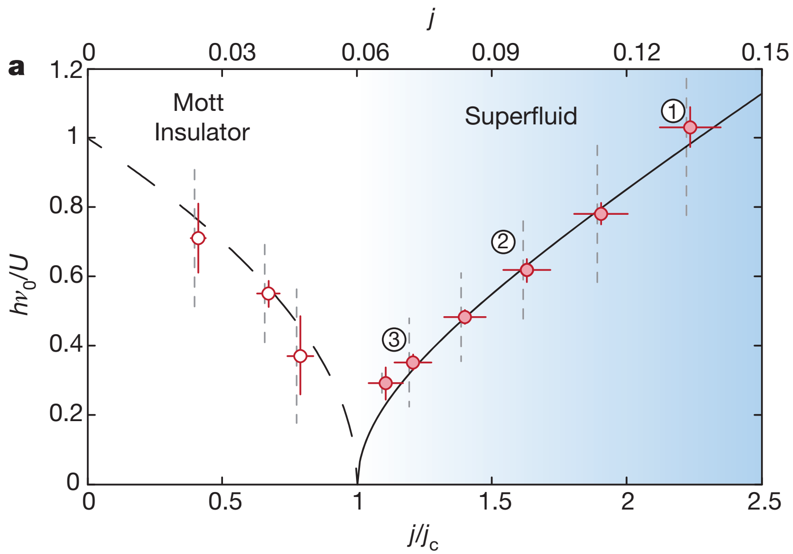
Ruegg et al. PRL 2008
Quantum spins - 3D



Blissbort et al. PRL 2008
Cold atoms - 3D

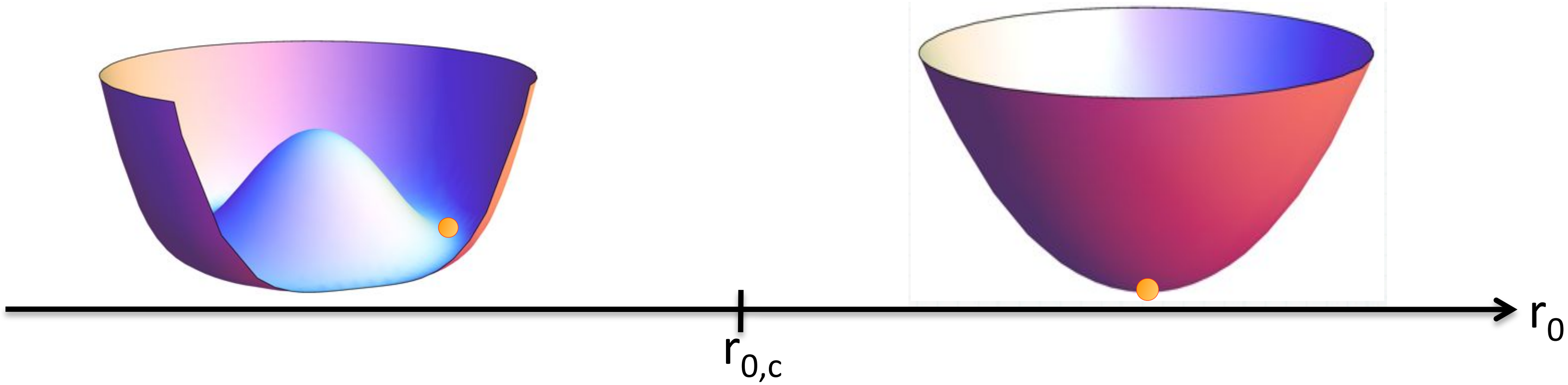


Enders et al. Nature 2012
Cold atoms – 2D



Quantum O(N) model

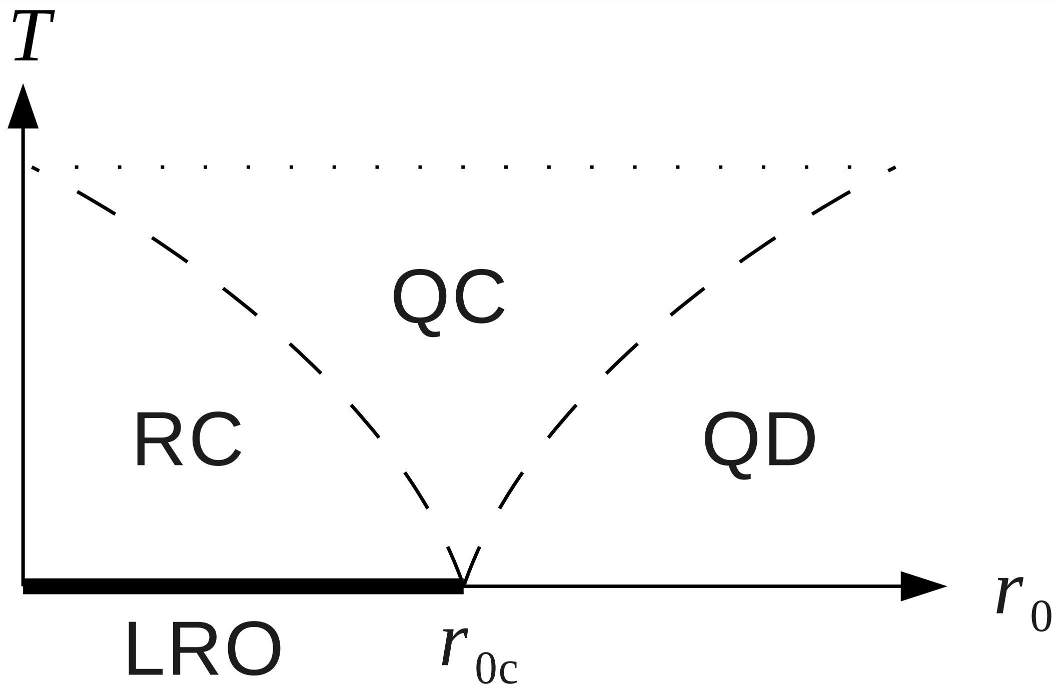
$$S = \int_0^\beta d\tau \int d^d x \left\{ \frac{1}{2}\psi_i(-c^{-2}\partial_\tau^2 - \nabla^2 + r_0)\psi_i + \frac{u_0}{4!N}(\psi_i^2)^2 \right\}$$



- Generalization of classical O(N) model
- at T=0, Lorentz symmetry
- Describes critical regime of a number of systems :
  - bosons in optical lattices
  - antiferromagnets
  - Josephson junction arrays
  - granular superconductors, …

Chakravarty et al. 1989
Chubukov et al. 1994
Sachdev's book, etc.

Typical phase diagram in 2D



Quantum critical point described by the classical O(N) model in 2+1 D

Non trivial critical exponent : $\eta,\ \nu$

One (quantum) exponent : dynamical exponent z = 1 (Lorentz symmetry)

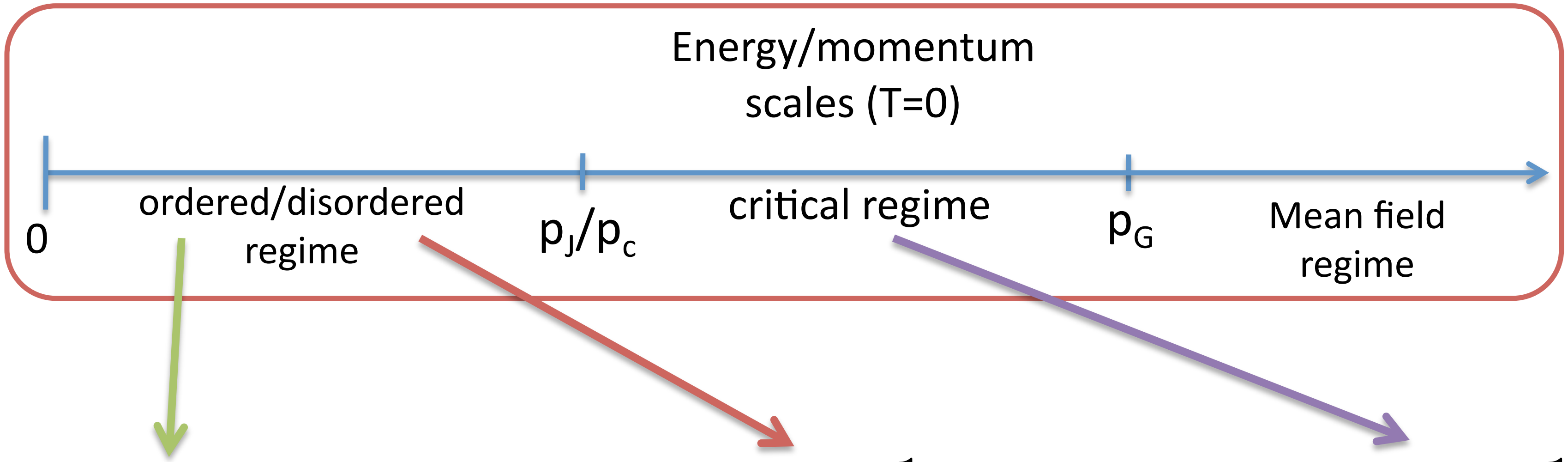
Vanishing energy scale close to the critical point (gap in the disordered phase)

$$\Delta \propto |r_0 - r_{0c}|^{z\nu}$$

Excitation spectrum in ordered phase ?

