Massive renormalization scheme and perturbation theory at finite temperature

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Based on work done with N. Wschebor, arXiv 1409.4795

European Research Council

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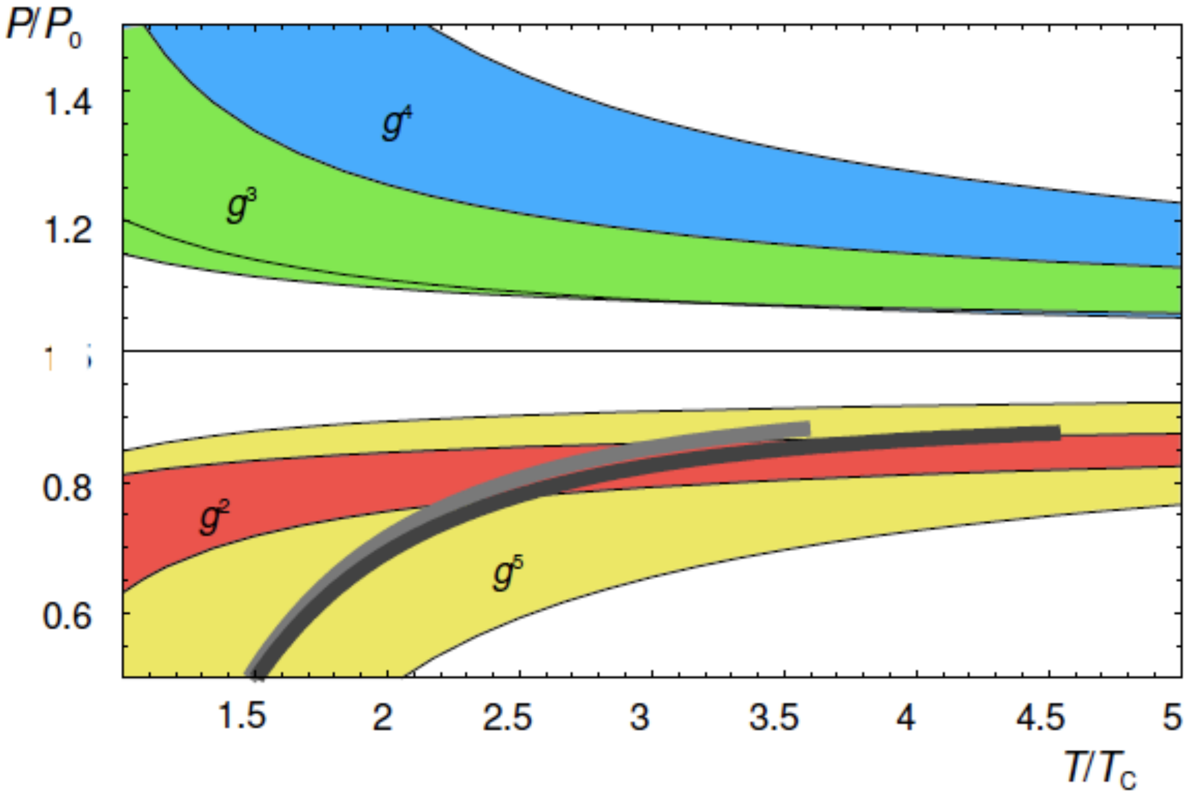
QCD at finite temperature

