ERG and gravity
recent developments

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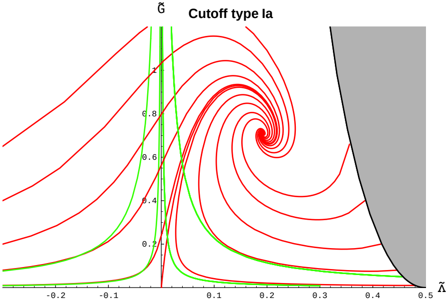
Einstein–Hilbert truncation

$$\Gamma_k = \int d^4x \sqrt{g} \frac{1}{16\pi G}(2\Lambda - R)$$



Fourth order gravity

$$\Gamma_k = \int d^4x \sqrt{g} \left[\frac{1}{16\pi G}(2\Lambda - R) + \alpha R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R^2 \right]$$

K.S. Stelle, Phys. Rev. **D16**, 953 (1977).
J. Julve, M. Tonin, Nuovo Cim. **46B**, 137 (1978).
E.S. Fradkin, A.A. Tseytlin,Phys. Lett. **104 B**, 377 (1981).
I.G. Avramidi, A.O. Barvinski,Phys. Lett. **159 B**, 269 (1985).

Four-derivative truncation of ERG

- O.Lauscher, M. Reuter, Phys. Rev. D 66, 025026 (2002) arXiv:hep-th/0205062
- A. Codello and R. P., Phys.Rev.Lett. **97** 22 (2006)
- D. Benedetti, P.F. Machado, F. Saueressig, Mod. Phys. Lett. A24, 2233-2241 (2009) arXiv:0901.2984 [hep-th] Nucl. Phys. B824, 168-191 (2010), arXiv:0902.4630 [hep-th]
- M. Niedermaier, Nucl. Phys. B833, 226-270 (2010)
- N. Ohta and R.P. Class. Quant. Grav. **31** 015024 (2014); arXiv:1308.3398

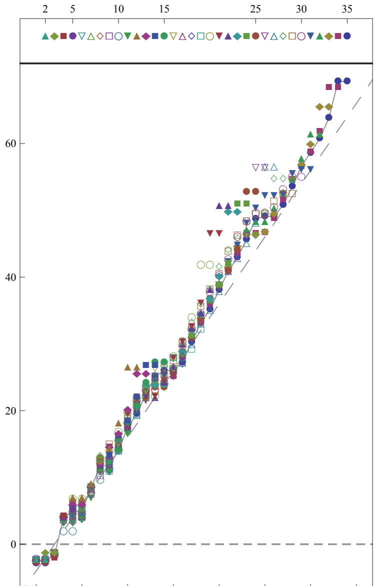
$f(R)$ gravity

$$\Gamma_k(g_{\mu\nu}) = \int d^4x \sqrt{g} f(R)$$

$$f(R) = \sum_{i=0}^{n} g_i(k) R^i$$

n=6
A. Codello, R.P. and C. Rahmede Int.J.Mod.Phys.A23:143-150 arXiv:0705.1769 [hep-th];
n=8
A. Codello, R.P. and C. Rahmede Annals Phys. 324 414-469 (2009) arXiv: arXiv:0805.2909;
P.F. Machado, F. Saueressig, Phys. Rev. D arXiv: arXiv:0712.0445 [hep-th]
n=35
K. Falls, D.F. Litim, K. Nikolakopoulos, C. Rahmede, arXiv:1301.4191 [hep-th]

$f(R)$ gravity $n = 35$



Outline

Anisotropic scaling a.k.a. Hořava-Lifschitz gravity

Fourth-order gravity actions renormalizable but have ghosts.
Avoid ghosts by having only two time derivatives.
Achieve renormalizability by having higher spatial derivatives.
Hints of asymptotic freedom. Challenge: recovery of Lorentz symmetry at low energy.

$\Longrightarrow$ Saueressig mon 16:00
$\Longrightarrow$ D'Odorico tue 16:40

Background cutoff

If the metric is dynamical, what are fast and the slow modes?
Write

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

and use the eigenmodes of $-\bar{\nabla}^2$ as a basis in field space. Define fast and slow modes using this reference basis.
To define the EAA introduce the cutoff

$$\Delta S_k(h;\bar{g}) = \frac{1}{2}\int d^d x\sqrt{-\bar{g}}\, h^{\mu\nu}\mathcal{R}_k(-\bar{\nabla}^2)h_{\mu\nu};$$

The EAA is a functional $\Gamma_k(h;\bar{g})$.