-Critical behavior and phase diagram well-known

-Excitation spectrum ? Much less clear in the ordered phase



$$G_t(p) = \frac{1}{p^2}$$

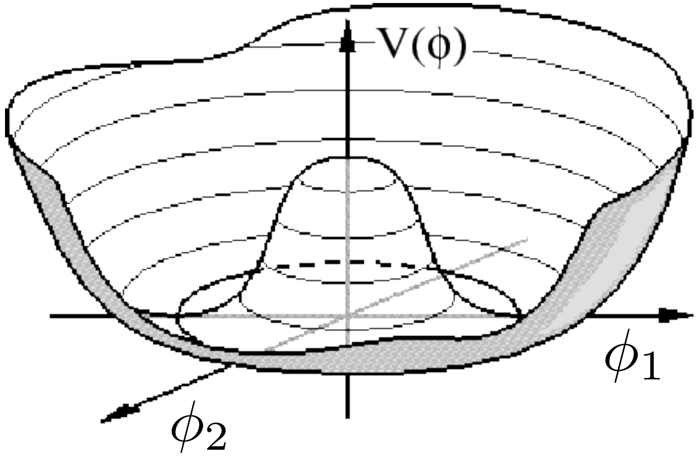
$$G(p) = \frac{1}{p^2 + \Delta^2}$$

$$G(p) = \frac{1}{p^{2-\eta}}$$

$$G_l(p) = ?$$

$$p^2 = \omega_n^2 + \mathbf{p}^2$$

Excitation spectrum : Mean Field



Broken symmetry phase $\langle \phi_i \rangle = \delta_{i1} \phi_0$

Mean-field picture : - N-1 transverse (massless) Goldstone modes $G_t(p) = \dfrac{1}{p^2}$

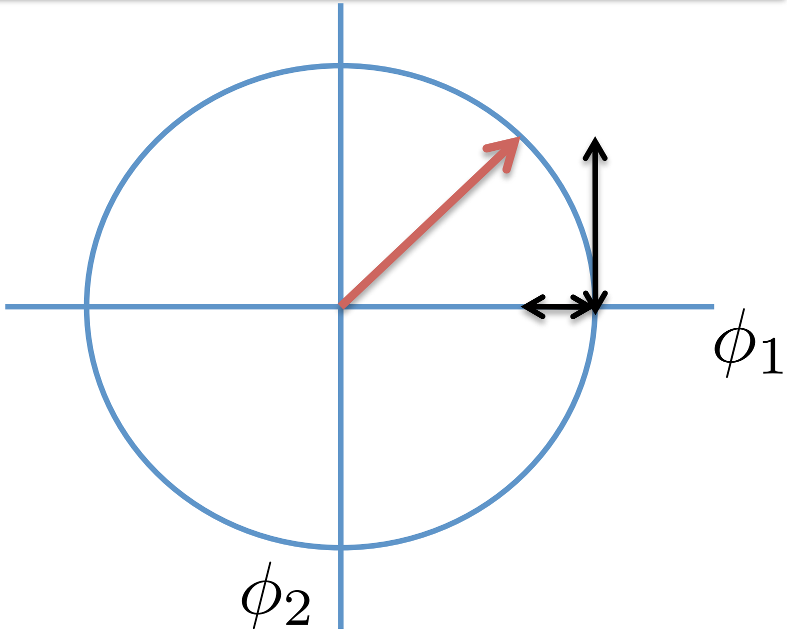
- 1 gapped longitudinal mode $G_l(p) = \dfrac{1}{p^2 + 2\Delta^2}$

Amplitude fluctuations = longitudinal fluctuations (in MF !)

MF spectrum has one gapped mode arbitrarily close to the QCP ($\Delta \to 0$) at energy $\omega = \sqrt{2}\Delta$

Spectrum : Goldstone regime

At low energy (dominated by the Goldstones), amplitude $\langle|\phi|\rangle$ constant.



Strong coupling between longitudinal and transverse fluctuations

$$G_l(p) \propto \frac{1}{p}$$ at low energy

Longitudinal (amplitude ?) fluctuations seem not to be a well defined mode close to criticality:
the 1/p divergence might hide the resonance as Δ goes to zero.

Zwerger 2004, Dupuis 2011

Amplitude mode : scalar fluctuations

Podolsky et al. 2011 : it depends on the correlation function !

$$\phi = \sqrt{\rho}\mathbf{n} \qquad G_\rho = \langle \rho\rho \rangle$$

magnitude

Not the same correlation function !
Example for antiferromagnets: - longitudinal = neutron scattering
- scalar (amplitude) = Raman scattering

At low energy, coupling for decay of an amplitude fluctuation into two Goldstone is small (derivative coupling – proportional to ω^2).

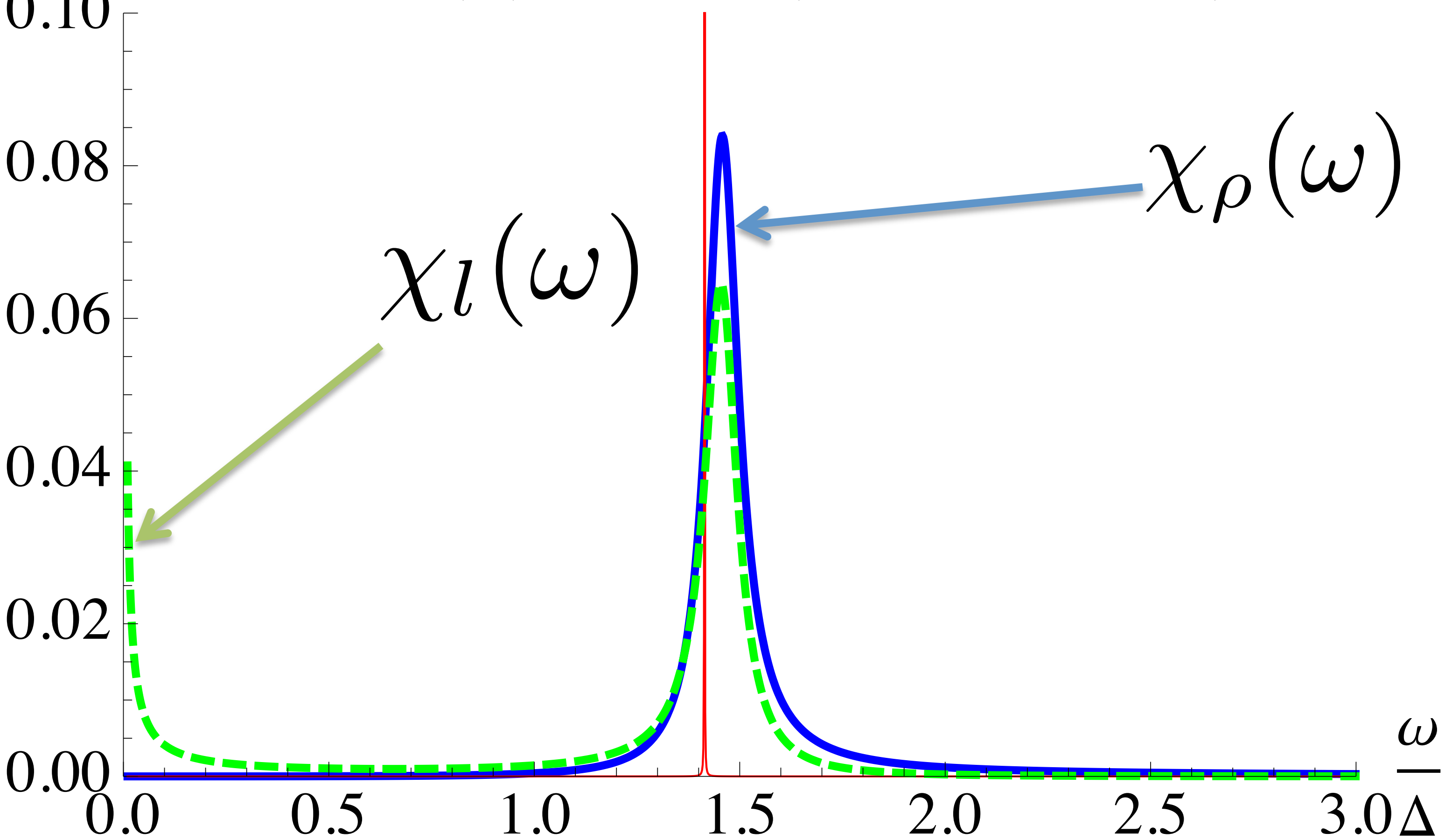
We expect the spectral function to be of order ω^{d+1} at small ω (no divergence), so the resonance might still be visible close to the QCP.

