Super Efimov effect in mass imbalanced systems

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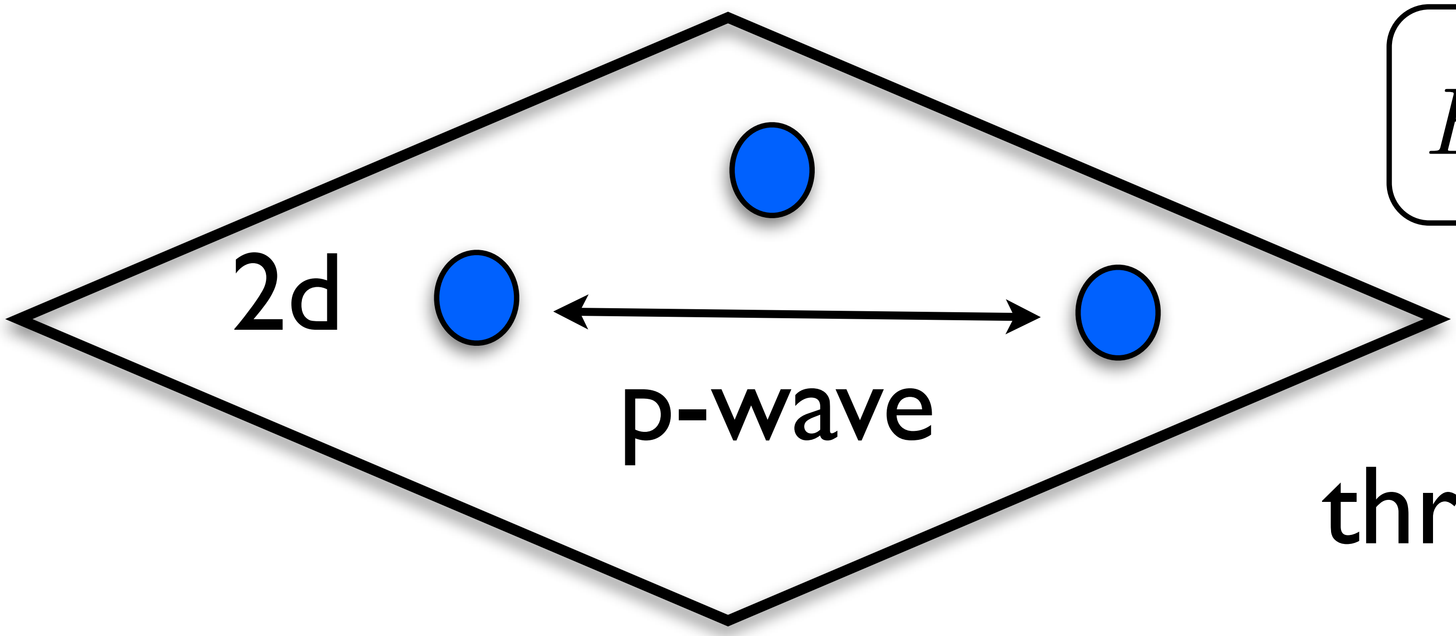
Super Efimov effect

At p-wave resonance near threshold:

Infinite tower of $l = \pm 1$ trimer bound states



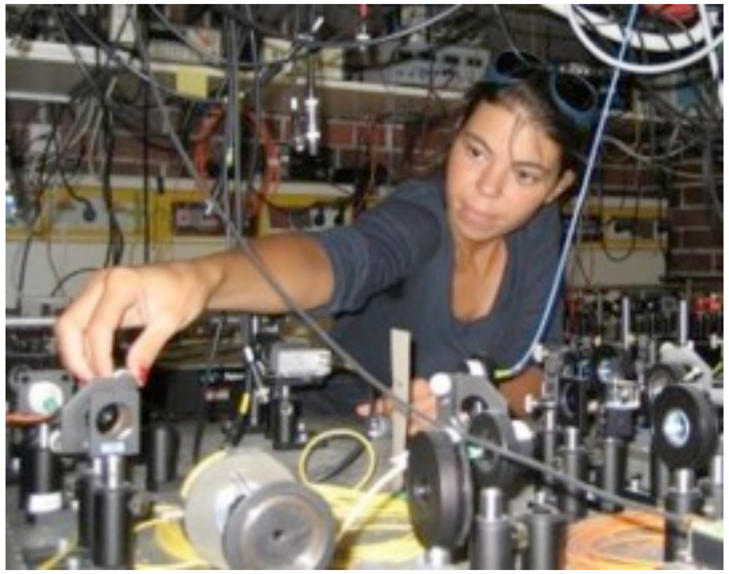
$$E_3^{(n)} \propto \exp\left(-2e^{3\pi n/4+\theta}\right)$$

three identical fermions

Super exponential scaling!

talk of
Yusuke Nishida

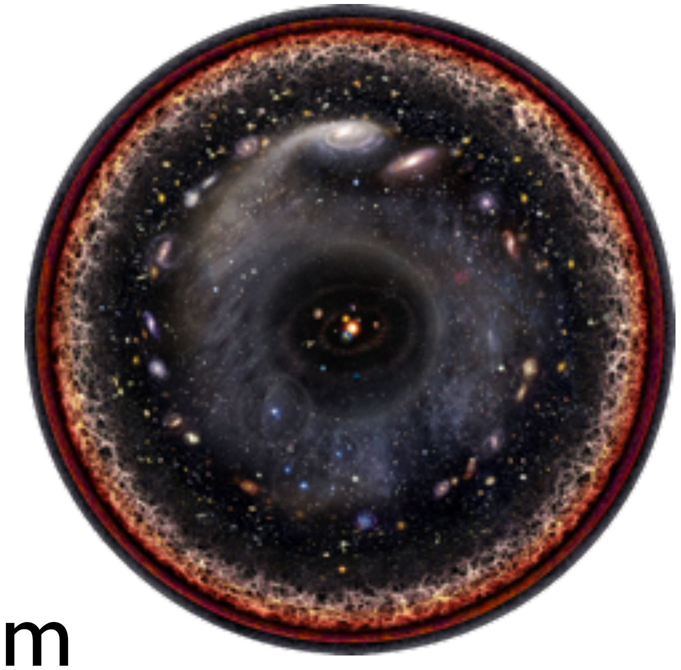
Experiments



- Efimov physics in cold atom experiments *since 2006*
- Quasi 2d fermions near p-wave resonance *ETH 2005*
- Trimers sizes: $a_3^{(n)} \propto \exp\left(e^{3\pi n/4+\theta}\right)$