Split symmetry is broken

$\Gamma(h; \bar{g})$ is not invariant under "split" transformations

$$\delta_\epsilon \bar{g}_{\mu\nu} = m_{\mu\nu}, \qquad \delta_\epsilon h_{\mu\nu} = -m_{\mu\nu}.$$

This source of split-symmetry breaking vanishes when $k \to 0$.
Should recover this property in the IR.

The background field method

Another source of split-symmetry breaking is the background gauge fixing condition e.g.

$$S_{GF}(h;\bar{g}) = \frac{1}{2}\int d^d x \sqrt{-\bar{g}}\, \bar{g}^{\mu\nu}\chi_\mu\chi_\nu \; ; \qquad \chi_\mu = \bar{\nabla}^\nu h_{\nu\mu} - \frac{1}{2}\bar{\nabla}_\mu h$$

Since S_{GF} and ΔS_k are invariant under coordinate transformations, so is $\Gamma_k(h;\bar{g})$.

Comma and semicolon

The EAA is a bi-metric action.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\Gamma_k(g, \bar{g}) = \Gamma_k(h; \bar{g})$$

Double Einstein-Hilbert truncation

$$\Gamma_k(g,\bar{g}) = \frac{1}{16\pi G^{\mathrm{Dyn}}} \int d^4x \sqrt{g}(2\Lambda^{\mathrm{Dyn}} - R) + \frac{1}{16\pi G^{\mathrm{B}}} \int d^4x \sqrt{\bar{g}}(2\Lambda^{\mathrm{B}} - \bar{R})$$

Split-symmetry recovered on a two-parameter subset of RG trajectories.

⇒ D. Becker mon 16:40

Level expansion

Expand

$$\Gamma_k(h;\bar{g}) = \bar{\Gamma}_k(\bar{g}) + \Gamma_k^{(1)}(h;\bar{g}) + \Gamma_k^{(2)}(h;\bar{g}) + \dots$$

$$\bar{\Gamma}_k(\bar{g}) = \Gamma_k(0;\bar{g})$$

Single-metric truncations: neglect $\Gamma_k^{(n)}(h;\bar{g})$, $n > 0$.

Example: single-metric truncation with 2-derivative terms

$$
\begin{aligned}
\Gamma_k(h;\bar{g}) &= -\frac{1}{16\pi G}\int d^dx\sqrt{\bar{g}}\,\bar{R} \\
&\quad +\frac{1}{16\pi G}\int d^dx\sqrt{\bar{g}}\left(\bar{R}^{\mu\nu}-\frac{1}{2}\bar{g}^{\mu\nu}\bar{R}\right)h_{\mu\nu} \\
&\quad +\frac{1}{16\pi G}\int d^dx\sqrt{\bar{g}}\,h_{\mu\nu}\Delta^{\mu\nu\rho\sigma}h_{\rho\sigma}+\dots
\end{aligned}
$$

Example: bi-metric truncation with 2-derivative terms

$$
\begin{aligned}
\Gamma_k(\bar{g},h) &= -\frac{1}{16\pi G^{(0)}}\int d^dx\sqrt{\bar{g}}\bar{R} \\
&+\frac{1}{16\pi G^{(1)}}\int d^dx\sqrt{\bar{g}}\left(\bar{R}^{\mu\nu}+\frac{1}{2}\bar{g}^{\mu\nu}\bar{R}\right)h_{\mu\nu} \\
&+\frac{1}{16\pi G^{(2)}}\int d^dx\sqrt{\bar{g}}\,h_{\mu\nu}\Delta^{\mu\nu\rho\sigma}h_{\rho\sigma}+\dots
\end{aligned}
$$

Example: bi-metric truncation with 2-derivative terms

$$
\begin{aligned}
\Gamma_k(\bar{g}, h) &= -\frac{1}{16\pi G^{(0)}} \int d^d x \sqrt{\bar{g}}\, \bar{R} \\
&\quad + \frac{1}{16\pi G^{(1)}} \int d^d x \sqrt{\bar{g}} \left(\bar{R}^{\mu\nu} + \frac{1}{2} \bar{g}^{\mu\nu} \bar{R} \right) h_{\mu\nu} \\
&\quad + \frac{1}{16\pi G^{(2)}} \int d^d x \sqrt{\bar{g}}\, h_{\mu\nu} \Delta^{\mu\nu\rho\sigma} h_{\rho\sigma} + \dots
\end{aligned}
$$

