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Deconfinement @FRG

- QCD phase transition
- FRG @ deconfinement
- Summary & outlook

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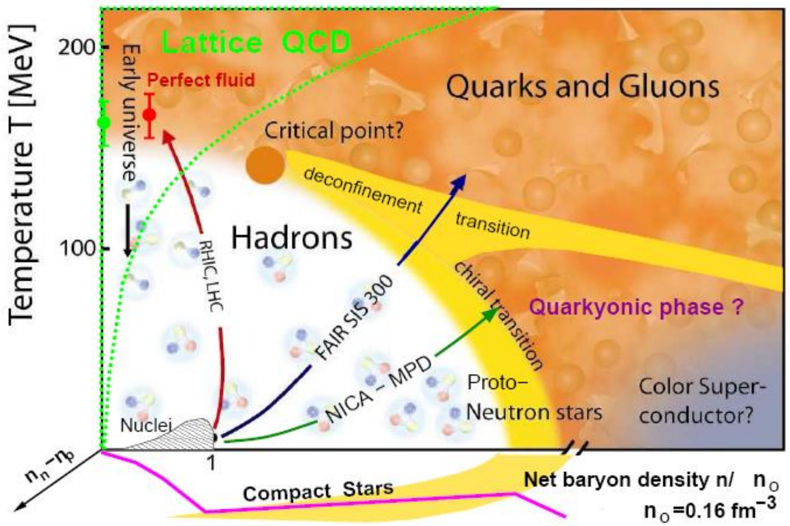
(1) QCD phase transition

QCD Phase Transition

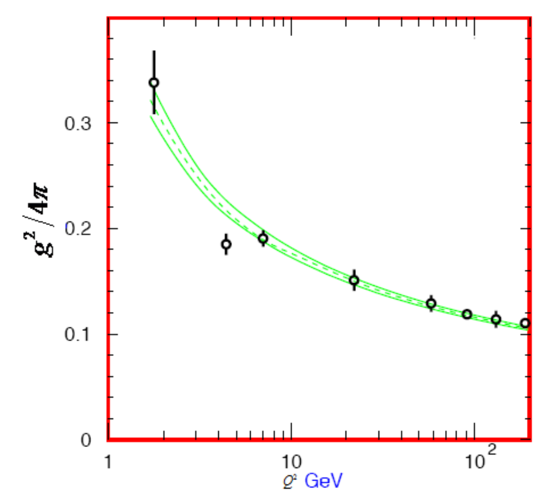


QCD: $$L_{QCD} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_q - g\gamma^\mu A_\mu^a T_a)\psi - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a$$

Symmetry: $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_B$

Chiral, Deconfinement,
Color SC, Pion superfluid

Asymptotic freedom



Partition function: $$Z = \int [d\Phi]\ e^{-S_\Lambda[\Phi, g]}$$

- **Lattice QCD**
- **Perturbation theory,**

- **Mean Field theory,**
- **beyond MF: Gaussian, RPA, HTL, …**

✓ **Wilsonian Renormalization Group method (Nobel Prize 1982)**

FRG (Wetterich 1993, Morris 1994)

non-perturbative flow Eq connecting classical action and effective action

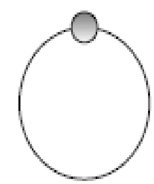
Average action:

$$\Gamma_k[M] - F_k[J] = \int_x JM - \frac{1}{2}\int_q R_{k,q} M_q M_{-q} M_x = -\frac{\delta F_k}{\delta J_x} = \langle \phi_x \rangle$$

IR cutoff

Wetterich Eq:

$$\partial_k \Gamma_k[M] = \frac{1}{2}\mathrm{Tr}\left(\partial_k R_k \left[\Gamma_k^{(2)} + R_k \right]^{-1}\right),$$

