Fixed points of critical scalar field theories

Edouard Marchais
University of Sussex

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*"Infrared Properties of Scalar Field Theories" - PhD Thesis

General Motivations

- Fixed points are fundamental structures that underpin the efficiency of field-theoretic methods
- A clear determination of them is essential to a deep understanding of physical problem at hand as well as the setting up of efficient and convergent calculations scheme
- We focus here on the fixed point search for O(N) symmetric scalar field theories which cover a large spectrum of physical situations via the effective field theories apporach
- To do so we use the functional renormalisation group technology within the local potential approximation at leading order

Flow equation

- The LPA leads to a single flow equation for the dimensionless effective potential $u(\rho)$

$$\partial_t u = (d-2)\rho u' - du + v_d (N-1)\,\ell(u') + v_d\,\ell(u' + 2\rho u'')$$

with $t = \ln k/\Lambda$, $u = k^{-d} U$ and $\rho = k^{2-d}\phi^2/2$.

- The fluctuation part is controlled by the threshold function

$$\ell(\omega) = -\int_0^\infty dy\, y^{d/2-1}\,\frac{y\,r'(y)}{y[1+r(y)]+\omega}$$

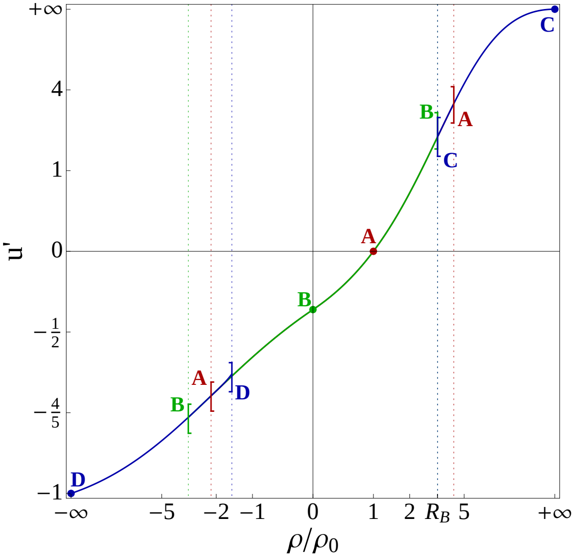
$$r(y) = (1/y - 1)\theta(1-y) \quad \Rightarrow \quad \ell(\omega) \propto \frac{1}{1+\omega}$$

- This equation encapsulated all the physical information about the infrared scaling of the effective potential and is our principal subject of investigation

Fluctuations

- The analytical structure of the scale invariant-potential ($\partial_t u' = 0$) is a key ingredient for the existence of genuine fixed point solution
- When transverse modes are dominating (N large) a complete analytical solution is computable for arbitrary regularization scheme and dimensions
- The large N shows a extremely rich fixed point structure that serves as a landmark for computation at finite N
- Due to the effect of the longitudinal mode we are reduced to local expansions only

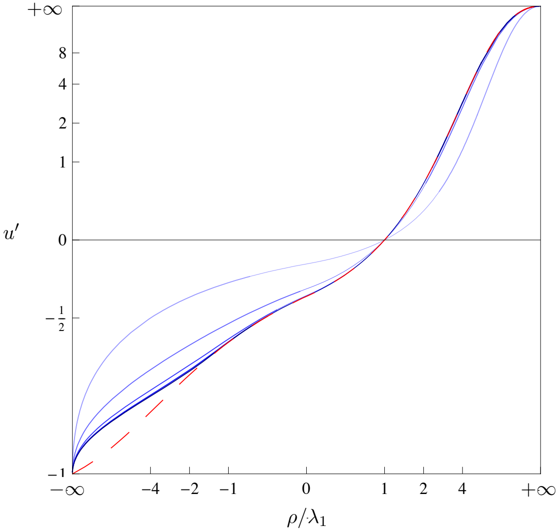
Wilson-Fisher potential reconstruction at large N