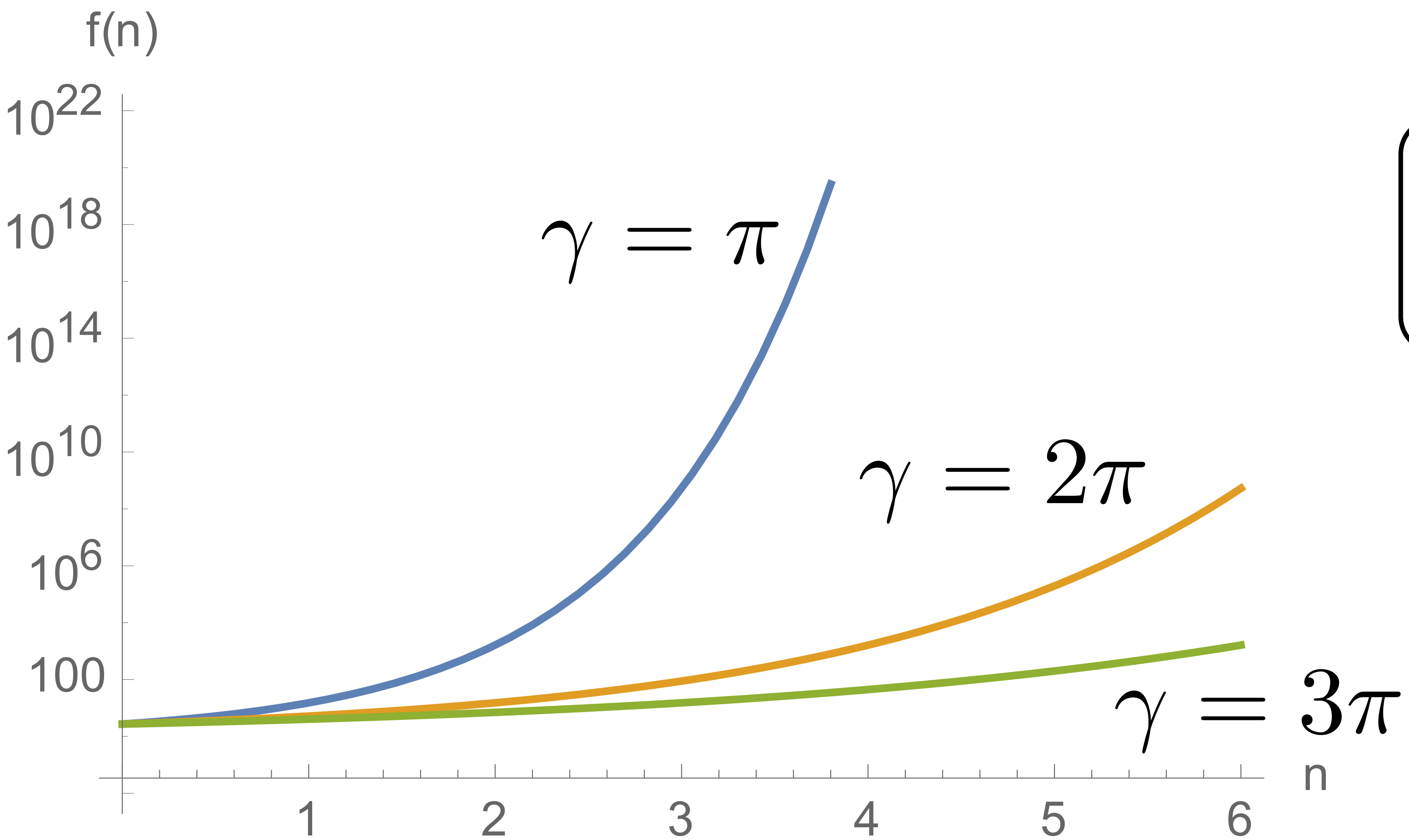
$\theta = 0$

n	GS	1	2	3
size	Å	μ	10^{38}m	10^{499}m



Radius of observable Universe 10^{26}m

Super exponential scaling



Set θ to zero

$$f(n) = \exp\left(e^{\pi n/\gamma}\right)$$

Slow function for

$$n < \gamma/\pi$$

Can we somehow increase γ ?