diagrammatic form $\partial_k \Gamma_k = \frac{1}{2}$  ,

$\Gamma_\Lambda[M] = S[\phi = M]$ **(classical action),**

$\Gamma_0[M] = \Gamma[M]$ **(effective action).**

(2) Deconfinement @FRG

Friedberg-Lee Model

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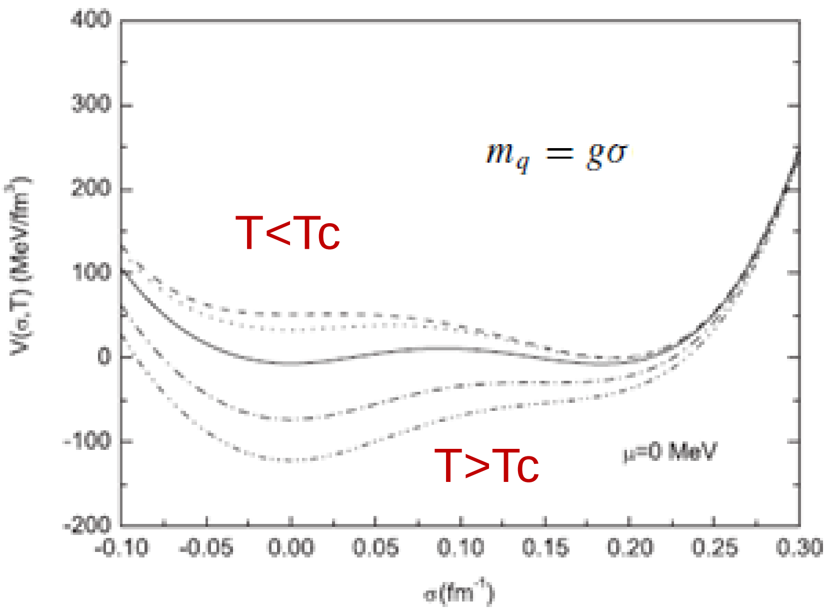
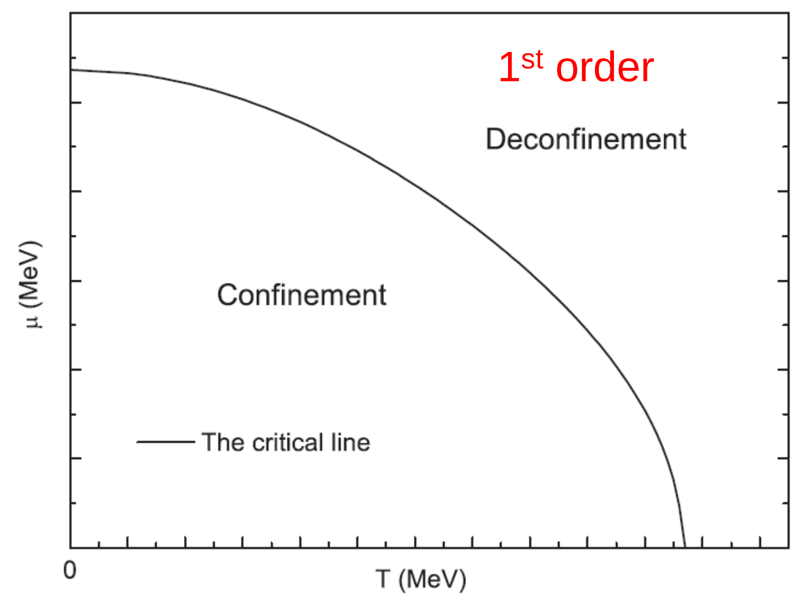
$$\mathcal{L} = \overline{\psi}(i\partial - g\sigma)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma), \quad U(\sigma) = \frac{a}{2!}\sigma^2 + \frac{b}{3!}\sigma^3 + \frac{c}{4!}\sigma^4 + B,$$

$m_\sigma \approx 2GeV$

$$a = 17.70\,\mathrm{fm}^{-2}, b = -1457.4\,\mathrm{fm}^{-1}, c = 20000, g = 12.$$





Tc=103.7MeV; μ_c=263.4MeV.

Mean field calculation @ Mao, Yao, Zhao, PRC 2008

- FRG for 1st order phase transition ?
- Fluctuation effects on T_c, μ_c ?
- Universal behavior ?

FL@FRG

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FL model: $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - g\sigma)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) \qquad U(\sigma) = \frac{a}{2!}\sigma^2 + \frac{b}{3!}\sigma^3 + \frac{c}{4!}\sigma^4$

Wetterich Eq: $\partial_k\Gamma_k[\sigma,\psi] = \frac{1}{2}\mathrm{Tr}\left([\Gamma_k^{(2,0)}[\sigma,\psi] + R_{kB}]^{-1}\frac{\partial R_{kB}}{\partial k}\right) - \mathrm{Tr}\left([\Gamma_k^{(0,2)}[\sigma,\psi] + R_{kF}]^{-1}\frac{\partial R_{kF}}{\partial k}\right)$

Vacuum IR cutoff: $R_{B,k} = (k^2 - q^2)\theta(k^2 - q^2),\; R_{F,k} = q\!\!\!/\left(\sqrt{\frac{k^2}{q^2}} - 1\right)\theta(k^2 - q^2).$

T, μ IR cutoff: $R_{kF} = (q\!\!\!/ + i\mu\gamma^0)\left(\sqrt{\frac{(q_0+i\mu)^2+k^2}{(q_0+i\mu)^2+\mathbf{q}^2}} - 1\Theta(k^2 - \mathbf{q}^2)\right) \quad R_{kB} = (k^2 - \mathbf{q}^2)\Theta(k^2 - \mathbf{q}^2)$

