The Role of Fluctuations in the Phase Diagram of QC$_2$D

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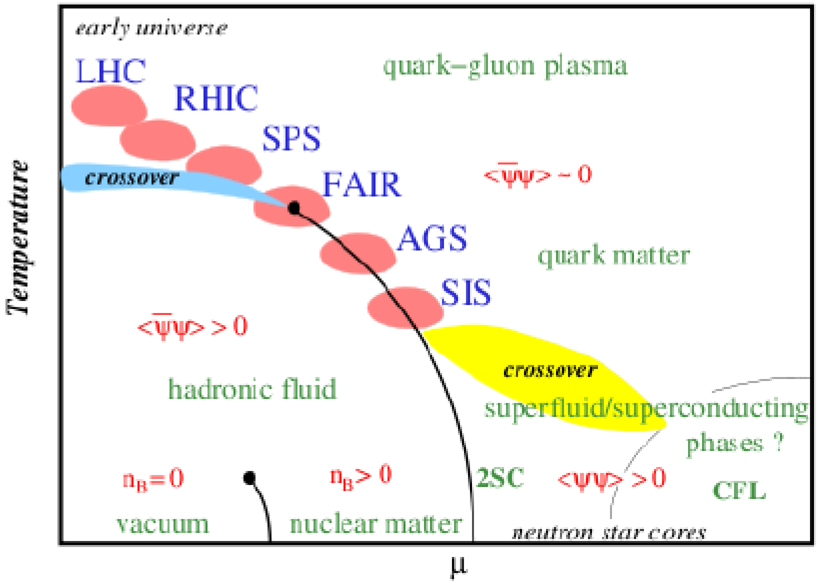
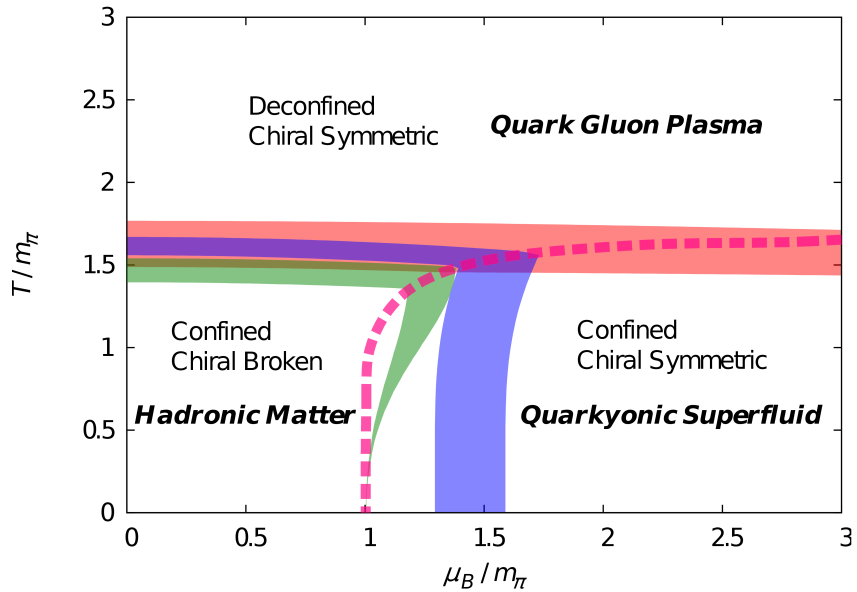
Motivation



[Schaefer, Delta Meeting '10]



[Brauner, Fukushima & Hidaka '09]

QC_2D

- No sign problem: Effective models $\Longleftrightarrow$ lattice calculations $\mu > 0$
- Diquarks $\Delta \sim \psi\psi$ are color neutral, bosonic baryons
- Impact of baryonic dof on the phase diagram
- Relativistic analog of ultra cold atoms (BEC-BCS Crossover)

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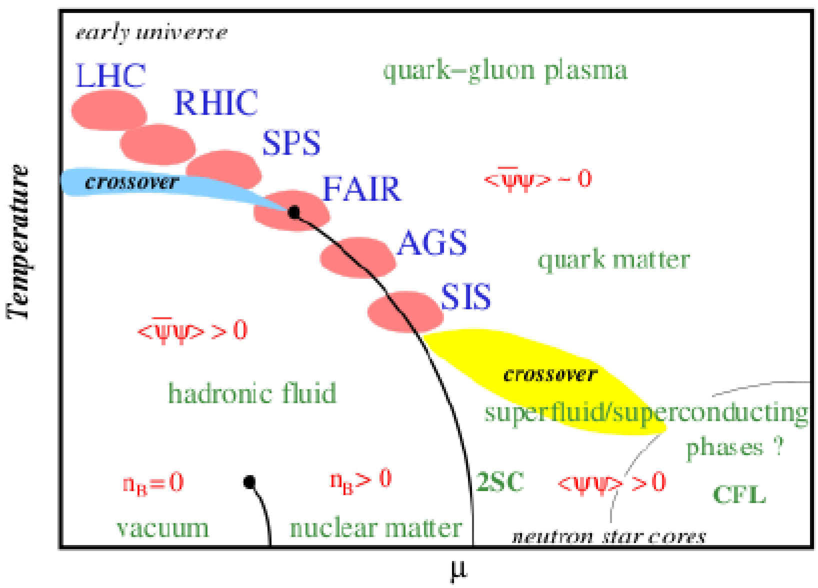

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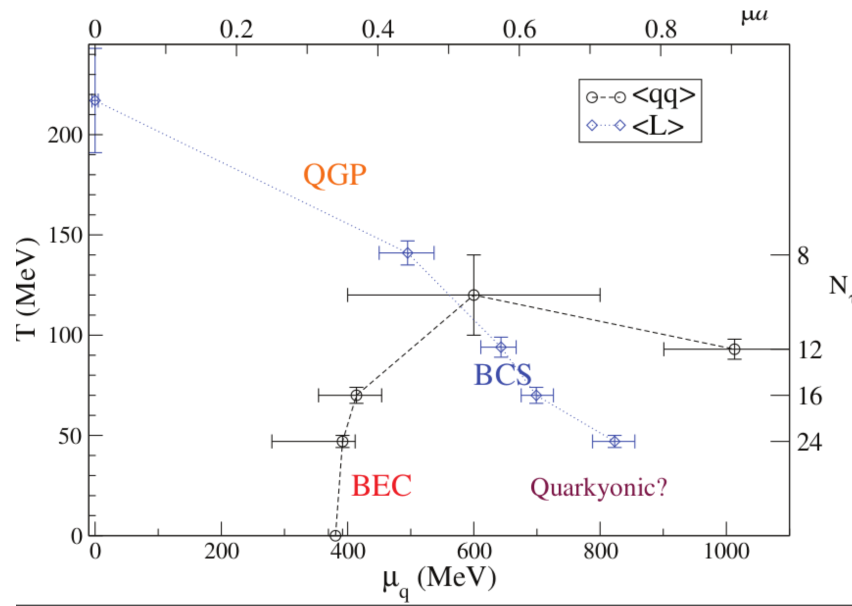