Perturbation theory is ill behaved

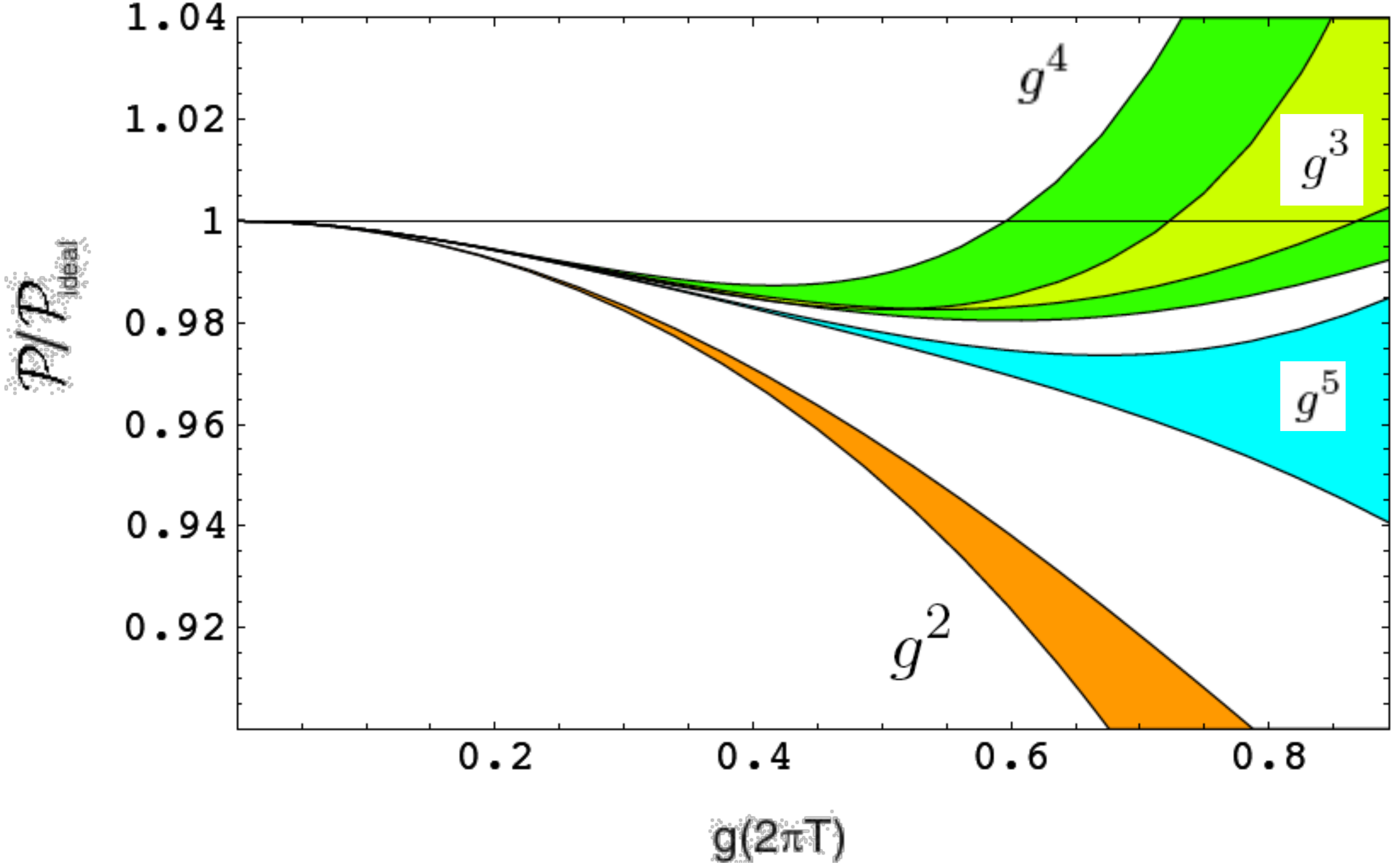


Perturbation theory:

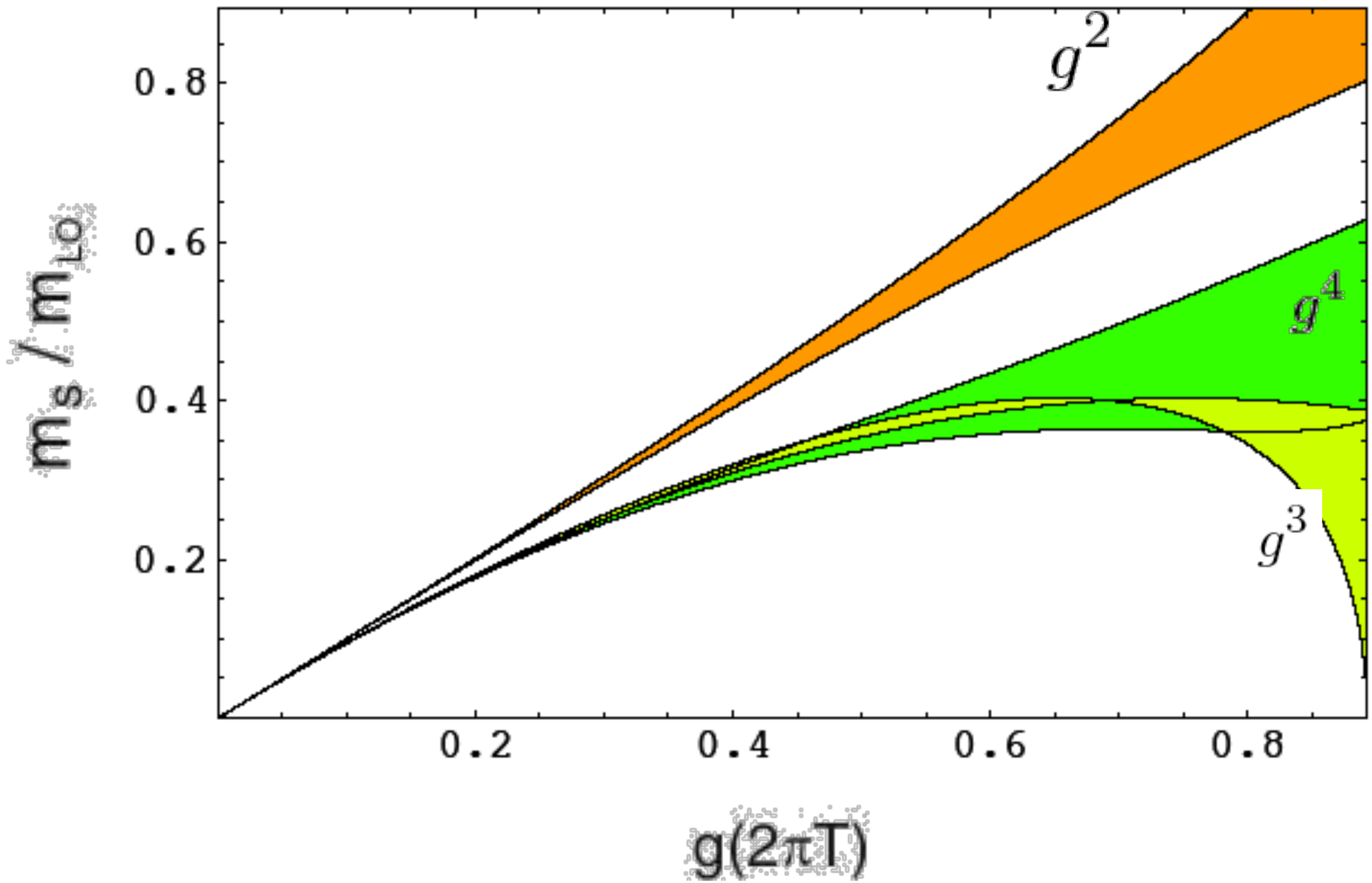
g^2: Shuryak; Chin (1978)
g^3: Kapusta (1979)
g^4 ln g: Toimela (1983)
g^4 : Arnold, Zhai (1994)
g^5 : Zhai, Kastening (1995), Braaten, Nieto (1996)
g^6 ln g: Kajantie, Laine, Rummukainen, Schröder (2002)
g^6 (partly): Di Renzo, Laine, Miccio, Schröder, Torrero (2006)