1. Derivative expansion:

$$\Gamma_k = \int d^4x\left(Z_{\psi,k}\bar{\psi}(i\partial\!\!\!/ - g\sigma)\psi + \frac{1}{2}Z_{\sigma,k}\partial_\mu\sigma\partial^\mu\sigma + U_k(\sigma)\right)$$

2. Assuming unitary field , and neglecting Z_k, g_k :

$$\partial_k\Gamma_k = \partial_k U_k = \partial_k U_{kF} + \partial_k U_{kB}$$

$$\partial_k U_{kF} = -4N_f N_c T\sum_n \frac{k^4}{6\pi^2}\frac{1}{\omega_n^2 + k^2 + g^2\sigma^2} = -4N_f N_c\frac{k^4}{6\pi^2}\left(\frac{1}{2E_f} - \frac{n_f(E_f+\mu) + n_f(E_f-\mu)}{2E_f}\right)$$

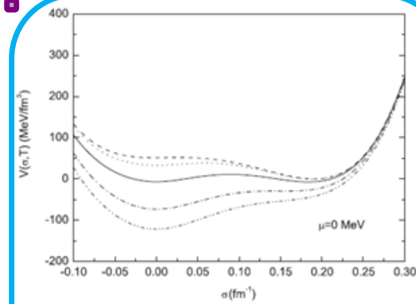
$$\partial_k U_{kB} = T\sum_m \frac{k^4}{6\pi^2}\frac{1}{\omega_m^2 + k^2 + \frac{\partial^2 U_k}{\partial\sigma^2}} = \frac{k^4}{6\pi^2}\left(\frac{1}{2E_B} + \frac{n_B(E_B)}{E_B}\right)$$

$$E_f = \sqrt{k^2 + g^2\sigma^2};\; E_B = \sqrt{\frac{\partial^2 U_k}{\partial\sigma^2} + k^2}$$

3. Solving flow Eq.: grid, potential expansion

Solving flow Equation

1. Two points expansion method for 1st order phase transition:



$m_\sigma \approx 2GeV$

$$U_k(\sigma) = \frac{a_k}{2!}\sigma^2 + \frac{b_k}{3!}\sigma^3 + \frac{c_k}{4!}\sigma^4 + B_k$$

Local min: $\sigma=0$, $\sigma=\sigma_{k,v}$, $m_\sigma^2 = \frac{\partial^2 U_k}{\partial \sigma^2} \geq 0$。 Local max: $\sigma=\sigma_{\max}, (0<\sigma_{\max}<\sigma_{k,v}), \frac{\partial^2 U_k}{\partial \sigma^2} \leq 0$

$$-\delta < \sigma < \delta,\ \sigma_{k,v} - \delta < \sigma < \sigma_{k,v} + \delta$$

Small parameter σ

Flow eqs: $\partial_k a_k = f_a; \partial_k b_k = f_b;$

$\partial_k c_k = f_c; \partial_k B_k = f_{B.}$

$U_0(0)$

Small parameter $\chi_k = \sigma - \sigma_{k,v}$

Flow eqs: $\partial_k a_k = h_a; \partial_k b_k = h_b;$

$\partial_k c_k = h_c; \partial_k B_k = h_{B.}$

$U_0(\sigma_{0,v})$

2. Grid calculation: $U_k(\sigma) = f(\sigma)$ $\quad -\delta\sigma < \sigma < \delta\sigma,\ \sigma_v - \delta\sigma < \sigma < \sigma_v + \delta\sigma$

discretization $\sigma_i = s(i+0.1), s = 0.01$

derive U" with 7-point formula, boundaries with 3-point formula;

Solve $\partial_k U_k$ with 4-order Runge-Kutta method --- Fukushima, 1010.6226

Solving flow Eq. in vacuum

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - g\sigma)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma),$$