But is the amplitude mode still well defined close to the quantum critical point ?

Amplitude mode : Large N

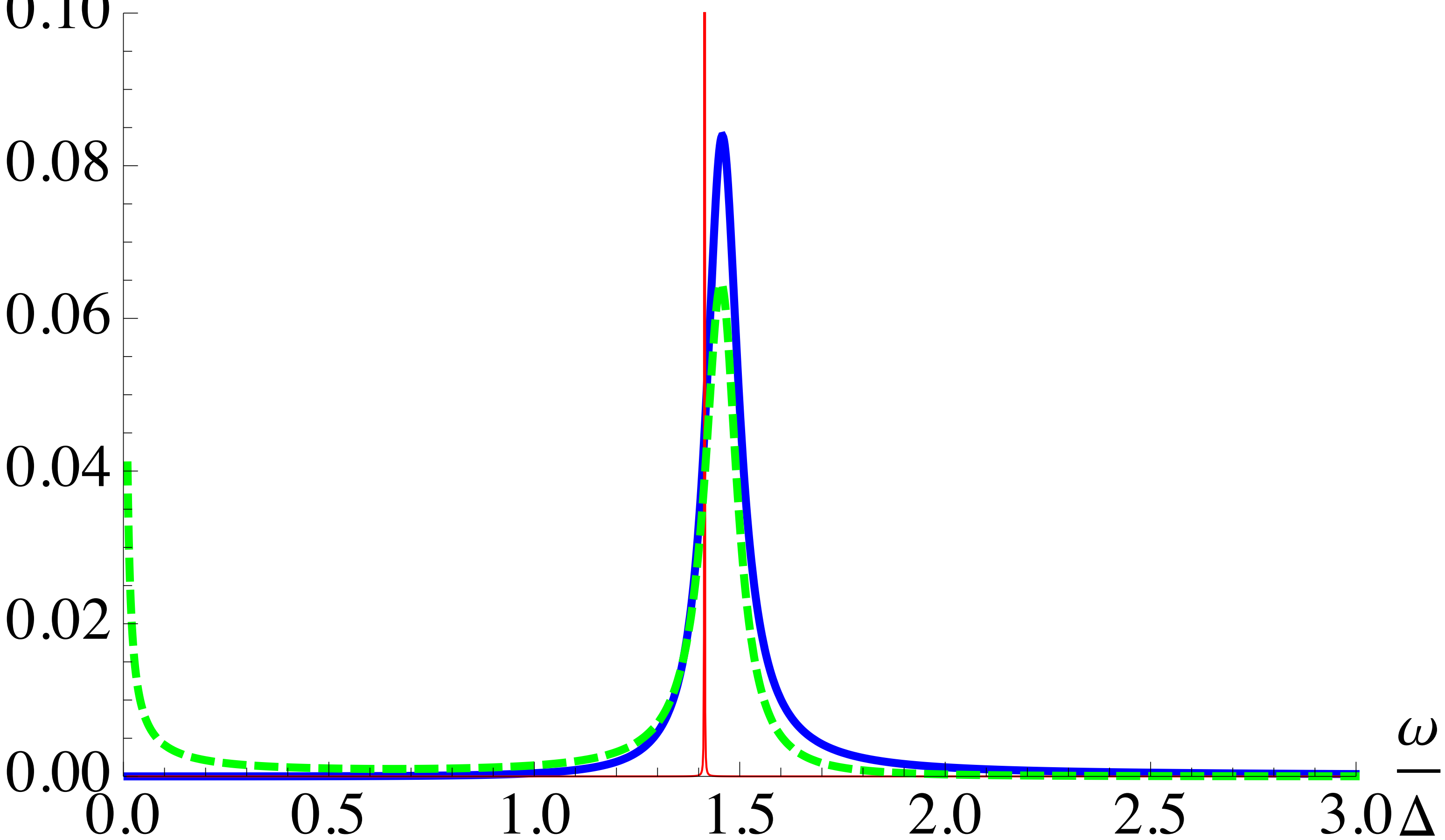
Spectral functions : far from criticality