Lattice data: G. Boyd et al. (1996); M. Okamoto et al. (1999).

Scalar field theory with quartic coupling



Scalar field theory with quartic coupling



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- Calculate higher orders....

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J.O. Andersen, L. Kyllingstad and L.E. Leganger, arXiv:0903.4596

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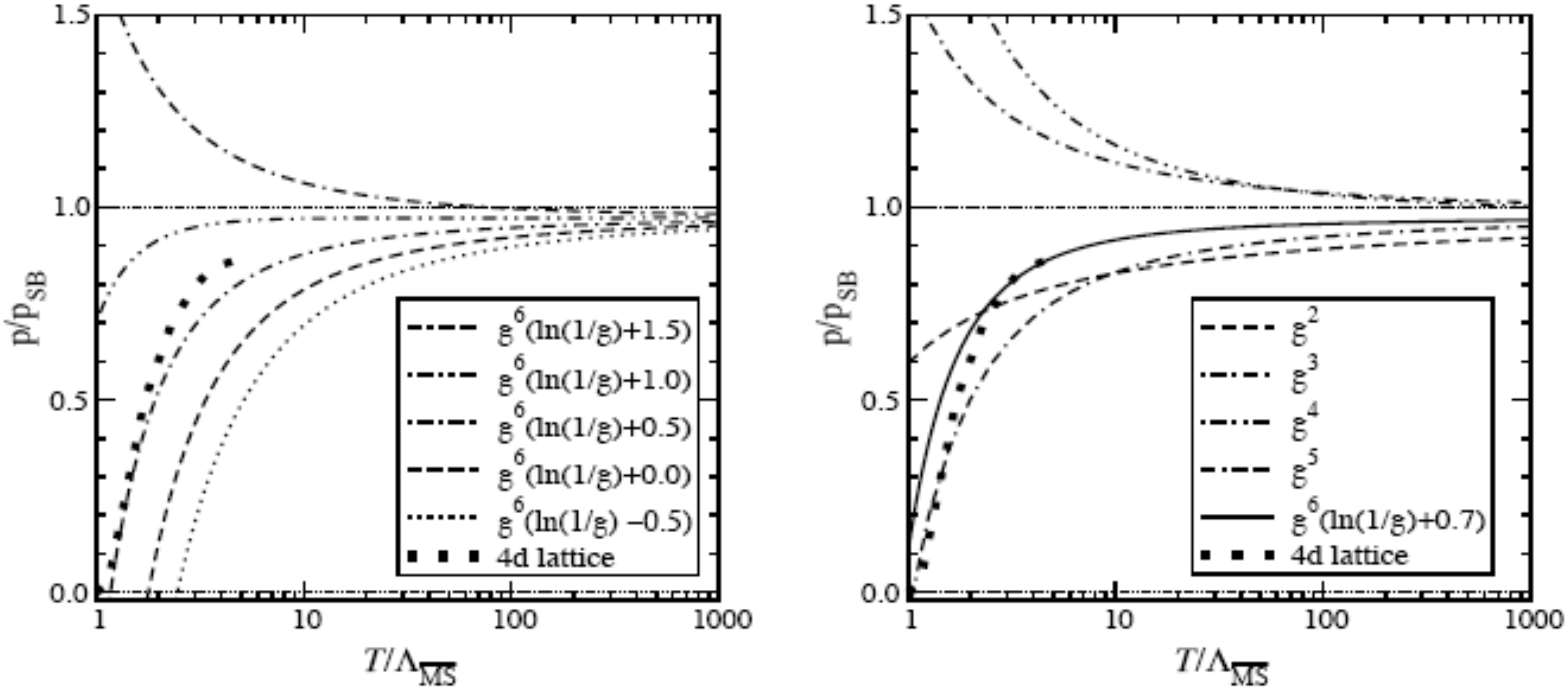
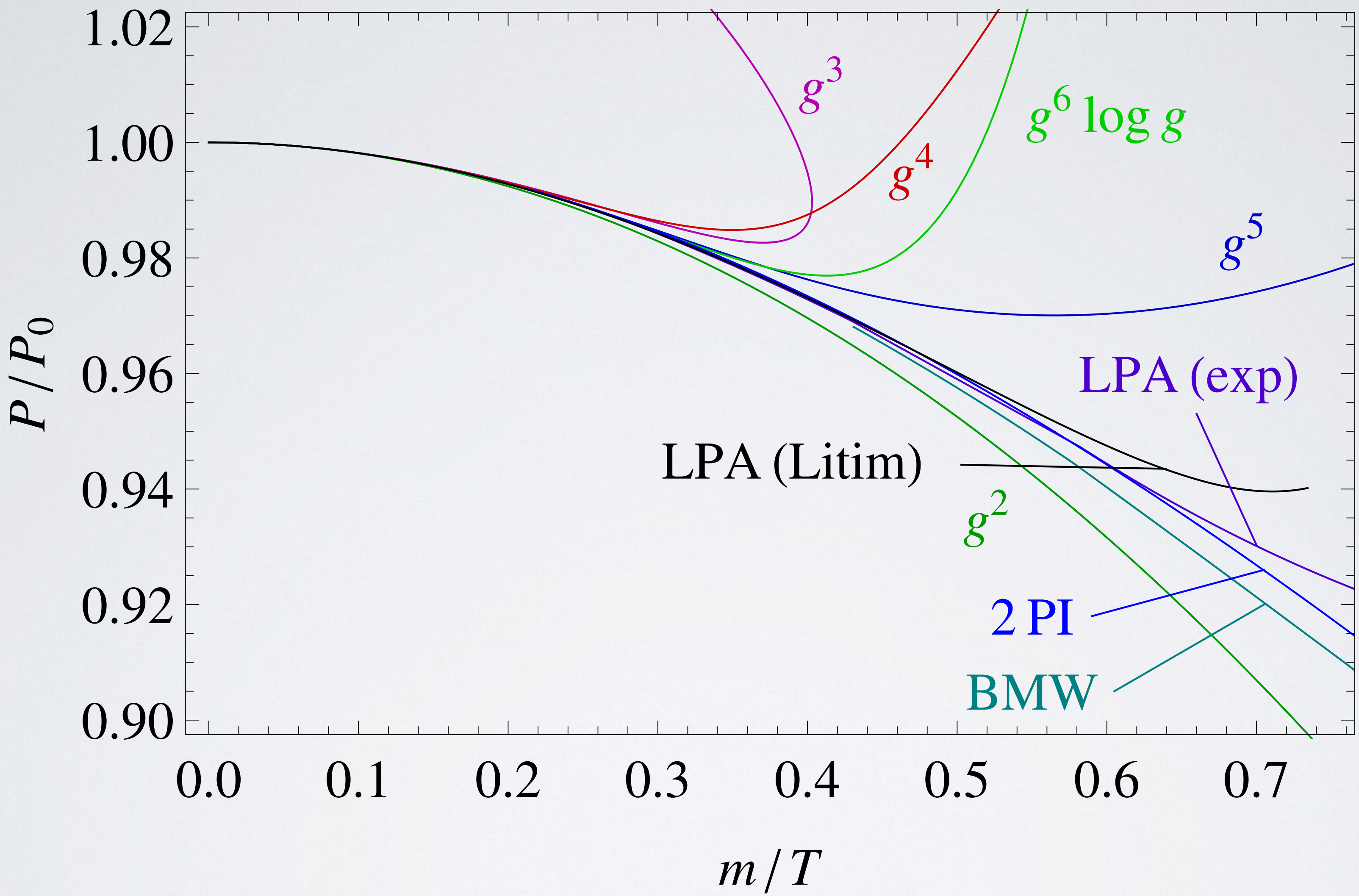
- reorganize perturbation theory, resum, 2PI, NPRG, etc)

$$p = T^4 \left[c_0 + c_2\, g^2 + c_3\, g^3 + \left(c_4' \ln g + c_4 \right) g^4 + c_5\, g^5 + c_6\, g^6 \right]$$

$$c_6 = N_c^3 \, \frac{N_c^2 - 1}{(4\pi^4)} \left[\left(\frac{215}{12} - \frac{805}{768}\, \pi^2 \right) \ln \frac{1}{g} + 8\, \delta \right]$$



K. Kajantie, M. Laine, K. Rummukainen, and Y. Schröder, *Phys. Rev. Lett.* 86 (2001) 10 , *Phys. Rev.* D 65 (2002) 045008 , *Phys. Rev.* D 67 (2003) 105008 , *JHEP* 0304 (2003) 036



JPB, A. Ipp, N. Wschebor, arXiv:1007.0991

JPB, A. Ipp, R. Mendez Galain, N. Wschebor, arXiv: hep-ph/0610004

.....perseverare diabolicum !

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Why is pertubation theory so bad ?