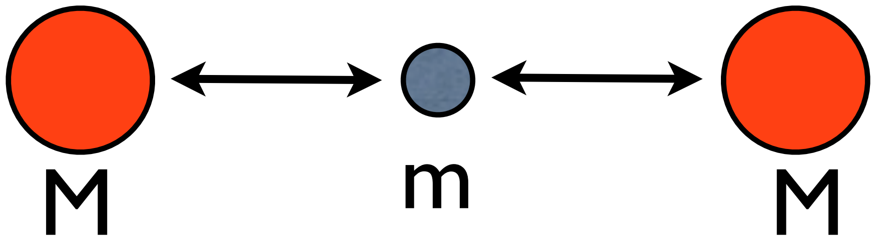
Mass imbalanced system

- Efimov problem for heavy-heavy-light system

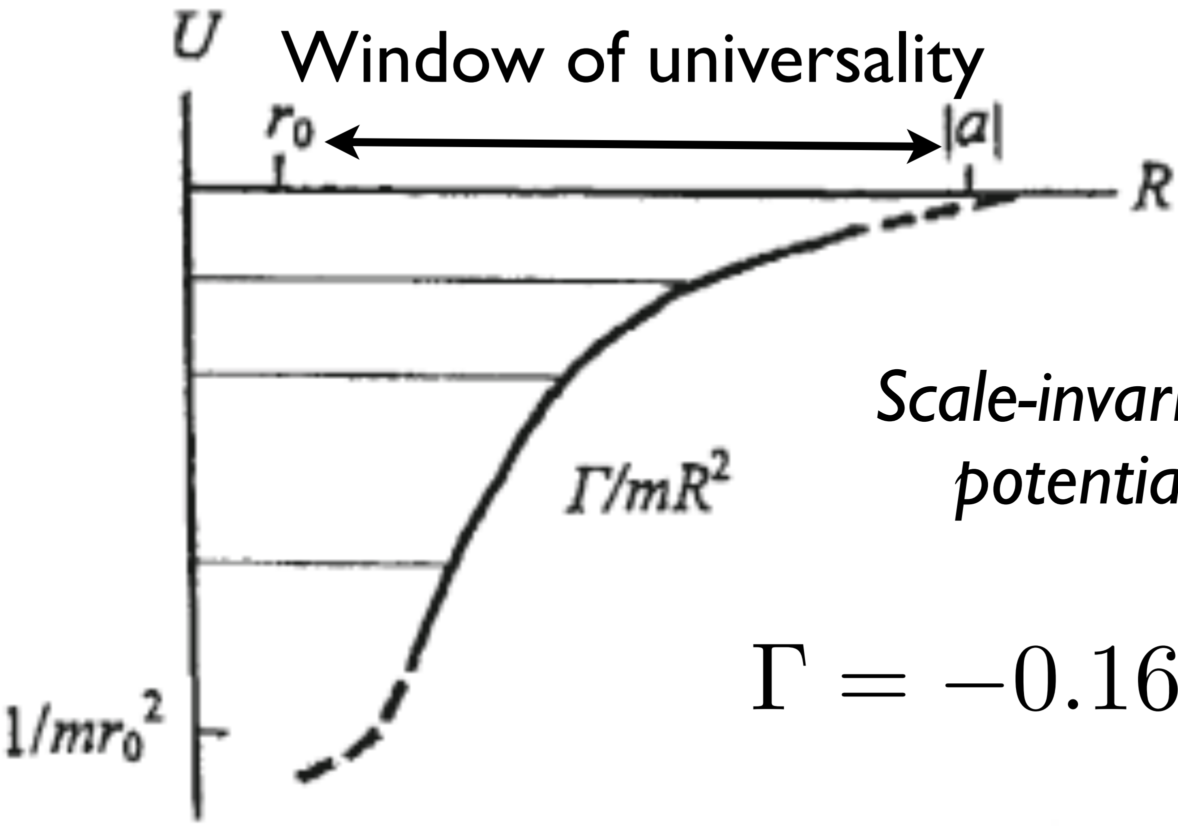
Efimov 1972
Amado&Noble 1972

- Born-Oppenheimer approximation: first freeze heavy particles

s-wave resonance in 3d



identical heavy bosons

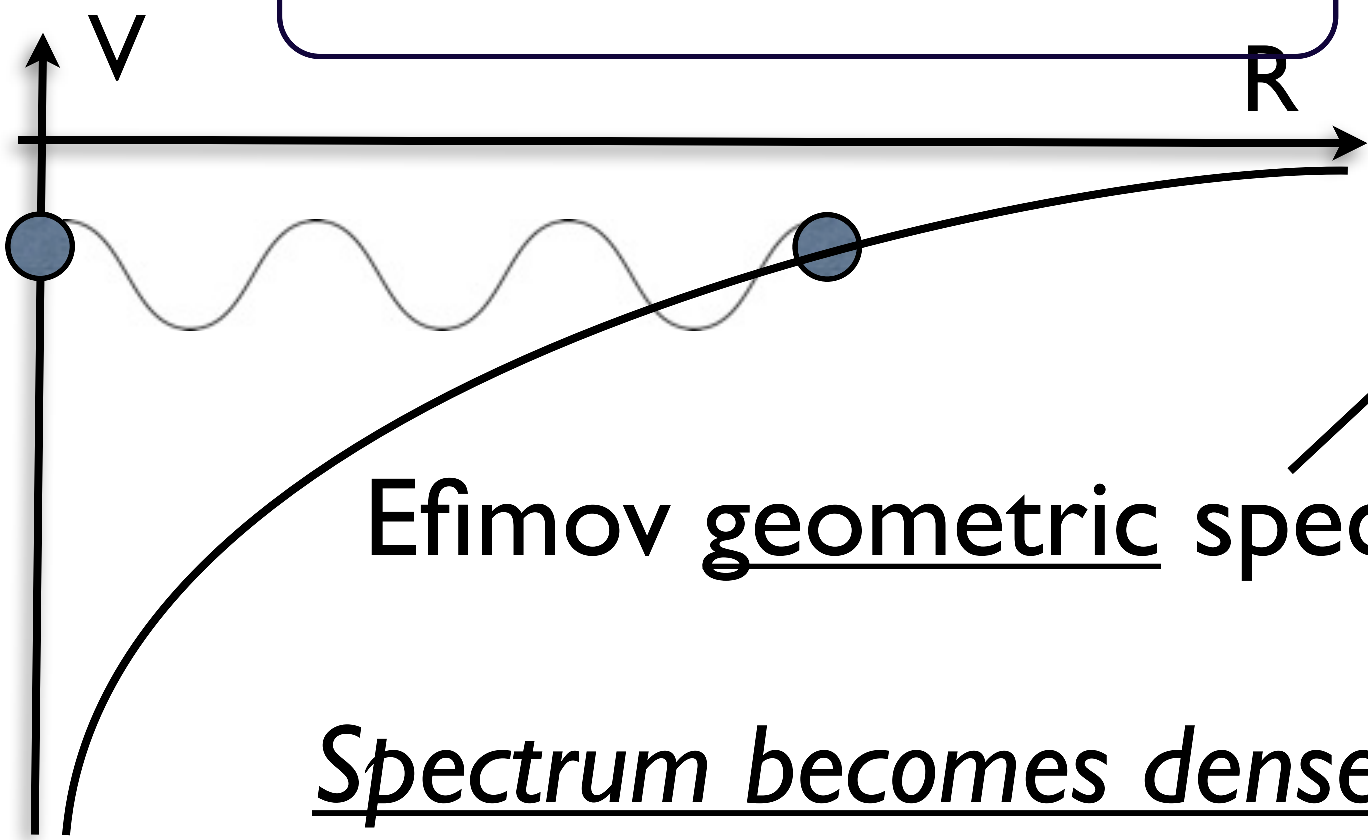


Scale-invariant potential

$$\Gamma = -0.16$$

Heavy-heavy spectrum

$$V(R) = \frac{\Gamma}{mR^2}$$



Landau&Lifshitz:
Fall to center for strong attraction

$$E_n \sim \exp\left(-\frac{2\pi n}{s} + \theta\right)$$