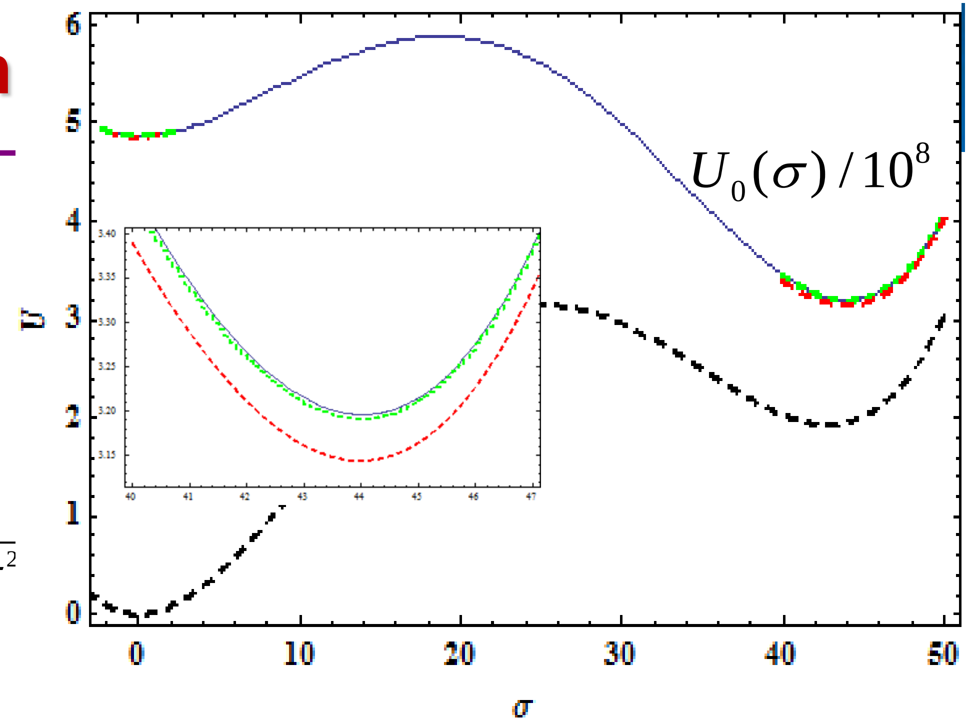
$$\partial_k U_k = \partial_k U_{kF} + \partial_k U_{kB};$$

$$\partial_k U_{kB} = \frac{k^5}{32\pi^2}\cdot\frac{1}{k^2+U_k''};\partial_k U_{kF} = -8N_c\frac{k^5}{32\pi^2}\cdot\frac{1}{k^2+g^2\sigma^2}$$

$$U_\Lambda(\sigma) = \frac{a_\Lambda}{2!}\sigma^2 + \frac{b_\Lambda}{3!}\sigma^3 + \frac{c_\Lambda}{4!}\sigma^4 + B_\Lambda$$

$$R_{B,k} = (\mathrm{k}^2-\mathrm{q}^2)\theta(\mathrm{k}^2-\mathrm{q}^2),$$

$$R_{F,k} = q\!\!\!/\left(\sqrt{\frac{k^2}{q^2}}-1\right)\theta(\mathrm{k}^2-\mathrm{q}^2).$$



Green line----Grid;
Red line------2-point expansion;
Blue line-----without scalar field ;
Black line----initial value.

Λ=400 MeV, $g=12.416$, $a_\Lambda = 4.22*10^6\,\mathrm{MeV}^2$,
$b_\Lambda = -532845.58\,MeV$, $c_\Lambda = 23533.07$, $B_\Lambda = 0$.
$m_\sigma(\sigma=0) = 2.05\,\mathrm{GeV}$; $m_\sigma(\sigma=\sigma_v) = 1.93\,\mathrm{GeV}$; $m_\sigma > \Lambda$.

- Two points expansion method is OK.
- Scalar's contribution is negligible, and so for Z_k, g_k

Deconfinement Phase transition

1. Determine initial coupling constants a_Λ , b_Λ , c_Λ
2. Obtain phase transition: T_c, μ_c

Λ (MeV)	T_c (MeV)	μ_c (MeV)	L_t (fm^{-4})	L_μ (fm^{-4})
1500	103.8	263.5	0.59	0.64
1000	104	263.5	0.58	0.64
800	105	263.5	0.55	0.64
600	109.7	263.5	0.47	0.64
500	117.5	263.5	0.40	0.64
400	141.3	263.5	0.26	0.64
300	No phase	transition	in medium	

MF approximation
T_c=103.7MeV;
μ_c=263.4MeV.

1st deconfinement phase transition depends on Λ.

Large Λ, FRG results becomes saturated.

Small Λ, FRG predicts no phase transition.

Λ=??
Λ>300,
$\Lambda > m_q$,
$\Lambda ? m_\sigma$

(3) Flow diagram

Flow diagram(1)

Dimensional reduction:

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - g\sigma)\psi + \frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma - U(\sigma),$$

$$\partial_k U_k = \partial_k U_{kF} + \partial_k U_{kB}; \qquad \partial_k U_{kB} = \frac{k^4}{6\pi^2}\cdot\frac{1}{k^2+U_k''}; \partial_k U_{kF} = -8N_c\frac{k^4}{6\pi^2}\cdot\frac{1}{k^2+g^2\sigma^2};$$

2nd phase transition

$$U_k = \frac{a_k}{2!}\sigma^2 + \frac{c_k}{4!}\sigma^4 \qquad \partial_k a_k = f_a; \partial_k c_k = f_c; \partial_k g_k = f_g \equiv 0;$$

