The 3d O(N) model in the large N limit

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Contents

Flow Equation

Flow equation for the O(N) symmetric *effective action*

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\int \frac{d^D q}{(2\pi)^D}\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi(q)\delta\phi(-q)} + R_k(q)\right)^{-1} \partial_t R_k(q)$$

Ansatz for the solution → Local Potential Approximation (LPA)

$$\Gamma_k = \int d^D x \left[\frac{1}{2}(\partial\phi)^2 + U_k(\phi^a \phi_a)\right]$$

This gives the flow equation for the effective *potential*

$$\partial_t U_k = \frac{1}{2}(2\pi)^{-3}\int_q \partial_t R_k \left(\frac{N-1}{M_0} + \frac{1}{M_1}\right)$$

$$M_0 := q^2 + R_k + U_k'$$
$$M_1 := q^2 + R_k + U_k' + 2\rho U_k''$$
$$(.)' := \tfrac{\delta}{\delta\rho}$$

$$D = 3$$
$$\bar{\rho} := \phi_a \phi^a / 2$$

Flow Equation

$$\partial_t U_k = \frac{1}{2}(2\pi)^{-3}\int_q \partial_t R_k \left(\frac{N-1}{M_0}+\frac{1}{M_1}\right)$$

$M_0 := q^2 + R_k + U_k'$
$M_1 := q^2 + R_k + U_k' + 2\rho U_k''$
$(.)' := \frac{\delta}{\delta\rho}$

- Using the optimized regulator: $R_k = (k^2 - q^2)\theta(k^2 - q^2)$ the integral is analytic
- The flow for the dimensionless effective potential for finite N

Dimensionless quantities

$$\begin{cases} u' \equiv U'/k^2 \\ u'' \equiv U''/k \\ \rho \equiv \bar{\rho}/k \end{cases}$$

$$\partial_t u = (D-2)\rho u' - Du + (N-1)A_D \frac{1}{1+u'} + A_D \frac{1}{1+u'+2\rho u''}$$

$A_3 = \frac{1}{6\pi^2}$ in 3D

Flow Equation

$$\partial_t U_k = \frac{1}{2}(2\pi)^{-3}\int_q \partial_t R_k \left(\frac{N-1}{M_0} + \frac{1}{M_1}\right)$$

$M_0 := q^2 + R_k + U_k'$

$M_1 := q^2 + R_k + U_k' + 2\rho U_k''$

$(.)' := \frac{\delta}{\delta\rho}$

- Using the optimized regulator: $R_k = (k^2 - q^2)\theta(k^2 - q^2)$ the integral is analytic
- Taking the large *N*-limit (η=0 and LPA becomes exact)

The flow for the dimensionless effective potential (actually for its derivative)

$$\partial_t u = (D-2)\rho u' - Du + \frac{1}{1+u'}$$

$$\partial_t u' = -2u' + \rho u'' - \frac{u''}{(1+u')^2} \quad \text{in 3D}$$

Dimensionless quantities

$u' \equiv U'/k^2$

$u'' \equiv U''/k$

$\rho \equiv \bar{\rho}/k$

Local Flow

Expanding the potential in Taylor-series around the min.

$$u = \sum_{n=1}^{n_{trunc}} \frac{a_n}{n!}(\rho - \rho_0)^n$$

$$u'(\rho_0) = 0 \quad \lambda \equiv a_2 \quad \tau \equiv a_3$$

		Running	Fixed Points
β–functions	*(decoupled)* $\partial_t \rho_0 = 1 - \rho_0$	$\rho(t) = 1 + c_\rho e^{-t}$	$\rho_{0*} = \rho_{0kr} = 1$
	$\partial_t \lambda = -\lambda(1 - 2\lambda)$	$\lambda(t) = \dfrac{1}{2 + c_\lambda\, e^t}$	$\lambda_* = 0,\ \lambda_{**} = \frac{1}{2}$
	$\partial_t \tau = -6\lambda(\lambda^2 - \tau)$	$\tau(t) = \dfrac{2 + c_\tau\, e^{3t}}{(2 + c_\lambda\, e^t)^3}$	$\tau_* = \tau,\ \tau_{**} = \frac{1}{4}$
	$(\partial_t a_n = f(\lambda, \tau), n > 3)$		Tricritical (UV attractive); Wilson-Fisher (IR attractive)

Constants from initial value $c_\rho = \rho_{0,\Lambda} - 1,\ c_\lambda = 1/\lambda_\Lambda - 2,\ c_\tau = \tau_\Lambda / \lambda_\Lambda^3 - 2$

Local Flow

Expanding the potential in Taylor-series around the min.

$$u = \sum_{n=1}^{n_{trunc}} \frac{a_n}{n!}(\rho - \rho_0)^n$$

$$u'(\rho_0) = 0 \quad \lambda \equiv a_2 \quad \tau \equiv a_3$$

So in summary: from Taylor expansion 3 type of fixed points:

Gauss

$$\rho_0* = 1$$
$$\lambda_* = 0$$
$$\tau_* = 0$$

← $\tau \to 0$

Tricritical

$$\rho_0* = 1$$
$$\lambda_* = 0$$
$$\tau_* = \tau$$

Where τ is real.

Wilson-Fisher

$$\rho_0* = 1$$
$$\lambda_* = \frac{1}{2}$$
$$\tau_* = \frac{1}{4}$$

A half interval of fixed points can be observed beside the isolated W-F.

Global Flow

The flow equation for *u'* can be solved ANALITICALLY in the large N

$$\frac{\rho-1}{\sqrt{u'}} - \frac{1}{2}\frac{\sqrt{u'}}{1+u'} - \frac{3}{2}\arctan\sqrt{u'} = G(u' e^{2t})$$

$$u' \geq 0$$

$$\partial_t u' = -2u' + \rho u'' - \frac{u''}{(1+u')^2}$$

Global Flow

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$$\frac{\rho-1}{\sqrt{u'}} - \frac{1}{2}\frac{\sqrt{u'}}{1+u'} - \frac{3}{2}\arctan\sqrt{u'} = G(u'e^{2t})$$

$u' \geq 0$

analytic continuation *

$u' \leq 0$

$$\frac{\rho-1}{\sqrt{-u'}} + \frac{1}{2}\frac{\sqrt{-u'}}{1+u'} - \frac{3}{4}\ln\frac{1-\sqrt{-u'}}{1+\sqrt{-u'}} = \bar{G}(u'e^{2t})$$

$$^{*}\frac{1}{i}\arctan ix = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

Here *G*-s on the RHS are depending on the initial conditions

Global Flow

$$\partial_t u' = -2u' + \rho u'' - \frac{u''}{(1+u')^2}$$

$$\frac{\rho-1}{\sqrt{u'}} - \frac{1}{2}\frac{\sqrt{u'}}{1+u'} - \frac{3}{2}\arctan\sqrt{u'} = G(u'e^{2t}) \qquad u' \geq 0$$

$$\frac{\rho-1}{\sqrt{-u'}} + \frac{1}{2}\frac{\sqrt{-u'}}{1+u'} - \frac{3}{4}\ln\frac{1-\sqrt{-u'}}{1+\sqrt{-u'}} = \bar{G}(u'e^{2t}) \qquad u' \leq 0$$

Fixed points are described by scaling solutions

$$G(u'e^{2t}) \to c$$

$$\bar{G}(u'e^{2t}) \to \bar{c}$$

constants

0

Global Flow

$$\partial_t u' = -2u' + \rho u'' - \frac{u''}{(1+u')^2}$$

$$\frac{\rho-1}{\sqrt{u'}} - \frac{1}{2}\frac{\sqrt{u'}}{1+u'} - \frac{3}{2}\arctan\sqrt{u'} = G(u'e^{2t}) \qquad u' \geq 0$$

$$\frac{\rho-1}{\sqrt{-u'}} + \frac{1}{2}\frac{\sqrt{-u'}}{1+u'} - \frac{3}{4}\ln\frac{1-\sqrt{-u'}}{1+\sqrt{-u'}} = \bar{G}(u'e^{2t}) \qquad u' \leq 0$$

Fixed points are described by scaling solutions

$$G(u'e^{2t}) \to c$$

$$\bar{G}(u'e^{2t}) \to \bar{c}$$

constants

0

So the equations to be solved:

$$\frac{\rho-1}{\sqrt{u'}} - \frac{1}{2}\frac{\sqrt{u'}}{1+u'} - \frac{3}{2}\arctan\sqrt{u'} = c$$