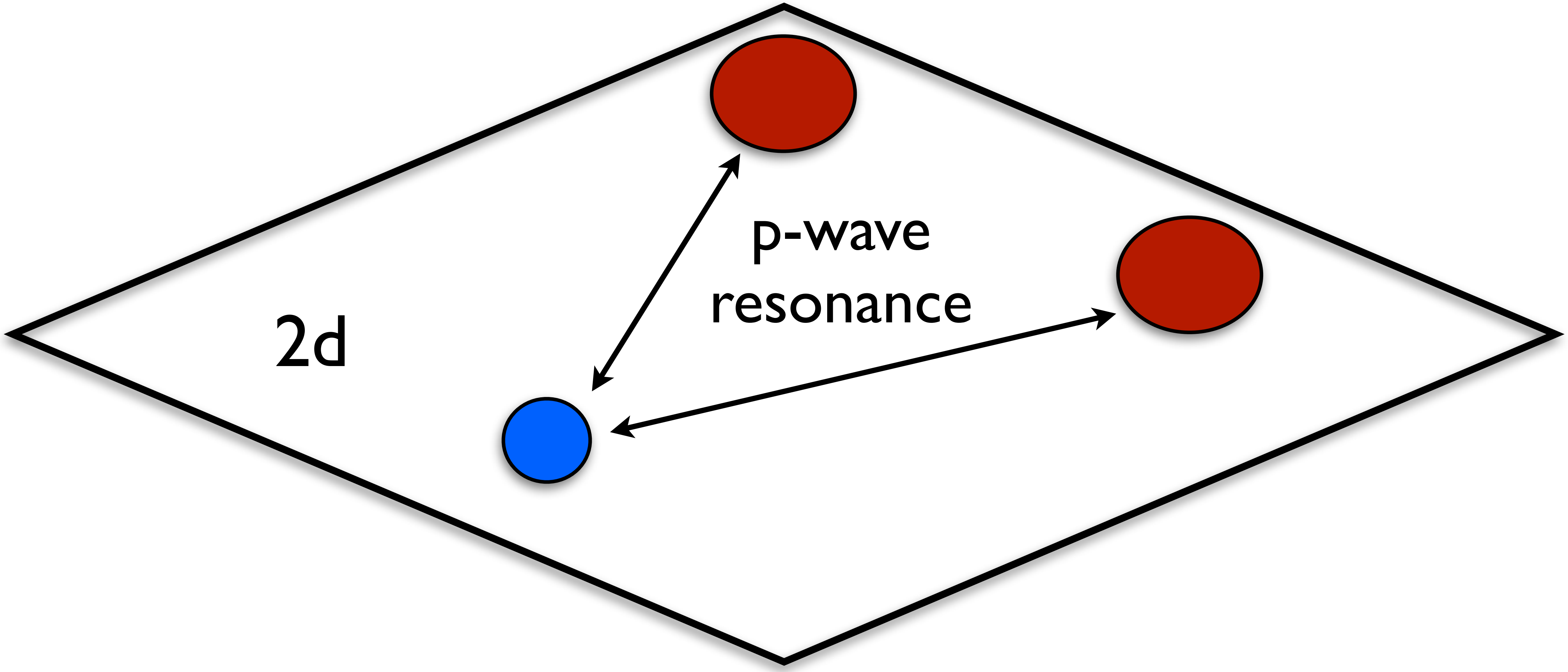
Efimov geometric spectrum $s = \sqrt{0.16M/m}$

Spectrum becomes denser as mass ratio grows!

Multiple Efimov states observed in Li-Cs mixture

Chicago and Heidelberg 2014

Mass imbalanced system



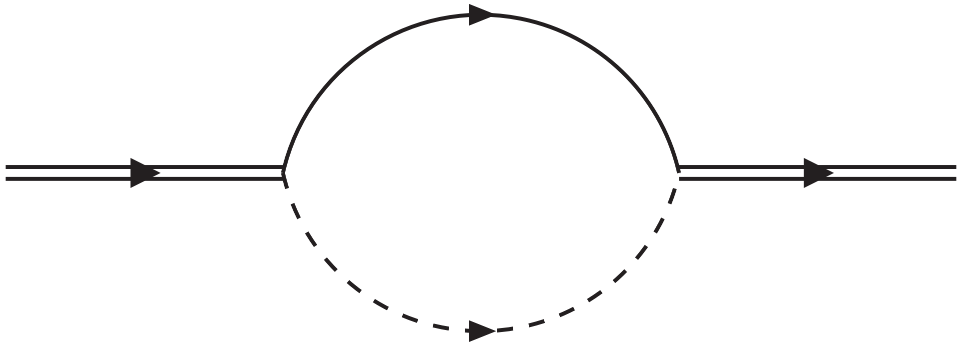
Identical particles are fermions or bosons

Non-resonant s-wave interactions allowed

Two-body from RG

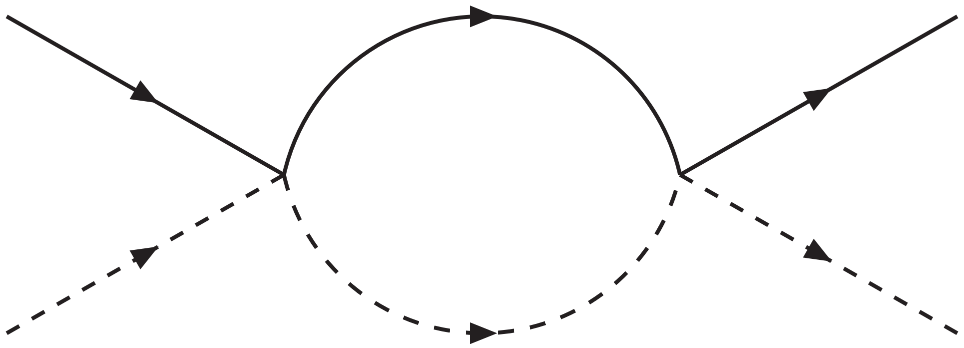
$$s = \ln \Lambda / k \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

