Asymptotic safety beyond polynomial approximation

ERG2014, 22/9/14

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Juergen A. Dietz & TRM, JHEP 1301 (2013) 108, arXiv:1211.0955.
Juergen A. Dietz & TRM, JHEP 1307 (2013) 64, arXiv:1306.1223.
I. Hamzaan Bridle, Juergen A. Dietz & TRM, JHEP 1403 (2014) 093, arXiv:1312.2846.

- Use Exact Renormalisation Group
- Project on four-sphere background $(R \geq 0)$
- Effective action: $\Gamma = \int d^4x \sqrt{g}\, f(R,t)$
- Get non-linear PDE flow equation for $f(R,t)$

Versions:

- P. F. Machado and F. Saueressig, Phys. Rev. D 77 (2008) 124045, arXiv:0712.0445. (MS)
- A. Codello, R. Percacci and C. Rahmede, Annals Phys. 324 (2009) 414, arXiv:0805.2909. (CPR)
- D. Benedetti and F. Caravelli, JHEP 1206 (2012) 017, arXiv:1204.3541. (BC)

arXiv:1211.0955: How to solve?

Fixed points: $f(R,t) \mapsto f(R)$

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Fixed singularities and parameter space

Suppose we have normal form: $$f'''(R) = \frac{F(f, f', f'', R)}{R}$$

with fixed singularity at R=0.

Substitute: $$f(R) = a_0 + a_1 R + \frac{1}{2} a_2 R^2 + \cdots$$

=> $$\text{regular in } R \;= \frac{u(a_0, a_1, a_2)}{R} + \text{regular in } R$$

$u(a_0, a_1, a_2)$ is non-trivial constraint on parameters a_0, a_1, a_2

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BC 04/12:

$$768\pi^2\left(2f - Rf'\right) = \frac{40\left(Rf'' - 4f'\right)}{(R-2)f' - 2f} - 48 - 5R^2$$

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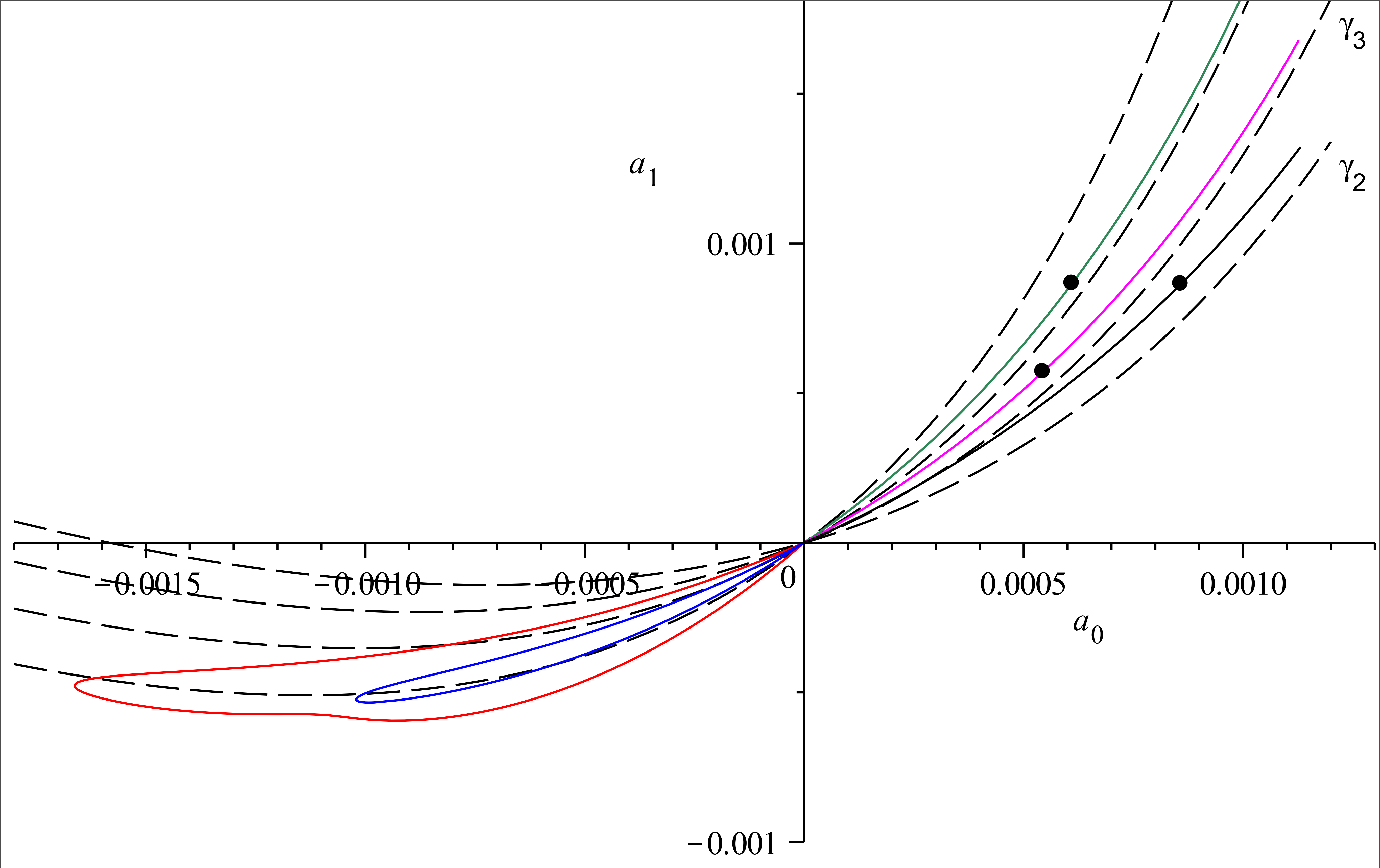