Dimensionless couplings

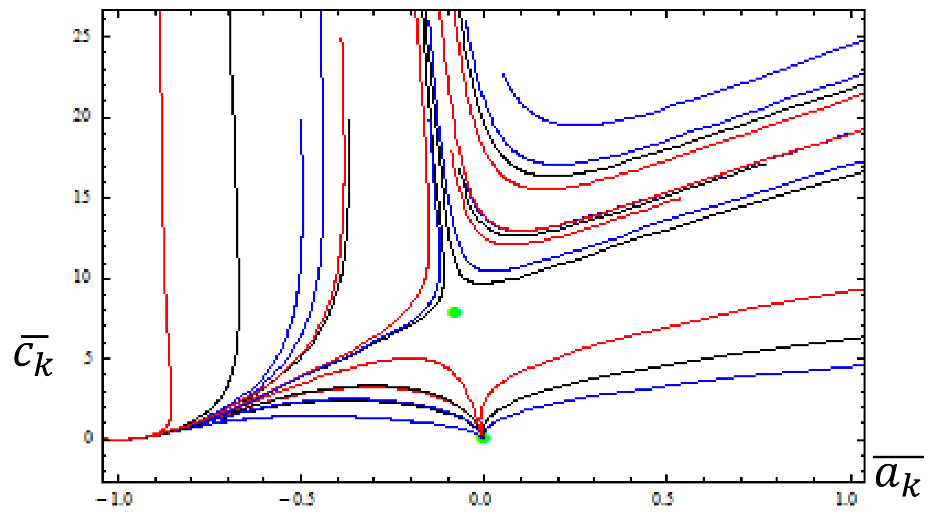
$$\overline{a_k} = \frac{a_k}{k^2}; \overline{c_k} = \frac{c_k}{k}; \quad \overline{g_k} = \frac{g_k}{k^{1/2}};$$

Fixed points: $\overline{g_k} = 0$; (-0.08, 7.76); (0,0)

Critical exponent:

$$\partial_t \begin{pmatrix} \bar{a} - \bar{a}_* \\ \bar{c} - \bar{c}_* \end{pmatrix} = M \begin{pmatrix} \bar{a} - \bar{a}_* \\ \bar{c} - \bar{c}_* \end{pmatrix}$$

$$M = \begin{pmatrix} -2 + \frac{c}{3(1+a)^3\pi^2} & -\frac{1}{6(1+a)^2\pi^2} \\ -\frac{3c^2}{(1+a)^4\pi^2} & -1 + \frac{2c}{(1+a)^3\pi^2} \end{pmatrix} \cong \begin{pmatrix} -1.8426 & 0 \\ 0 & 1.1759 \end{pmatrix}$$



$\nu = 0.5427$

large $\overline{g_k}$ $\Leftrightarrow$ $\overline{g_k} = 0$

Flow diagram(2)

1st phase transition $U_k = \frac{a_k}{2!}\sigma^2 + \frac{b_k}{3!}\sigma^3 + \frac{c_k}{4!}\sigma^4;$

4 fixed points: $\overline{g_k} = 0;$ large $\overline{g_k} \Leftrightarrow \overline{g_k} = 0$

(0,0,0); (-0.08, 0, 7.76);