(a)



$$g^2(s) = \frac{1}{\frac{1}{g^2(0)} + \frac{2\mu^2}{\pi} s}$$

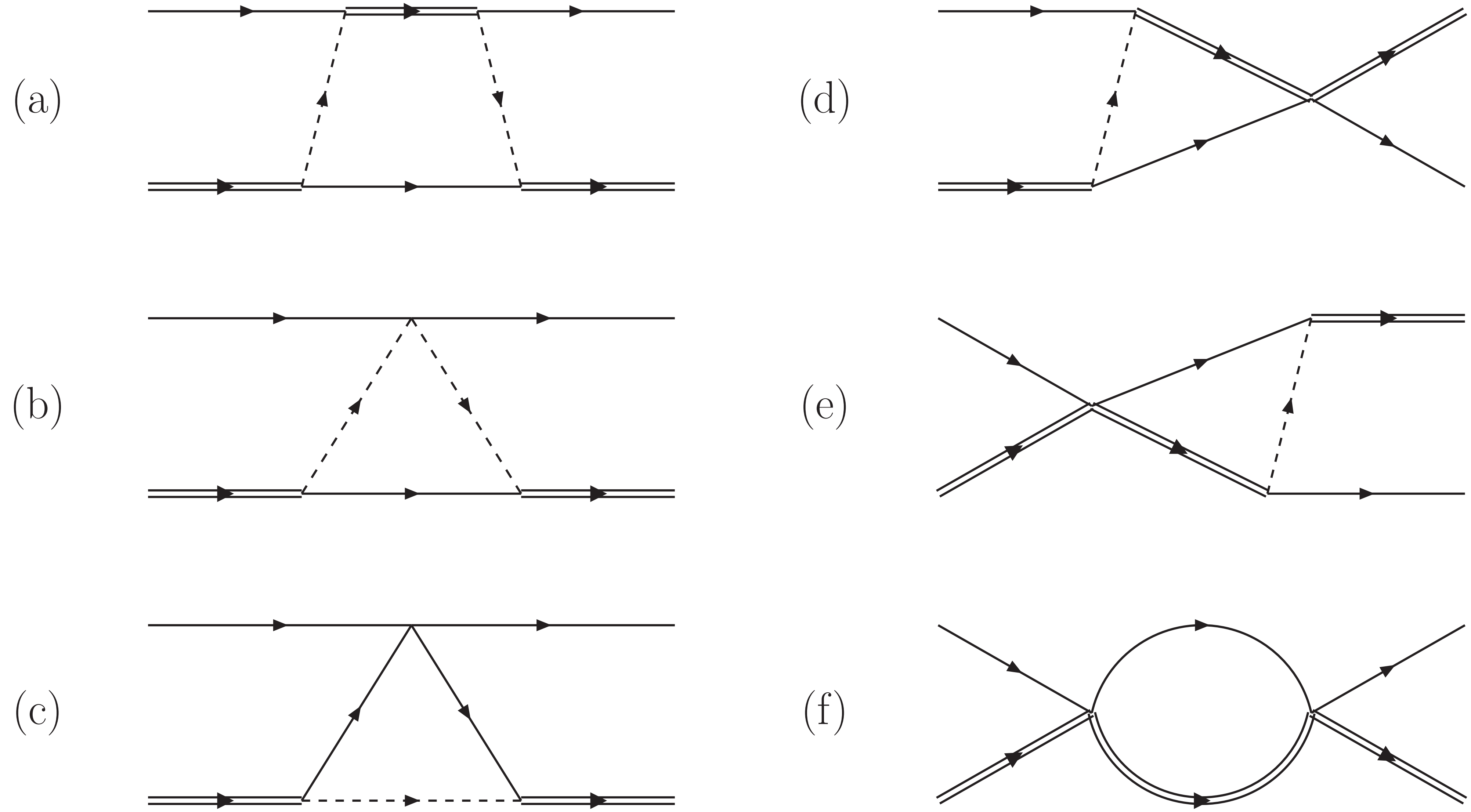
(b)