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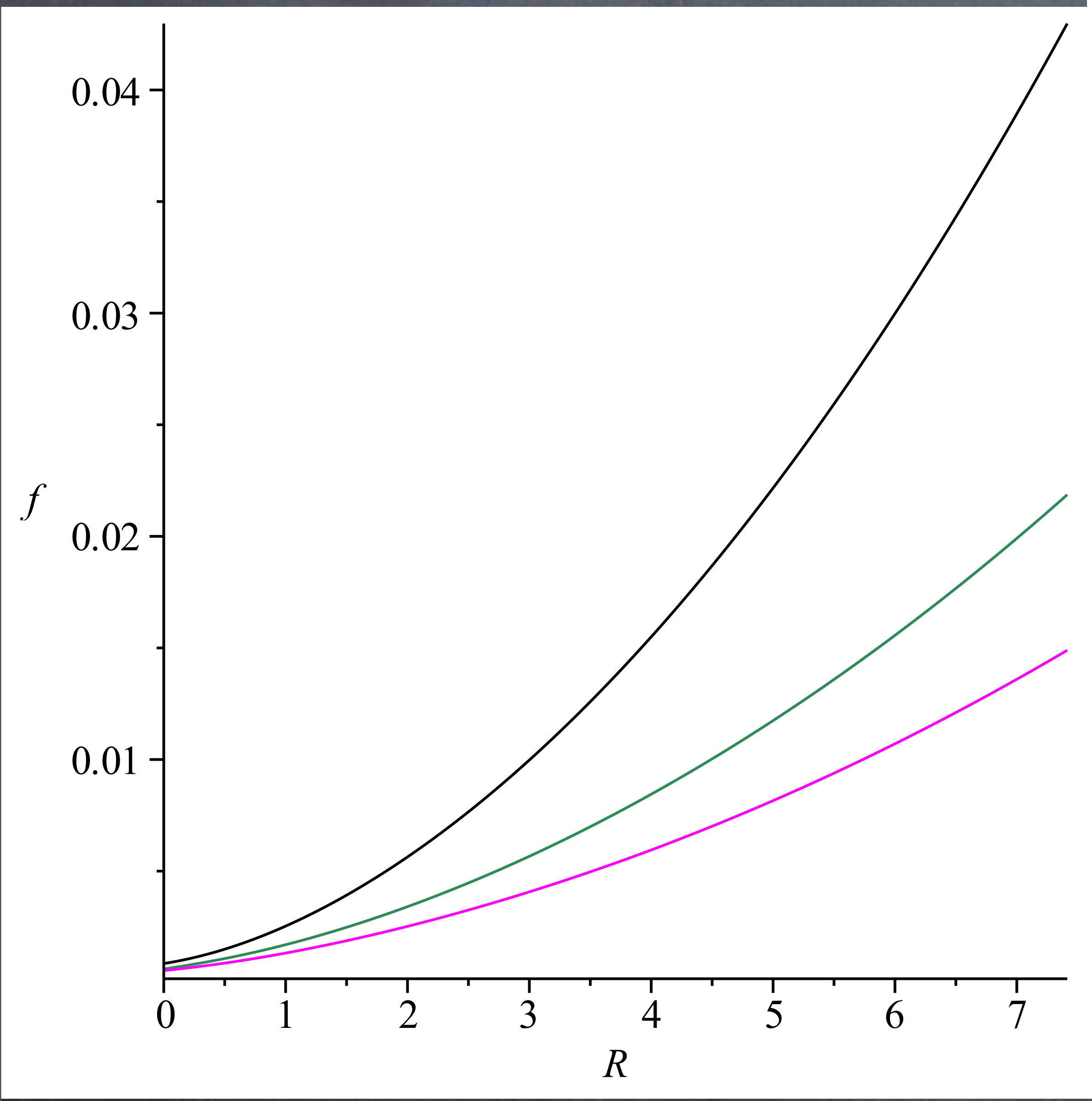
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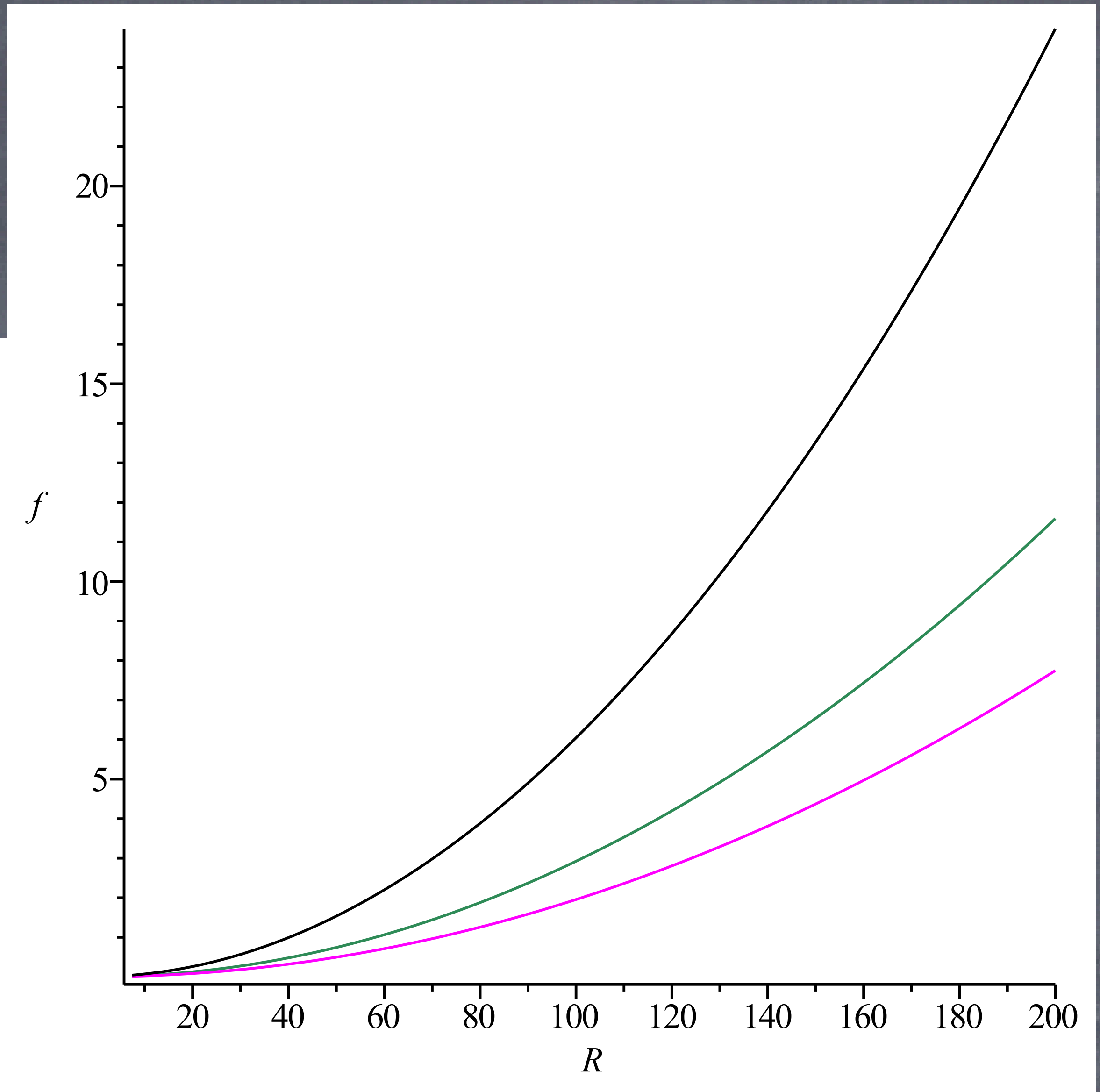
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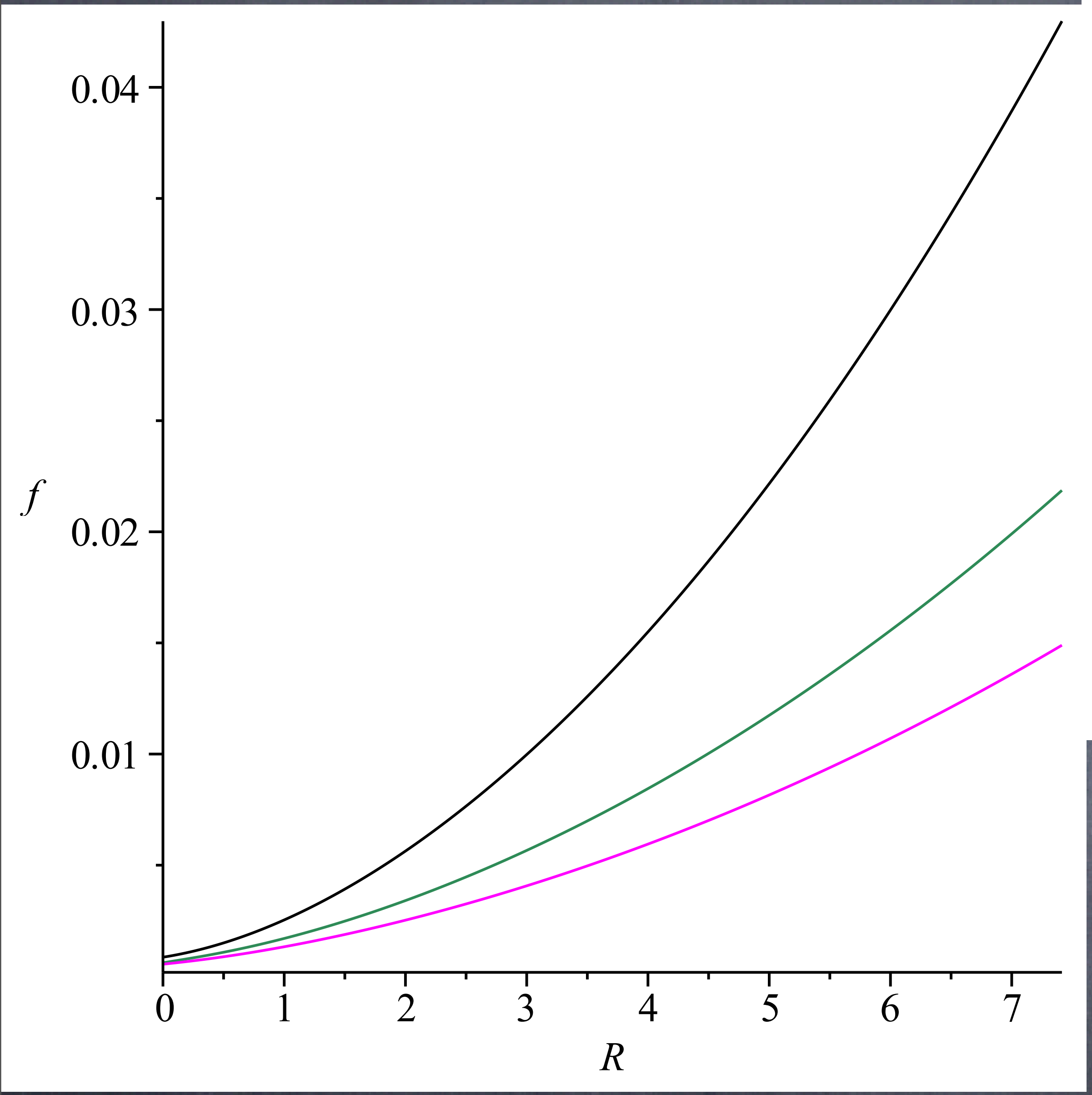