2nd Phase transition

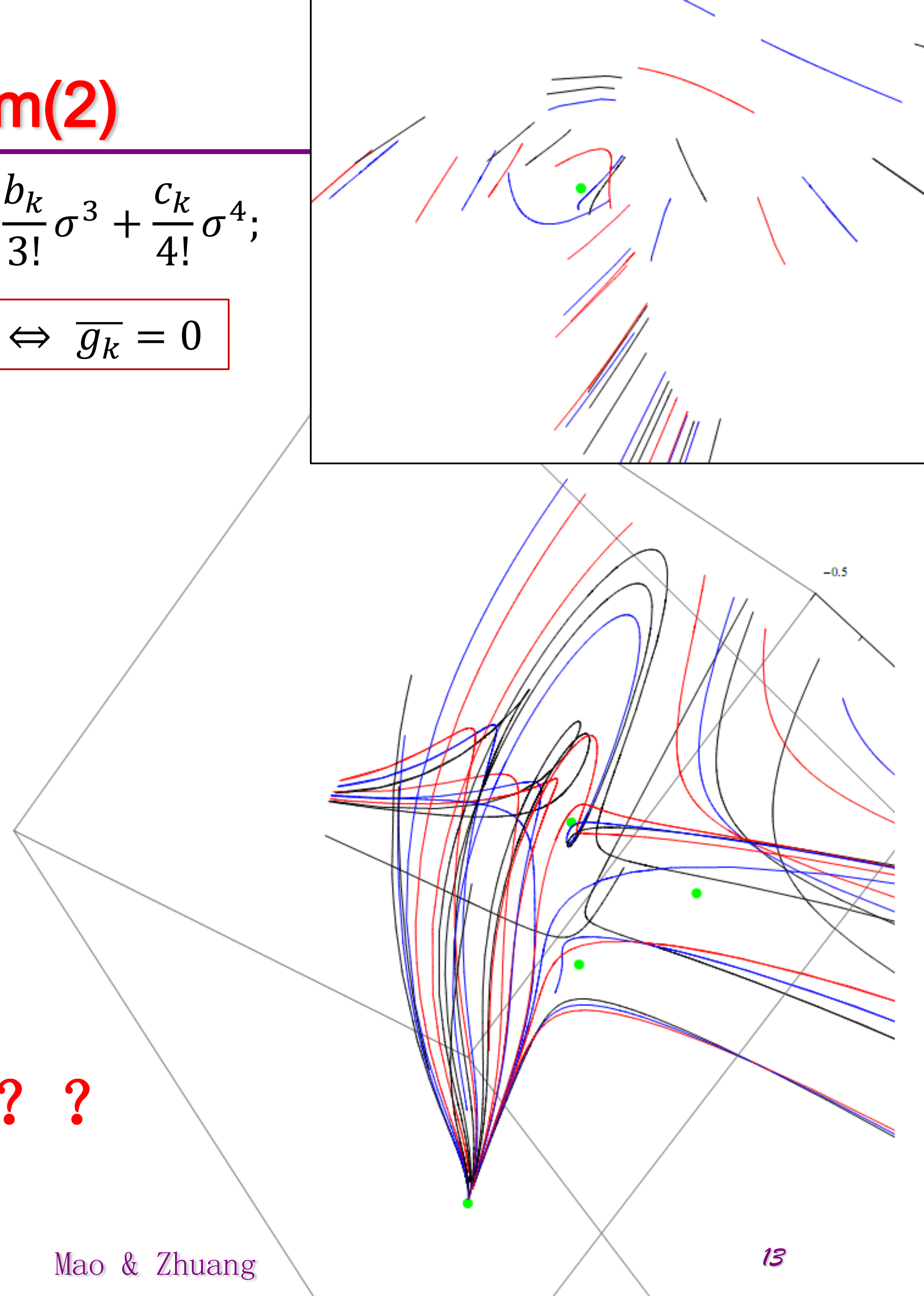
$\nu = 0.5427$

(-0.10, ±0.82, 11.39);

$$\partial_t(\bar{a} - \overline{a_*}, \bar{b} - \overline{b_*}, \bar{c} - \overline{c_*})^T = M(\bar{a} - \overline{a_*}, \bar{b} - \overline{b_*}, \bar{c} - \overline{c_*})^T$$

Eigenvalues: {-1.79493, 0.795625 + 1.12263 i, 0.795625 - 1.12263 i}

? ?



Flow diagram(3)

1st phase transition

$$U_k = \frac{a_k}{2!}\sigma^2 + \frac{b_k}{3!}\sigma^3 + \frac{c_k}{4!}\sigma^4 + \frac{d_k}{5!}\sigma^5 + \frac{e_k}{6!}\sigma^6$$

4 fixed points: $\overline{g_k} = 0;$

large $\overline{g_k} \Leftrightarrow \overline{g_k} = 0$

$(0,0,0,0,0);$

$(-0.143,0,12.431,0,540.813)$ → $2nd, \nu = 0.5932$

$(-0.186, \pm 1.047, 17.299, 61.541, 1184.115)$? ?

Eigenvalues: {12.0818 + 8.08018 i, 12.0818 - 8.08018 i, -1.5845, 0.750471 + 1.18035 i, 0.750471 - 1.18035 i}