$$v_{12}(s) = \frac{1}{\frac{1}{v_{12}(0)} - \frac{\mu}{\pi} s}$$

<u>Logarithmic running to free fixed point!</u>

Three-body from RG

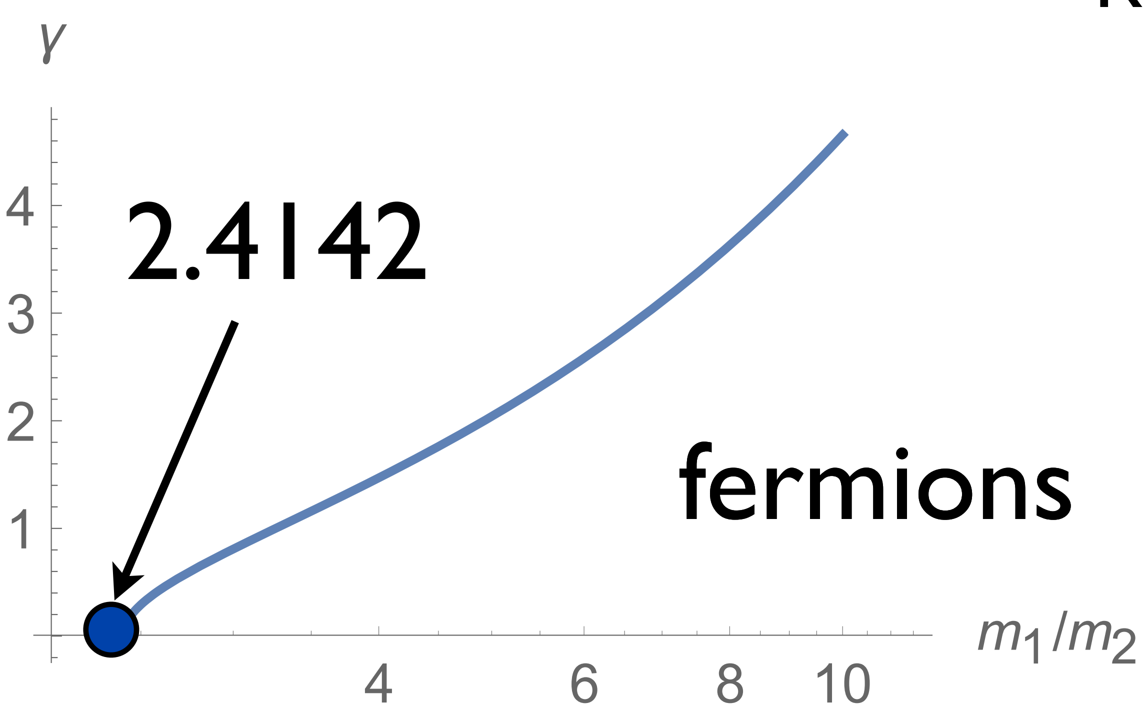


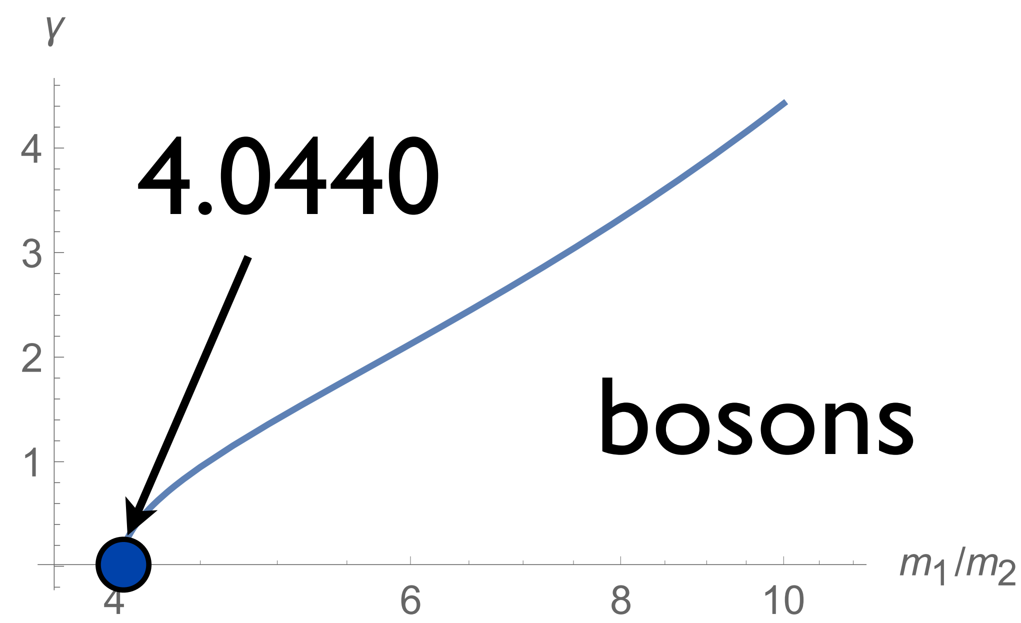
RG flow of heavy-heavy-light coupling u_1

Three-body from RG

Double log solution: $su_1(s) \quad \to \mp\frac{\pi}{m_2} - \frac{\pi\gamma}{\nu_1}\cot[\gamma(\ln s - \theta)]$

RG divergences= trimer bound states





in Li-Cs mixture $\gamma \approx 10.7$

<u>Super Efimov spectrum</u>

$$E_n \propto \exp\left(-2e^{\pi n/\gamma+\theta}\right)$$

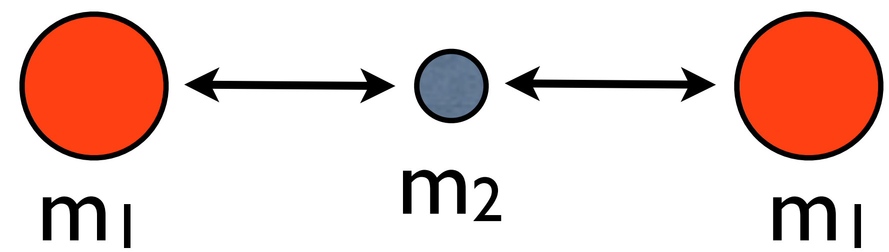
Born-Oppenheimer approximation

Induced heavy-heavy potential:

$$V(R) = -\frac{1}{m_2 R^2 \ln(R\Lambda)}$$

$$E_n^{(\mathrm{BO})} \propto \exp\left(-\frac{m_2 \pi^2}{2m_1} n^2\right)$$

p-wave resonance in 2d



identical heavy bosons/fermions

BO spectrum **differs** from super Efimov spectrum!

$$E_n \propto \exp\left(-2e^{2\frac{m_2}{m_1}\pi n+\theta}\right)$$

Failure of BO approximation

Heavy particles time scale: $T_{\text{heavy}} \sim m_1 R^2$

Light particles time scale: $T_{\text{light}} \sim m_2 R^2 \ln(R\Lambda)$

Can not use adiabatic approximation if

$$T_{\text{light}} \gtrsim T_{\text{heavy}}$$

BO approximation breaks down for large distances

$$\boxed{R\Lambda \gtrsim e^{m_1/m_2}}$$

Conclusion

- Super Efimov effect was extended to mass imbalanced three-body systems
- Mixtures with high mass imbalance are favorable for experimental verification
- Intermediate Born-Oppenheimer scaling?