[Hands '12]

QC$_2$D

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Quark-Meson-Diquark Model

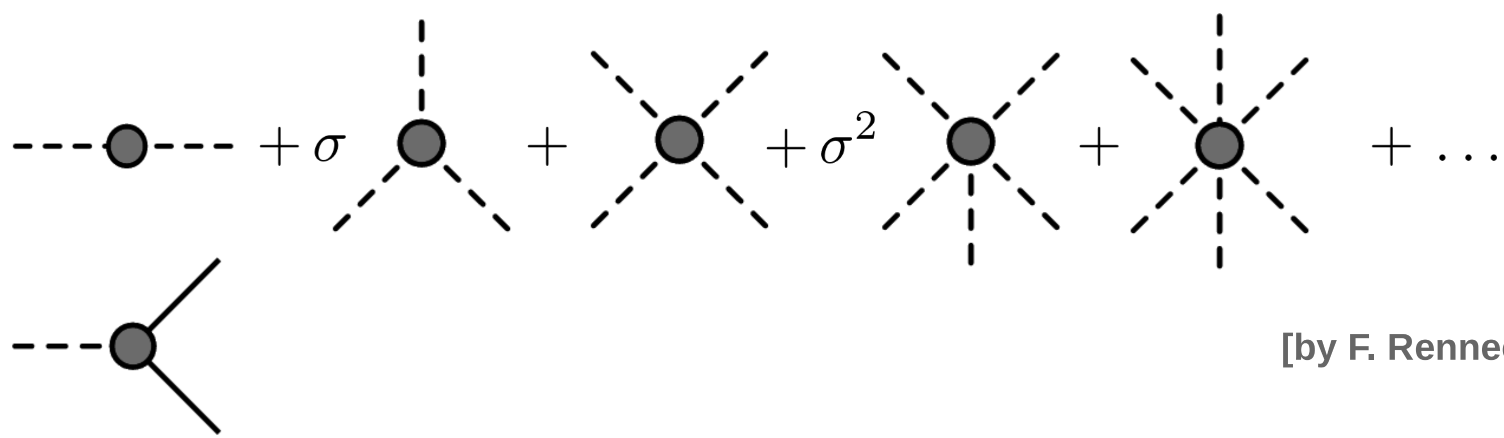
- Extended flavor symmetry: $SU(4) \simeq SO(6)$

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix} \quad , \qquad \phi = \begin{pmatrix} \vec{\pi} \\ \sigma \\ \Delta \\ \Delta^* \end{pmatrix} \sim \begin{pmatrix} \bar{\psi}\vec{\tau}\gamma_5\psi \\ \bar{\psi}\psi \\ \psi\psi \\ \bar{\psi}\bar{\psi} \end{pmatrix}$$

- Interactions:

$V(\phi^2) \sim$

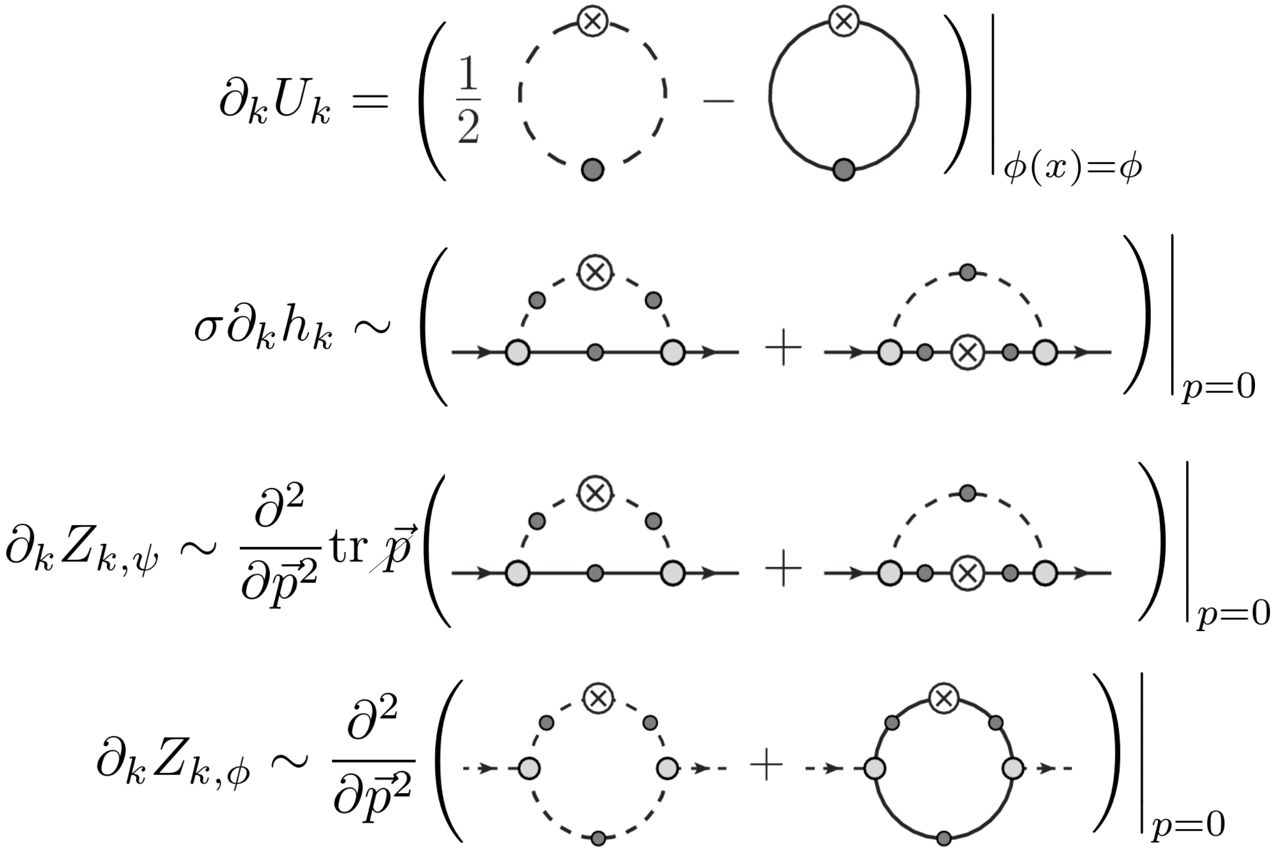
$h \sim$



[by F. Rennecke]