Summary

1. Two-point expansion method to solve flow equation for 1^{st} phase transition;

2. Flow diagram for 1^{st} order phase transition.

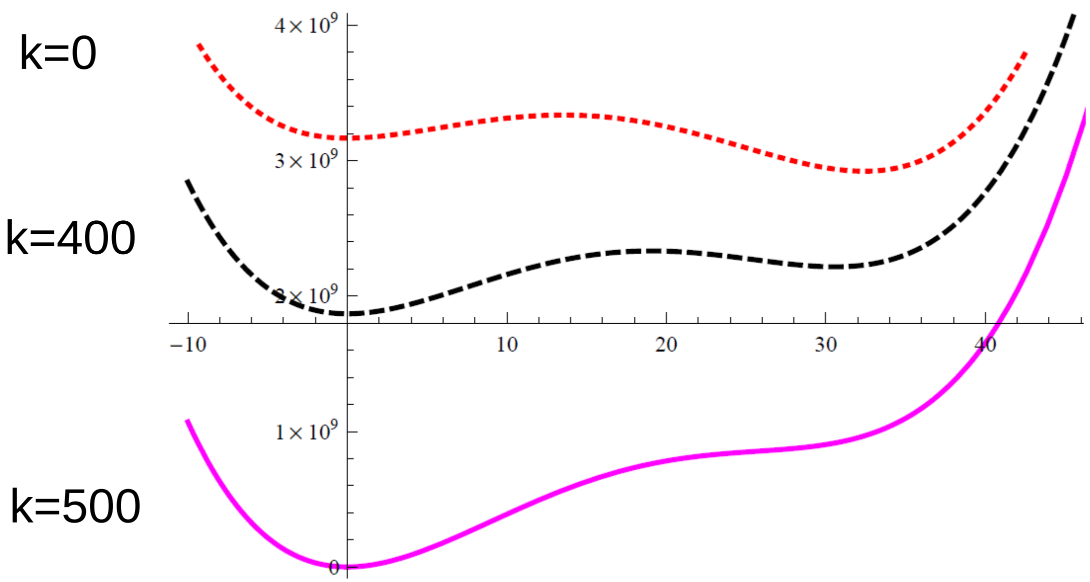
Outlook

3. Grid calculation of flow diagram, critical exponents.

谢谢！

BACK UP

真空中的相变是否与有限温度密度情形的相变有某种对应关系呢？



$\mu = 0$, $T = 0$, $T = \infty$。$T = 0$时，$n_f(x > 0) = 0$，流方程Eq.17退化为真空流方程Eq.19，U_0对应图中的点线。$T = \infty$时，$n_f(x > 0) = 1/2$，流方程右边为零，此时U_0对应图中的实线。这意味着，从$T = 0$到$T = \infty$的相变，对应于体系的有效势从点线变为了实线。若$\Lambda < k_c$，点线与实线之间没有相变，则$T = 0$到$T = \infty$也不会发生相变。

$$\begin{aligned}\partial_k U_k &= \partial_k U_{kF} \\ &= -4N_f N_c \frac{k^4}{6\pi^2}\left(\frac{1}{2E_f} - \frac{n_f(E_f+\mu) + n_f(E_f-\mu)}{2E_f}\right)\end{aligned}$$

Wilsonian Renormalization Group method

Step 1: Mode elimination

$$\Phi = \Phi^{<} + \Phi^{>}$$

$$e.g.\ \ \Phi^{<}(q) = \Theta(k - |q|)\Phi(q)$$
$$\Phi^{>}(q) = \Theta(|q| - k)\Phi(q)$$

$$Z = \int [\mathrm{d}\,\Phi^{<}] \int [\mathrm{d}\,\Phi^{>}]\, \mathrm{e}^{-S_{\Lambda}^{<}[\Phi^{<}+\Phi^{>};\mathrm{g}]}$$
$$= \int [\mathrm{d}\,\Phi^{<}]\, \mathrm{e}^{-S_{k}^{<}[\Phi^{<};\mathrm{g}^{<}]}$$

consecutive inclusion of fluctuations

✓ Phase transition including fluctuation: T_c, μ_c ← **Functional RG;**

Step 2: Rescaling

$$b = \frac{\Lambda}{k}$$

$$q' = bq; \Phi'(q') = \zeta_b \Phi^{<}(q'/\mathrm{b}) \longrightarrow \vec{g}\,' = f_G(\mathrm{b}, \vec{\mathrm{g}})$$

$S_k^{<}[\Phi'; \mathrm{g}']$ has the same form as $S_{\Lambda}[\Phi, g]$

※ Critical behavior: critical exponents, universality

Flow diagram(1)

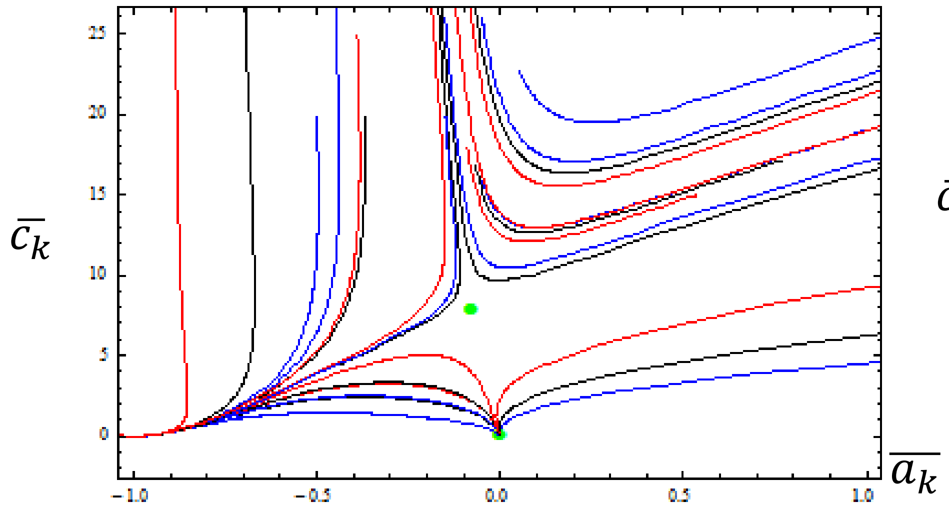
Dimensional reduction:

$$\partial_k U_k = \partial_k U_{kF} + \partial_k U_{kB};$$

$$\partial_k U_{kB} = \frac{k^4}{6\pi^2} \cdot \frac{1}{k^2 + U_k''};\partial_k U_{kF} = -8N_c \frac{k^4}{6\pi^2} \cdot \frac{1}{k^2 + g^2\sigma^2};$$