Ignore the level 1 couplings and redefine

$$
\frac{1}{16\pi G^{(2)}} = \frac{Z_h}{16\pi G^{(0)}} \; ; \qquad h_{\mu\nu} \to \sqrt{32\pi G^{(0)}}\, h_{\mu\nu}
$$

Bi-metric level 2 Einstein–Hilbert truncation

Including also cosmological term, gauge fixing and ghost terms

$$
\begin{aligned}
\Gamma_k(h;\bar{g}) \;=\; & \frac{1}{16\pi G}\int d^dx\sqrt{\bar{g}}\,(2\Lambda-\bar{R}) \\
& +\frac{Z_h}{2}\int d^dx\sqrt{\bar{g}}\,h_{\mu\nu}(\Delta^{\mu\nu\rho\sigma}+M^2K^{\mu\nu\rho\sigma})h_{\rho\sigma} \\
& -\sqrt{2}Z_c\int d^dx\sqrt{\bar{g}}\;\bar{c}_\mu\Big(\bar{D}^\rho\bar{g}^{\mu\kappa}g_{\kappa\nu}D_\rho+\bar{D}^\rho\bar{g}^{\mu\kappa}g_{\rho\nu}D_\kappa-\bar{D}^\mu\bar{g}^{\rho\sigma}g_{\rho\nu}D_\sigma\Big)c^\nu
\end{aligned}
$$

The anomalous dimensions

We include a factor Z_h, Z_c in the definition of the cutoff:

$$\Delta S_k(h;\bar{g}) = \frac{Z_h}{2}\int d^dx\sqrt{\bar{g}}\,h_{\mu\nu}K^{\mu\nu\alpha\beta}R_k(-\bar{D}^2)h_{\rho\sigma}$$

This leads to beta functions of the form

$$A(\tilde{\Lambda},\tilde{G}) + B(\tilde{\Lambda},\tilde{G})\,\eta_h\ ; \qquad \eta_h = -\partial_t Z_h/Z_h$$

To close flow equations need a formula for η.

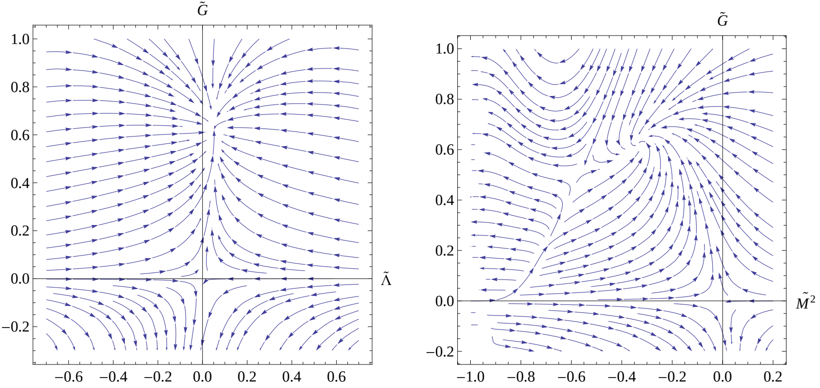
Anomalous dimensions

Generally three options:

- $\eta = 0$ (one loop)
- In the single-metric calculations, identify Z_h with $\frac{1}{16\pi G}$. Consequently identify η_h with $k\frac{d\log G}{dk}$
- in bi-metric level-2 calculations compute η from two point function of graviton

[A. Codello, G. d'Odorico, C. Pagani, Phys.Rev. D89 (2014) 081701, arXiv:1304.4777 [gr-qc]]
[N. Christiansen, B. Knorr, J.M. Pawlowski, A. Rodigast, arxiv:1403.1232 [hep-th]]

Bi-metric flow in $\tilde{\Lambda}$-$\tilde{G}$-$\tilde{M}^2$ space



[P.Donà, A. Eichhorn, R.P., unpublished]