Expansion parameter and thermal fluctutations

$$\langle \varphi^2 \rangle_\kappa \approx \int^\kappa \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{n_{\boldsymbol{p}}}{E_{\boldsymbol{p}}} \approx T\kappa$$

$$n_{\boldsymbol{p}} = \frac{1}{\mathrm{e}^{E_{\boldsymbol{p}}/T} - 1}$$

$$\gamma_\kappa \sim \frac{g^2 \langle \varphi^2 \rangle_\kappa}{\kappa^2} \sim \frac{g^2 T}{\kappa}$$

Suggests a breakdown of perturbation theory when $\kappa \lesssim g^2 T$

But !

- Dimensional reduction at high temperature

$$\kappa \frac{d\gamma_\kappa}{d\kappa} = -\gamma_\kappa + \frac{3}{16}\gamma_\kappa^2$$

- Dynamical generation of a thermal mass

$$m \sim gT$$

Massive, decoupling, scheme

$$S[\varphi] = \int_0^\beta d\tau \int d^d x \left\{ \frac{1}{2} \partial_\mu \varphi \partial_\mu \varphi + \frac{{m_{\mathrm{B}}}^2}{2} \varphi^2 + \frac{g_{\mathrm{B}}^2}{4!} \varphi^4 \right\}$$

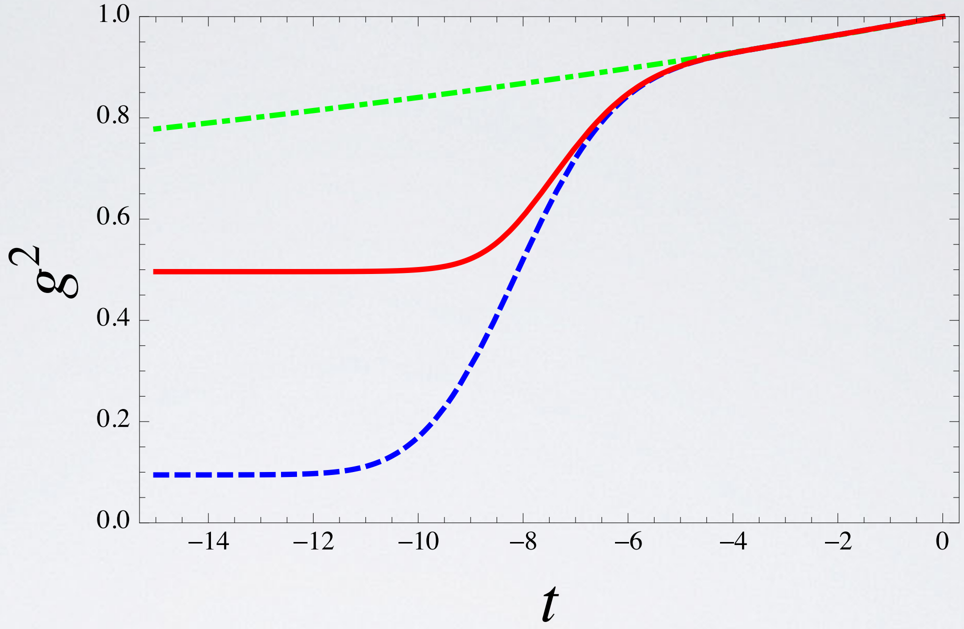
Renormalization conditions

$$m^2 = \Gamma^{(2)}(\boldsymbol{p} = \boldsymbol{0}, \omega = 0, T)$$

$$1 = \frac{d\Gamma^{(2)}}{d\boldsymbol{p}^2}(\boldsymbol{p}^2 = \mu^2, \omega = 0, T)$$

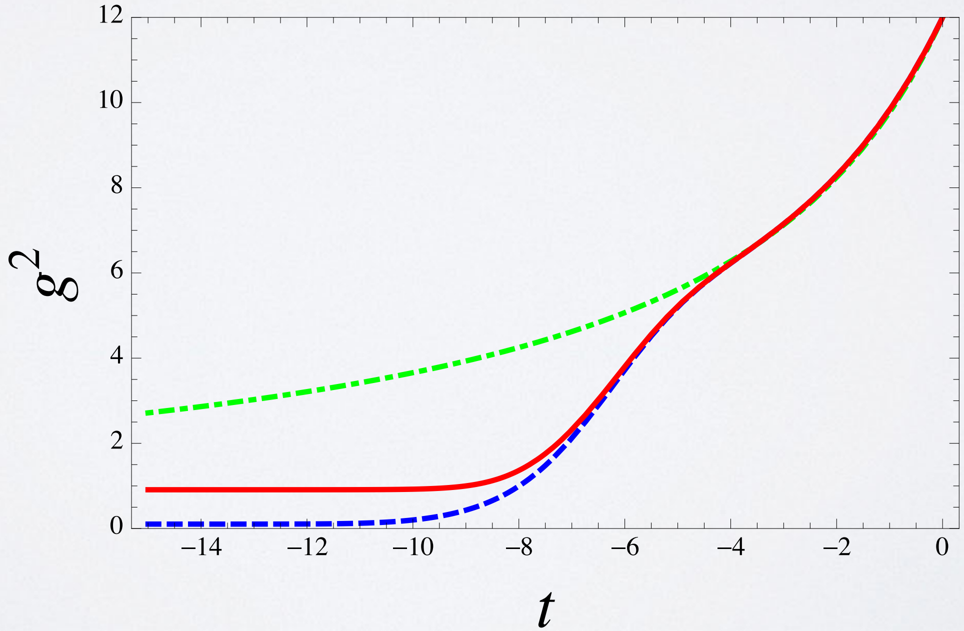
$$g^2 = \Gamma^{(4)}(\boldsymbol{p}_{sym}^2 = \mu^2, \omega_i = 0, T)$$