Extra slides

p-wave in 2d

Square well solution

$$\frac{dJ_l(kr)/dr}{J_l(kr)} = \frac{dK_l(\kappa r)/dr}{K_l(\kappa r)}$$

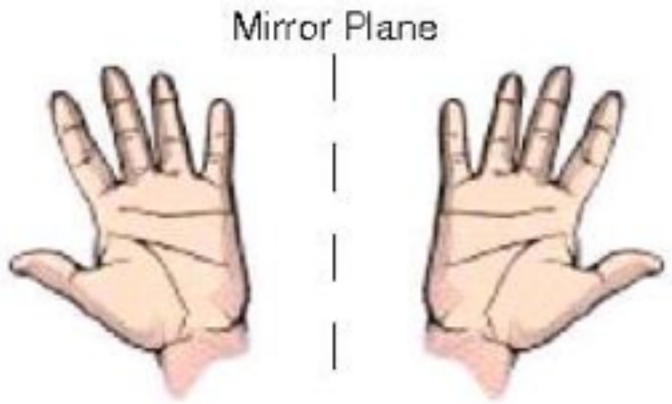
In p-wave critical attraction needed $\quad V_0 r_0^2 = 5.784$

Normalized wave-function

$$\psi(r) = \frac{\kappa}{\sqrt{2\pi}} \frac{K_1(\kappa r)}{\sqrt{\ln(\kappa r_0)}}$$

- No scale invariance
- Point-like boson as $r_0 \to 0$

P-wave superfluids



From mean-field: *Volovik, Read, Green,...*

- Chiral condensate $\Delta_{\mathbf{p}} = (p_x \pm ip_y)\hat{\Delta}$ preferred
- Topological phase transition at $\mu = 0$
- Chiral Majorana modes on boundaries
- Toy model for a film of ^{3}He

Sometimes mean-field is not good enough near resonance!

P-wave model in d=2

TAKE ACTION!

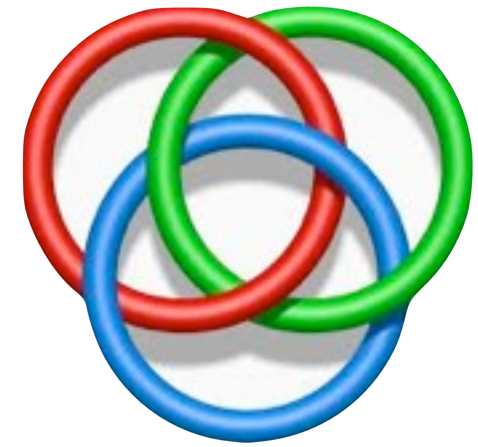
$$
\begin{aligned}
\mathcal{L} \;\; = \;\; & \psi^\dagger\left(i\partial_t + \frac{\nabla^2}{2}\right)\psi + \phi_a^\dagger\left(i\partial_t + \frac{\nabla^2}{4} - \varepsilon_0\right)\phi_a \\
& + g\,\phi_a^\dagger\psi(-i\nabla_a)\psi + g\,\psi^\dagger(-i\nabla_{-a})\psi^\dagger\phi_a \\
& + v_3\,\psi^\dagger\phi_a^\dagger\phi_a\psi + v_4\,\phi_a^\dagger\phi_{-a}^\dagger\phi_{-a}\phi_a + v_4'\,\phi_a^\dagger\phi_a^\dagger\phi_a\phi_a
\end{aligned}
$$

spinless fermion

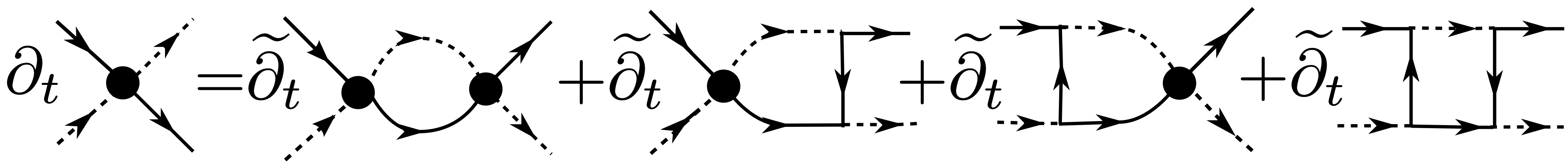
composite $l = \pm 1$ boson

- P-wave resonance ⟷ zero energy bound state
- All dimensionless couplings are included