$\Rightarrow$ Christiansen mon 17:00

Enter matter

- because it's there
- because it may help (large N limit)
- because pure gravity has no local observables
- because experimental constraints more likely

[L. Griguolo, R.P. Phys. Rev. D 52, 5787 (1995)]

[R.P., D.Perini, Phys.Rev. D 67 081503 (2003)]

[R.P., D.Perini, Phys. Rev. D68, 044018 (2004)]

[G. Narain. R.P. Class. Quant. Grav. 27 075001 (2010)]

[P. Donà, A. Eichhorn, R.P. arXiv:1311.2898 [hep-th](2013)]

Bimetric EH with minimally coupled matter

$$\begin{aligned}\Gamma_k(\bar{g},h) &= \frac{1}{16\pi G}\int d^dx\sqrt{\bar{g}}\,(-\bar{R}+2\Lambda)\\ &+\frac{Z_h}{2}\int d^dx\sqrt{\bar{g}}\,h_{\mu\nu}K^{\mu\nu\alpha\beta}((-\bar{D}^2-2\Lambda)\mathbf{1}^{\rho\sigma}_{\alpha\beta}+W^{\rho\sigma}_{\alpha\beta})h_{\rho\sigma}\\ &-\sqrt{2}Z_c\int d^dx\sqrt{\bar{g}}\;\bar{c}_\mu\Big(\bar{D}^\rho\bar{g}^{\mu\kappa}g_{\kappa\nu}D_\rho+\bar{D}^\rho\bar{g}^{\mu\kappa}g_{\rho\nu}D_\kappa-\bar{D}^\mu\bar{g}^{\rho\sigma}g_{\rho\nu}D_\sigma\Big)c^\nu\end{aligned}$$

$$\begin{aligned}S_S &= \frac{Z_S}{2}\int d^dx\sqrt{g}\,g^{\mu\nu}\sum_{i=1}^{N_S}\partial_\mu\phi^i\partial_\nu\phi^i\\ S_D &= iZ_D\int d^dx\sqrt{g}\sum_{i=1}^{N_D}\bar{\psi}^i\nabla\!\!\!/\,\psi^i,\\ S_V &= \frac{Z_V}{4}\int d^dx\sqrt{g}\sum_{i=1}^{N_V}g^{\mu\nu}g^{\kappa\lambda}F^i_{\mu\kappa}F^i_{\nu\lambda}+\dots\end{aligned}$$

Gravitational beta functions

$$
\begin{aligned}
\frac{d\tilde{\Lambda}}{dt} &= -2\tilde{\Lambda} + \frac{8\pi\tilde{G}}{(4\pi)^{d/2} d(d+2)\Gamma[d/2]}\Bigg[\frac{d(d+1)(d+2-\eta_h)}{1-2\tilde{\Lambda}} - 4d(d+2-\eta_c) \\
&\qquad +2N_S(2+d-\eta_S) - 2N_D 2^{[d/2]}(2+d-\eta_D) + 2N_V(d^2-4-d\,\eta_V)\Bigg] \\
&\quad - \frac{4\pi\tilde{G}\tilde{\Lambda}}{3d(4\pi)^{d/2}\Gamma[d/2]}\Bigg[\frac{d(5d-7)(d-\eta_h)}{1-2\tilde{\Lambda}} + 4(d+6)(d-\eta_c) \\
&\qquad -2N_S(d-\eta_S) - N_D 2^{[d/2]}(d-\eta_D) + 2N_V(d\,(8-d)-(6-d)\eta_V)\Bigg] \\
\frac{d\tilde{G}}{dt} &= (d-2)\tilde{G} - \frac{4\pi\tilde{G}^2}{3d(4\pi)^{d/2}\Gamma(d/2)}\Bigg[\frac{d(5d-7)(d-\eta_h)}{1-2\tilde{\Lambda}} + 4(d+6)(d-\eta_c) \\
&\qquad -2N_S(d-\eta_S) - N_D 2^{[d/2]}(d-\eta_D) + 2N_V(d(8-d)-(6-d)\eta_V)\Bigg]
\end{aligned}
$$