One-loop running in massive scheme



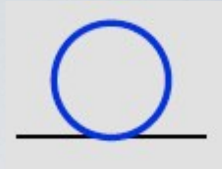
$g^2 = 1$

$t = \ln(\mu/\Lambda)$



$g^2 = 12$

Leading order calculation



$$\Gamma^{(2)}(\boldsymbol{p},\omega,T) = m^2 + \delta m^2 + \boldsymbol{p}^2 + \frac{g^2 T}{2}\sum_n \int \frac{\mathrm{d}^d q}{(2\pi)^d}\frac{1}{\omega_n^2 + \boldsymbol{q}^2 + m^2}$$

$$I(m) \equiv T\sum_n \int_{\boldsymbol{q}} \frac{1}{\omega_n^2 + \boldsymbol{q}^2 + m^2} = \int_{\boldsymbol{q}} \frac{1 + 2n_{\boldsymbol{q}}}{2E_{\boldsymbol{q}}} \equiv I_0(m) + I_T(m)$$

$$\Gamma^{(2)}(\boldsymbol{p}=0,\omega=0,T) = m^2 + \delta m^2 + \frac{g^2}{2}I(m)$$

The renormalization condition implies $\quad \delta m^2 = -\frac{g^2}{2}I(m)$

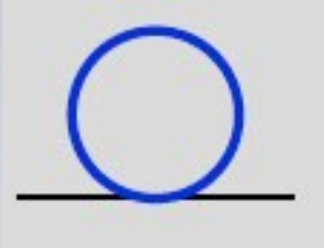
Relate thermal mass to zero temperature mass

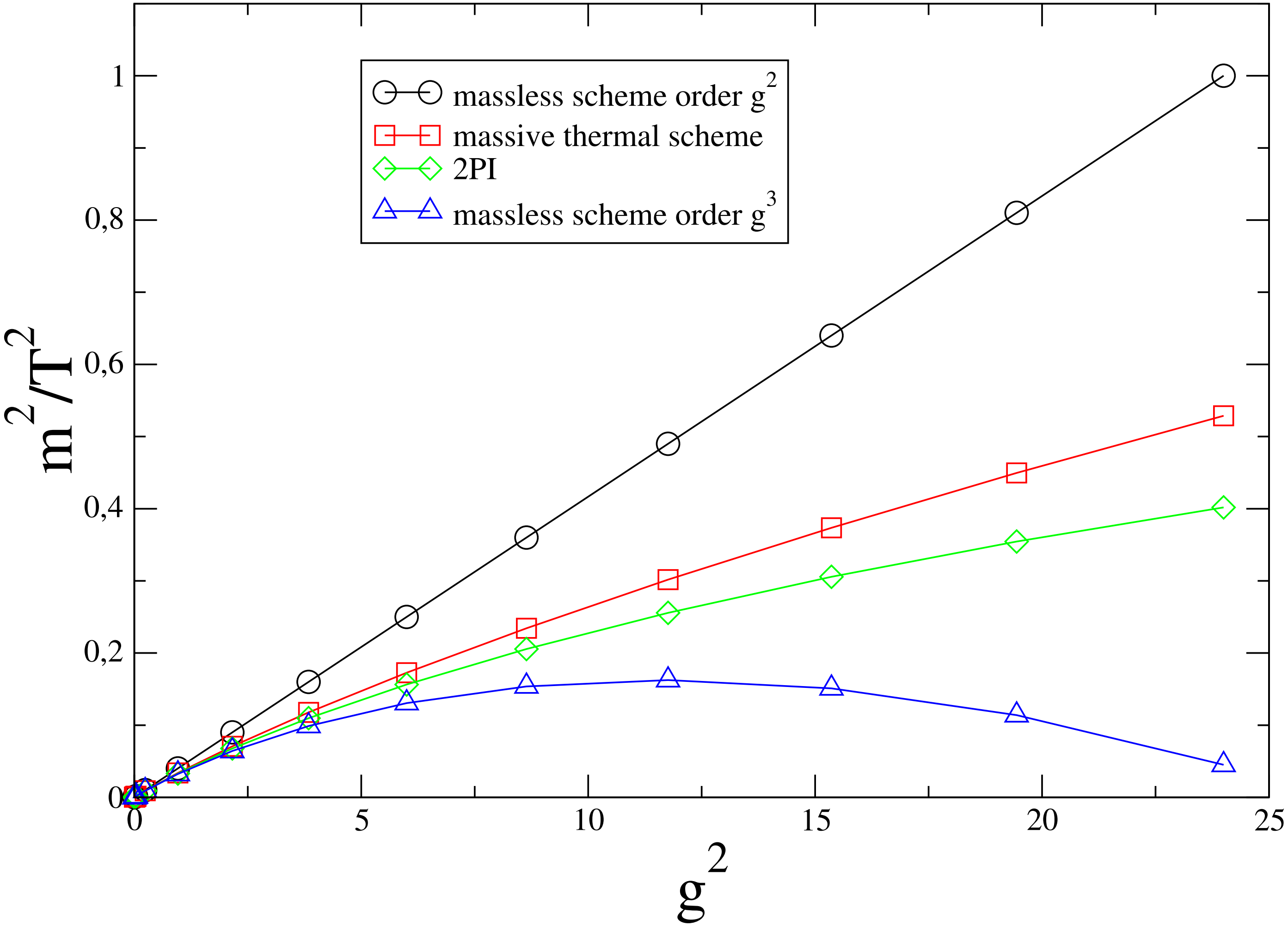
$$\Gamma^{(2)}(\boldsymbol{p}=0,\omega=0,T=0) = m^2 + \delta m^2 + \frac{g^2}{2} I_0(m) = m^2 - \frac{g^2}{2} I_T(m)$$