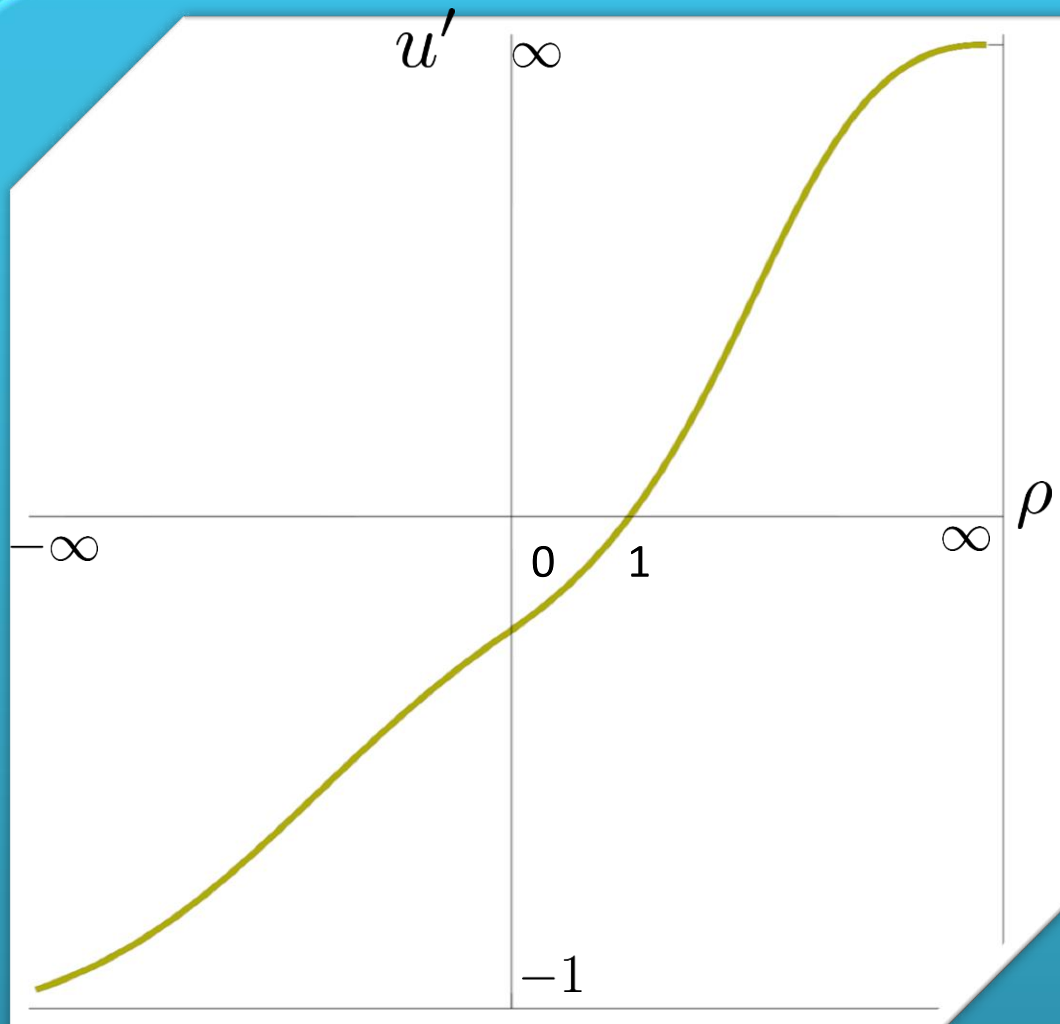
$$\frac{\rho-1}{\sqrt{-u'}} + \frac{1}{2}\frac{\sqrt{-u'}}{1+u'} - \frac{3}{4}\ln\frac{1-\sqrt{-u'}}{1+\sqrt{-u'}} = \bar{c}$$

Turns out $c=-\bar{c}$

Global Flow

Solutions are given in four branches.

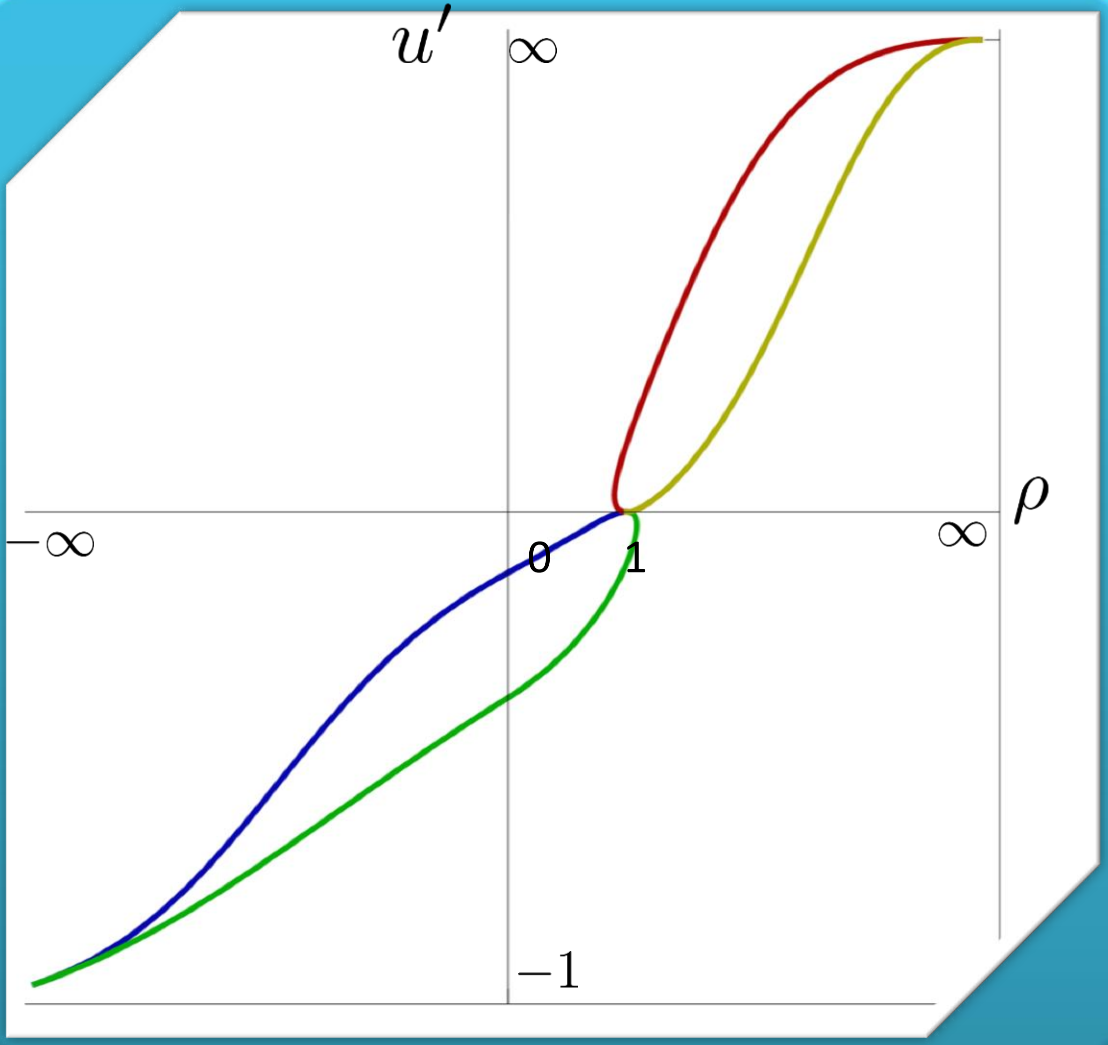
c=0



Global Flow

Solutions are given in four branches.