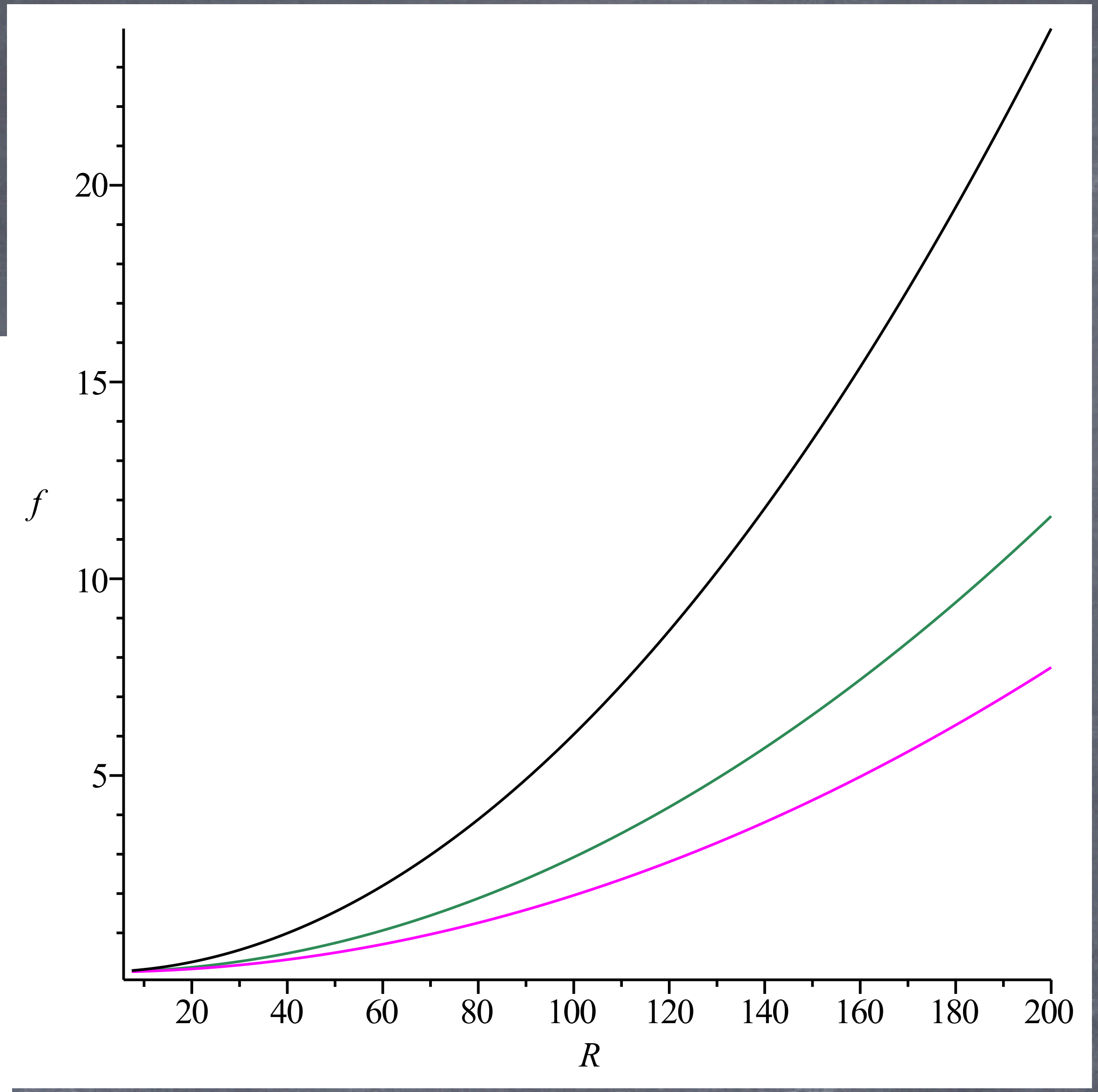

Continuous eigenspectrum!

J. A. Dietz & T.R. Morris, JHEP 1301 (2013) 108.

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Quantised Eigenoperator spectrum

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M. Demmel, F. Saueressig & O. Zanusso, arXiv:1401.5459

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Break-down of f(R) approximation

FP action $\Gamma = \int d^4x \sqrt{g}\, f(R)$

J.A. Dietz & T.R. Morris, JHEP **07** (2013) 064

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Wegner, J. Phys. C7 (1974) 2098.

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$$F_{\mu\nu} = \zeta(R)\, g_{\mu\nu} \quad \Longrightarrow \quad v(R) = \zeta(R) \left\{ 2f(R) - R f'(R) \right\} .$$

$$\boxed{v(R) = \zeta(R)\left\{2f(R) - Rf'(R)\right\}}$$

$$2f(R_*) - R_* f'(R_*) = 0 \quad \Longrightarrow \quad v(R_*) = 0$$

R=R* `vacuum' solution

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- FPs with BC eqns have no `vacuum' => ζ always has a soln => theory space collapses to a point!

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- FPs with BC eqns have no \`vacuum' => ζ always has a soln => theory space collapses to a point!

- BC eqns -∞<R<∞ would have had \`vacuum' => ζ has no solution => no redundant operators

Scalar field theory @ LPA

I.H. Bridle, J. Dietz & T.R. Morris, JHEP **03** (2014) 093

$$\Gamma[\phi] = \int d^d x \left\{ \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi) \right\}$$

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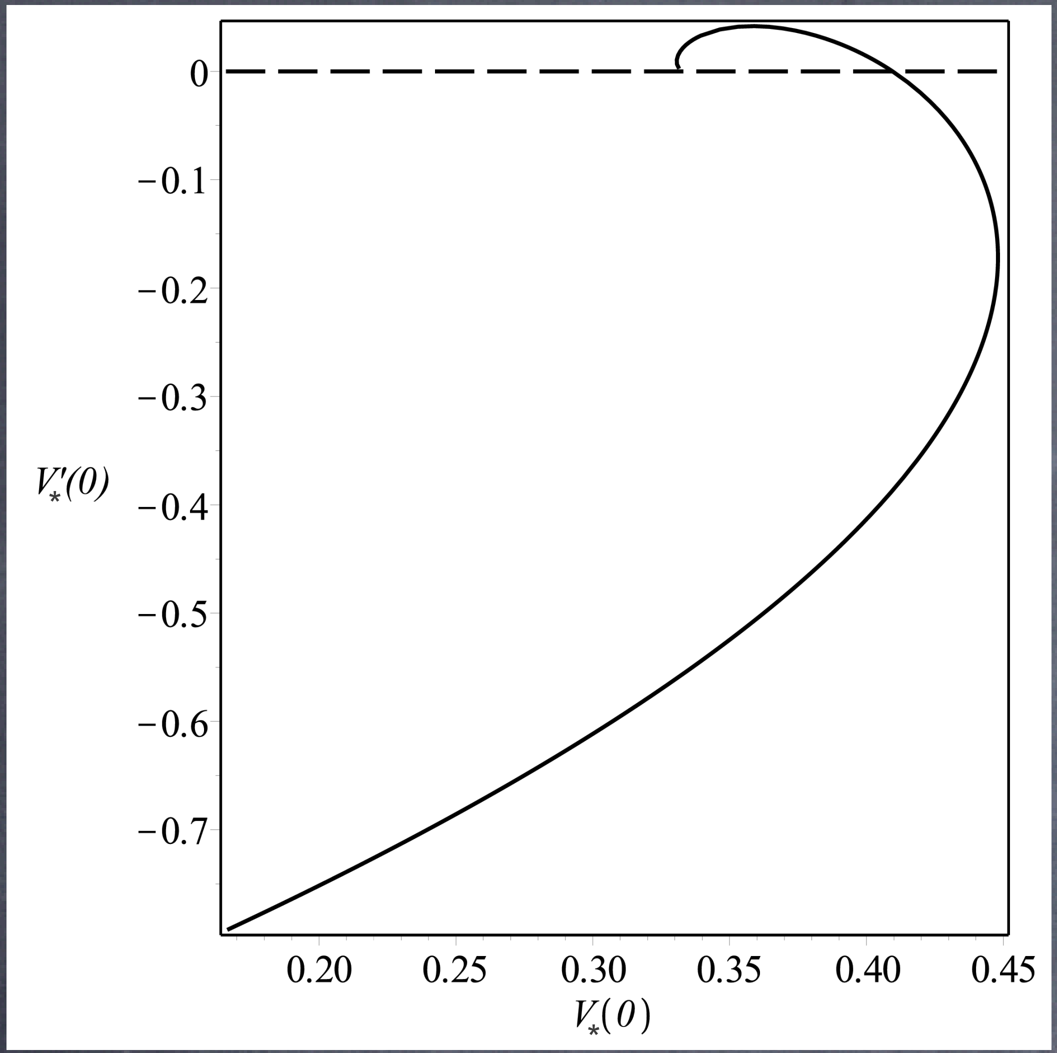
ϕ -> − ϕ symmetric

Fixed points: $V_*(\phi),\ V_*(-\phi)$

Fixed points: $\{V_*(0),\ \pm V_*'(0)\}$

h=0 (d=3):



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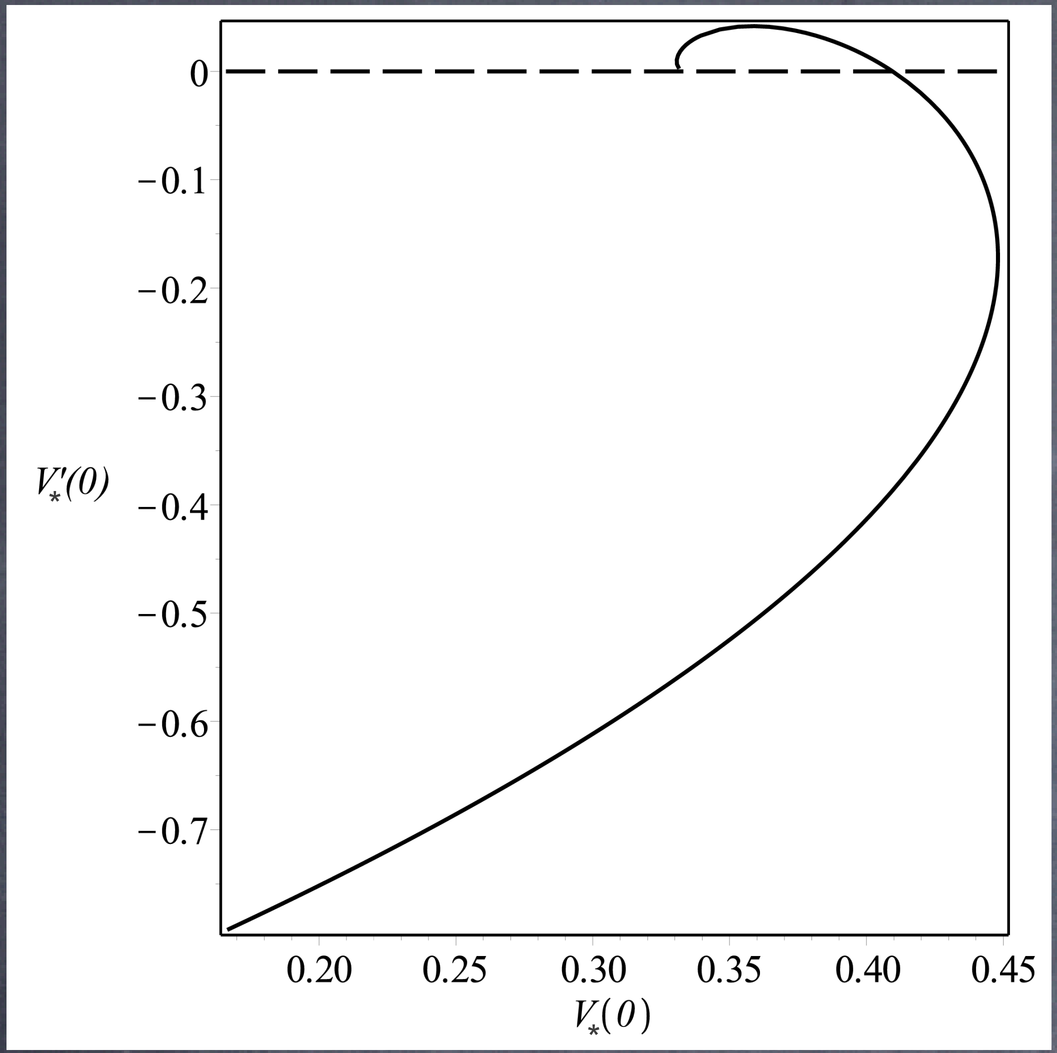