Relevant parameters and local solutions

- Local expansions are controlled by convergence limiting poles in the complex plane
- The calculation of expansions about finite field can be systematized to reach several hundreds orders of magnitude
- Expansion usually depends on one or two relevant parameters that controls completely the local solution. A unique value of these parameters corresponds to a fixed point solution
- Using large order expansions one can computed with a reasonable accuracy the critical value for the bare relevant parameter
- Alternatively, numerical integration of the flow at the fixed point can be use to compute this critical value

Critical potential in 3 dimensions for all ρ ($N = 10^n$ for $n = 0, 1, 2, 3, 5, \infty$)



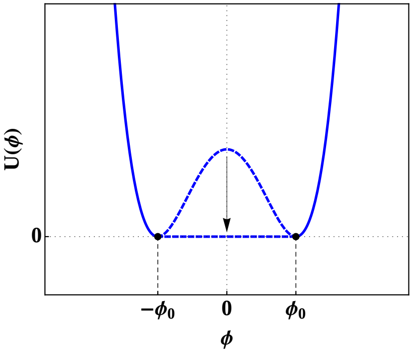
Wilson-Fisher critical exponents

- By linearizing the RG flow around the desired fixed point solution (stability matrix) we can easily computed the critical exponents
- The more precise is the critical value of the relevant parameter, better is the precision on the critical exponents corresponding to a wider parameter space investigation

N	M	ν	ω	ω_2
1	33	0.649561	0.655745	3.18001
10	33	0.9186051	0.871310	2.898458
100	33	0.992424	0.987849	2.986599
1000	31	0.9992492	0.998798	2.998650
10000	31	0.999924	0.999879	2.999865
∞		1	1	3

Convexity

- Expectation: A correct evaluation of the corrections to the microscopic potential should lead to a flattening of the function $U(\phi)$ between its minima



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- The effect of the longitudinal mode does not modify the infrared scaling and therefore the approach to convexity is super-universal. This fixed solution is also apparent in local expansions around vanishing field.

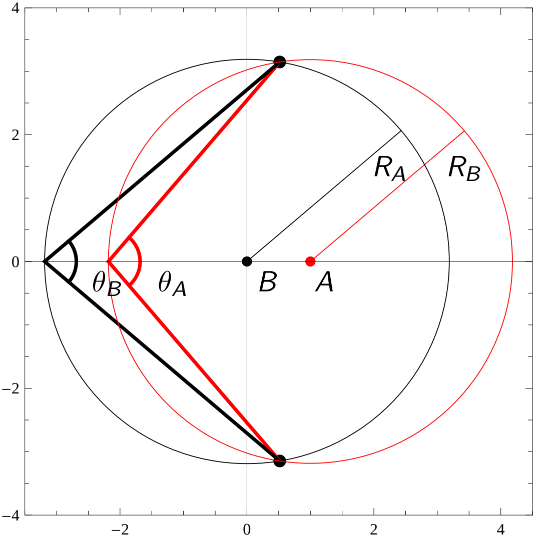
Conclusion and perspectives

- The large N limit gives a complete precise analytical picture of the scaling effective potential
- This information can be used to understand and improve the calculation of the effective potential at finite N
- The scaling potential in higher dimensions can also be studied along similar lines
- Fixed points that only exist for large N, dependence on the regularization scheme, multi-critical models, next to LPA approximation, etc ...

THANK YOU !

Back up

Convergence Radii of exp. A and B of the Wilson-Fisher fixed point solution, and location of the convergence-limiting poles (dots) in the complexified ρ-plane ($R_A = 3.18$, $\theta = 98.7^\circ$ and $R_B = 3.18$, $\theta_B = 80.68^\circ$)



Back up

Radius of convergence for the expansion C of the Wilson-Fisher fixed point solution and estimated location of the convergence-limiting pole (dots) in the complexified $\frac{1}{\rho}$-plane. ($R_C = 3.18...$ and $\theta_C = 80.68^\circ ...$)