$$\chi_\alpha(\omega) = \mathrm{Im} G_\alpha(i\omega_n \to \omega + i0^+)$$



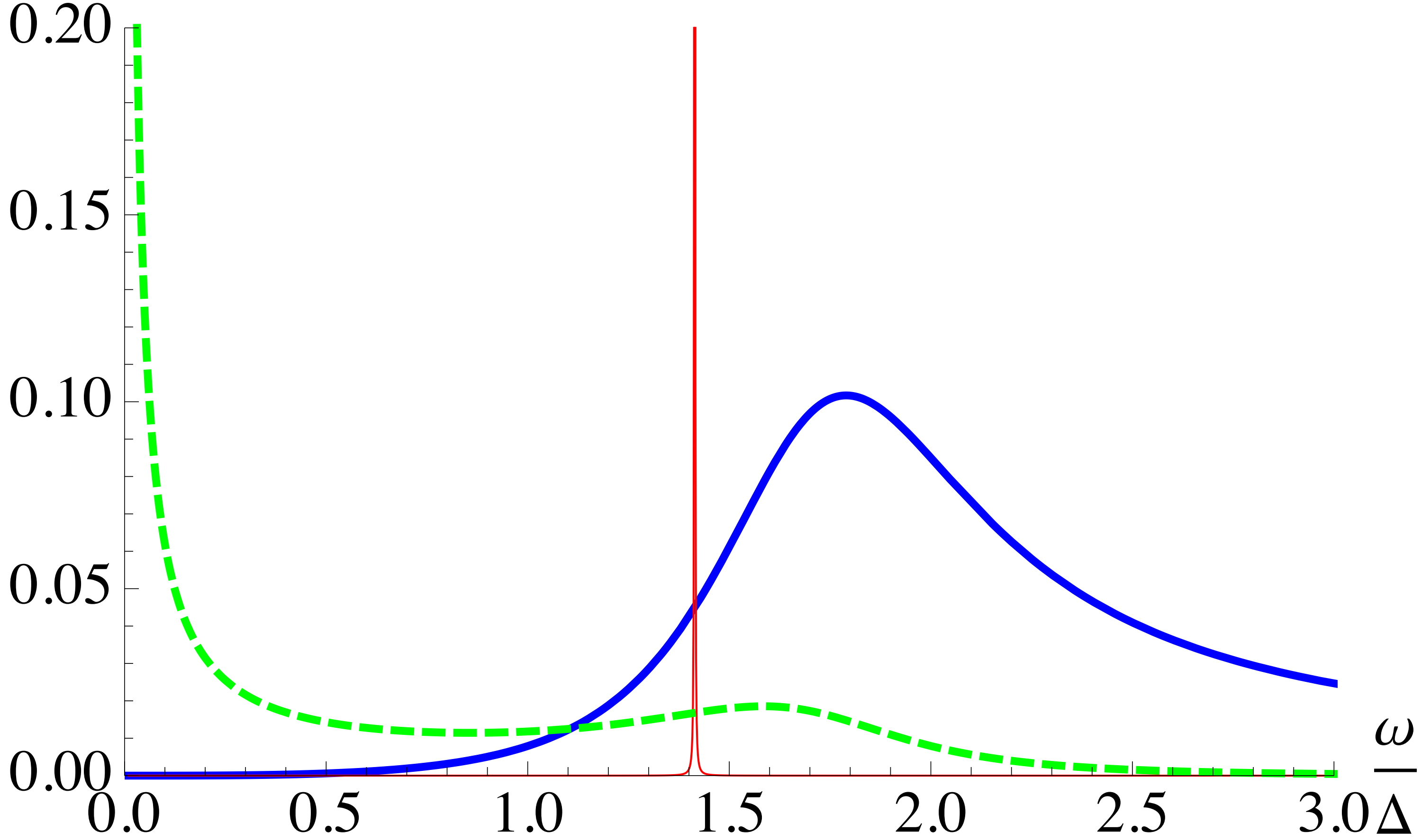
Amplitude mode : Large N

Spectral functions : far from criticality



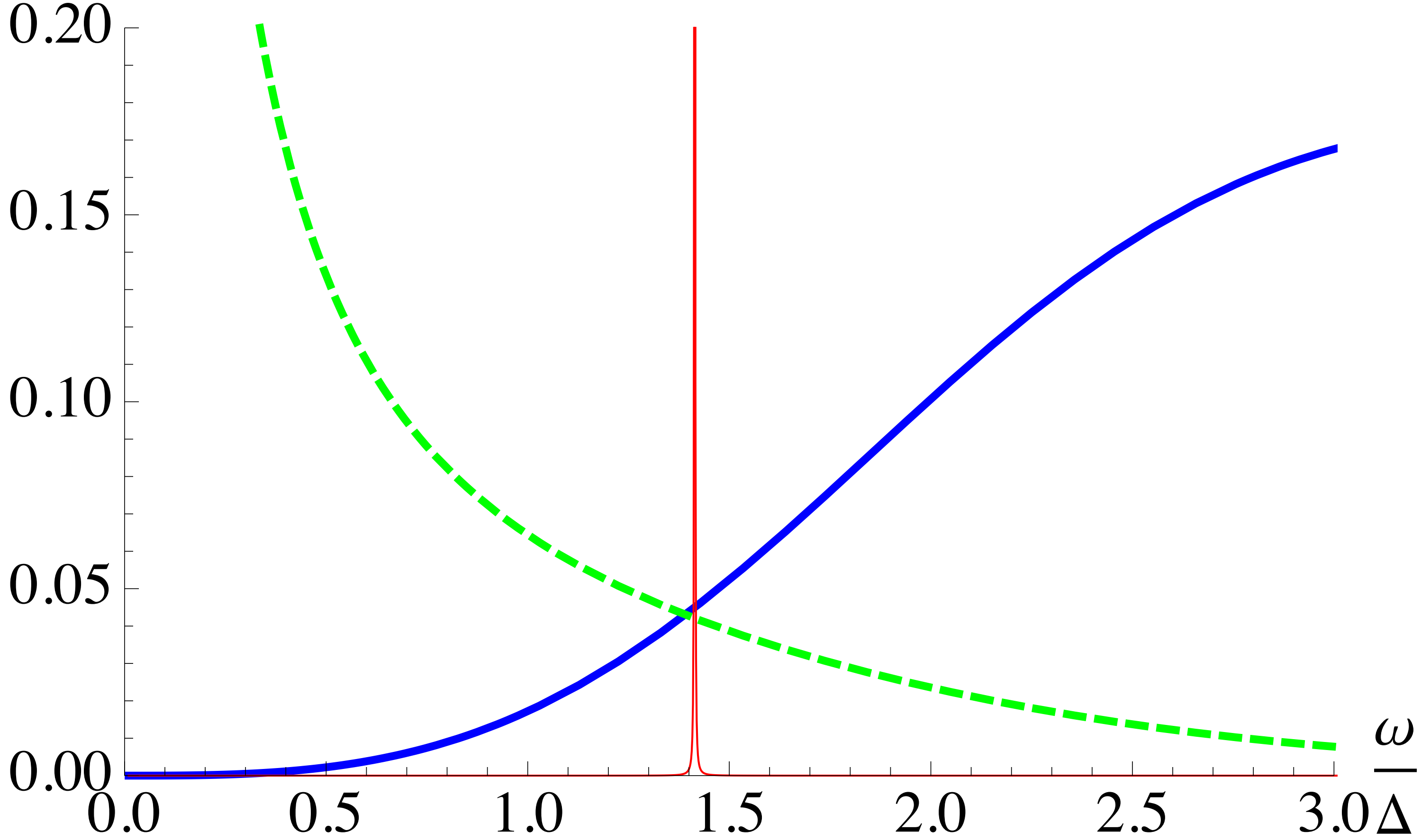
Amplitude mode : Large N

Spectral functions : closer to criticality



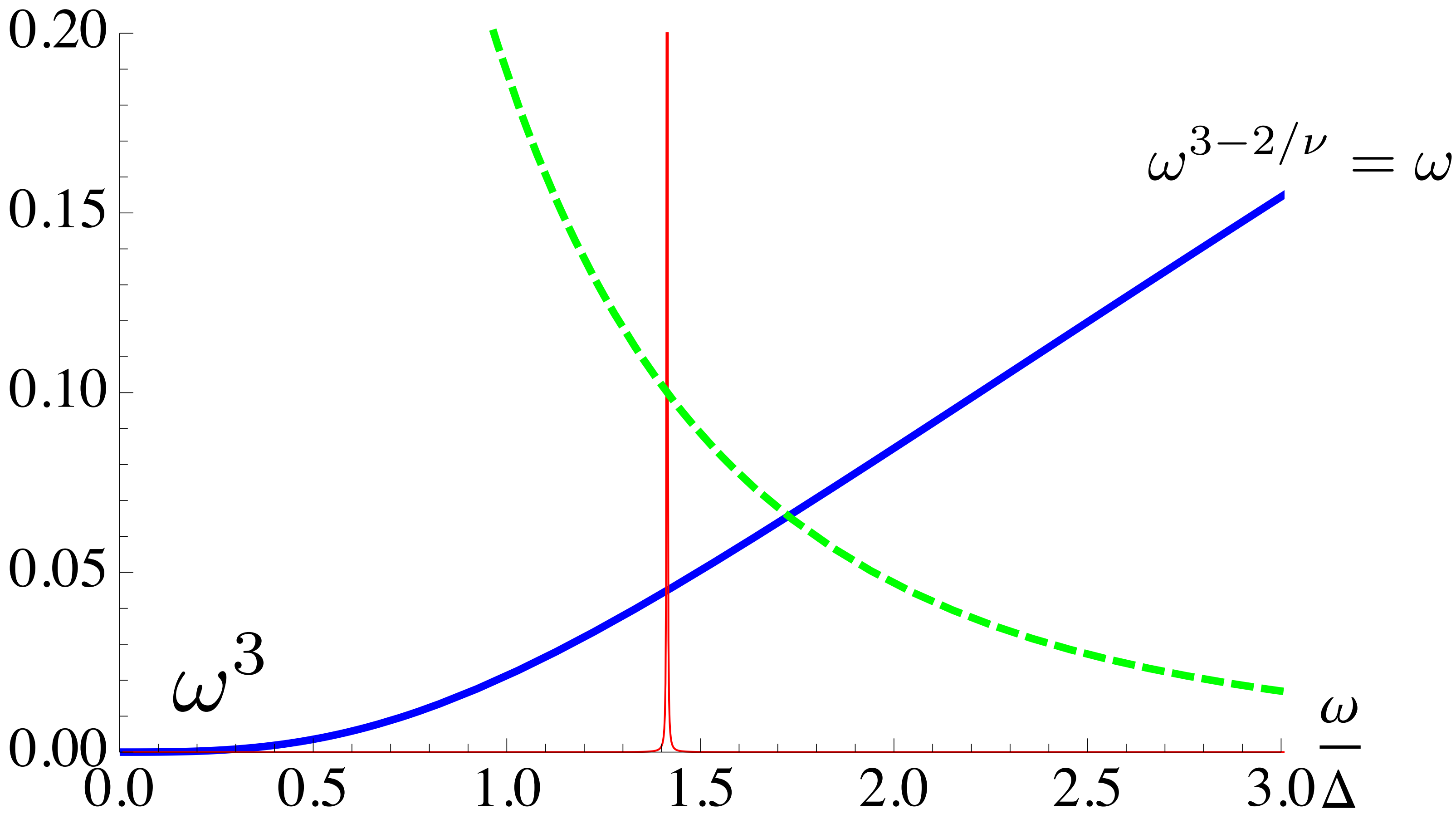
Amplitude mode : Large N

Spectral functions : closer to criticality



Amplitude mode : Large N

Spectral functions : critical regime – no amplitude mode



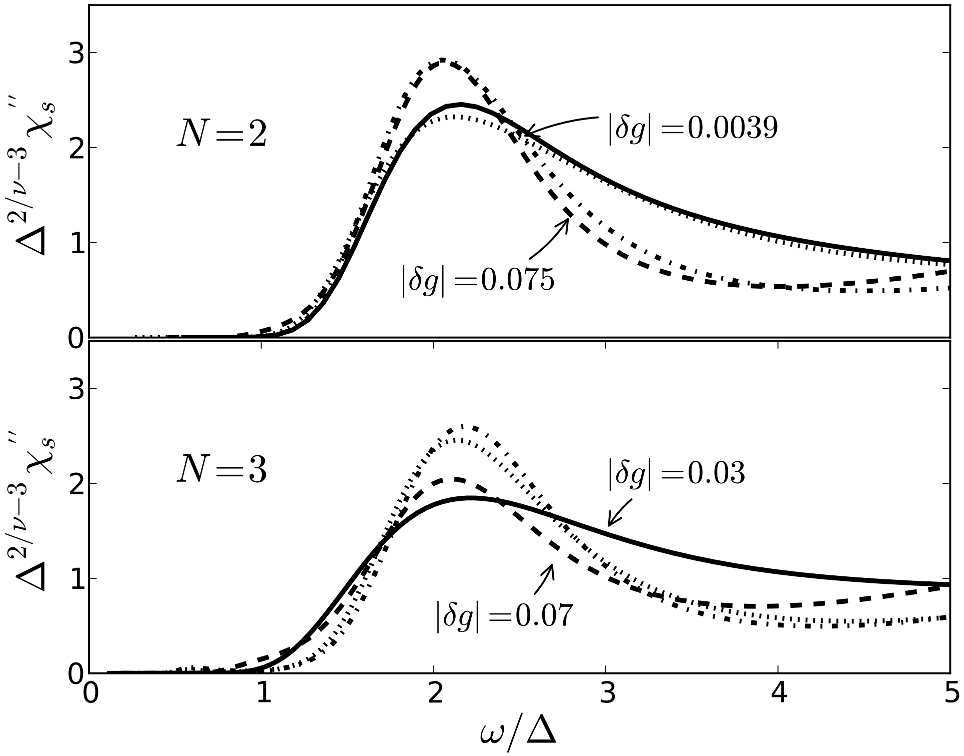
Ror 1/N corrections, see (if you dare) Podolsky & Sachdev, PRB2012