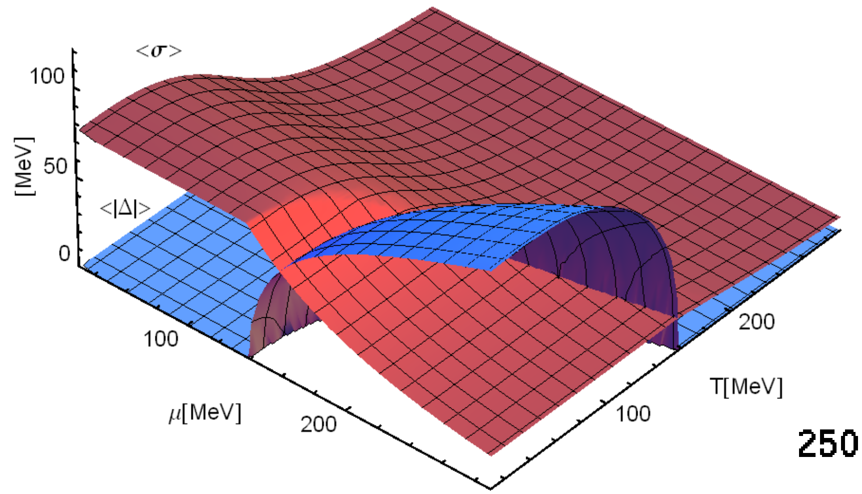
Functional Renormalization Group

- Flow equations:

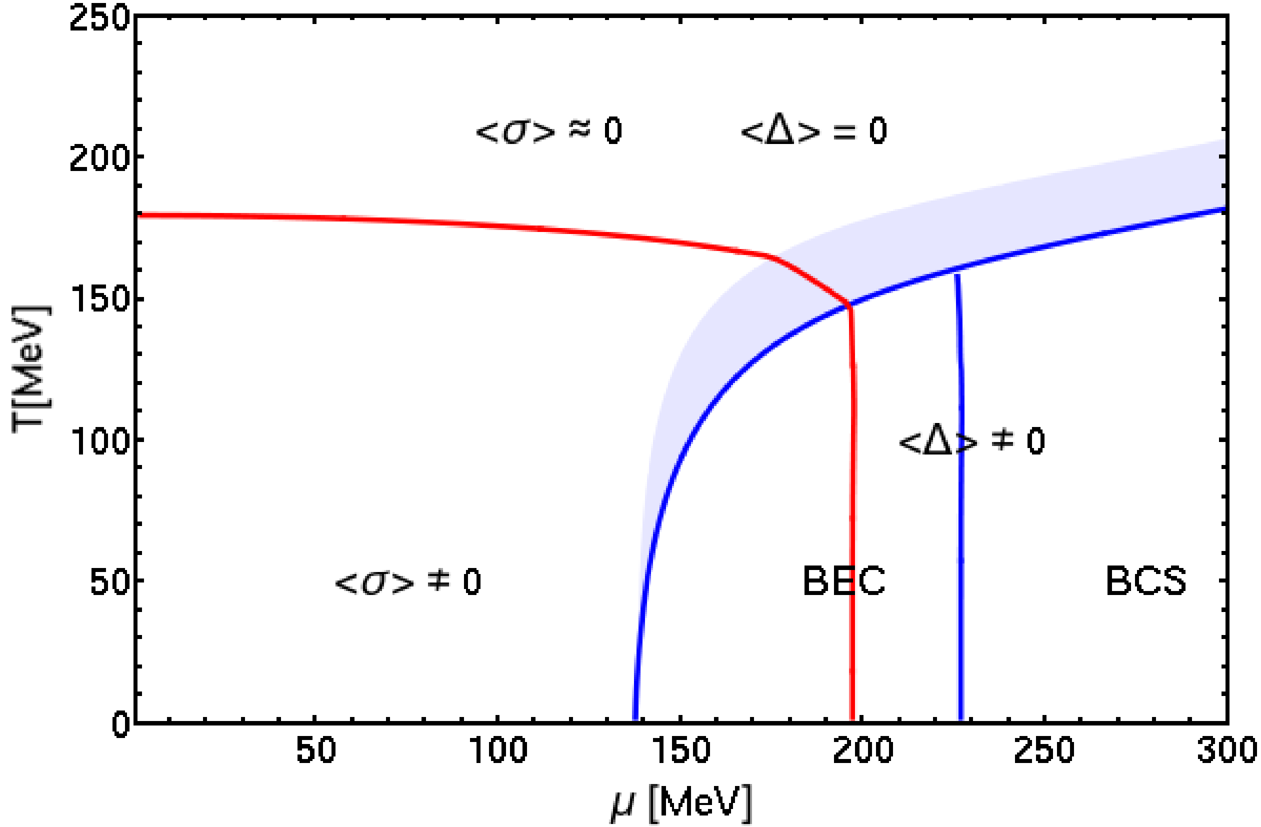


Results

The QC$_2$D Phase Diagram

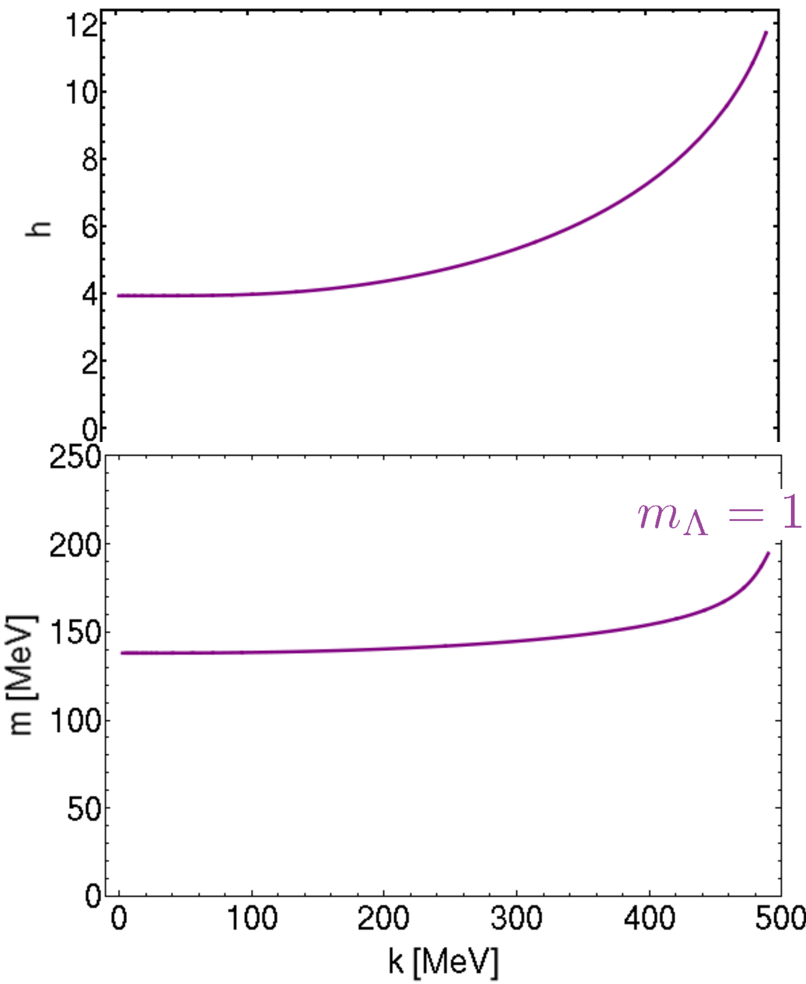


$$\phi^4, \eta = 0, \dot{h} = 0$$



UV cut-off

- ϕ^4 $1000\ \mathrm{MeV}$
- $\phi^4, \eta, \dot{h}$ $500\ \mathrm{MeV}$. UV cut-off of the "full" system is small, due to strong fluctuations



$m_\Lambda = 195\ \mathrm{MeV}$

→ diverges and system breaks down

→ restricts the range of the model

m : mass parameter in U

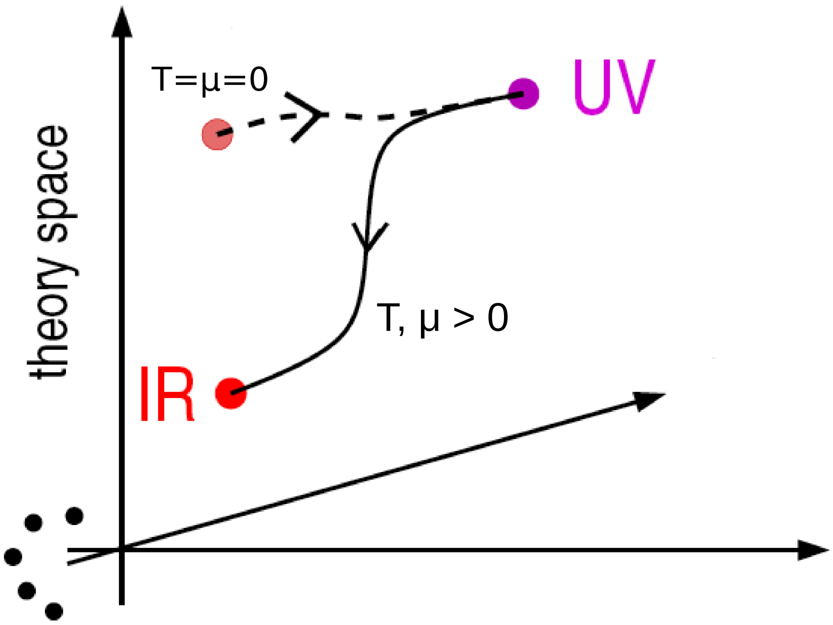
Initial Flow

- Question: In what range of T and μ is the flow independent of the cut-off?

- Define relative difference of initial flow:

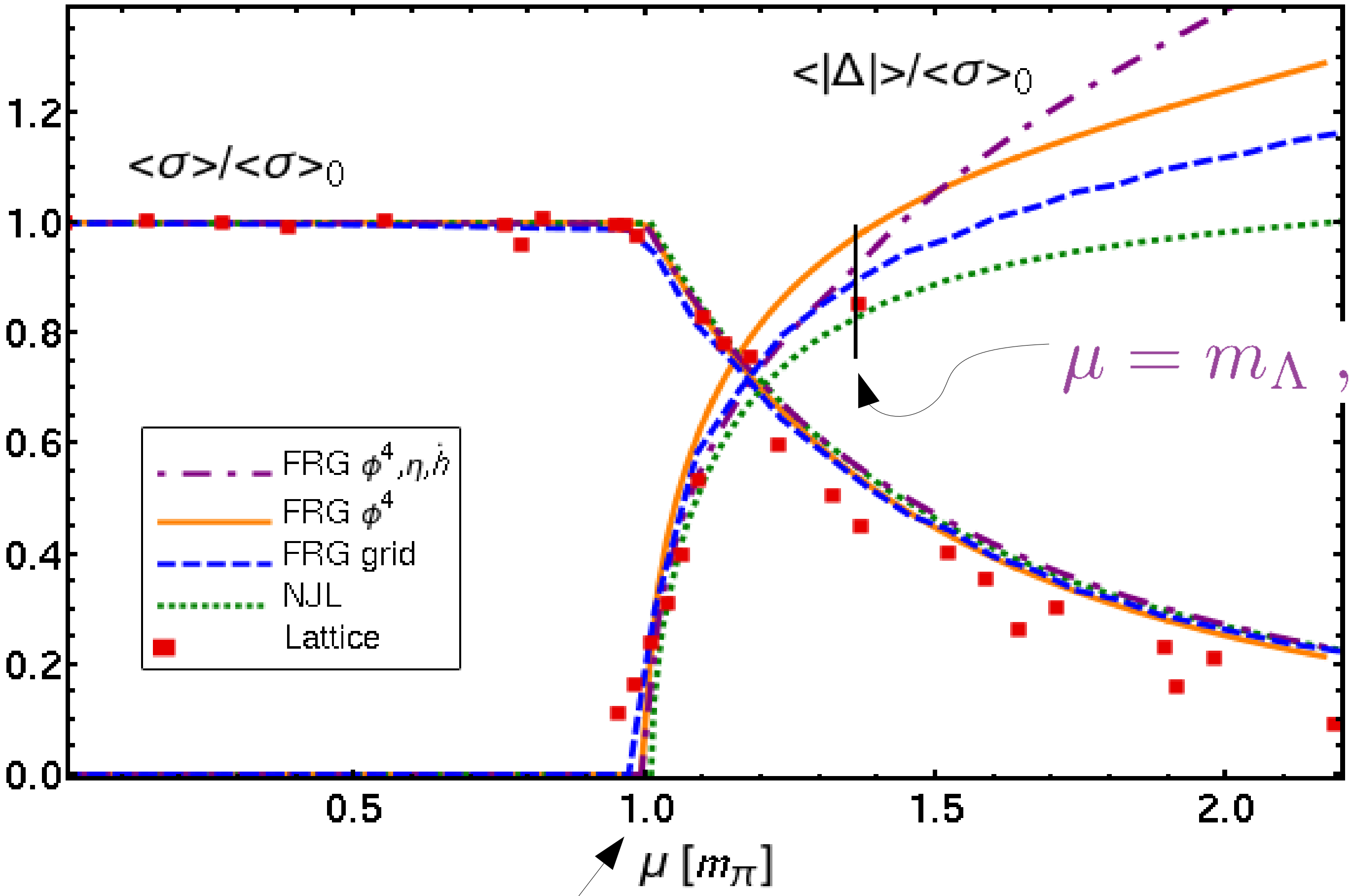
$$f_\Lambda = \frac{\dot{\sigma}_{k=\Lambda,T,\mu} - \dot{\sigma}_{k=\Lambda,0,0}}{\dot{\sigma}_{k=\Lambda,0,0}}$$