Note: unusual calculation !

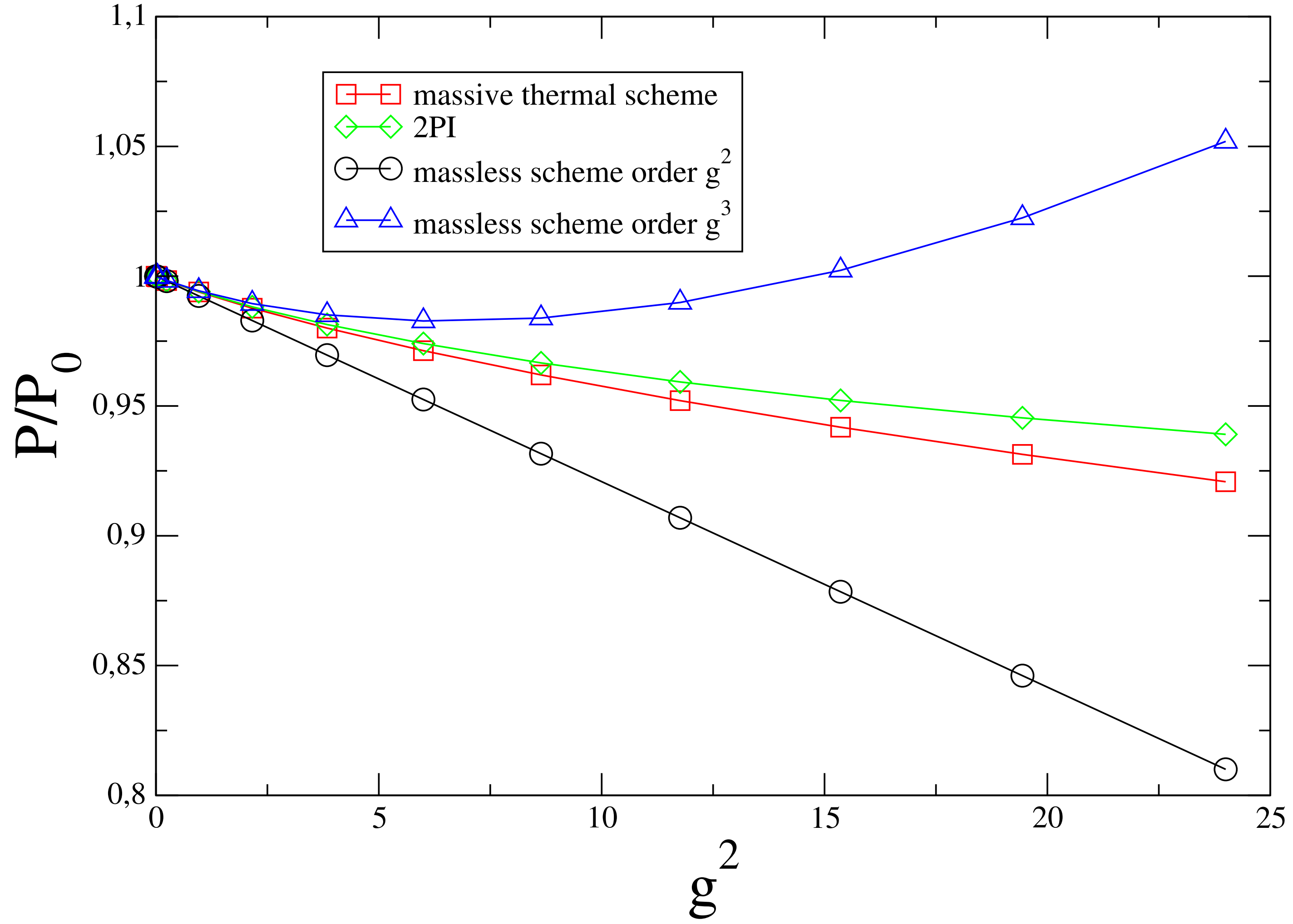
Self-consistent equation for the thermal mass