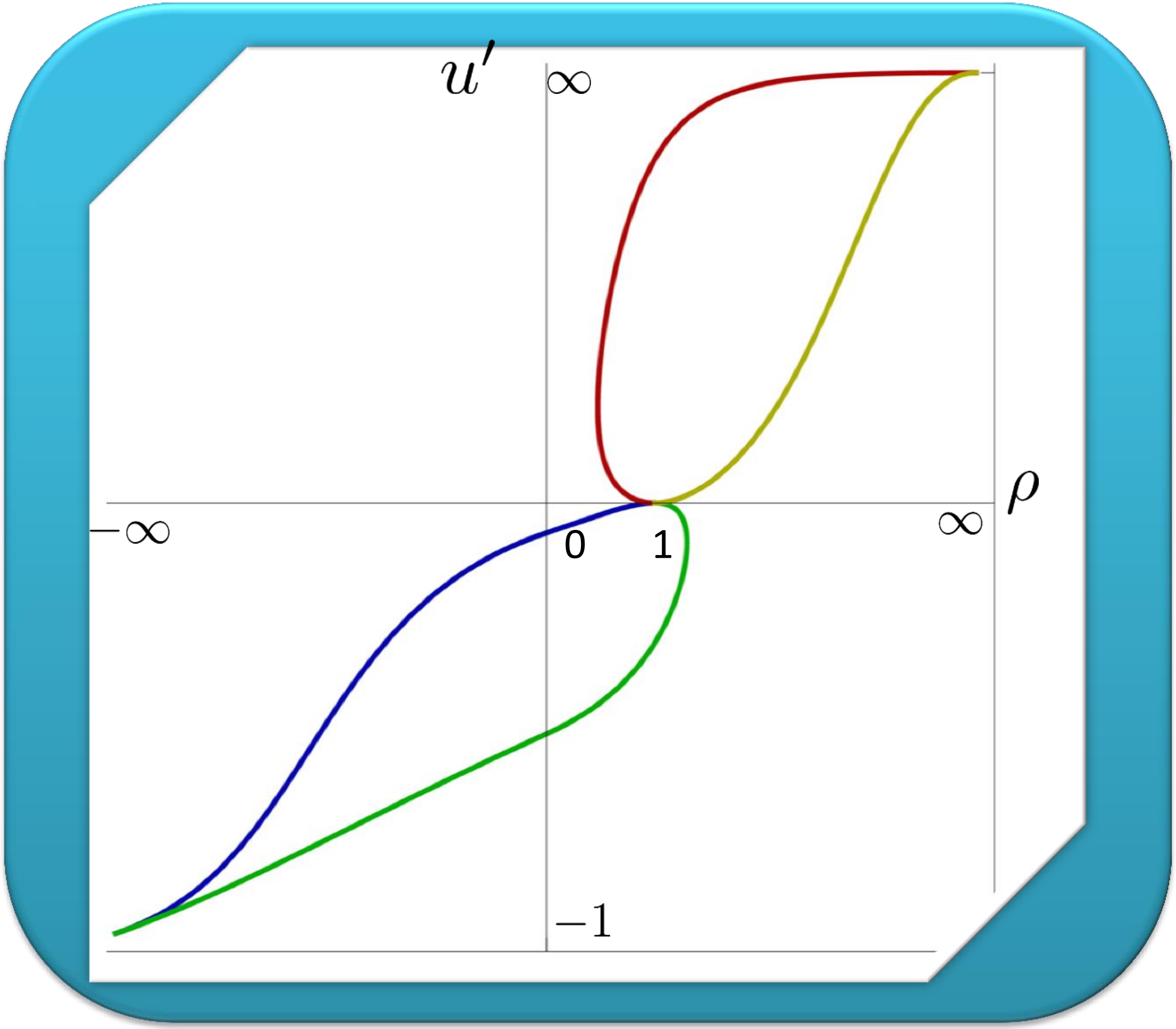
c=1



Global Flow

Solutions are given in four branches.

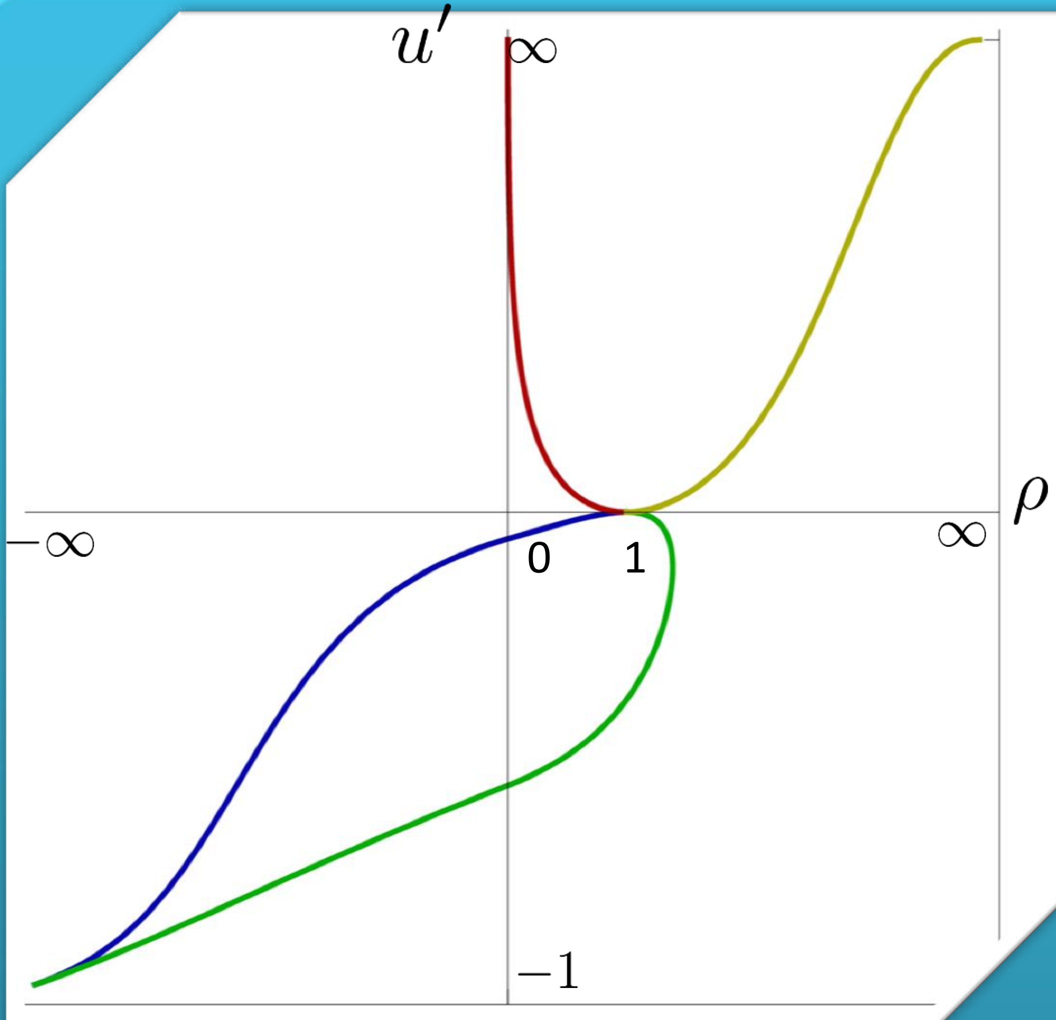
c=2



Global Flow

Solutions are given in four branches.

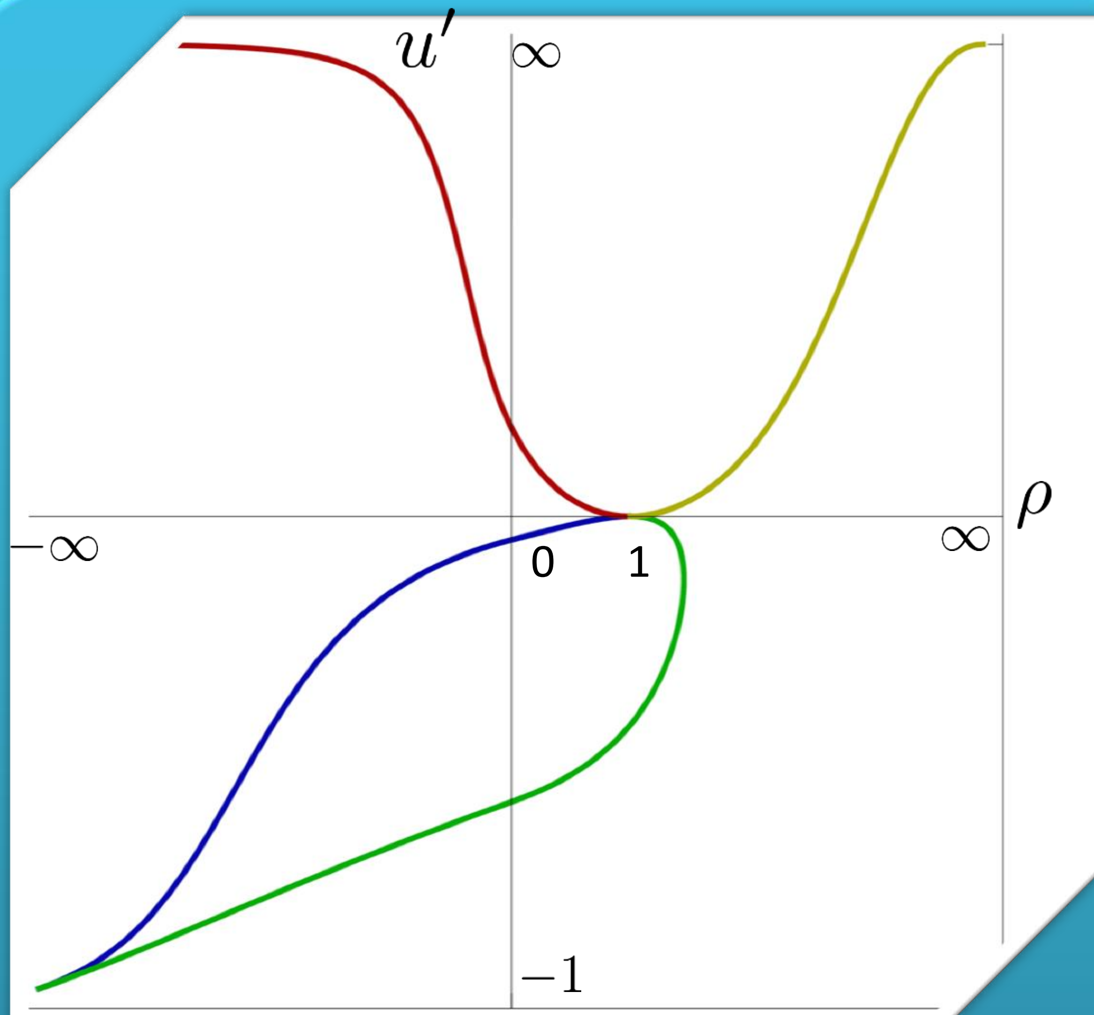
c=3/4 π



Global Flow

Solutions are given in four branches.