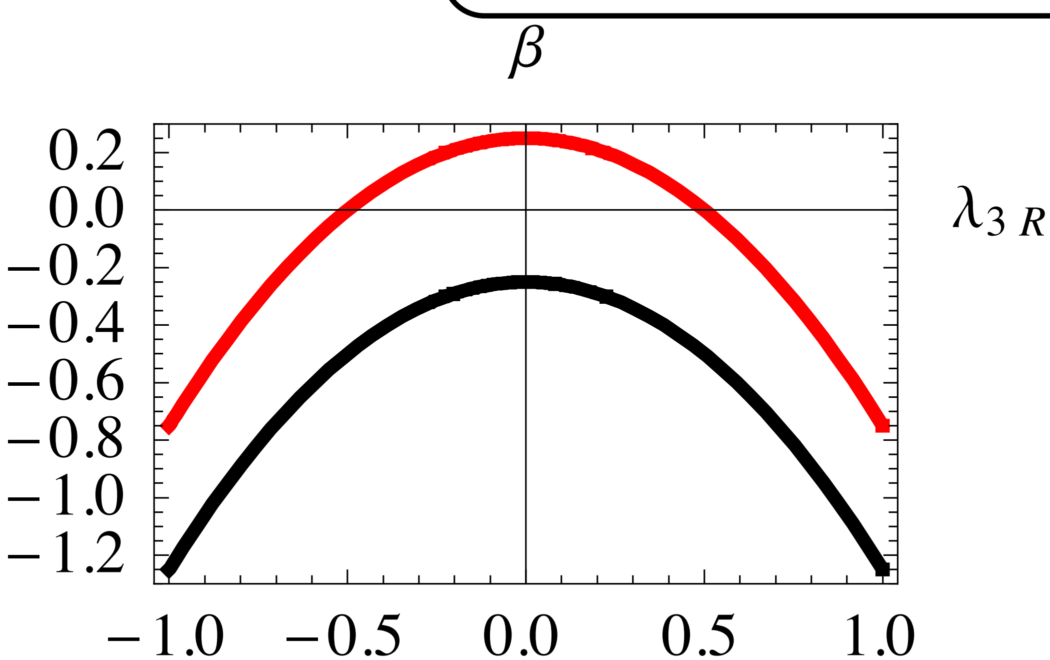
Efimov effect from RG



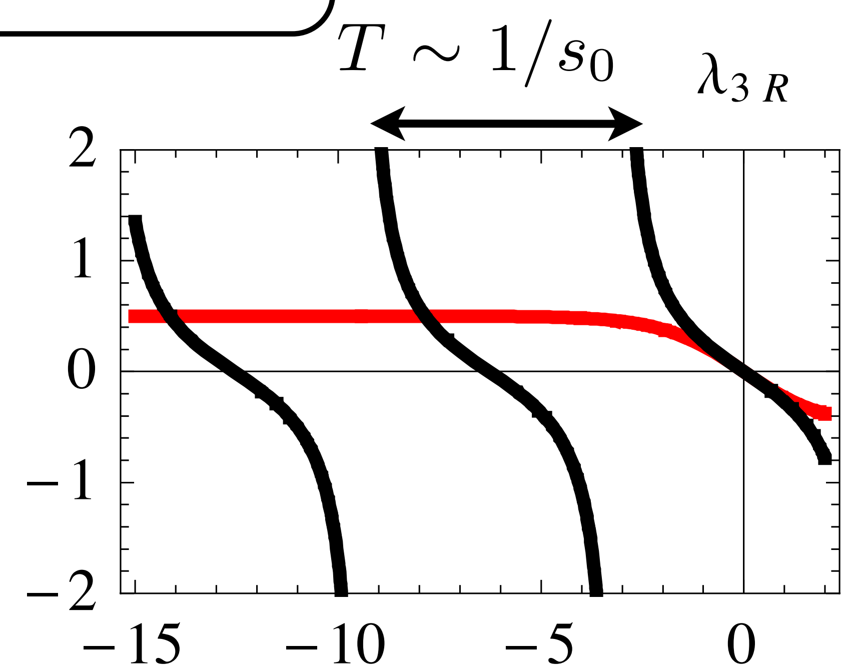
Flow of atom-dimer vertex: RG=one-loop diagrams



bosons vs fermions in 3d



Tetramers can be found from RG

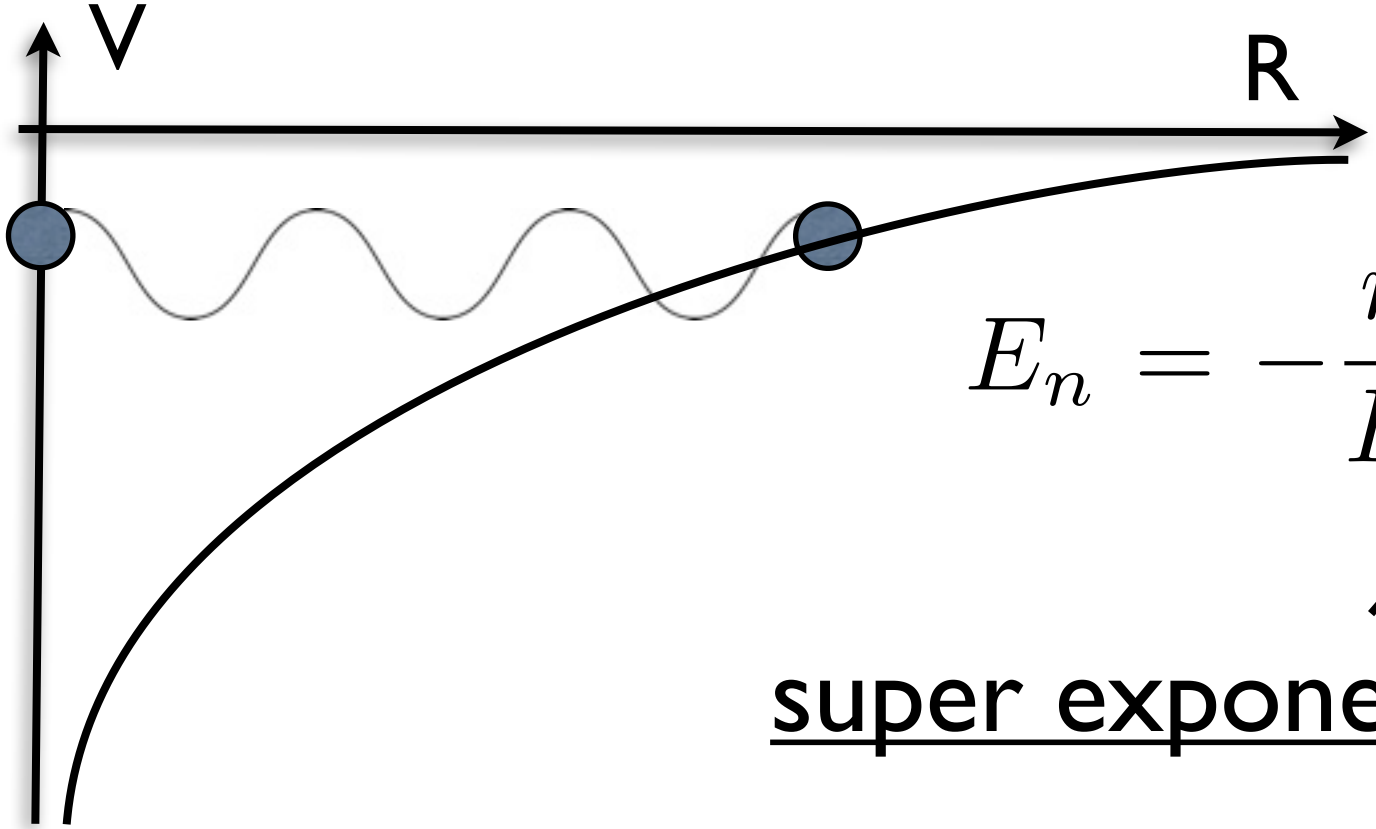


limit cycle

Effective potential

$$V(R) = -\frac{1}{4R^2} - \frac{1/4 + r^2}{(R \ln \frac{R}{R\,0})^2}$$

Semiclassical solution with double Langer correction



$$E_n = -\frac{r^2}{R_0^2} \frac{\exp\left(-2e^{\frac{\pi}{r}n+\theta}\right)}{e^{2\left(\frac{\pi}{r}n+\theta\right)}}$$

super exponential scaling