$$m_0^2 = m^2 - \frac{g^2}{2} I_T(m)$$

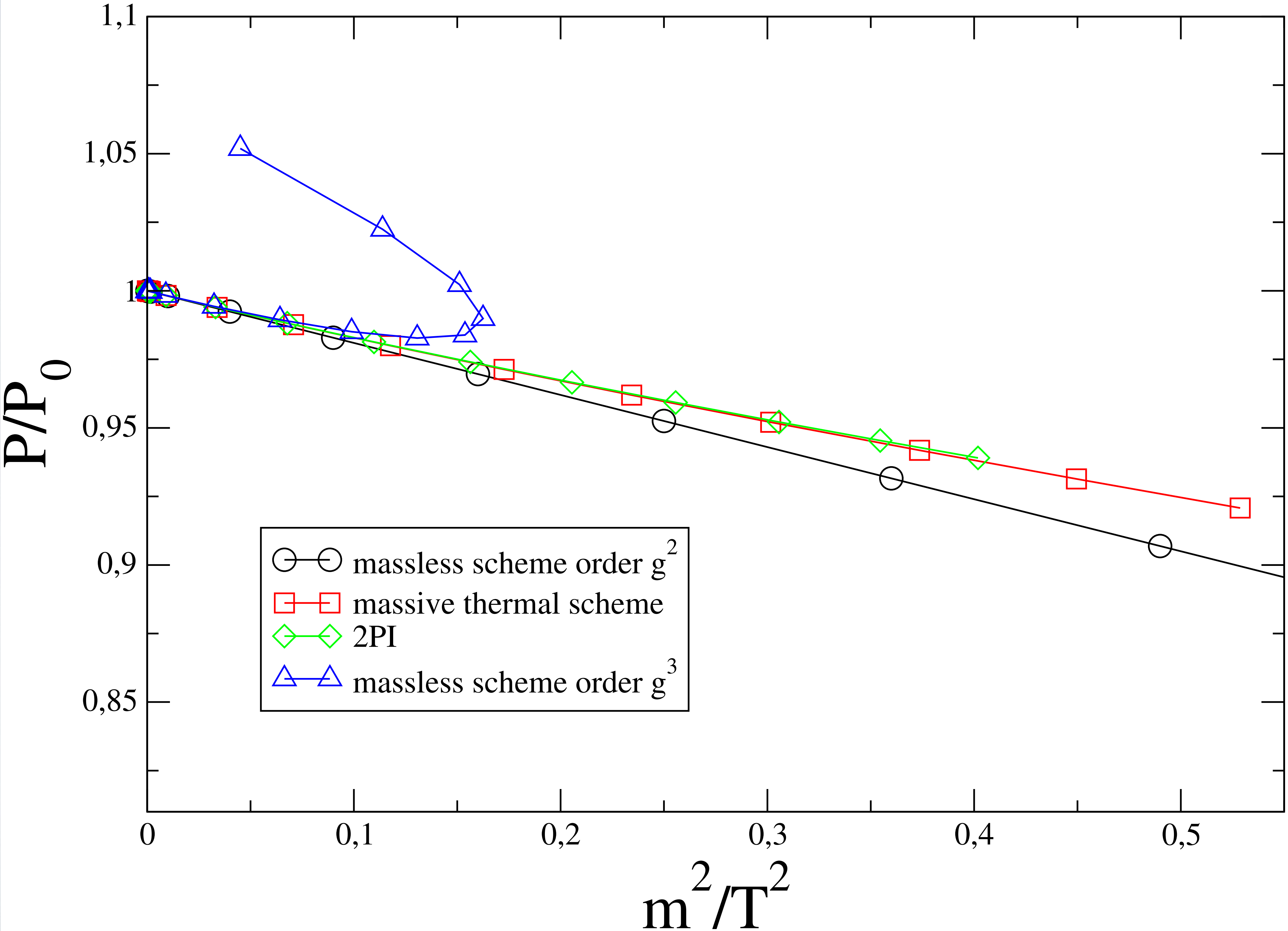




massless scheme order g^2
massive thermal scheme
2PI
massless scheme order g^3
m^2/T^2
g^2
0
0,2
0,4
0,6
0,8
1
5
10
15
20
25



massive thermal scheme
2PI
massless scheme order g^2
massless scheme order g^3
P/P_0
g^2
1,1
1,05
1
0,95
0,9
0,85
0,8
0
5
10
15
20
25



1,1
1,05
1
0,95
0,9
0,85
0
0,1
0,2
0,3
0,4
0,5
P/P0
m2/T2
massless scheme order g2
massive thermal scheme
2PI
massless scheme order g3

Summary

- An appropriate choice of renormalization scheme can greatly improve perturbation theory at finite temperature
- The proposed massive scheme leads to a well behaved perturbative expansion
- The idea of expanding around a massive theory is not new (screened perturbation theory, optimized perturbation theory, etc), but the present implementation is conceptually and technically simpler.