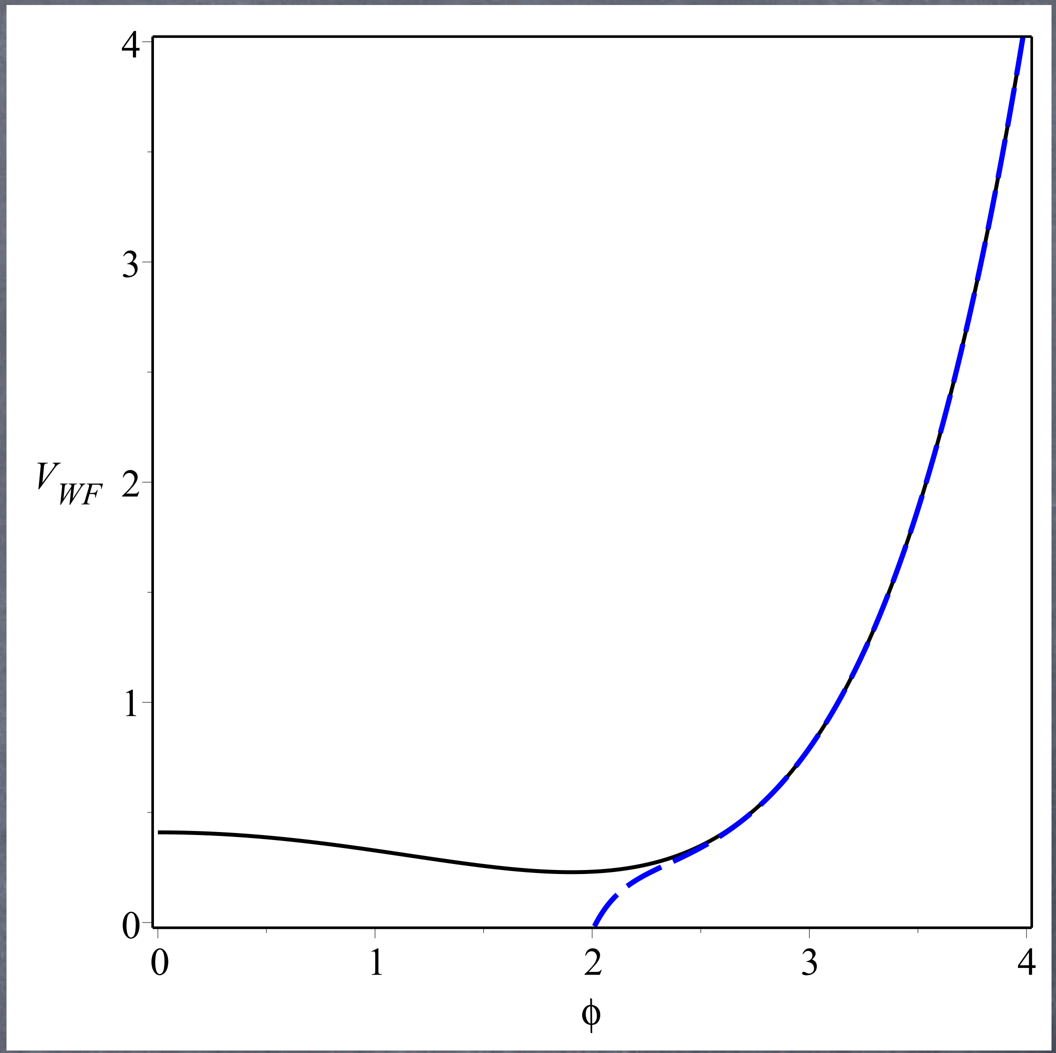

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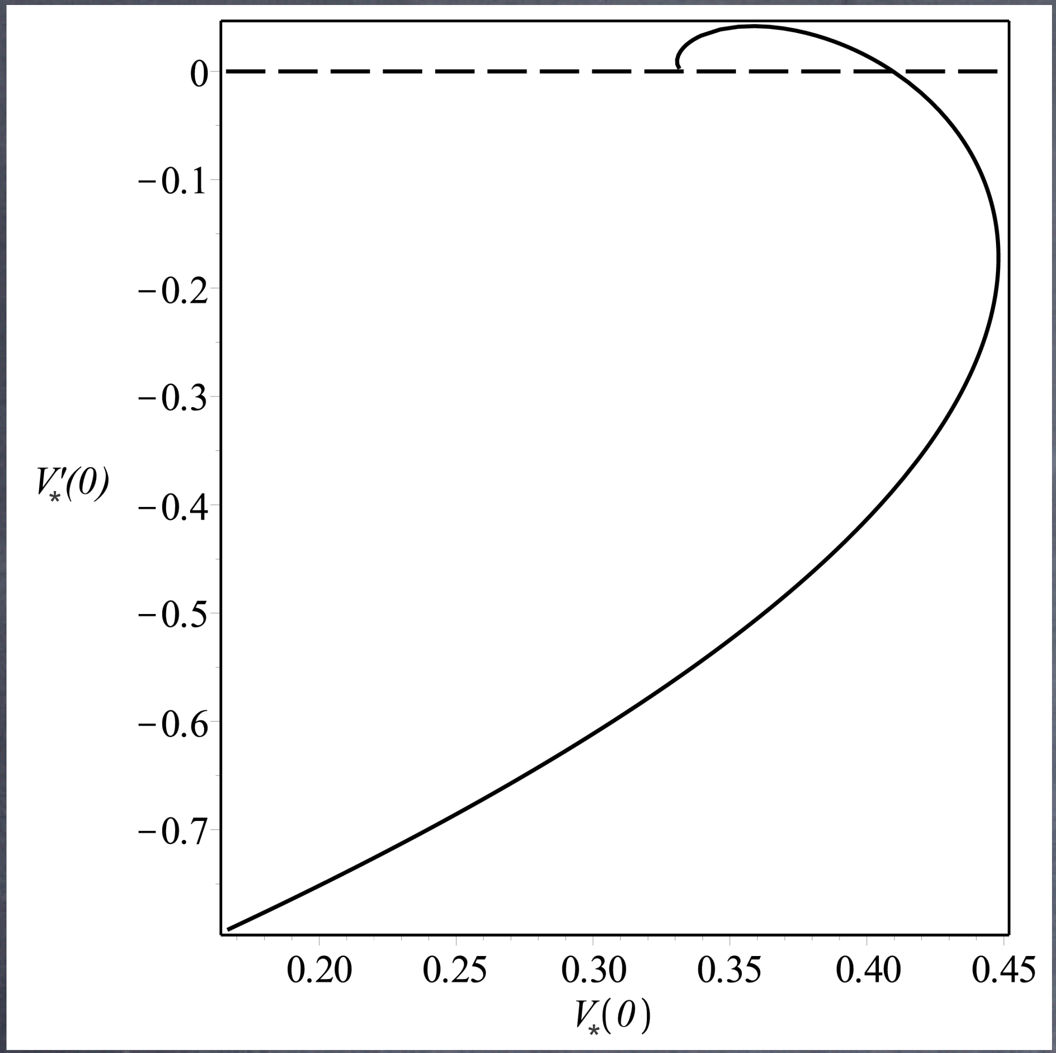