→ If f_Λ is too large, flow is cut off dependent



[Diehl *et* al '10]

Condensates at *T=0*



$\mu = m_\Lambda\ ,\ f_\Lambda > 0.1$

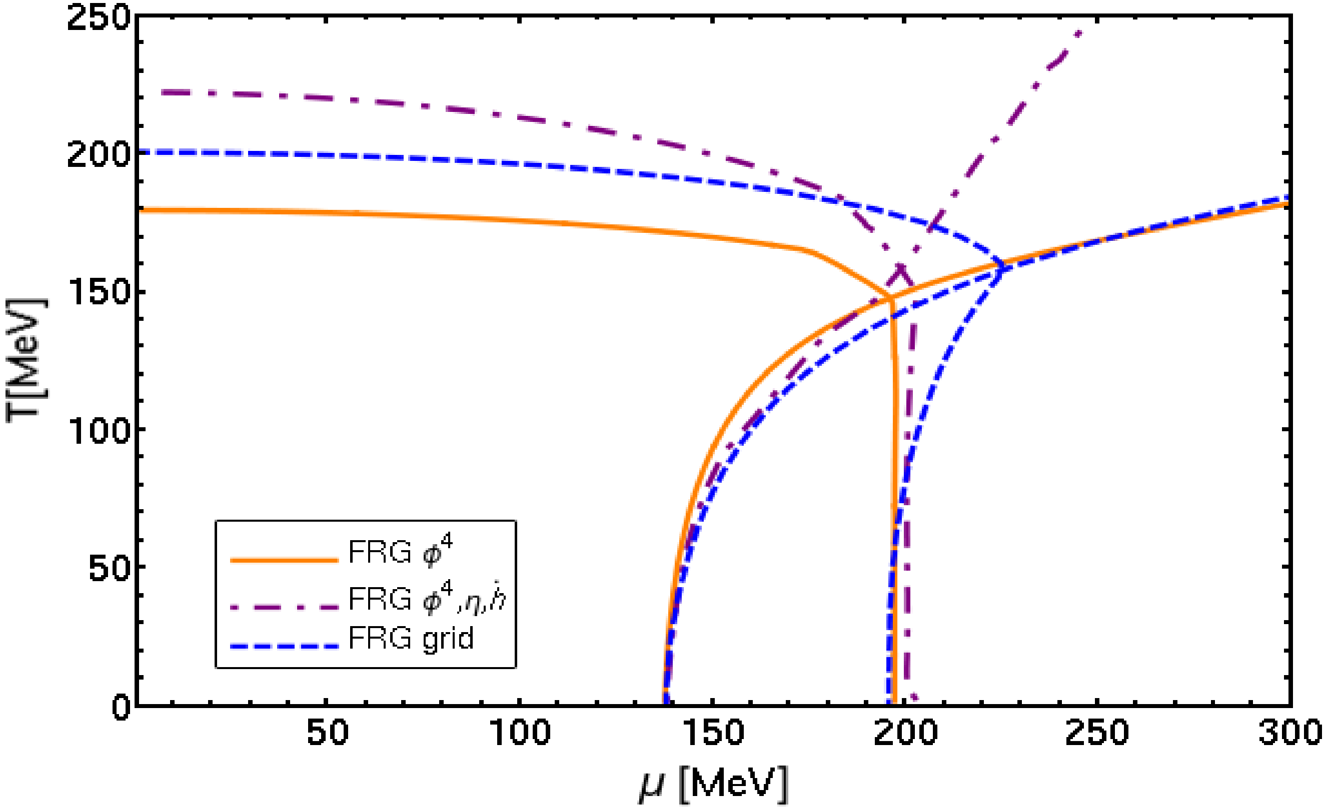
$m_\Delta^2 = m^2 - \mu^2 = 0$

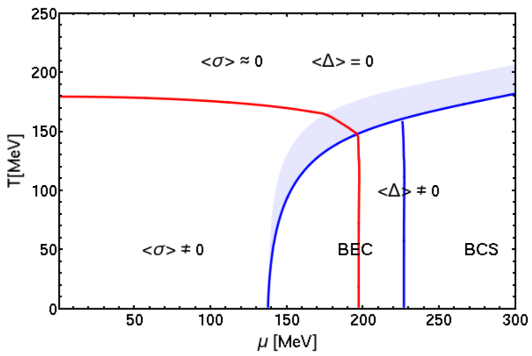
Lattice: [Hands et al. '00]