Amplitude mode : Small N Monte Carlo

Recent Monte Carlo simulations : Polet et al. PRL 2012, Gazit et al. PRL 2013, Chen et al. PRL 2013 for N=2 and 3

resonance between 2.1Δ and 3.3Δ

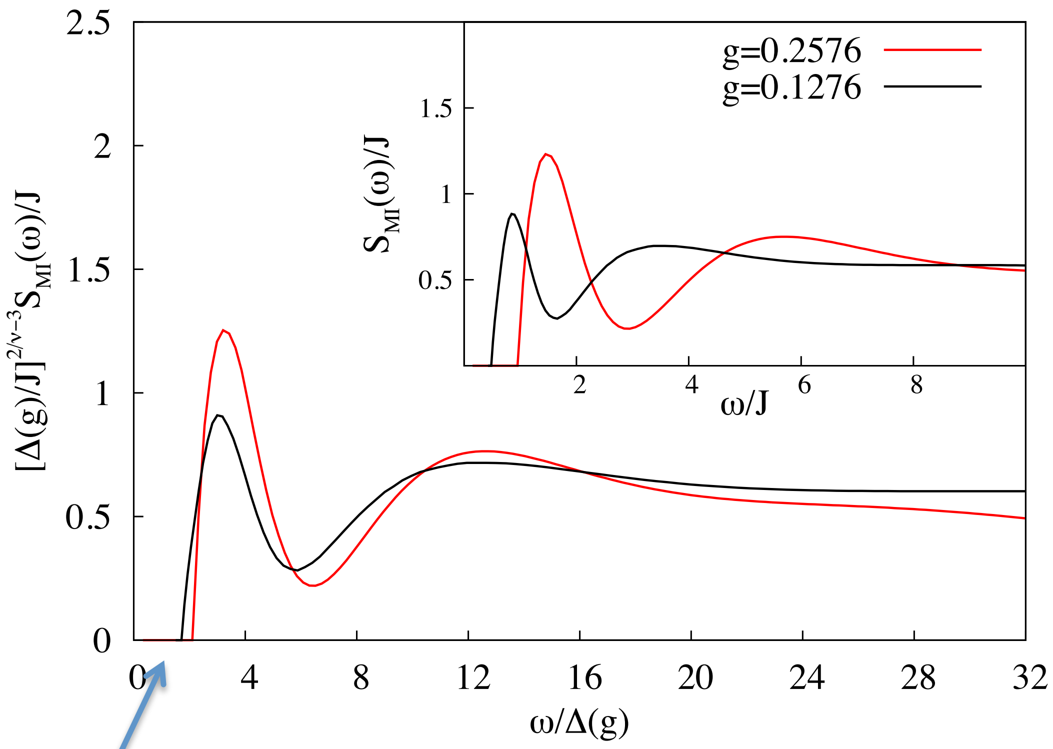
≈2.2Δ



Amplitude mode defined close to criticality, but how does it disappear as N increases ?

Amplitude mode : disordered phase ?

Chen et al. : amplitude mode in the symmetric (disordered) phase !?



Gapped system : threshold at ω=2Δ (ρ -> 2 massive bosons)

Gave an RG argument, but is it true ?

Non-Pertubative Renormalization Group

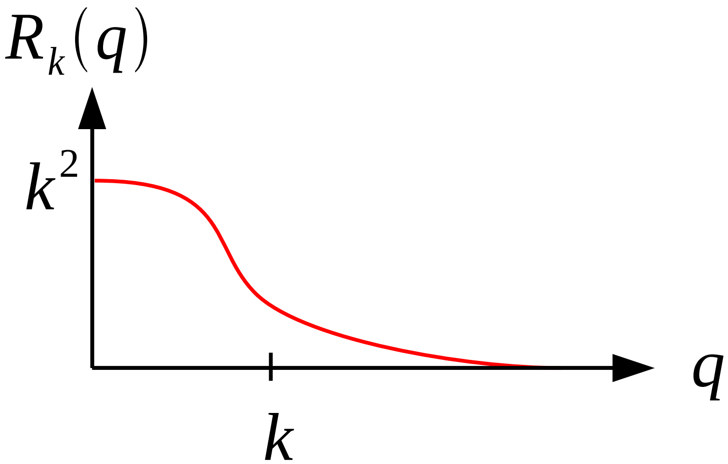
Family of actions indexed by momentum scale k

$$S+\Delta S_{\Lambda} \longrightarrow S+\Delta S_{k} \longrightarrow S+\Delta S_{k=0}=S$$

Reference system — System of interest

RG equation: $\partial_k \Gamma_k[\phi]$

$$\Delta S_k[\psi] = \sum_i \int_q R_k(q)\psi_i(q)\psi_i(-q)$$



Exact Flow equation (Wetterich `93) : $\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left\{ \partial_k R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1} \right\}$

Average Effective Action
("Gibbs" free energy)

Bare action :

$$S = \int_0^\beta d\tau \int d^d x \left\{ \frac{1}{2}\psi_i(-c^{-2}\partial_\tau^2 - \nabla^2 + r_0)\psi_i^2 + \frac{u_0}{4!N}(\psi_i^2)^2 \right\}$$

"Functional free energy" gives access to all the physics :

$$W_k[J,h] = \ln \int D\psi_i e^{-S-\Delta S_k + J_i\psi_i + h\rho}$$

modified Legendre transform $\quad \phi_i(\tau) = \langle \psi_i(\tau) \rangle$

$$\Gamma_k[\phi, h]$$

Source term for the amplitude

$$G_\rho(p) = -\Gamma^{(0,2)}(p) + \Gamma^{(1,1)}(p) G_l(p) \Gamma^{(1,1)}(p)$$

Ansatz

$$\Gamma[\phi,h]=\Gamma[\phi,0]+\int_{x,y}\frac{1}{2}(\phi^2(x)-\phi_0^2)H_1(x-y)h(y)+\int_{x,y}h(x)H_2(x-y)h(y)$$

Standard derivative and field expansion

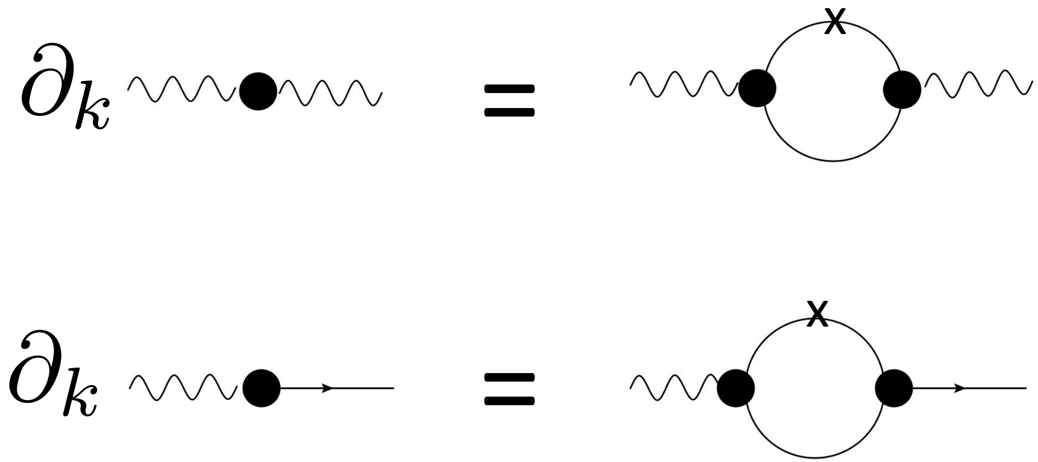
Parameterization of longitudinal propagator

$$\Gamma_l^{(2,0)}(p)=p^2+\Delta_A(p)+\phi_0^2\Delta_B(p)$$

Computed in a "poor man's" BMW approximation (Benitez et al. PRB 2008)