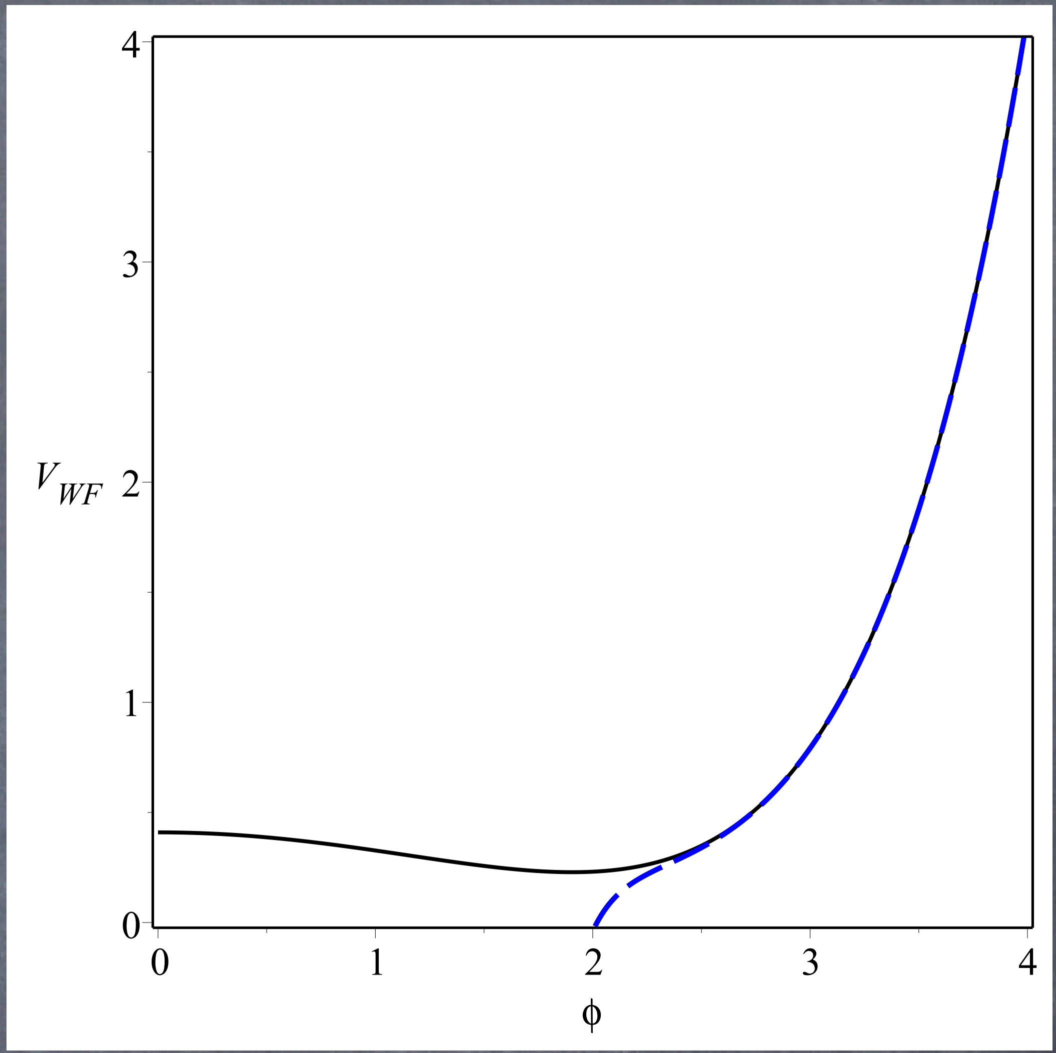

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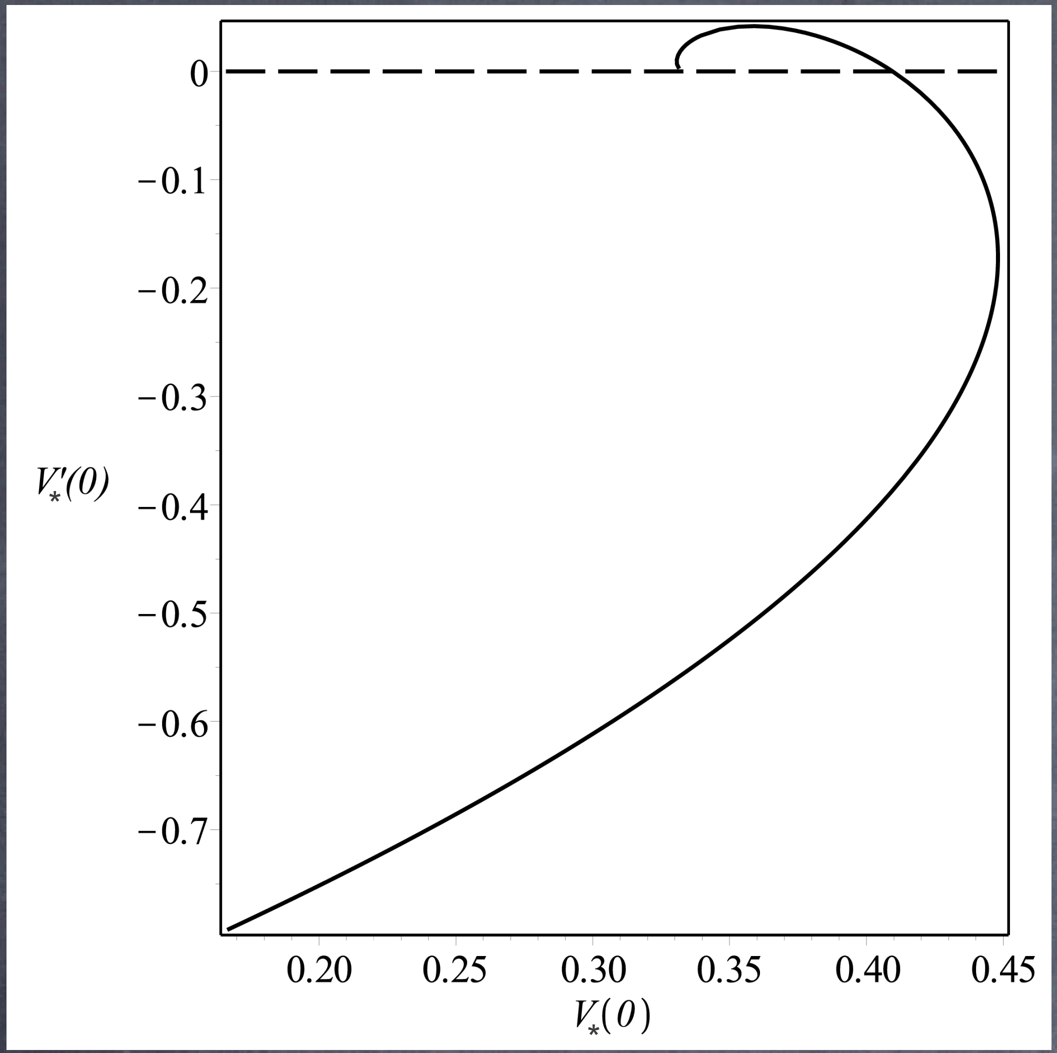

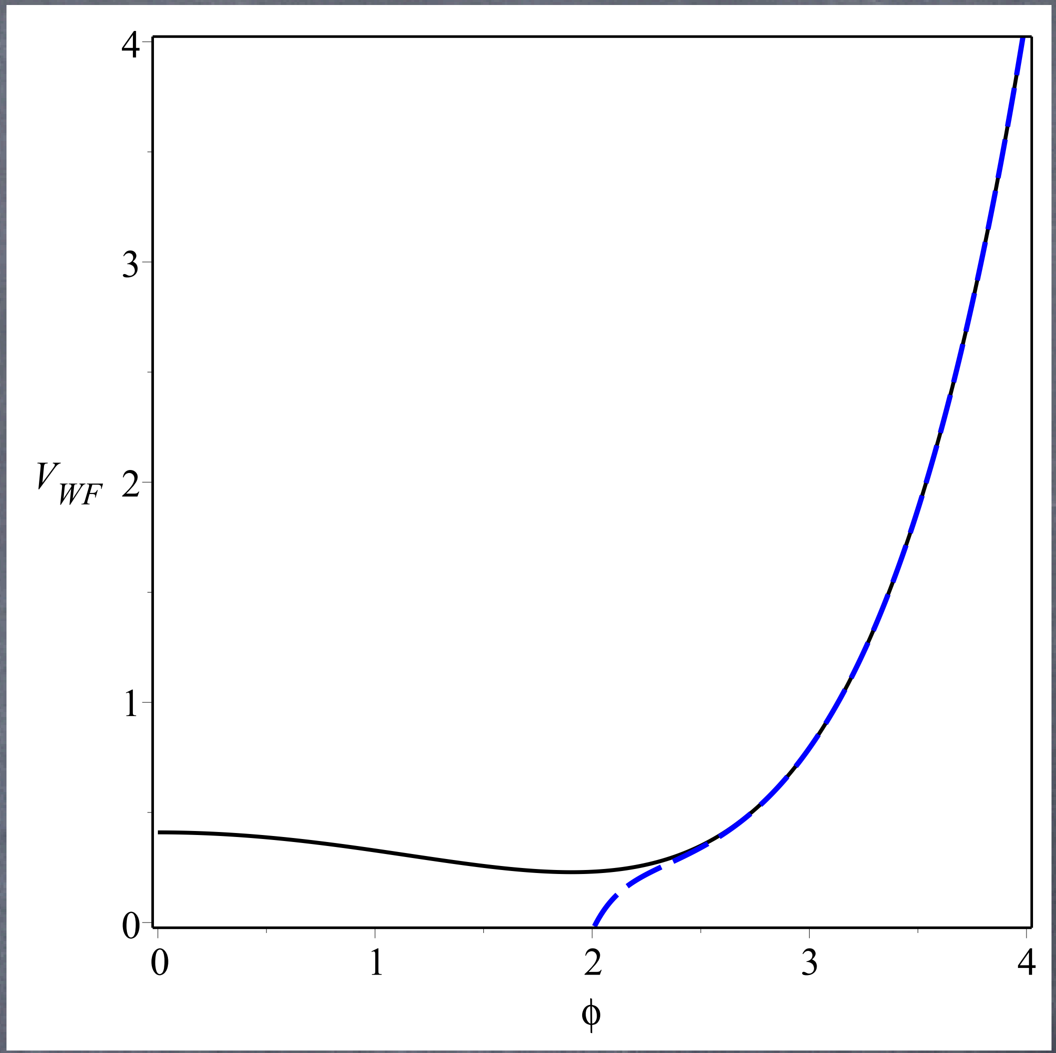

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one odd one $(\phi \mapsto \phi + \epsilon)$

Monday, 22 September 14

h=$\alpha\bar{\varphi}^2$ (d=3):