NJL: [Ratti, Weise '04]

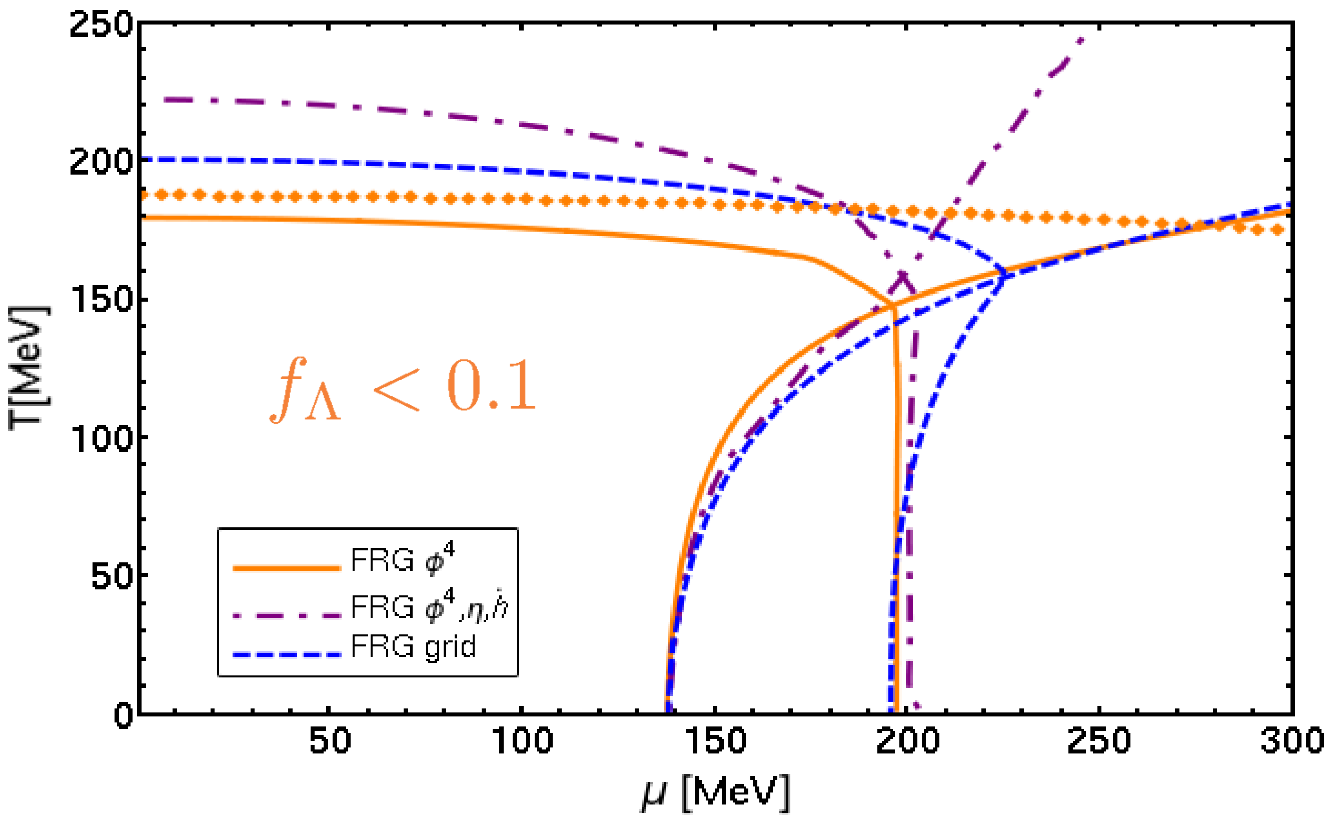
FRG grid: [Strodthoff, Schaefer, von Smekal '12]