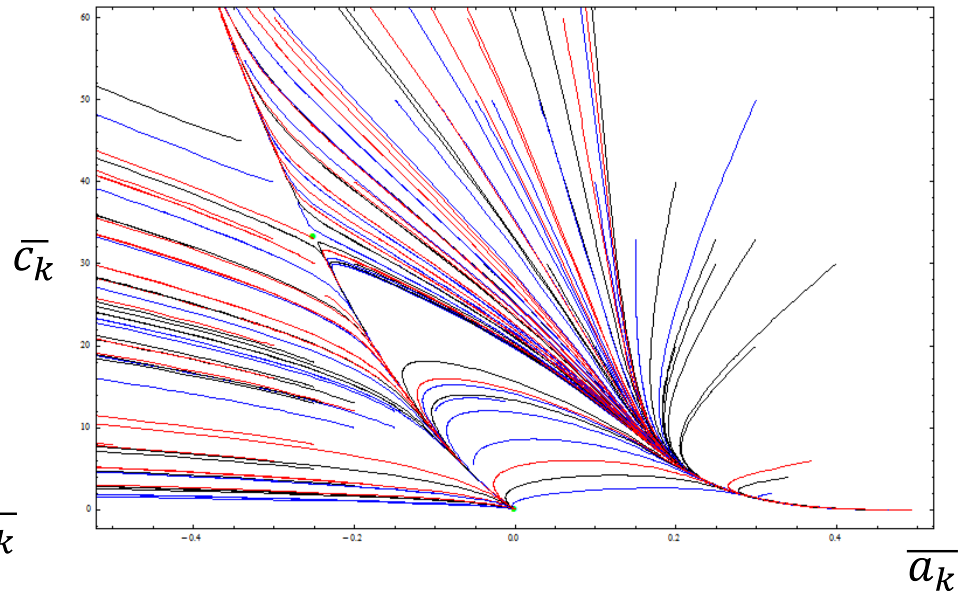
$$U_k = \frac{a_k}{2!}\sigma^2 + \frac{c_k}{4!}\sigma^4 \qquad \overline{a_k} = \frac{a_k}{k^2}; \overline{c_k} = \frac{c_k}{k}; \quad \overline{g_k} = \frac{g_k}{k^{1/2}};$$

(1)Small parameter σ



$\overline{g_k} = 0$; (-0.08, 7.76); (0,0)

(2)Small parameter $\chi_k = \sigma - \sigma_{k,\nu}$



$\overline{g_k} = 0$; $(-0.25, 33.3)$;(0,0)

夸克禁闭

严格表述：一切可观测的粒子都处于色单态。

唯象表述：夸克只能被囚禁于强子内部，而不能自由存在。

(1) Hagedorn —— 强子气体的极限

$$\ell n Z_R \approx \int_{m_\pi}^{\infty} dm\rho(m) \int \frac{d^3\vec{p}}{(2\pi)^3} e^{-\sqrt{m^2+\vec{p}^2}/T} \qquad \rho(m) \approx m^{-a} e^{bm} \qquad a, b > 0$$

当温度 $T > T_H = \frac{1}{b}$ 时，热力学势发散，强子气体不再存在。

(2) Bag 模型：用附加的边界条件(袋常数**B**)将夸克束缚在有限的空间区域，即袋内。

$$P_{HG}(T_c) = P_{QGP}(T_c, \mu_c) = -\Omega_0^q - \Omega_0^g - B$$

(3) 纯规范场：禁闭相变对应于中心对称性**Z(Nc)**,序参量是**Polyakov**圈

topological Solition (不懂)

$$L(\mathbf{x}) = \mathcal{P} \exp\left[-ig \int_0^\beta dx_4 A_4(\mathbf{x}, x_4) \right] \qquad e^{-V(\infty)/T} \sim \langle L \rangle = \begin{cases} 0 & V(\infty) = \infty, \quad \text{confinement} \\ \text{有限} & V(\infty) = \text{finite} \quad \text{deconfinement} \end{cases}$$

(4) 格点**QCD**：**Nc=2,**二级相变；**Nc>=3,**一级相变。

(5) PNJL, PQM 等：将手征相变与禁闭相变同时研究

(6) Soliton Bag Model：**non-topological (F-L model)**