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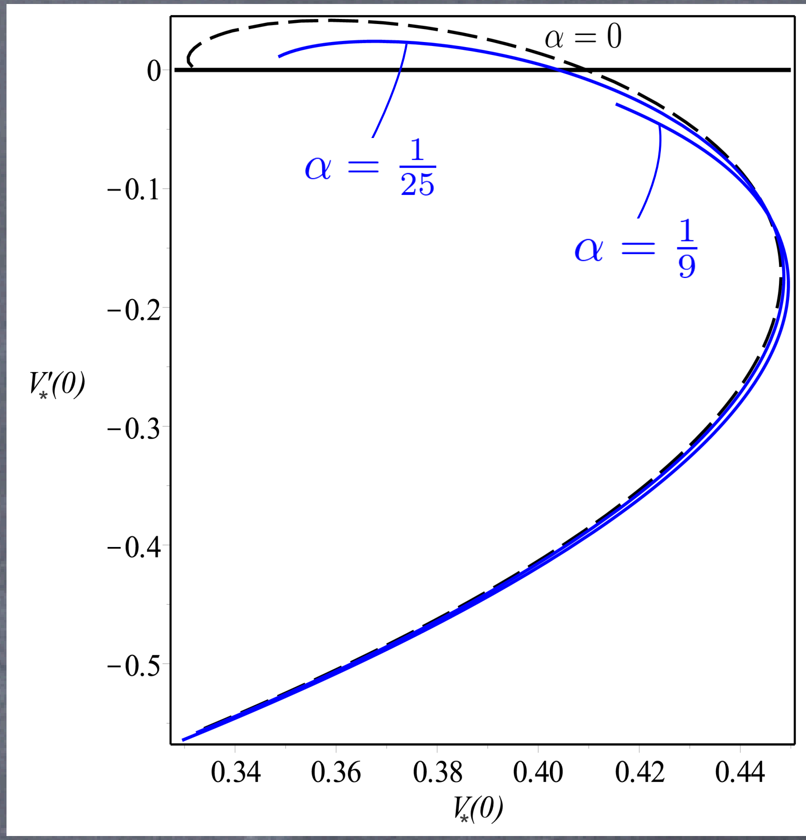

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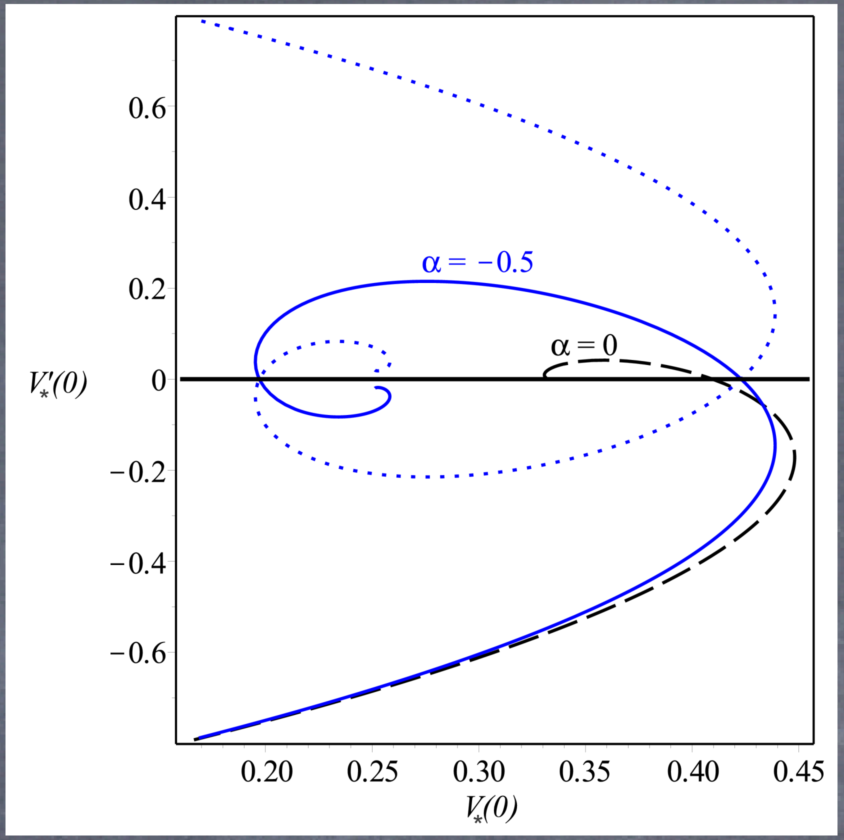

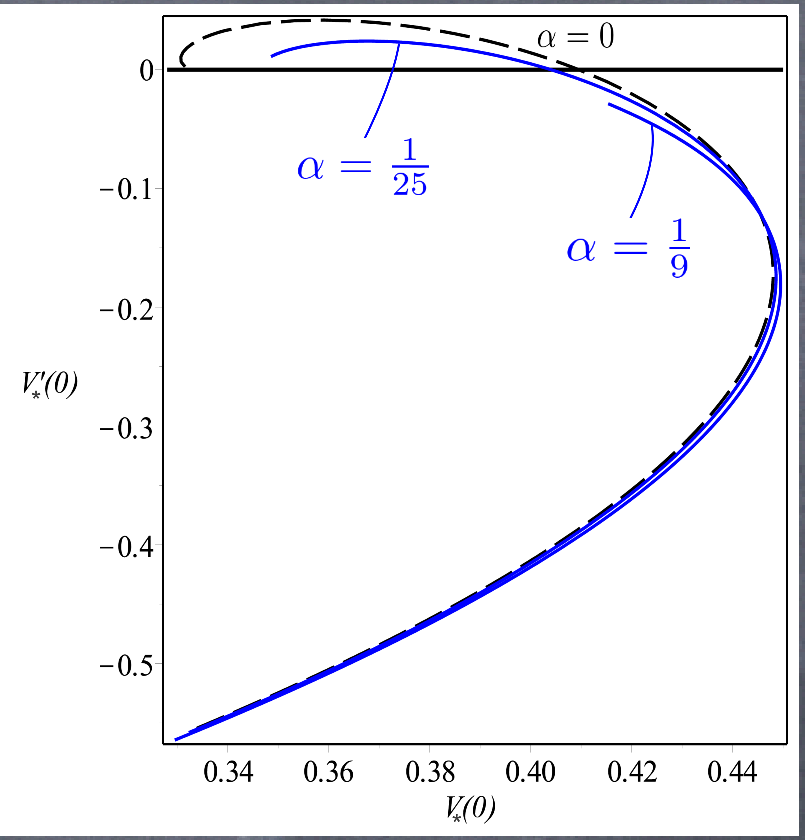

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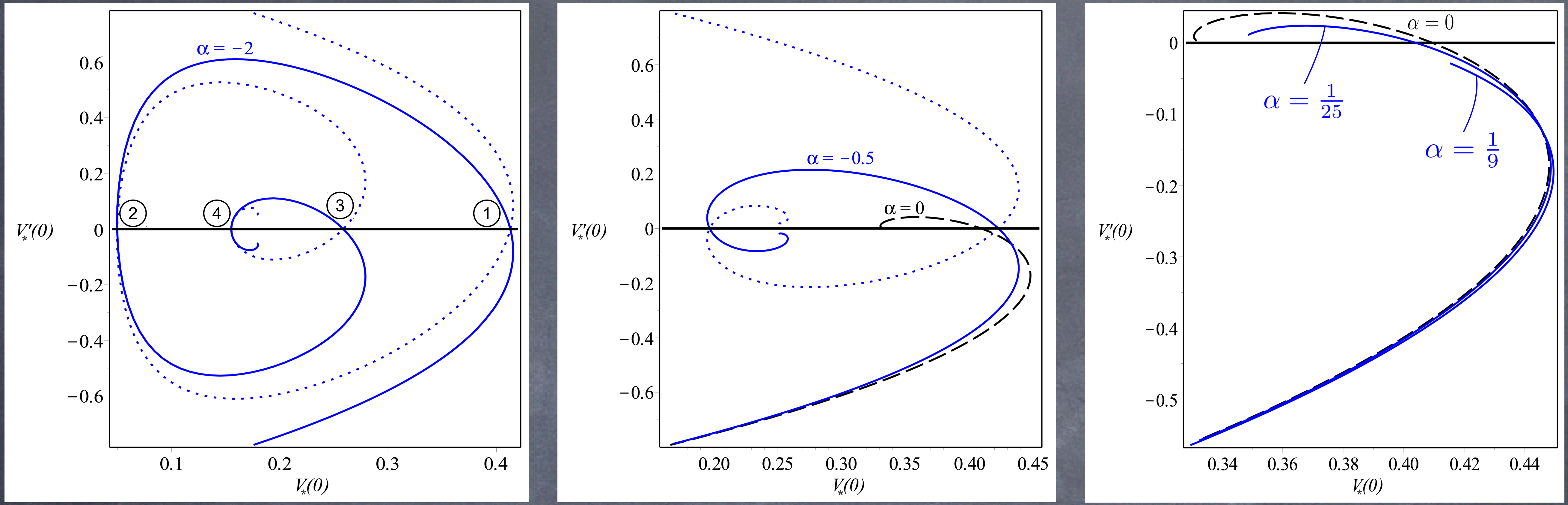

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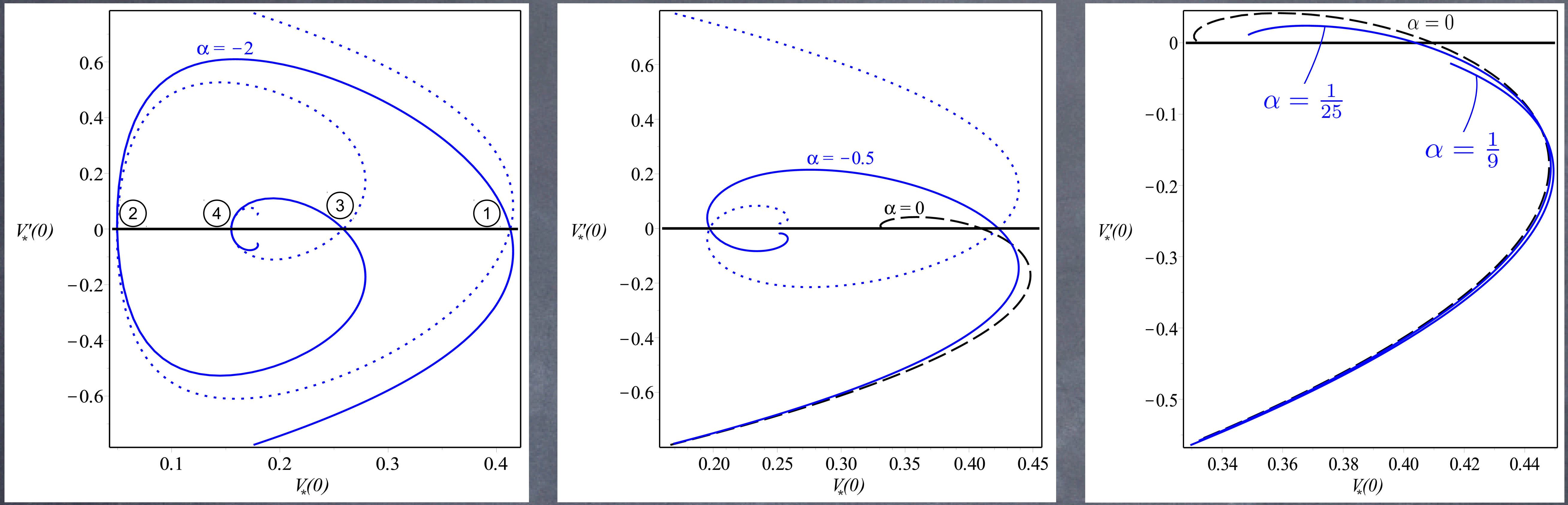