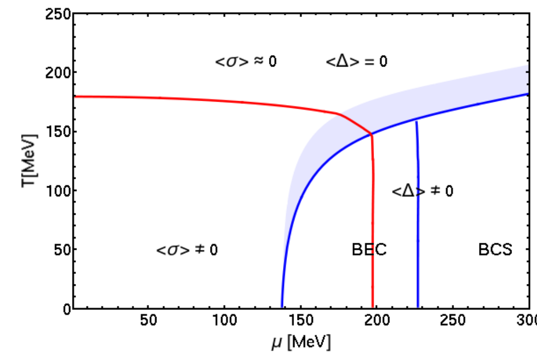
The QC$_2$D Phase Diagram



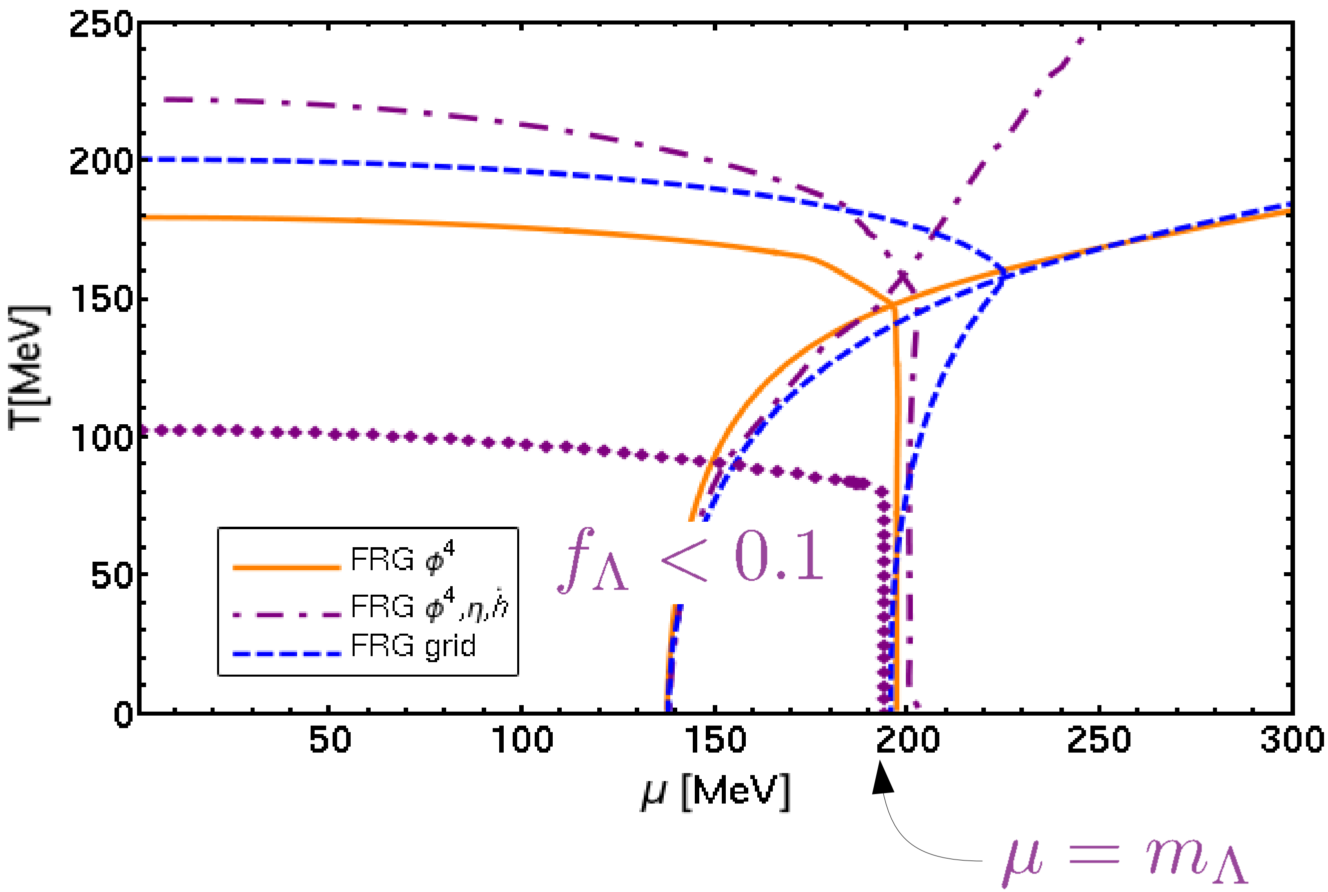


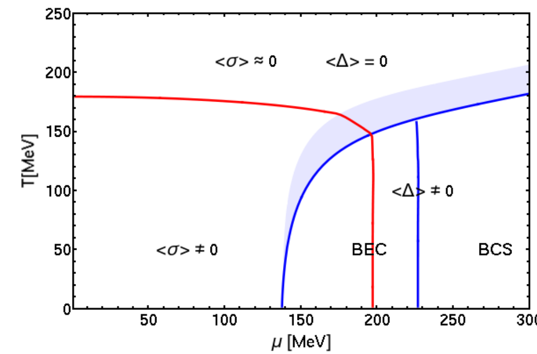
The QC$_2$D Phase Diagram





The QC_2D Phase Diagram





- Including fluctuations strongly constrains the model

→ all “lower” models are not reliable

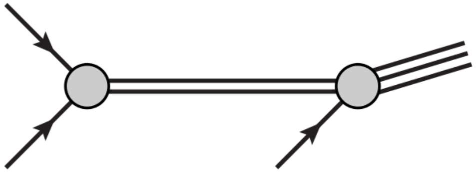
Summary and Outlook

Summary

- phase diagram of 2-color 2-flavor QCD
  with baryonic degrees of freedom
- FRG treatment in yields a precondensation phase
- Impact of fluctuations, running h and Z
- More work needs to be done, to have quantitative accuracy