Perturbative beta functions with matter

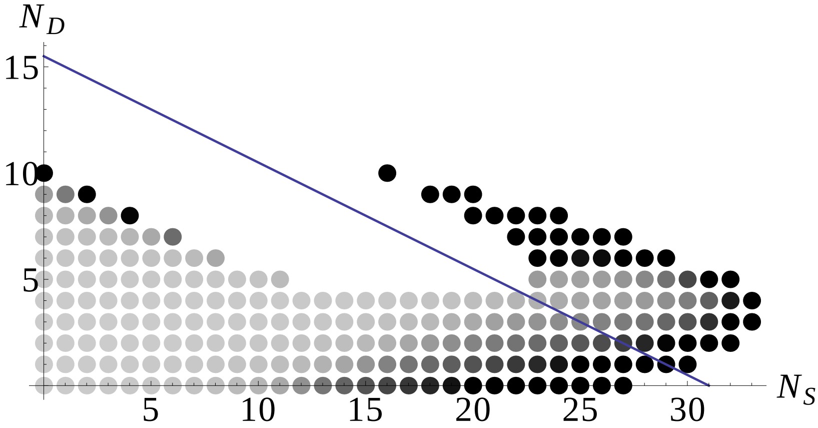
$$\beta_{\tilde{G}} = 2\tilde{G} + \frac{\tilde{G}^2}{6\pi}\left(N_S + 2N_D - 4N_V - 46\right),$$

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{\tilde{G}}{4\pi}\left(N_S - 4N_D + 2N_V + 2\right) + \frac{\tilde{G}\tilde{\Lambda}}{6\pi}\left(N_S + 2N_D - 4N_V - 16\right)$$

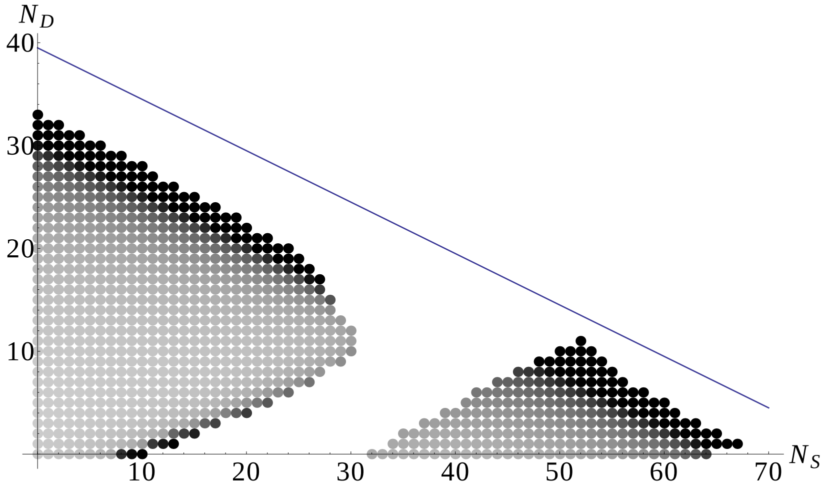
$$\tilde{\Lambda}_* = -\frac{3}{4}\frac{N_S - 4N_D + 2N_V + 2}{N_S + 2N_D - 4N_V - 31},$$

$$\tilde{G}_* = -\frac{12\pi}{N_S + 2N_D - 4N_V - 46}.$$

Exclusion plot $N_V = 0$



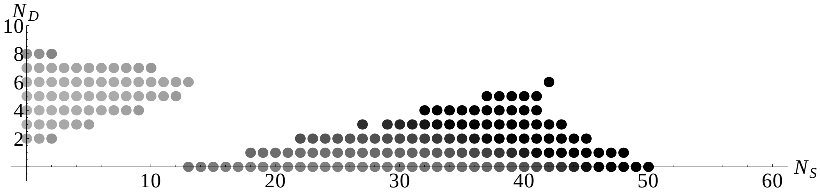
Exclusion plot $N_V = 12$



Specific models

model	N_S	N_D	N_V	$\tilde{G}_*$	$\tilde{\Lambda}_*$	θ_1	θ_2	η_h
no matter	0	0	0	0.77	0.01	3.30	1.95	0.27
SM	4	45/2	12	1.76	-2.40	3.96	1.64	2.98
SM +dm scalar	5	45/2	12	1.87	-2.50	3.96	1.63	3.15
SM+ 3 ν's	4	24	12	2.15	-3.20	3.97	1.65	3.71
SM+3ν's + axion+dm	6	24	12	2.50	-3.62	3.96	1.63	4.28
MSSM	49	61/2	12	-	-	-	-	-
SU(5) GUT	124	24	24	-	-	-	-	-
SO(10) GUT	97	24	45	-	-	-	-	-