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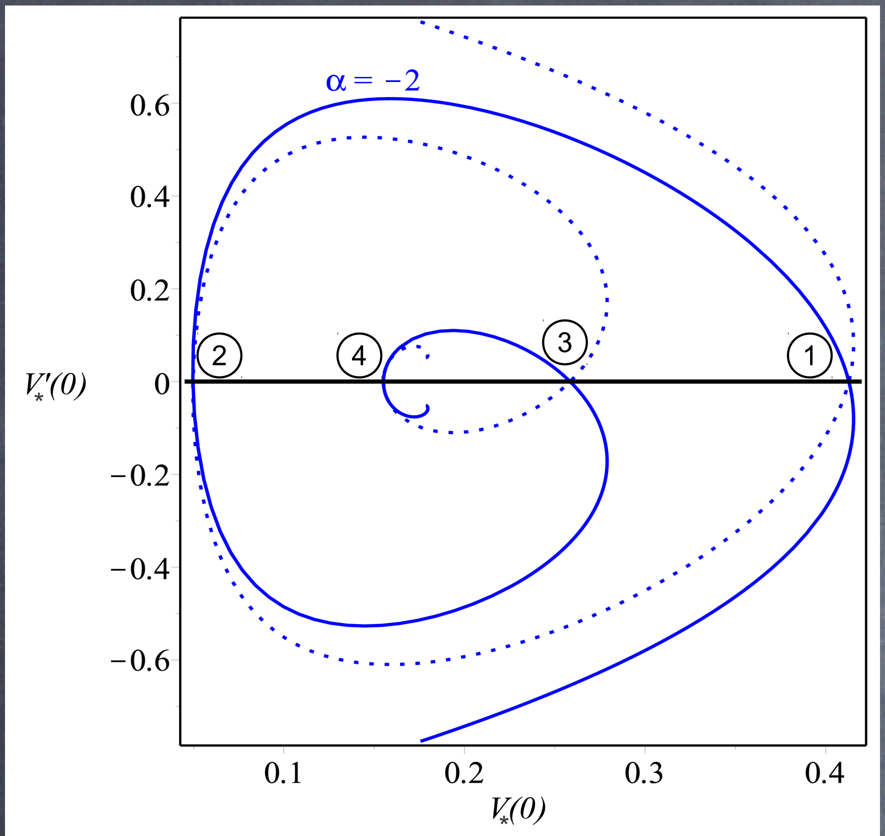

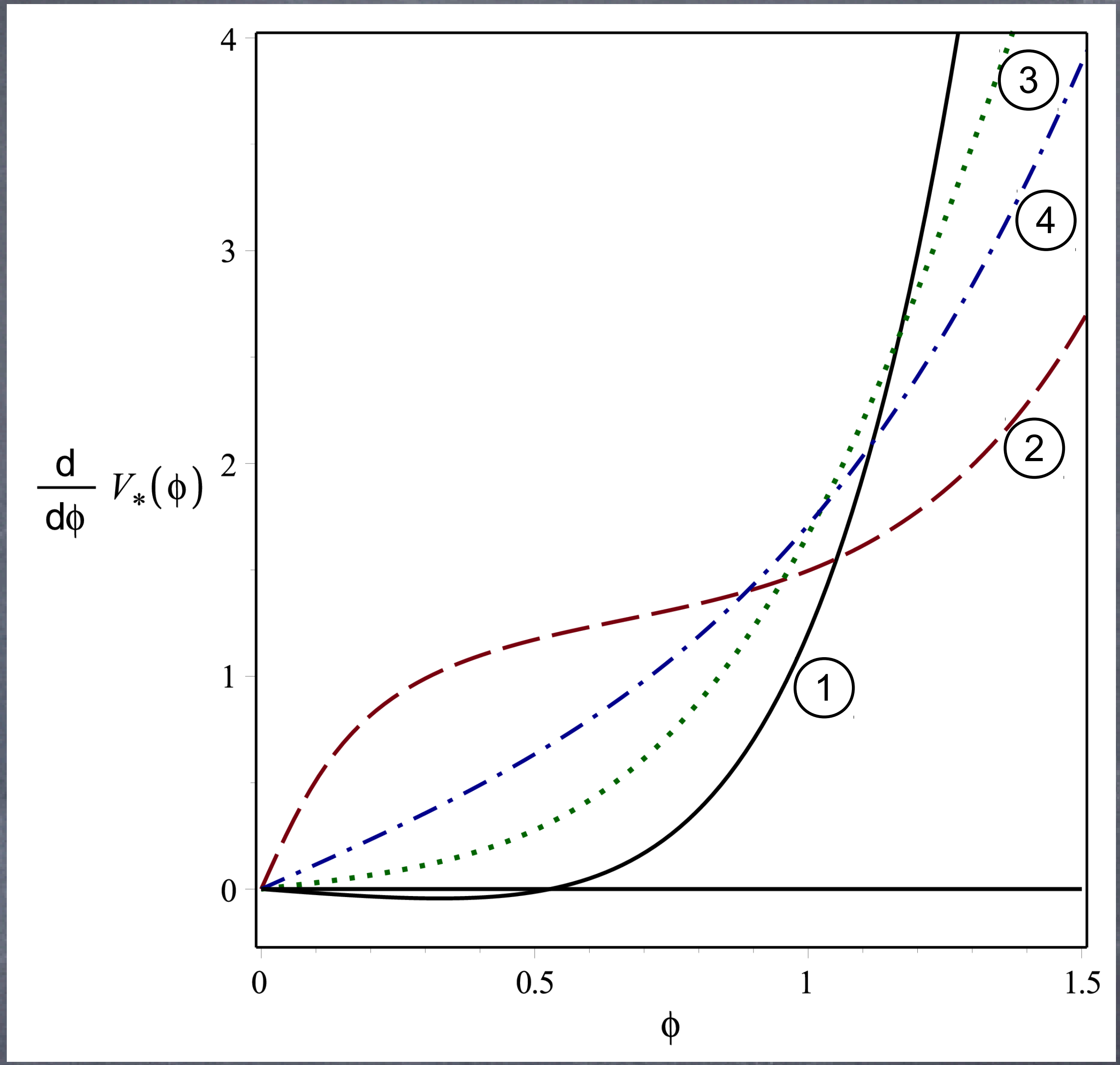

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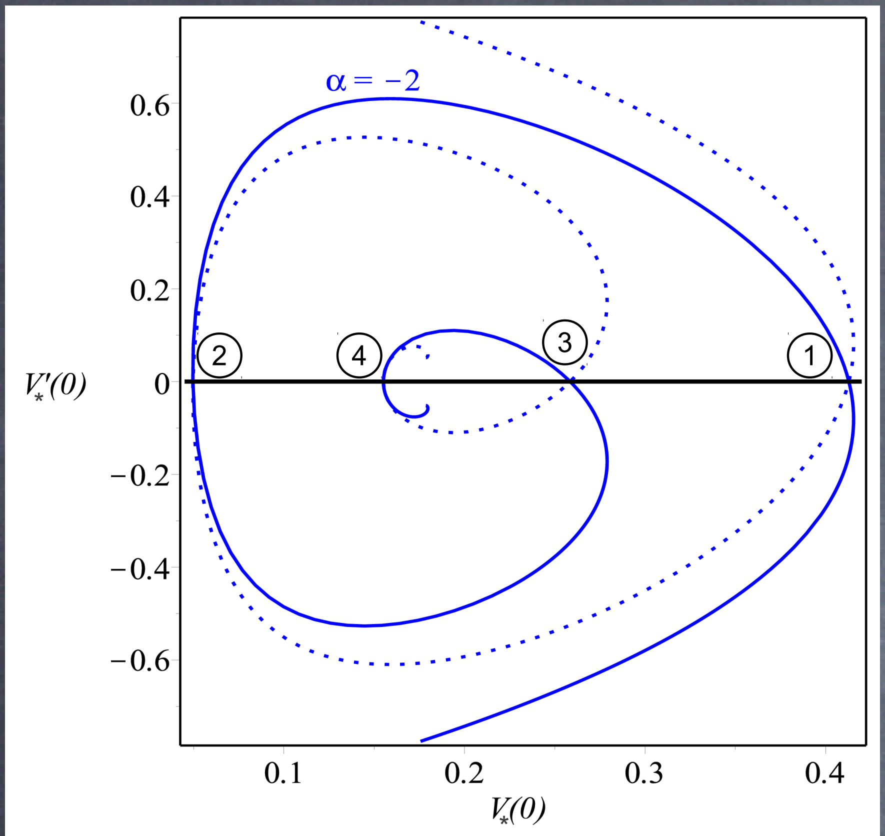