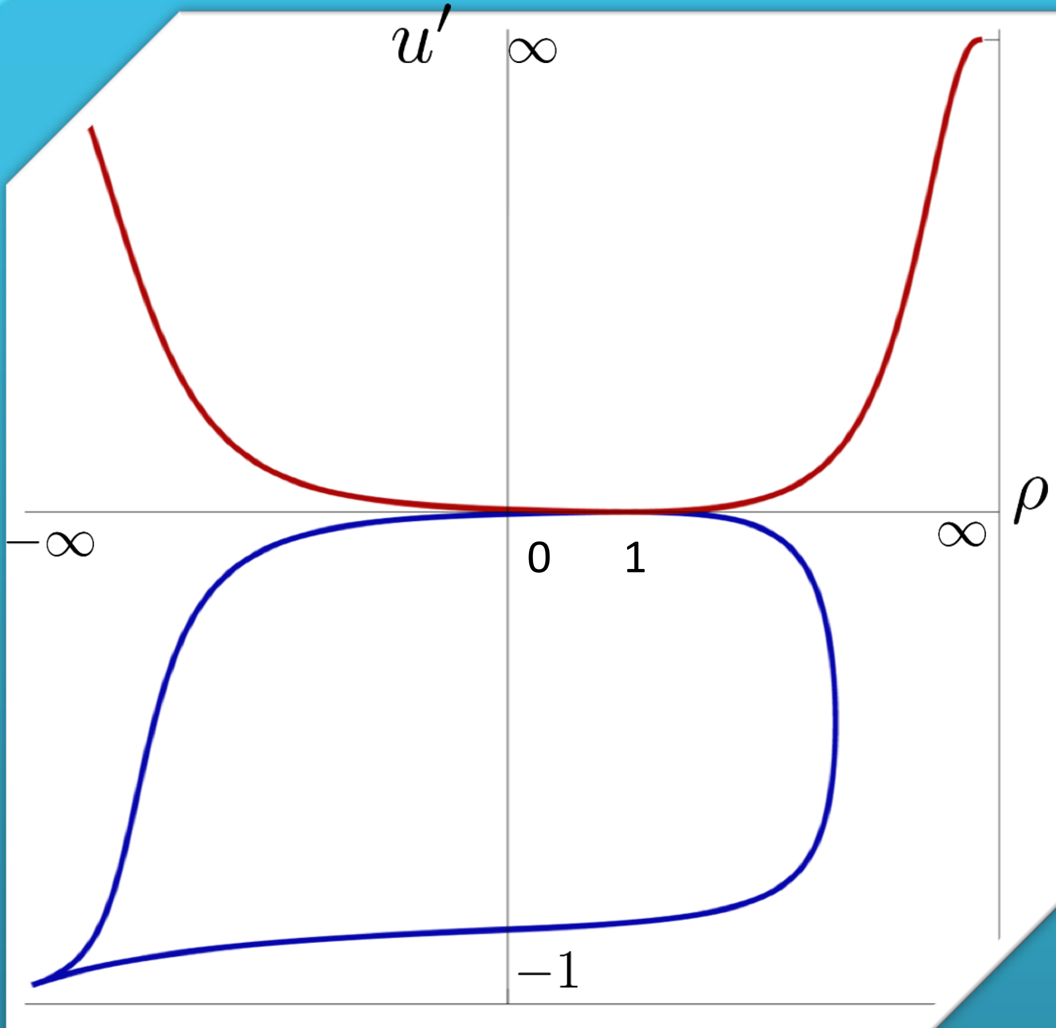
c=2.6



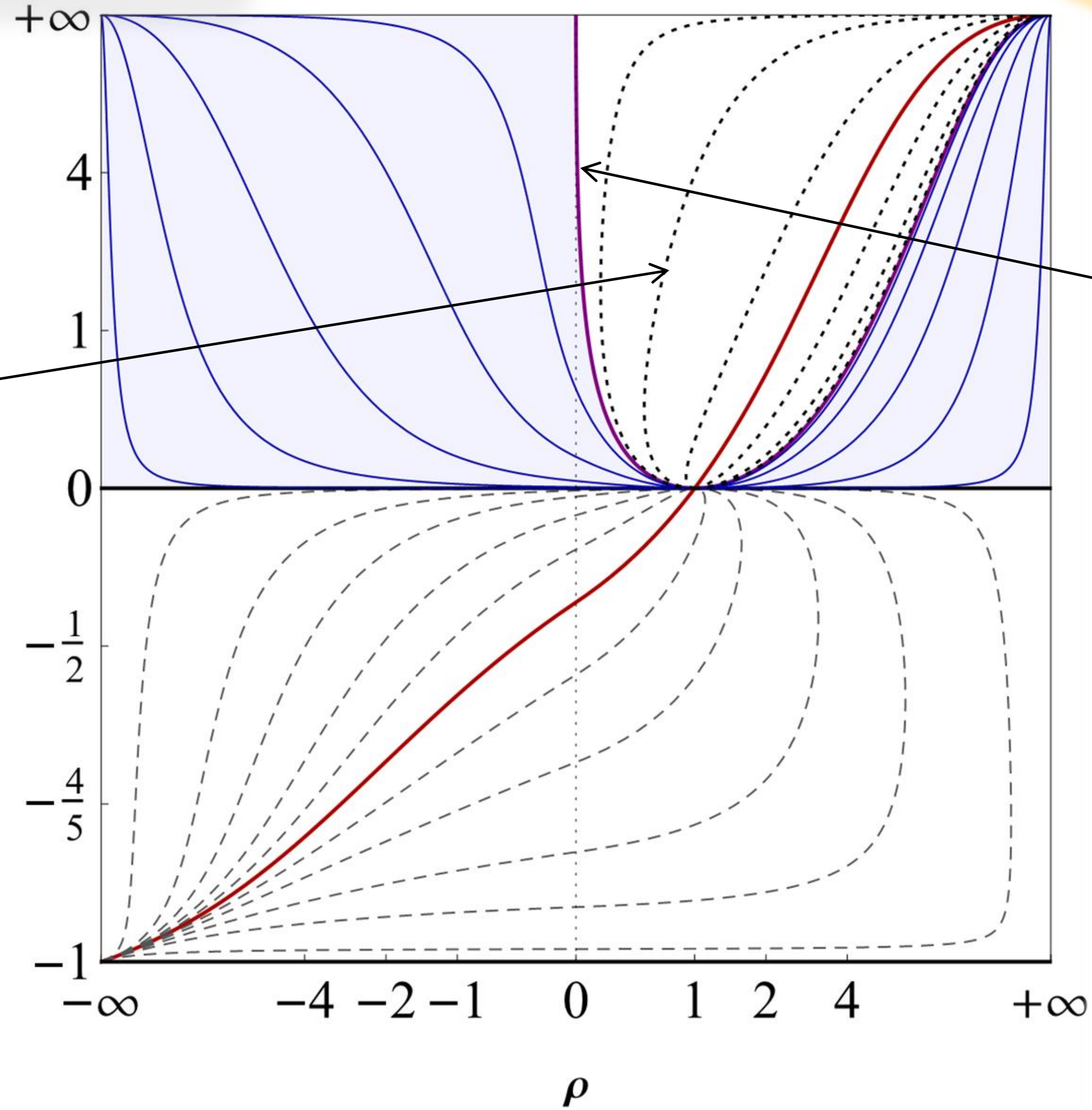
Global Flow

By analytical reasoning one can find the branches connected as:

- u’>0 to u’>0
- u’<0 to u’<0



The structure



Multi-valued functions considered as non-physical

$c_P = 3\pi/4$

The treshold value for the turning point

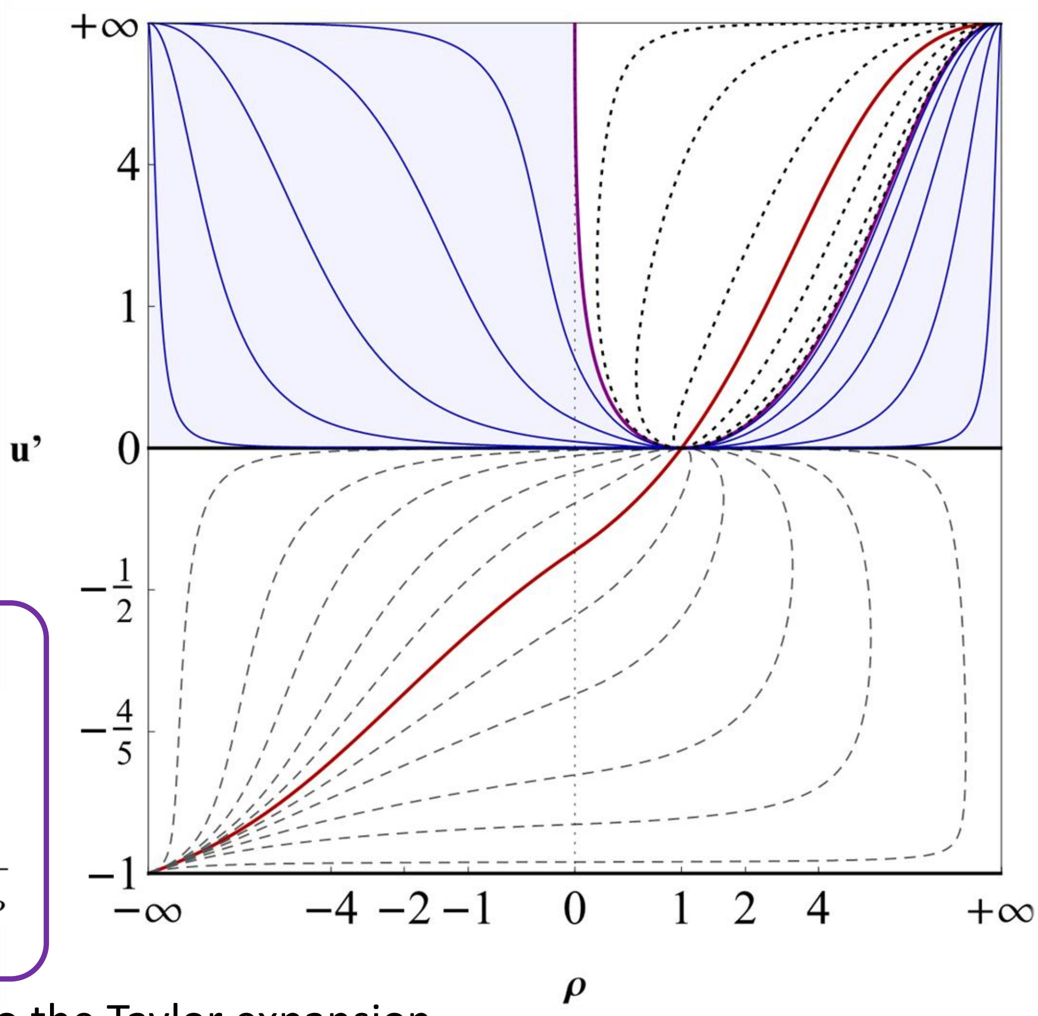
Global VS Local

$$\lambda \equiv u''(\rho_0)$$

$$\tau \equiv u'''(\rho_0)$$

Thus if we tune the VEV to its critical value we can distinguish different type of fixed point solutions

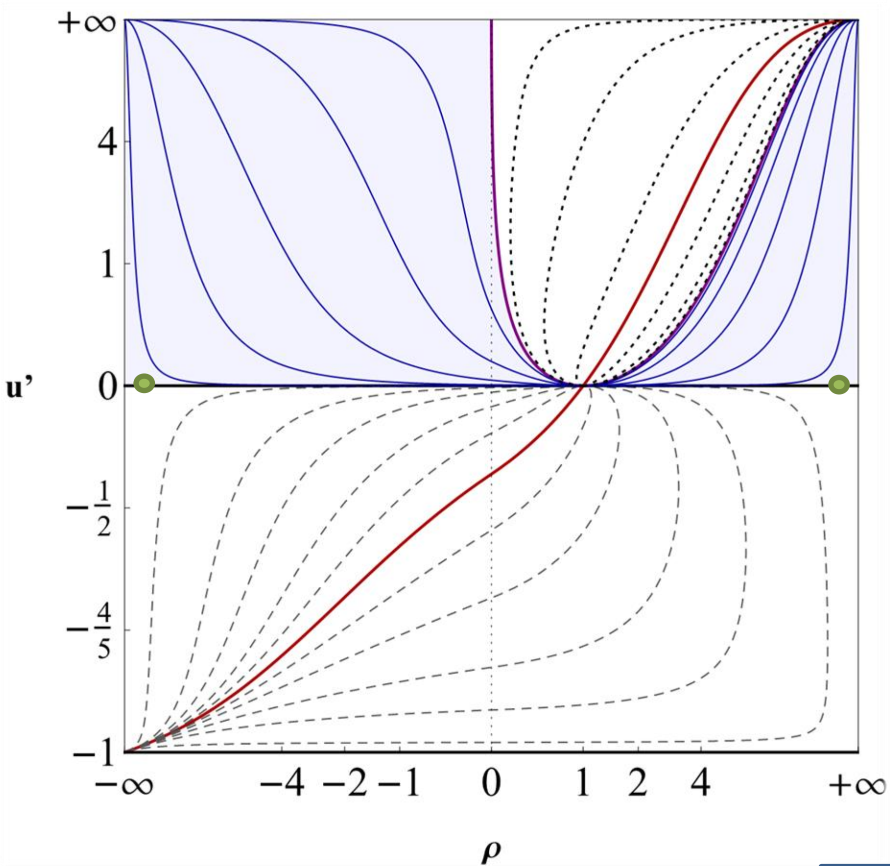
Gauss	Wilson-Fisher	Tricritical	BMB
$\rho_0{*} = 1$	$\rho_0{*} = 1$	$\rho_0{*} = 1$	$\rho_0{*} = 1$
$\lambda_* = 0$	$\lambda_* = \frac{1}{2}$	$\lambda_* = 0$	$\lambda_* = 0$
$\tau_* = 0$	$\tau_* = \frac{1}{4}$	$\tau_* = \frac{2}{c^2}$	$\tau_* = \frac{2}{c_P^2}$