Exclusion plot $N_V = 12,\ d = 6$



$f(R)$ gravity again

$$\Gamma_k(g_{\mu\nu}) = \int d^4x \sqrt{g} f(R)$$

$$f(R) = \sum_{i=0}^{\infty} g_i(k) R^i$$

Dario Benedetti, Francesco Caravelli, JHEP 1206 (2012) 017, Erratum-ibid. 1210 (2012) 157, arXiv:1204.3541[hep-th]
Dario Benedetti, Europhys. Lett. 102 (2013) 20007, arXiv:1301.4422[hep-th]
Juergen A. Dietz, Tim R. Morris, JHEP 1301 (2013) 108, arXiv:1211.0955[hep-th]
Juergen A. Dietz, Tim R. Morris, JHEP 1307 (2014) 064, arXiv:1311.1081[hep-th]
Maximilian Demmel, Frank Saueressig, Omar Zanusso, JHEP 1406 (2014) 026, arXiv:1401.5495[hep-th]
Maximilian Demmel, Frank Saueressig, Omar Zanusso, JHEP 1211 (2012) 131, arXiv:1208.2038[hep-th]

$\Longrightarrow$ Morris next talk

Gravity+scalar

$$\Gamma_k[g,\phi] = \int d^dx\,\sqrt{g}\left(V(\phi^2) - F(\phi^2)R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$

[G. Narain, R.P., Class. and Quantum Grav. 27, 075001 (2010)]

[T. Henz, J. Pawlowski, A. Rodigast and C. Wetterich, Phys. Lett. B727 (2013) 298]

[D. Benedetti and F. Guarnieri, New J. of Phys. (2014) 053051]

$\Longrightarrow$ Henz mon 18:10

$\Longrightarrow$ Wetterich fri 19:10

Functional flow of F, V

$$\partial_t V = \frac{k^4}{192\pi^2}\left\{6 + \frac{30\,V}{\Psi} + \frac{6(k^2\,\Psi + 24\,\phi^2\,k^2\,F'\,\Psi' + k^2\,F\Sigma_1)}{\Delta} + \text{terms containing } \partial_t F\right\}\ ,$$

$$\partial_t F = \frac{k^2}{2304\pi^2}\Bigg\{150 + \frac{120\,k^2\,F\,(3\,k^2 F - V)}{\Psi^2} - \frac{24}{\Delta}\left(24\,\phi^2\,k^2\,F'\,\Psi' + k^2\,\Psi + k^2\,F\Sigma_1\right)$$

$$-\frac{36}{\Delta^2}\Big[-4\,\phi^2\,(6\,k^4\,F'^2 + \Psi'^2)\,\Delta + 4\,\phi^2\,\Psi\,\Psi'\,(7\,k^2\,F' - V')\,(\Sigma_1 - k^2) + 4\,\phi^2\Sigma_1\,(7\,k^2\,F' - V')\,(2\,\Psi\,V' - V\,\Psi')$$

$$+2\,k^4\,\Psi^2\,\Sigma_2 + 48\,k^4\,F'\,\phi^2\,\Psi\,\Psi'\,\Sigma_2 - 24\,k^4 F\,\phi^2\,\Psi'^2\,\Sigma_2\Big] + \text{terms containing } \partial_t F$$

where we have defined the shorthands:

$$\Psi = k^2\,F - V\,; \quad \Sigma_1 = k^2 + 2\,V' + 4\,\phi^2\,V''\ ; \quad \Sigma_2 = 2\,F' + 4\,\phi^2\,F''\ ; \quad \Delta = \left(12\,\phi^2\,\Psi'^2 + \Psi\,\Sigma_1\right).$$

In $d = 3$ no trace of gravitationally dressed WF fixed point: in polynomial expansion, all coefficients of ϕ^2 are negative.