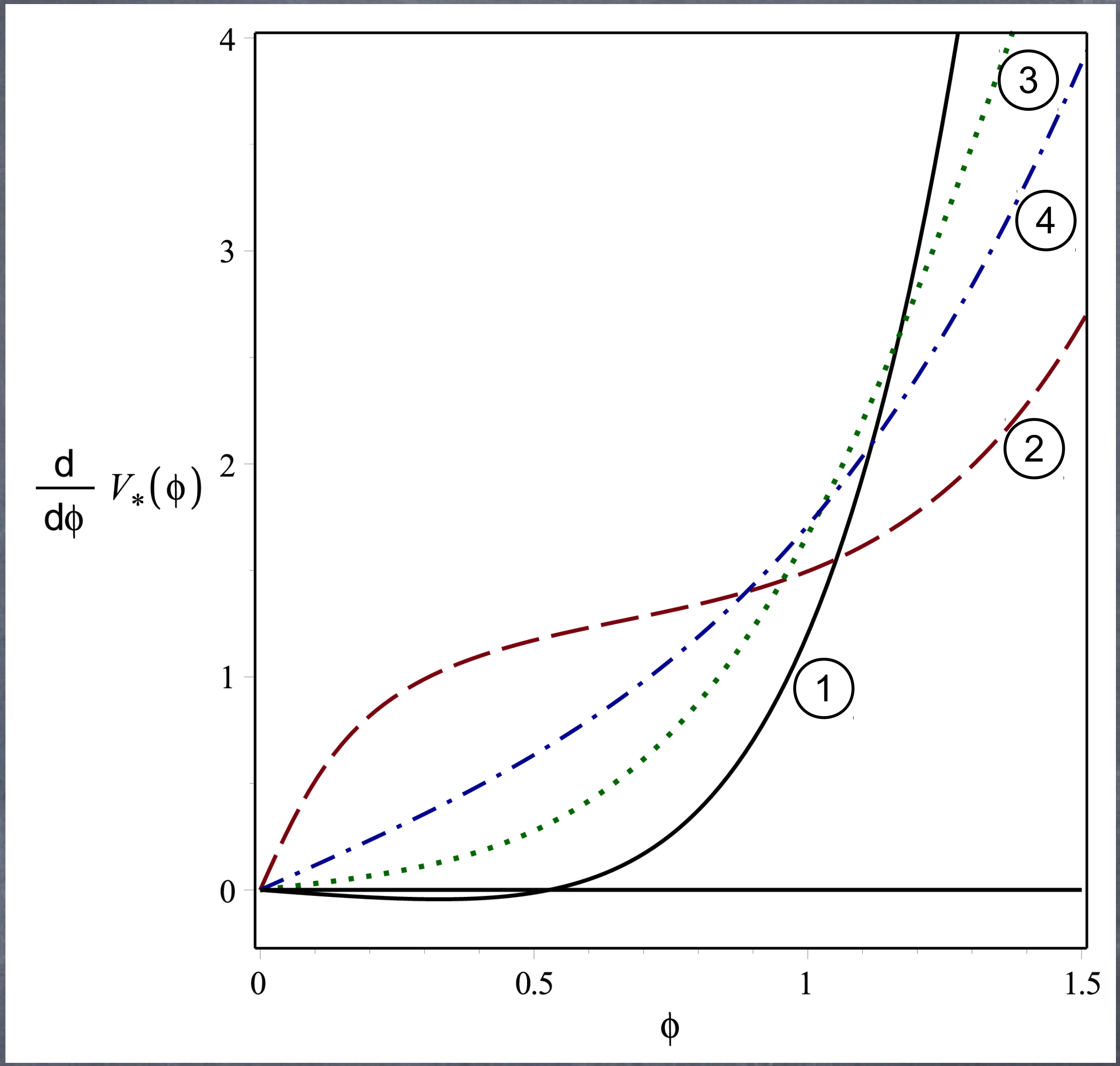

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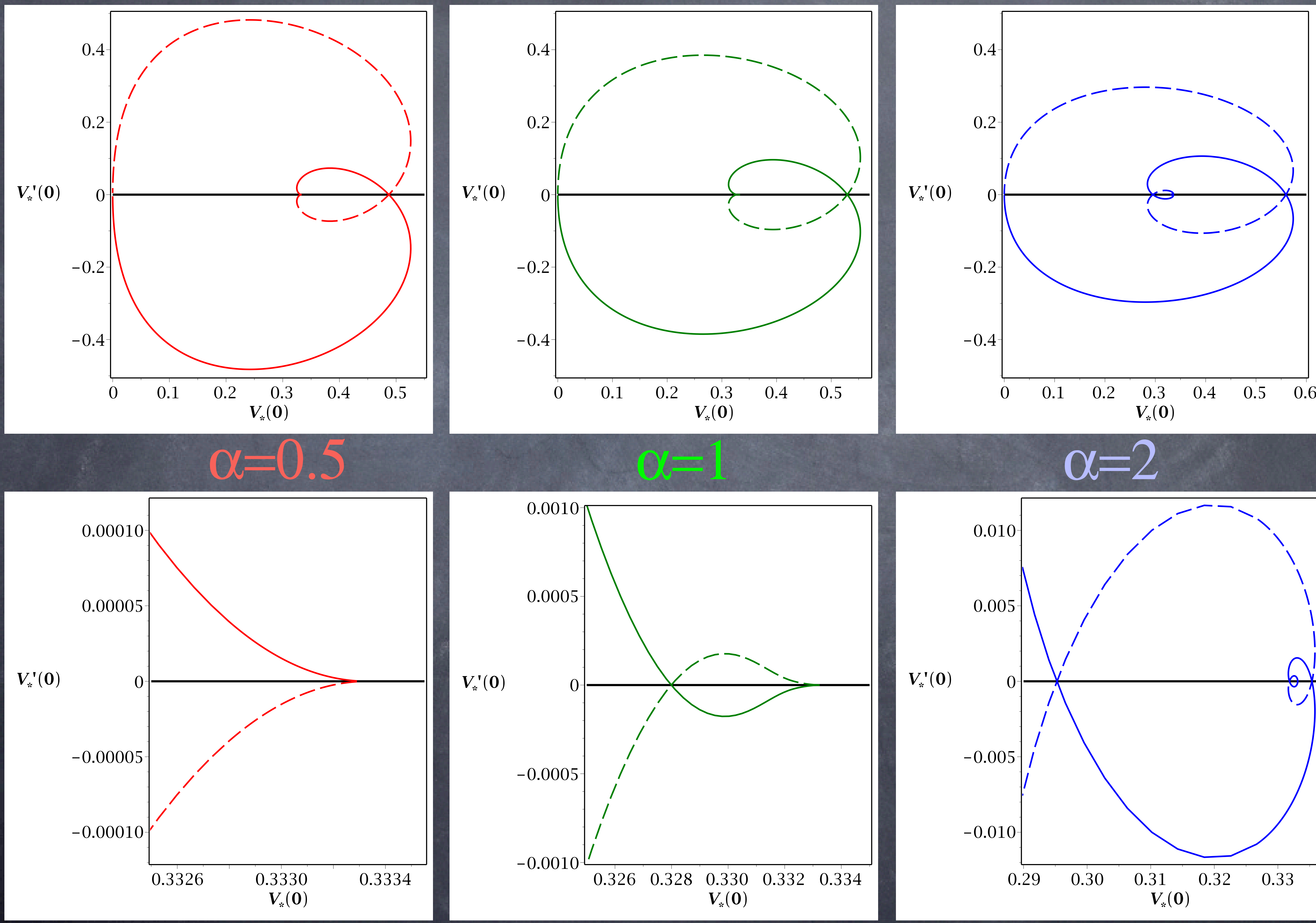
all the odd eigenoperators!

Spectral cutoff h=αV″(ϕ,t) (d=3):



Keep both fields

$$\partial_t V - \frac{1}{2}(d-2)\left(\varphi\partial_\varphi V + \bar{\varphi}\partial_{\bar{\varphi}} V\right) + dV = \frac{(1-h)^{d/2}}{1-h+\partial_\varphi^2 V}\left(1 - h - \frac{1}{2}\partial_t h + \frac{1}{4}(d-2)\bar{\varphi}h'\right)\theta(1-h).$$

I.H. Bridle, J. Dietz & T.R. Morris, JHEP **03** (2014) 093

Keep both fields & impose Ward Identity:

$$\phi = \varphi + \bar{\varphi} \qquad \bar{\varphi} \mapsto \bar{\varphi} + \varepsilon(x) \quad \text{and} \quad \varphi \mapsto \varphi - \varepsilon(x)$$

$$\frac{\delta \Gamma}{\delta \bar{\varphi}_a} - \frac{\delta \Gamma}{\delta \varphi_a} = \frac{1}{2} \mathrm{Tr} \left[\left(\mathcal{R} + \frac{\delta^2 \Gamma}{\delta \varphi \delta \varphi} \right)^{-1} \frac{\delta \mathcal{R}}{\delta \bar{\varphi}_a} \right].$$

Reuter, Wetterich, Litim, Pawlowski, Manrique, Saueressig, Becker...

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$$\Gamma \mapsto \Gamma + \Delta\Gamma[\bar{\varphi}] \quad \text{with} \quad \frac{\partial}{\partial k}\Delta\Gamma[\bar{\varphi}] = 0$$

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⟹implements background independence!

Conclusions.

- Technical advances can solve f(R) approximations
- New effects become visible in this regime & much more sensitive to issues with approximations.
- Need to work with less drastic approximations (inc. $h_{\mu\nu}$) Becker & Reuter; Codello, D'Odorico & Pagani; Dona, Eichhorn & Percacci; Christiansen, Knorr, Pawlowski & Rodigast; ...
- Does a flow equation exist which is good enough?