Flows

h-dependent part of the flow



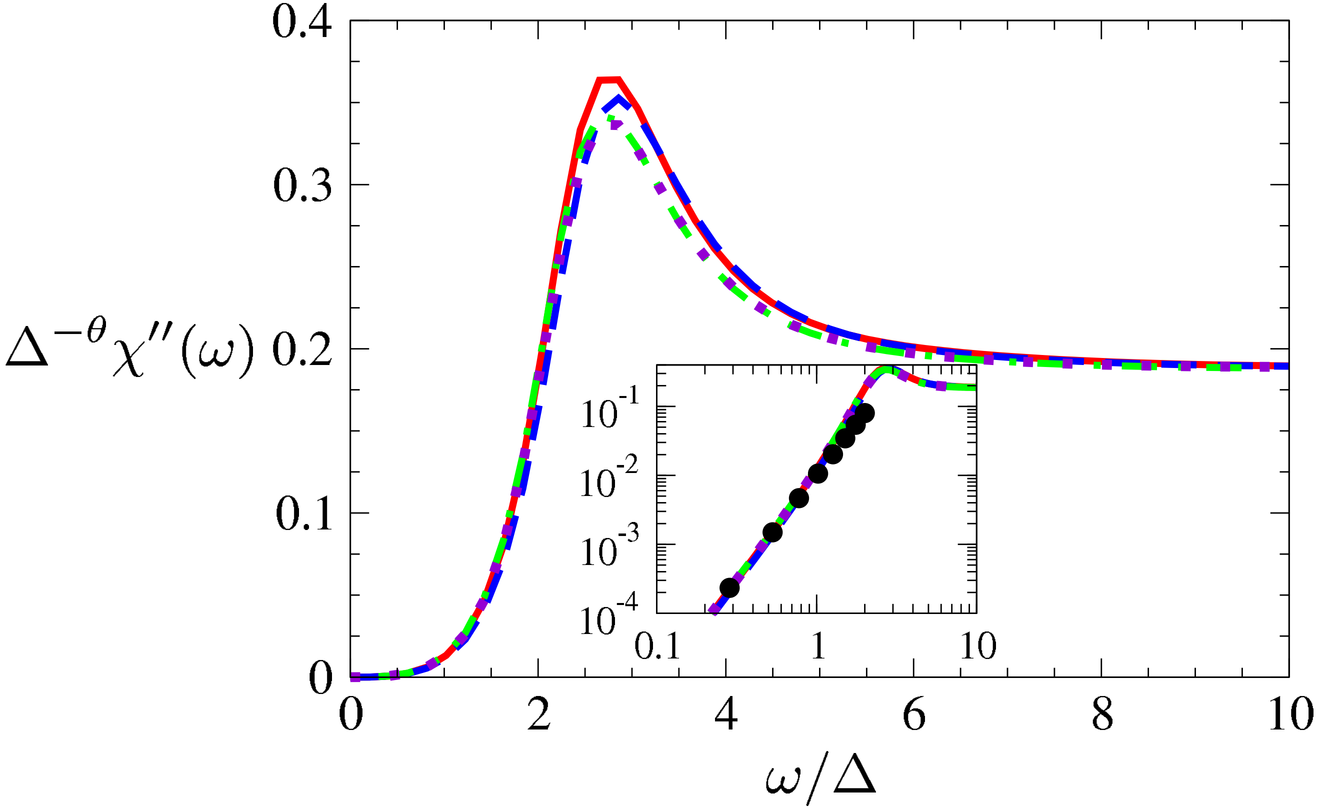
Propagators approximated by a derivative expansion but keep the Matsubara frequency dependence of the vertices

At the end of the day, numerical analytic continuation (Pade approximant)

Technical details in AR and N. Dupuis, PRB 2014

Amplitude mode : N=2

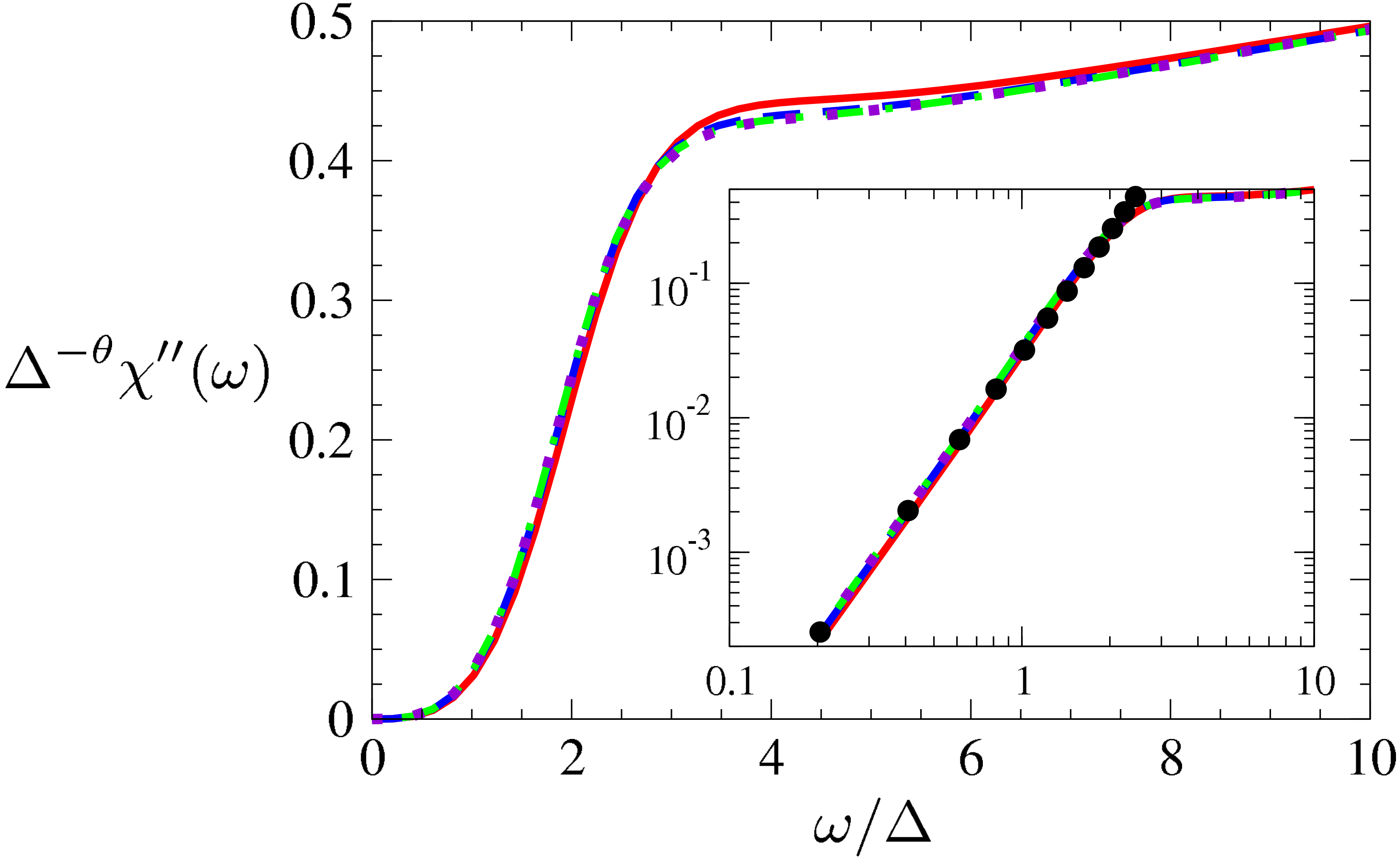
N=2, m≈2.4 Δ



Wrong scaling exponent
$\theta \neq 3 - 2/\nu$

AR & N. Dupuis, PRB 89 180501(R) (2014)