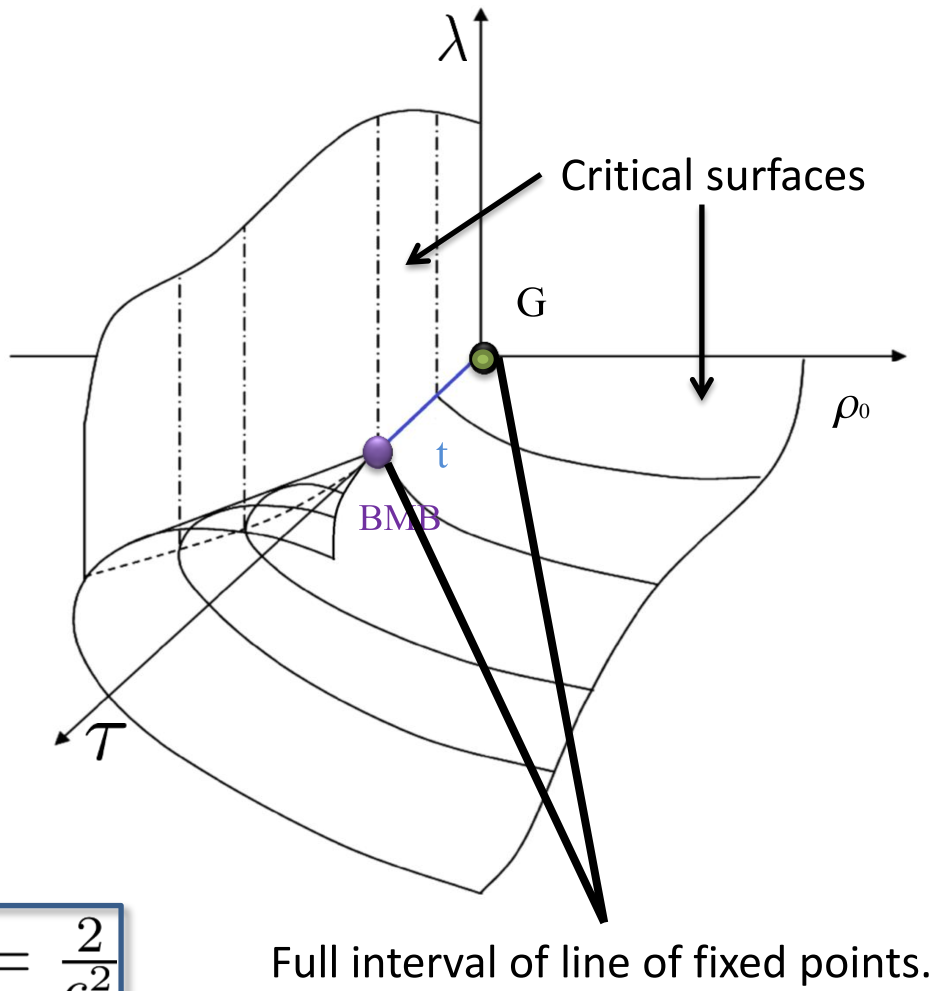
NEW compared to the Taylor expansion

Parameter space

Fixed point solutions



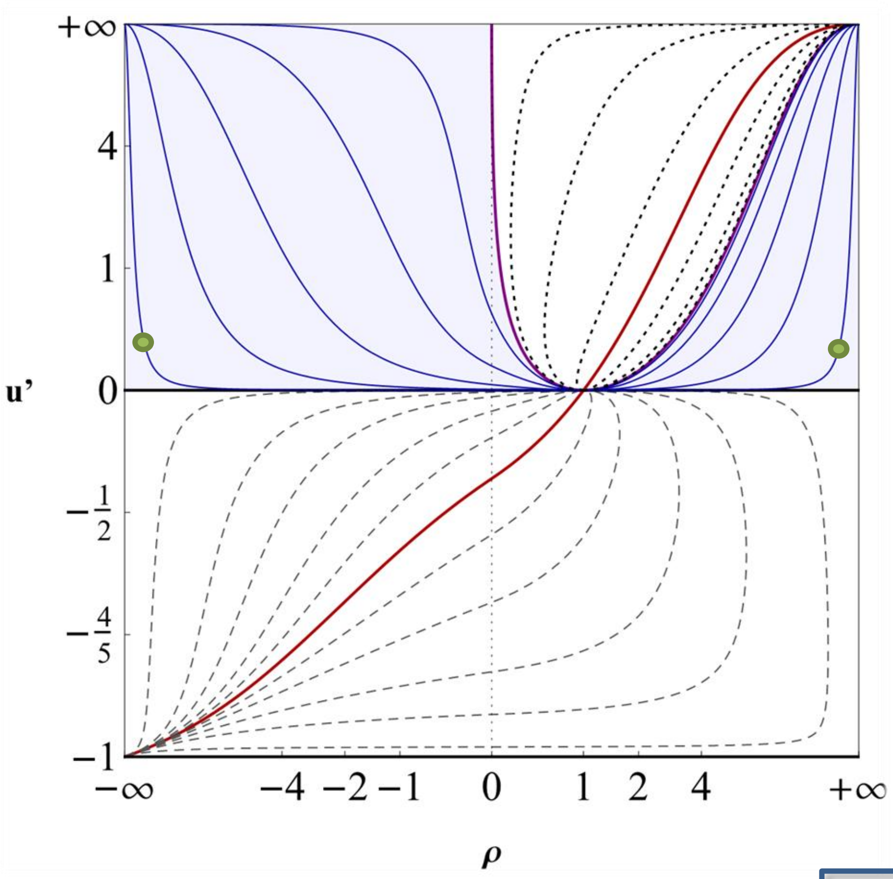
Topology of the parameter space



$$\tau = \frac{2}{c^2}$$

Parameter space

Fixed point solutions



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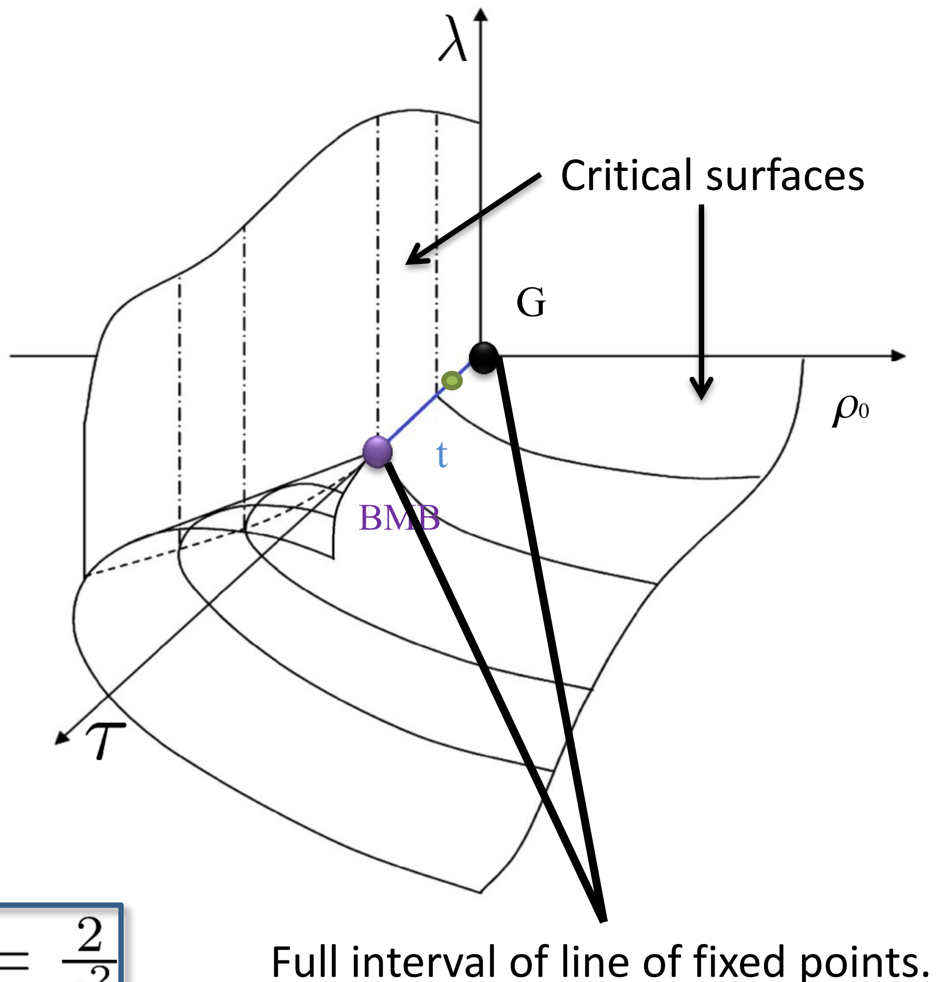

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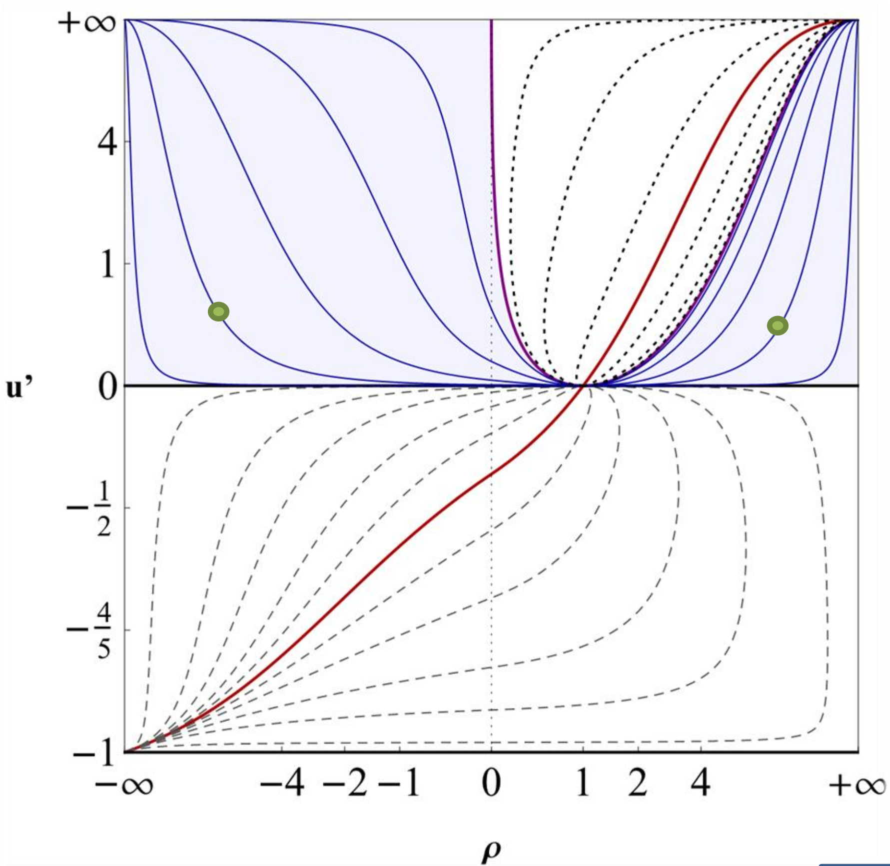
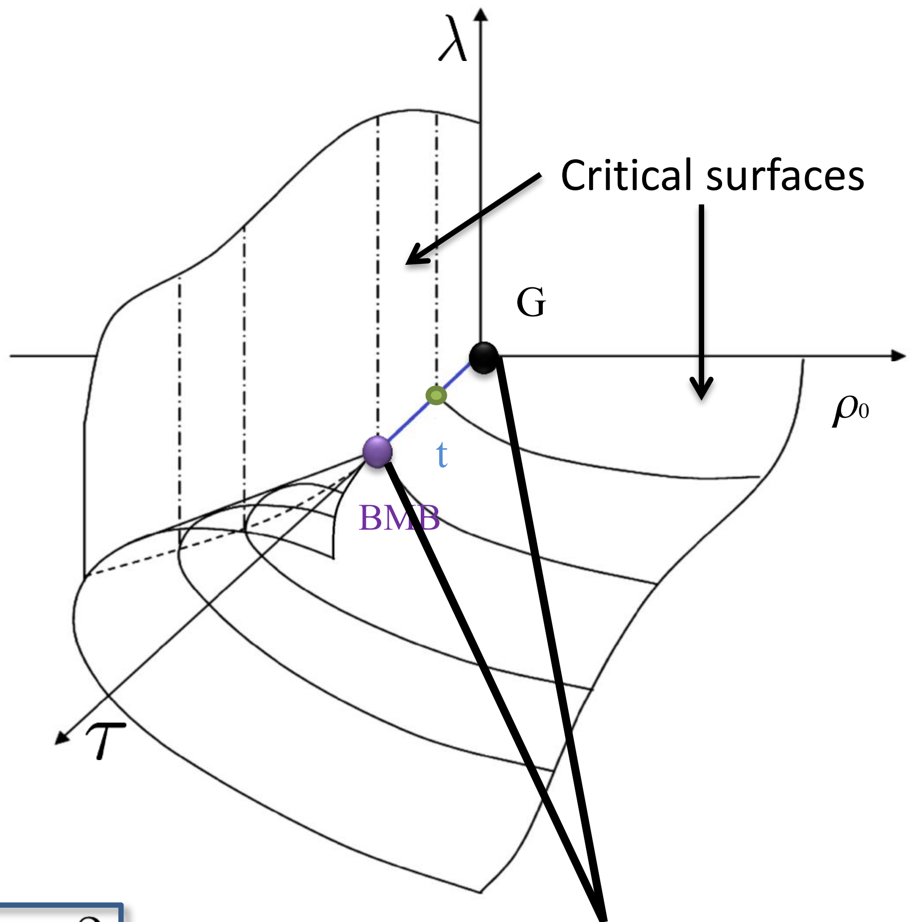
Parameter space