Scalar+gravity in exponential parametrization

- Exponential parametrization: $g_{\mu\nu} = \bar{g}_{\mu\rho}(e^h)^\rho{}_\nu$
- Unimodularity [A. Eichhorn (2013)] $\Longrightarrow$ Saltas mon 18:30
- Choice of gauge

$$\dot{v} = -3\,v + \frac{1}{2}\phi\,v' + \frac{f + 4f'^2}{6\pi^2\,(4f'^2 + f(1+v''))}$$

$$\dot{f} = -f + \frac{1}{2}\phi\,f' + \frac{25}{36\pi^2} + f\,\frac{(f+4f'^2)(1+3v''-2f'') + 2fv''^2}{12\pi^2(4f'^2 + f(1+v''))^2}$$

[G.P. Vacca and R.P., to appear]

Compare with equation for pure scalar in LPA

$$\dot{v} = -3\,v + \frac{1}{2}\phi\,v' + \frac{1}{6\pi^2(1+v'')}$$

Analytic solutions

Solution 1

$$v_* = \frac{1}{18\pi^2} \approx 0.00562 \; ; \qquad f_* = \frac{7}{9\pi^2} \approx 0.0788$$

$$\tilde{G}_* = \frac{9\pi}{112} \approx 0.252 \; ; \qquad \tilde{\Lambda}_* = \frac{1}{28} \approx 0.0357$$

Solution 2

$$v_* = \frac{1}{18\pi^2} \approx 0.00562 \; ; \qquad f_* = \frac{25}{36\pi^2} + \frac{1}{4}\phi^2$$

$$\tilde{G}_* = \frac{9\pi}{100} \approx 0.283 \; ; \qquad \tilde{\Lambda}_* = \frac{1}{25} = 0.04$$

Possible nontrivial solution

Match (Padé approximant to) polynomial soln of order $(\phi^2 - \kappa)^{12}$ to large ϕ solutions $v = A\phi^6 + \ldots$, $f = B\phi^2 + \ldots$ (up to ϕ^{-16})

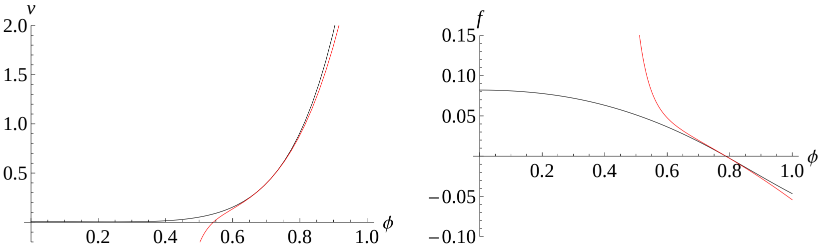


Figure : Black: approximate solution expanded around the minimum of the potential; red: approximate solution for large ϕ with $A = 3.38$, $B = -0.141$.

Comparison to pure scalar theory

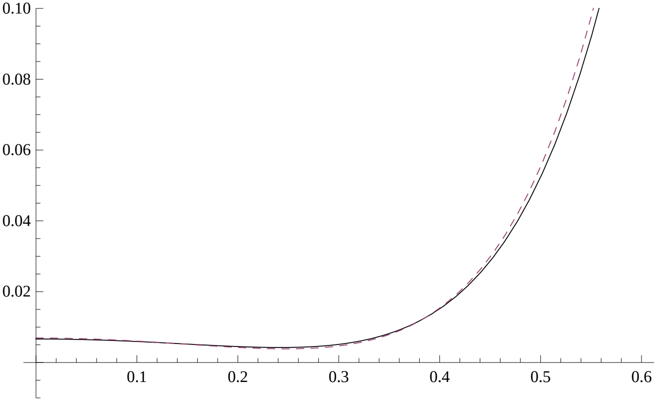


Figure : Solid curve: potential with gravity; dashed curve: LPA approximation of potential of Wilson-Fisher fixed point without gravity.

Conclusions

Perturbative renormalizability played a decisive role in the construction of the Weinberg-Salam model.
Still the SM is incomplete and nowadays is generally regarded as an EFT. The absence (or smallness) of nonrenormalizable couplings can be viewed as a consequence of scale separation.

It is perhaps time to use again renormalizability as a criterion to select a viable theory of the fundamental interactions, including gravity.

Conclusions

Perturbative renormalizability cannot play this role, but perhaps asymptotic safety can.

Most important gain: predictivity.

Calculations are becoming more complex, but appropriate techniques exist the way forward is relatively clear.

Expect to hear more at ERG2016.