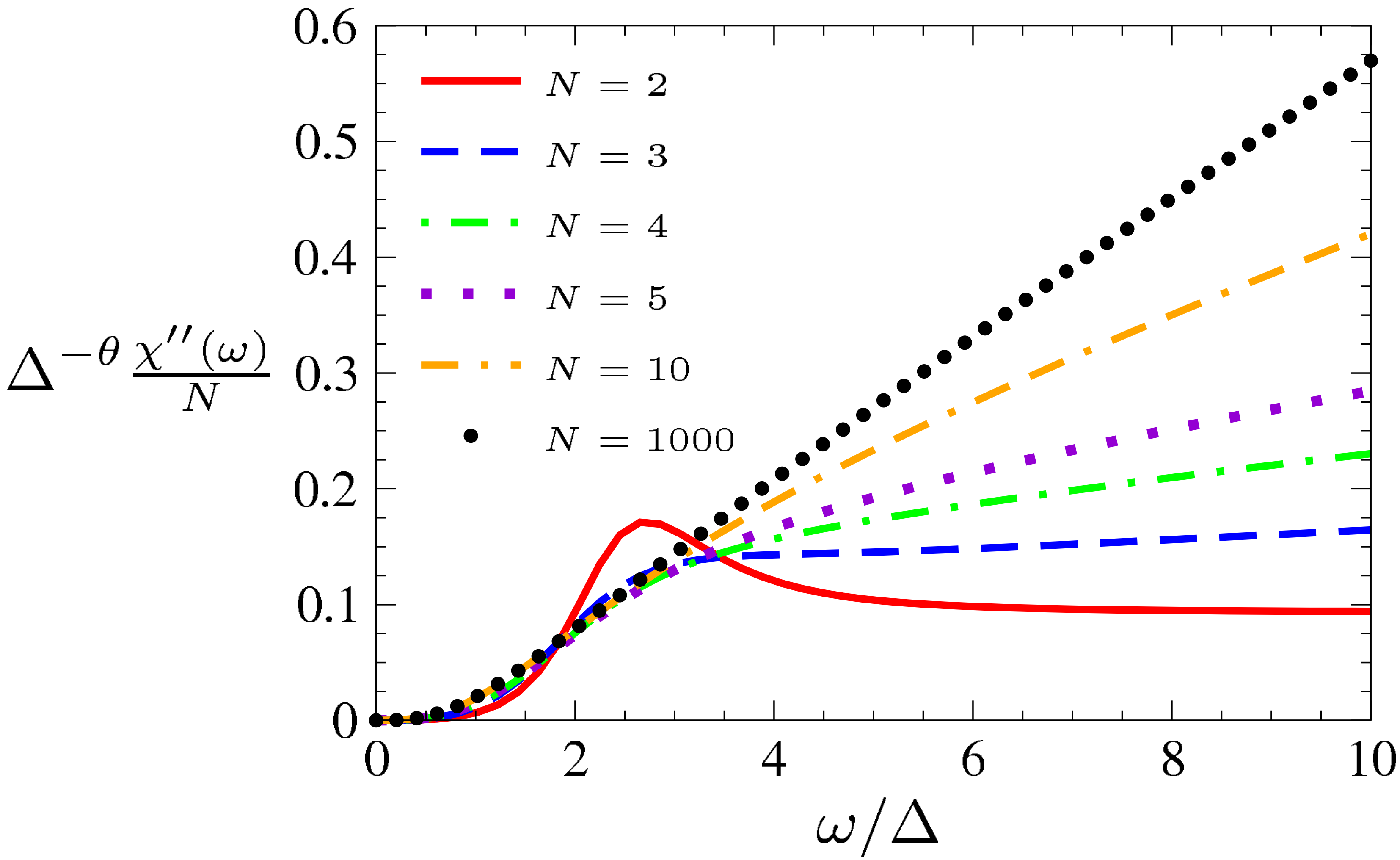
Amplitude mode : N=3



AR & N. Dupuis, PRB 89 180501(R) (2014)

Amplitude mode : all N

No resonance for N>2



AR & N. Dupuis, PRB 89 180501(R) (2014)

T=0 disordered phase - 1

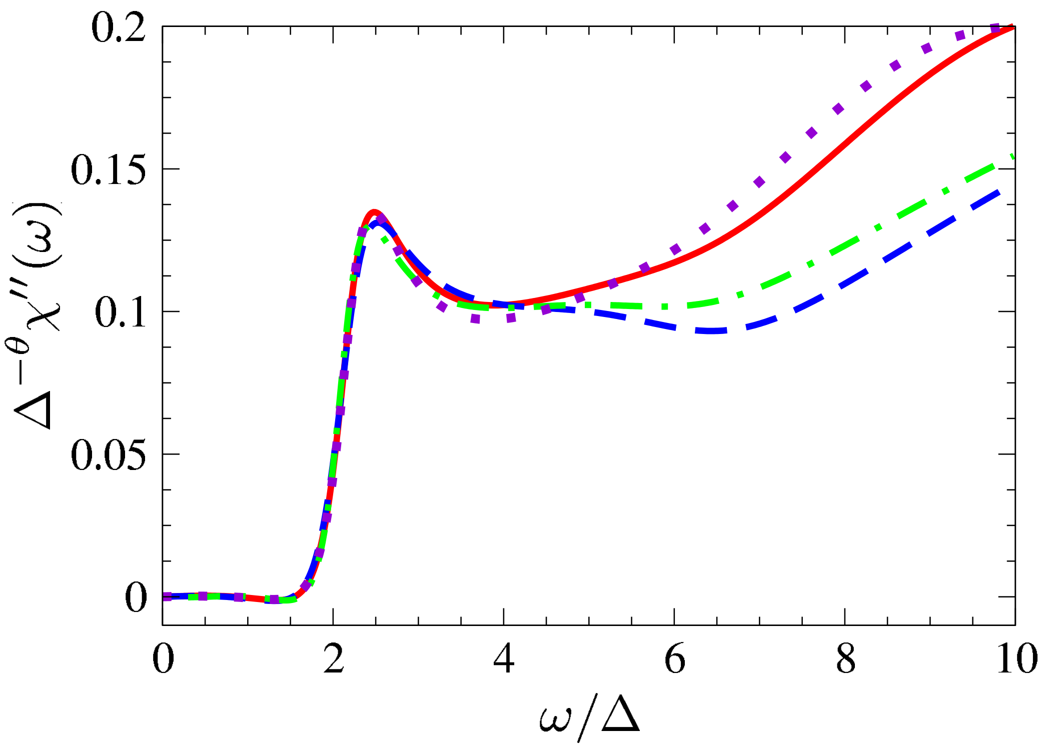
-“Resonance” close to gap at 2Δ : related to amplitude mode ?

-If so : resonance must appear in the critical regime (otherwise : not related)

-But in critical regime : scaling

$$\chi''_{\rm crit}(\omega) \propto \omega^{3-2/\nu}$$

-Implies no resonance in critical regime



- Thus resonance in ordered/ disordered regime are not related – not the same physics

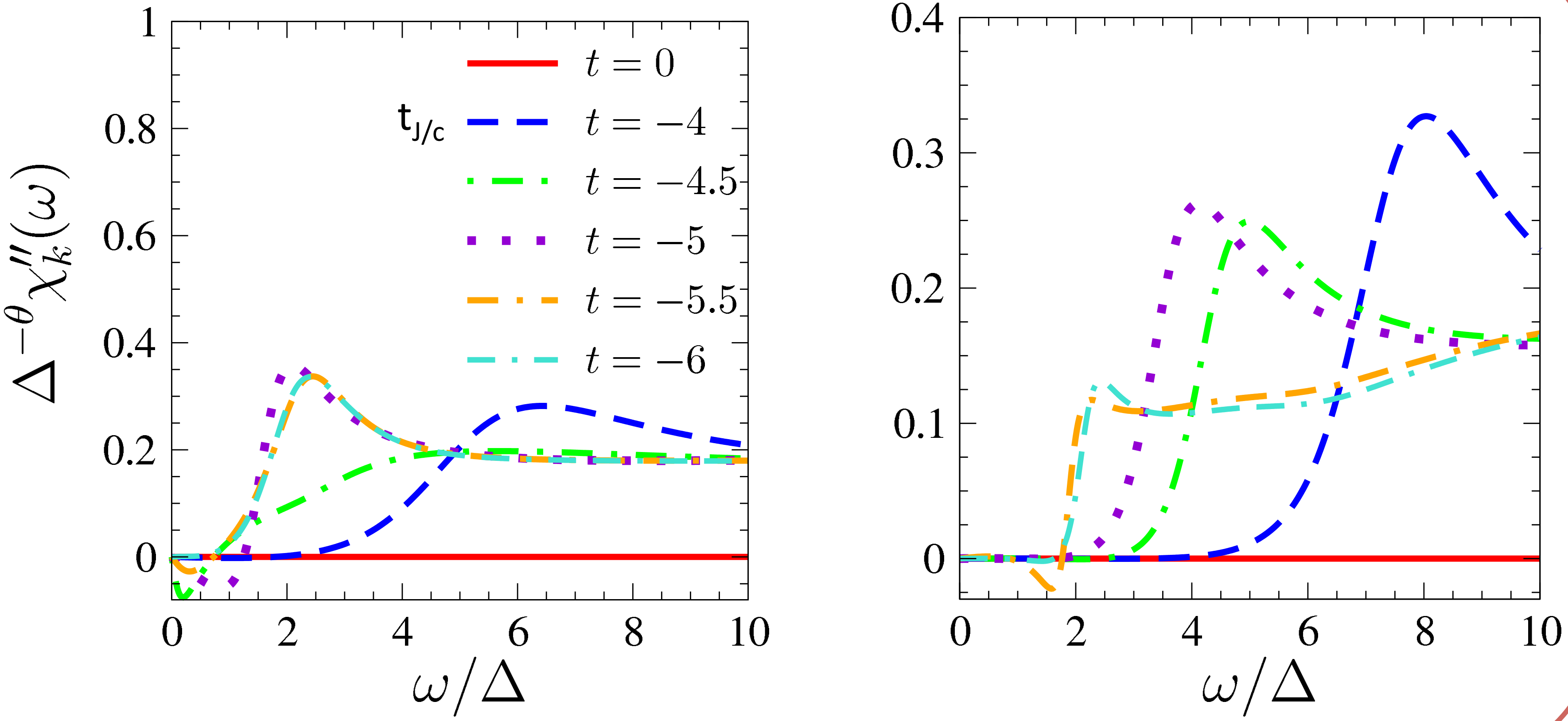
AR & N. Dupuis, PRB 89 180501(R) (2014)

Quantum disordered phase - 2

No resonance in the disordered phase : no amplitude mode precursor in the renormalization flow when the peak appears in the disordered phase.