Fixed point solutions



Topology of the parameter space

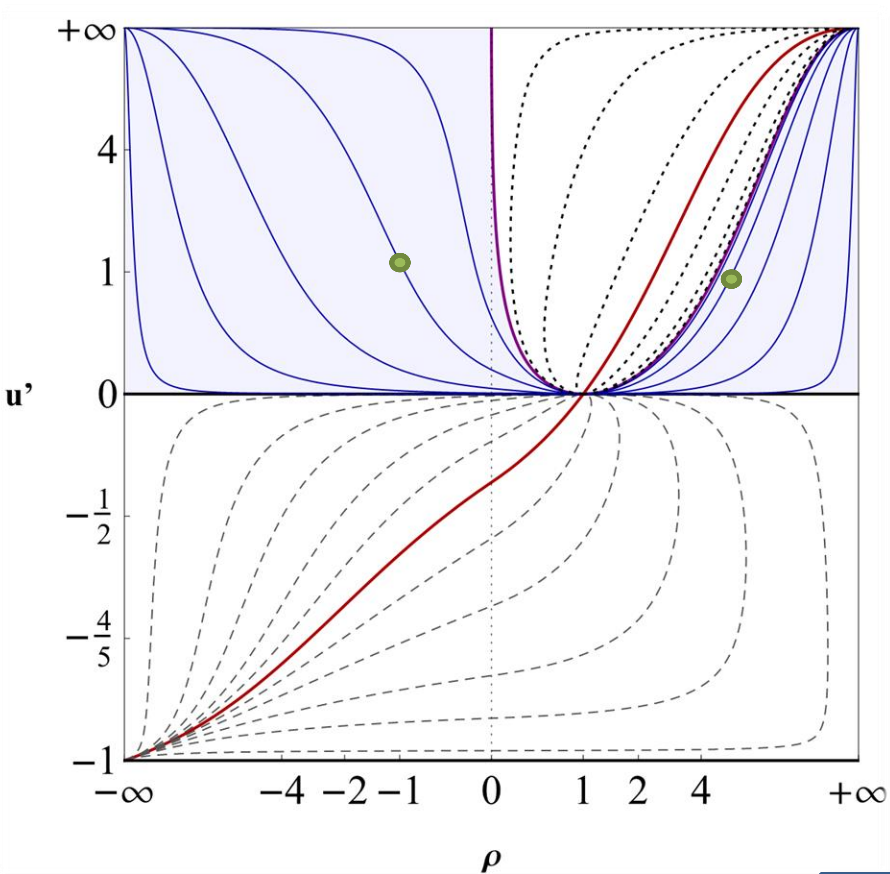


Full interval of line of fixed points.