Ordered phase

Disordered phase



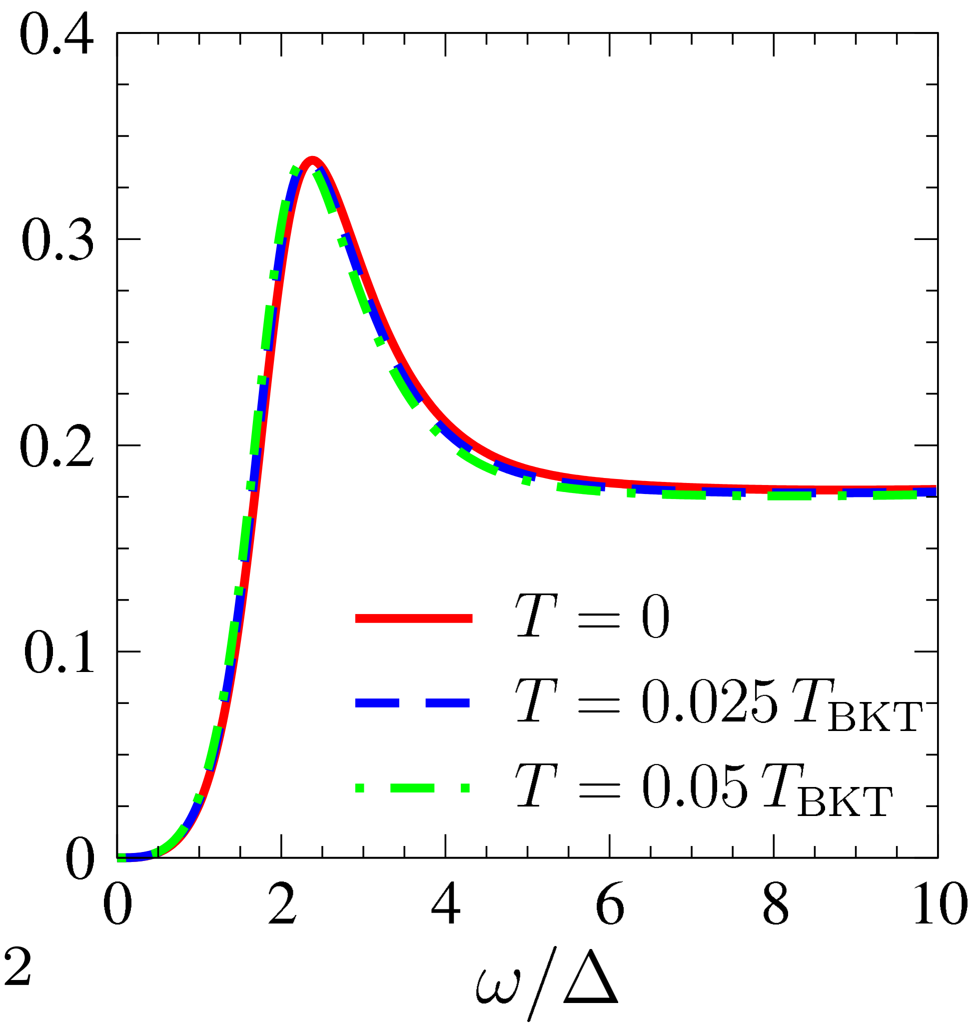
Energy scales during the flow

Amplitude mode : Kosterlitz-Thouless phase

N=2, T>0: BKT phase
(quasi-long range order)

$$T_{BKT} \simeq 0.42\Delta$$

AR et al. PRE 2013



Numerical analytic continuation hard
for ω < 2πT
Perturbative calculation :

$$\chi''(\omega) \propto \omega^3 \coth\left(\frac{\omega}{2T}\right) \to T\omega^2$$

AR & N. Dupuis, PRB 89 180501(R) (2014)

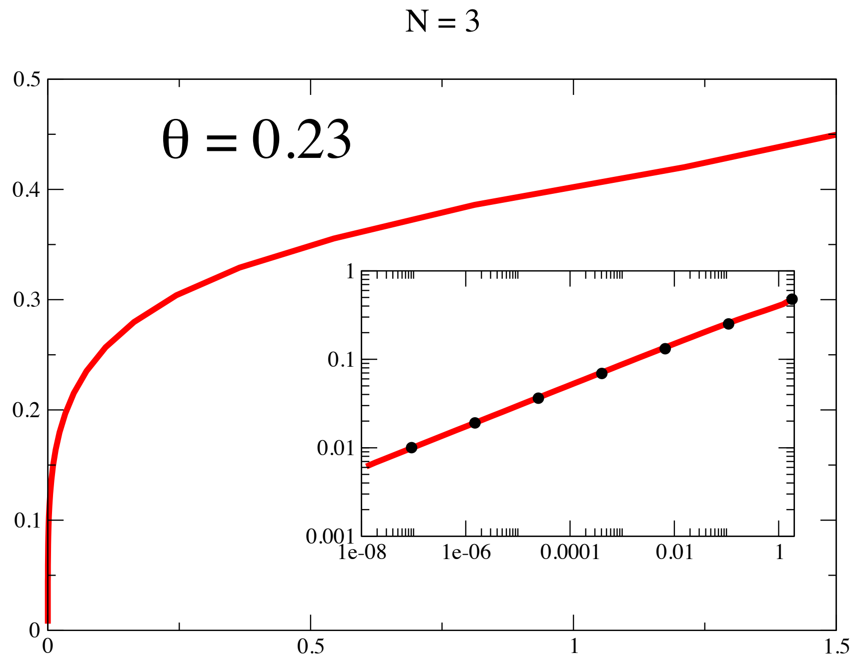
Conclusion and Perspectives

- NPRG study of amplitude mode close to a non-trivial critical point. Available approaches : Monte-Carlo, NPRG, others ?

- Study of the amplitude mode for all N. Nice agreement with Monte-Carlo for N=2. Case N=3 needs further studies :

F. Rose, F. Leonard, B. Delamotte and N. Dupuis :
Self-consistent BMW approach

Critical regime :
correct exponents !



Perspectives bis

- No amplitude mode resonance in the disordered regime

- Resonance exists in the BKT phase (vanishing order parameter !). Up to what temperature ?   ($T_{BKT} \approx m_H/5$)

-Numerical analytic continuation hard at large T : Strodthoff et al. scheme ?

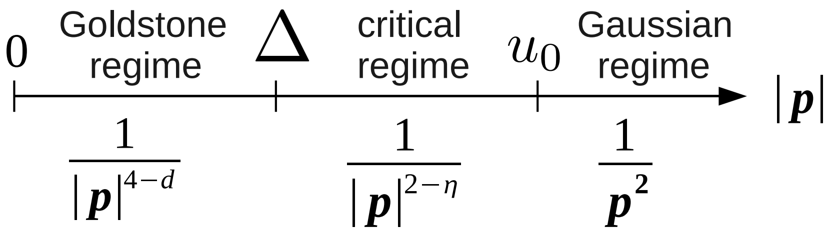
-Large N, resonance exists for $2.7 < d \leq 3$, and “high temperature” (fraction of T_c)

Typical energy scales

$$S = \int_0^\beta d\tau \int d^d x \left\{ \frac{1}{2}\psi_i(-c^{-2}\partial_\tau^2 - \nabla^2 + r_0)\psi_i^2 + \frac{u_0}{4!N}(\psi_i^2)^2 \right\}$$

Four energy scales : T , Δ , u_0 , $\omega_c \propto \sqrt{r_0 - r_{0,c}}$ (not all independent)

(a) Critical regime: $\omega_c \ll u_0$



(b) Non-critical regime: $\omega_c \gg u_0$