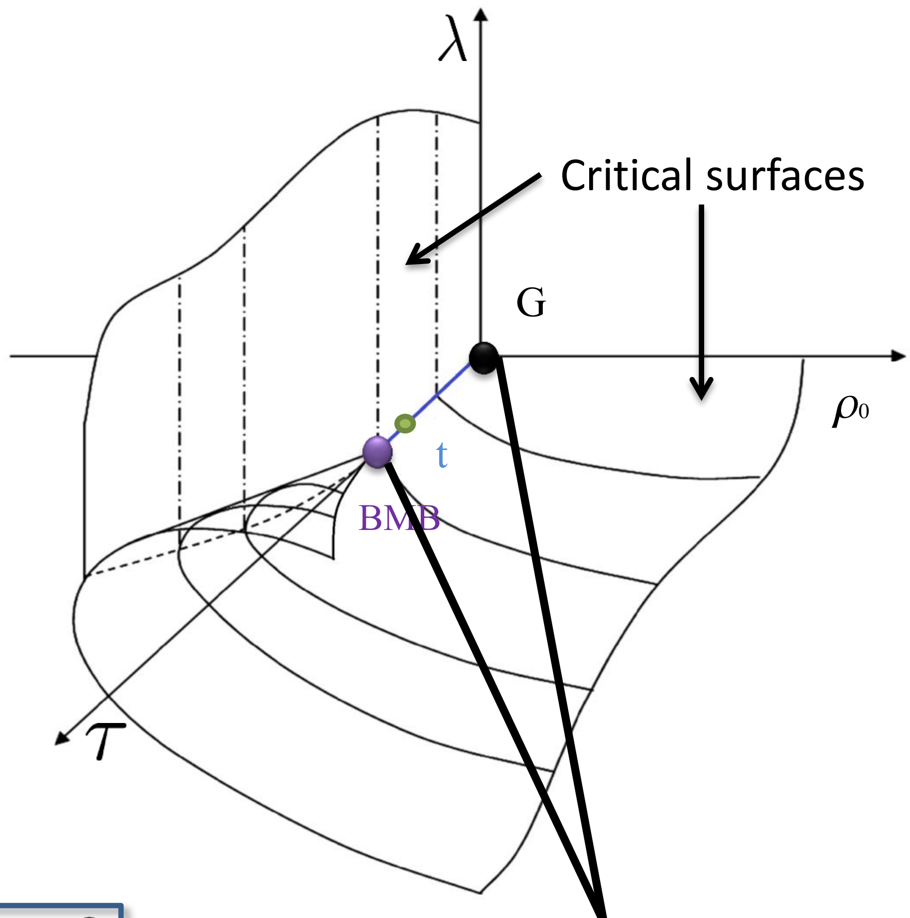
$$\tau = \frac{2}{c^2}$$

Parameter space

Fixed point solutions



Topology of the parameter space

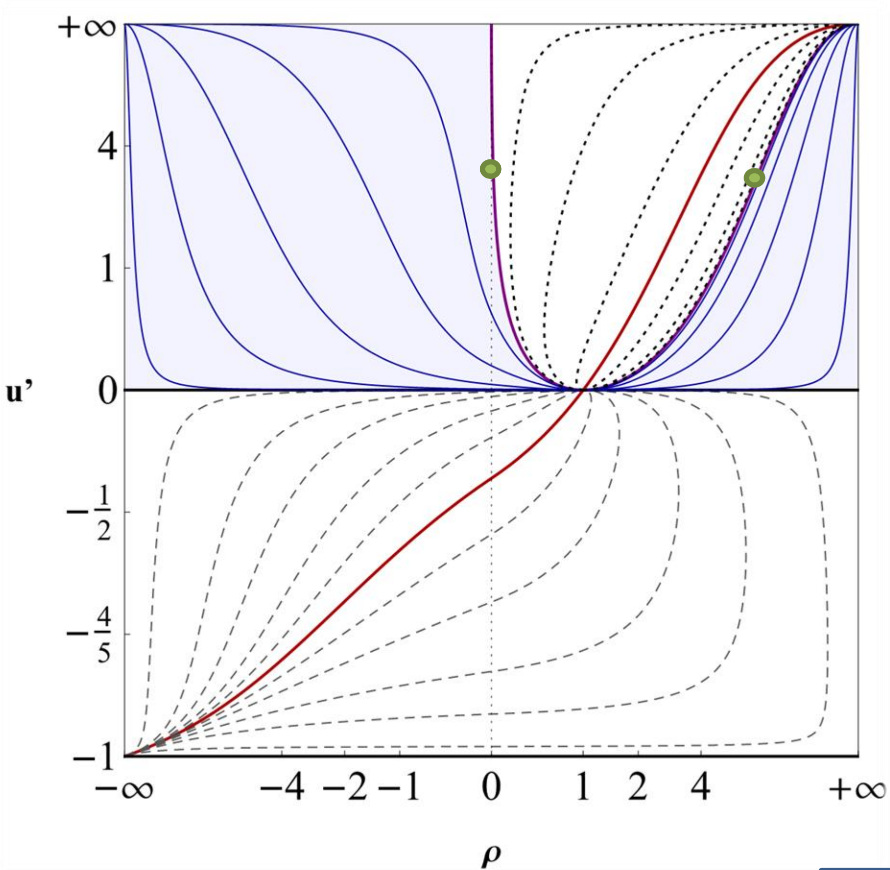


$$\tau = \frac{2}{c^2}$$

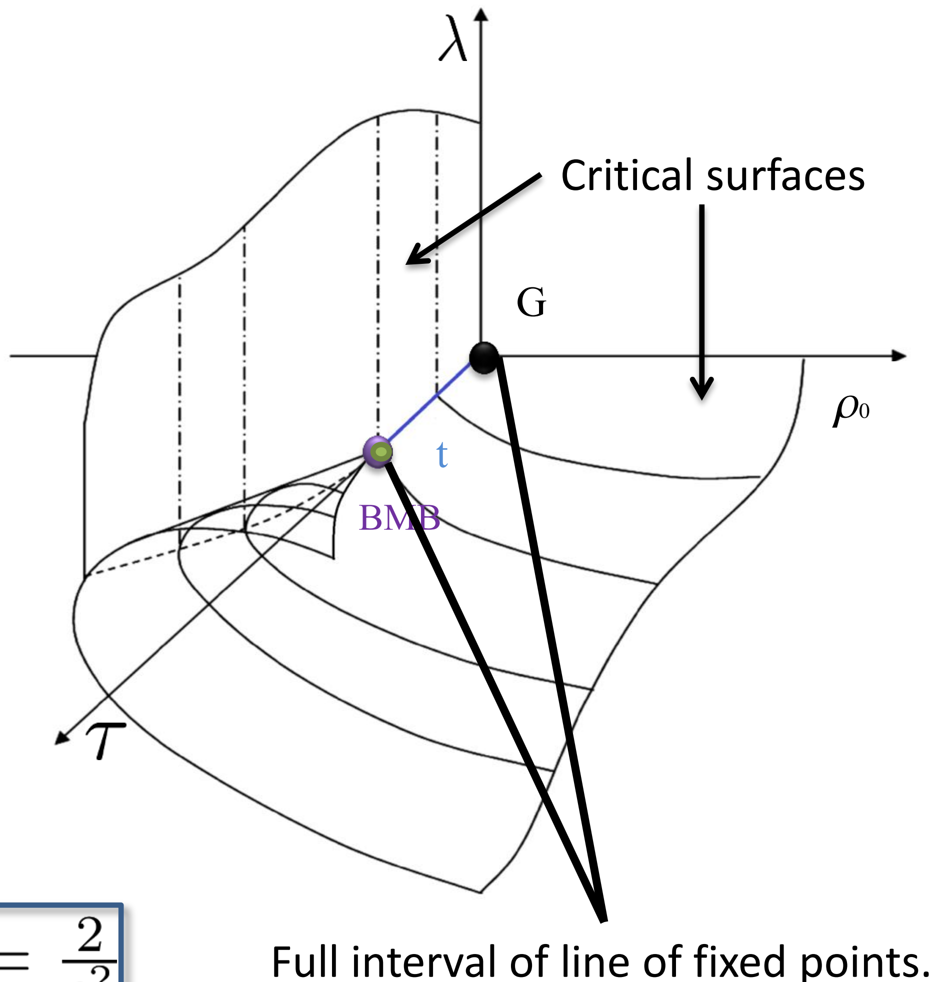
Full interval of line of fixed points.

Parameter space

Fixed point solutions



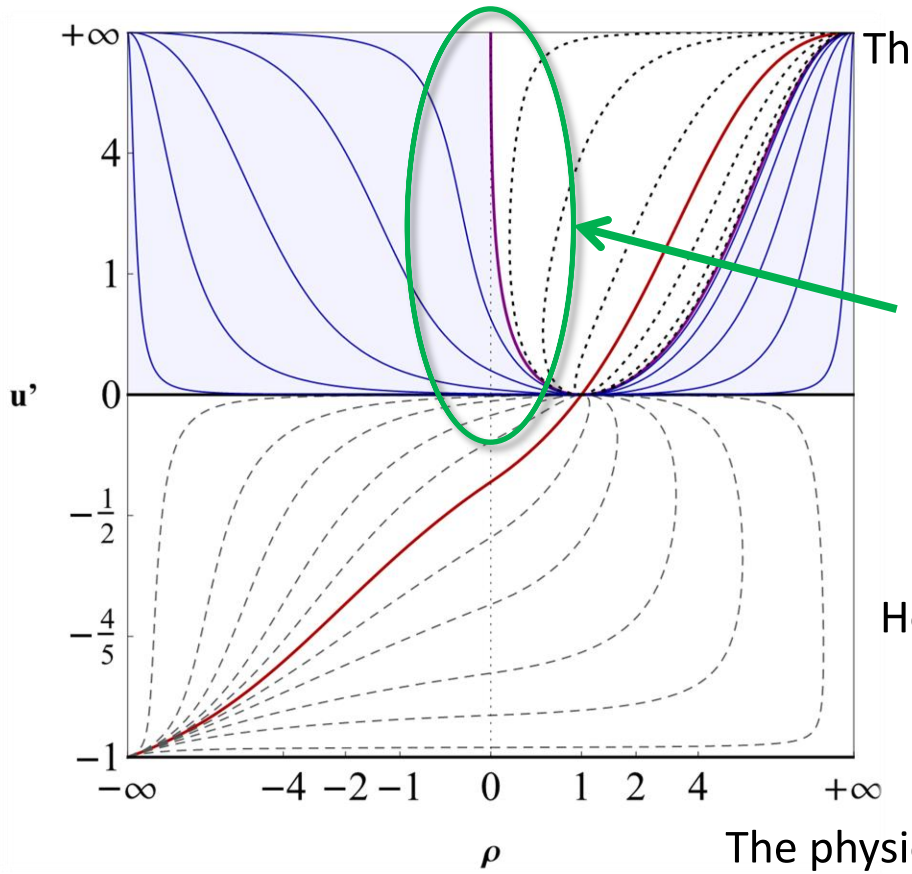
Topology of the parameter space



$$\tau = \frac{2}{c^2}$$

Breaking of scale invariance

The physical mass of the scalar field in the symmetric phase at the fixed point is then given by $m^2 = u'_*(0)\, k^2$



The physical mass vanishes in the limit $k \to 0$

$$m^2 = u'_*(0)\, k^2 \to 0 \qquad \text{(if } u'_*(0) \text{ finite)}$$

However at BMB

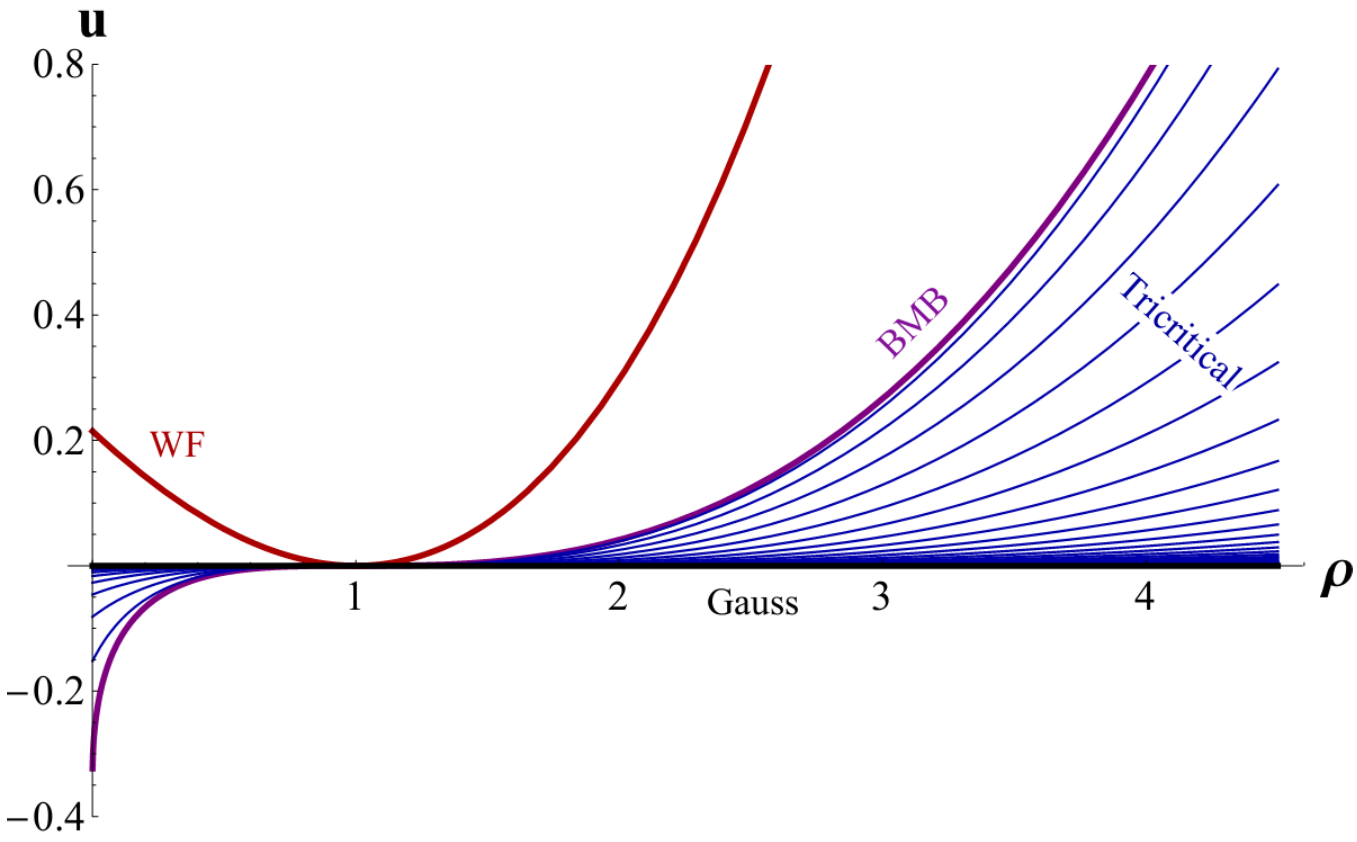
$$u'_* \propto \frac{1}{\sqrt{5}} \frac{1}{\sqrt{\rho}} \quad \text{for} \quad \rho \to 0 \qquad \text{(singular)}$$

Hence the physical mass undetermined for $k \to 0$

The physical mass remains a free parameter of the theory, which leads to breaking of scale invariance.

The Potential

Integrating u' respect to ρ



Conclusion

Non-perturbative solution to a 3d, *O(N)* symmetric quantum field theory theory in the large *N* limit

Study of the fixed point solutions and phase transitions (WF, Tricrit., BMB)

BMB: UV fixed point with breaking of the scale invariance

LITERATURE

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Theory space figure: Frencois David, Bardeen-Moshe-Bander FixedPoint and the Ultraviolet Triviality Phi^3_3

Τηανκ ψου!

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