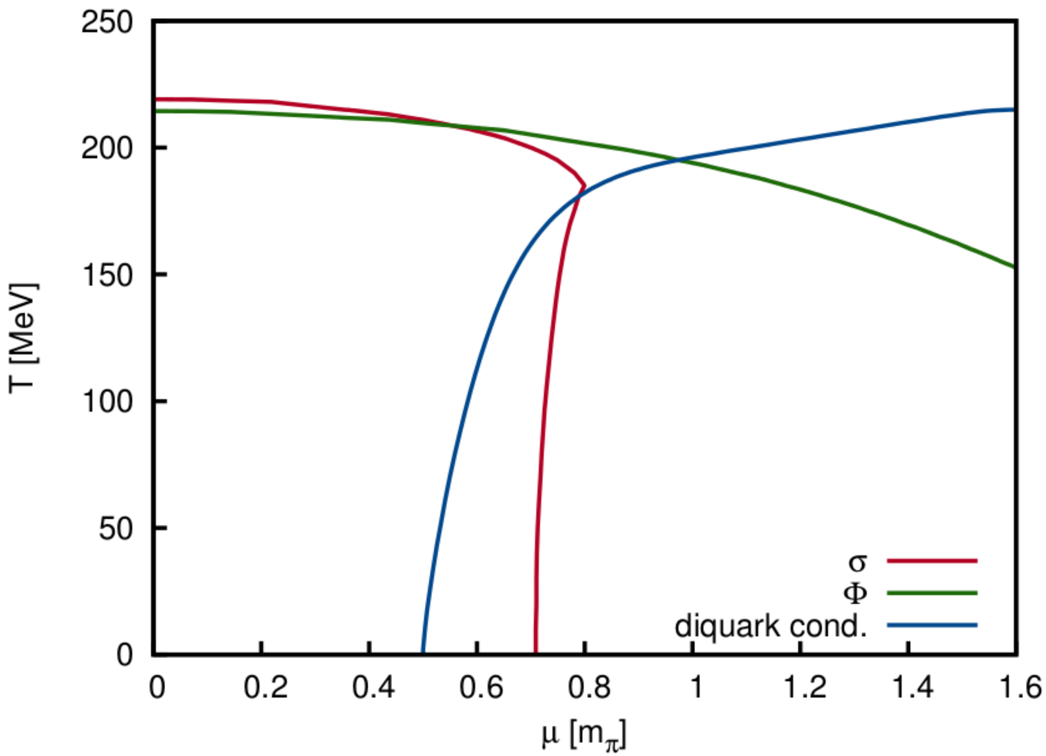
Outlook

- Include diquark and baryonic dof in QCD



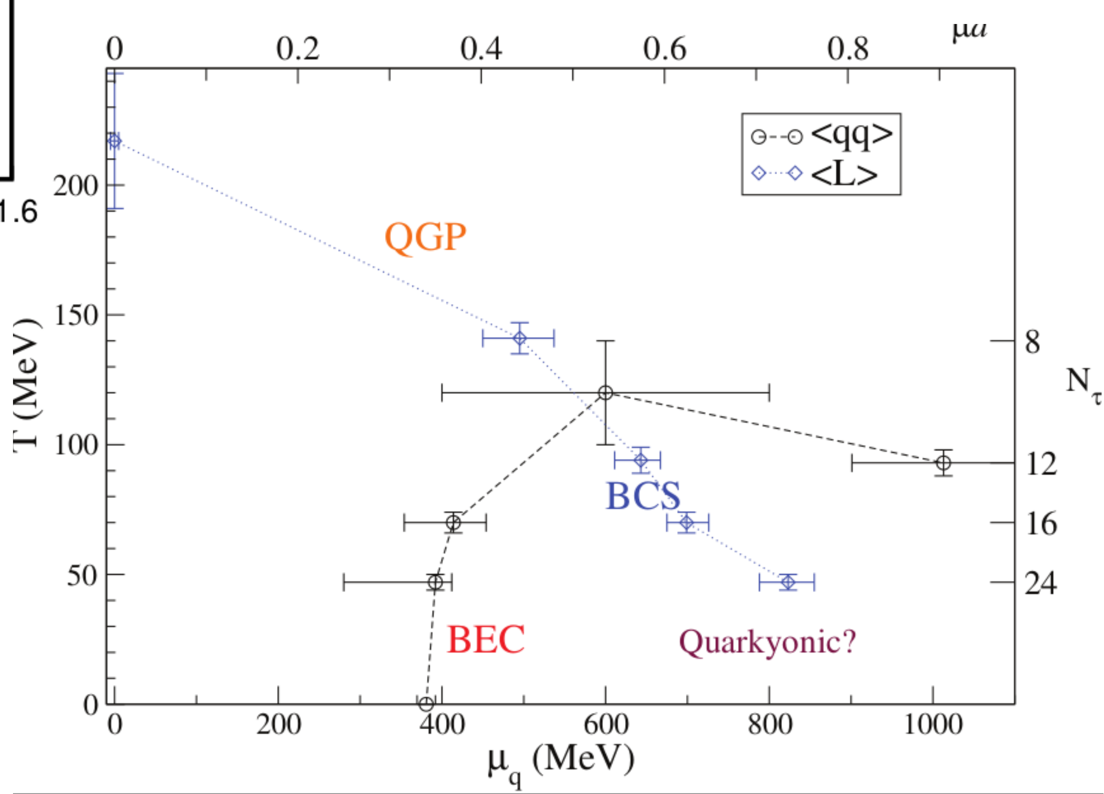
Thank you for your attention!

Confinement



[Strodthoff, von Smekal '14]

[Hands '12]



Features of QC$_2$D

[Kogut *et* al '99]

- Pseudoreality of $SU(2)_c$ gauge group generators:

$$t_a^* = t_a^T = -t_2 t_a t_2$$

- Antiunitarity of Dirac operator: $D^* = -t_2 C\gamma_5 D \gamma_5 C t_2$

No sign problem

- Extended flavor symmetry:

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \to SU(2N_f)$$

$$\mathcal{L}_{\mathrm{QC_2D}} = \psi_L^\dagger i\sigma_\mu D_\mu \psi_L + \psi_R^\dagger i\sigma_\mu^\dagger D_\mu \psi_R = \Psi^\dagger i\sigma_\mu D_\mu \Psi$$

$$\Psi = \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix}, \qquad \tilde{\psi}_R = \sigma_2 t_2 \psi_R^*$$

- allows us to rotate $\psi_L \to \tilde{\psi}_R$, similarly $\langle\bar{\psi}\psi\rangle \to \langle\psi\psi\rangle$

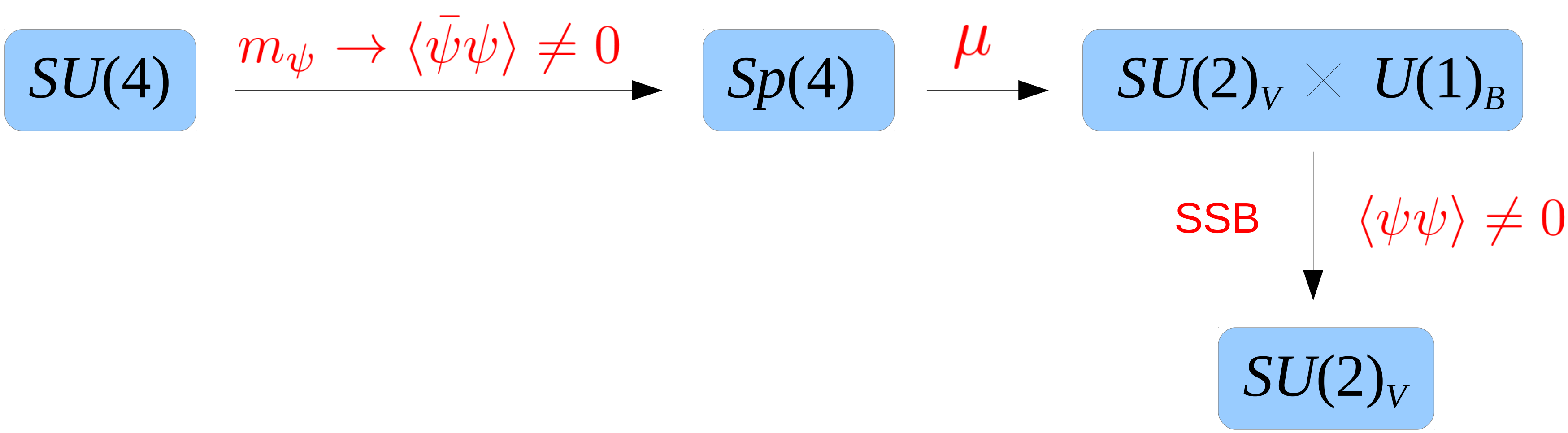
Features of QC_2D

[Kogut *et* al '99]

- 2 flavors: $SU(4)$

$$\mathcal{L}_{\mathrm{QC_2D}} = \Psi^\dagger i\sigma_\mu D_\mu \Psi \,, \qquad \Psi = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix}$$

- Symmetry breaking pattern:



Quark-Meson-Diquark Model

- Order parameter Potential: $O(6)$ + explixit breaking terms

$$U = V\left(\vec{\pi}^2 + \sigma^2 + \Delta_1^2 + \Delta_2^2\right) - c\sigma - \mu^2|\Delta|^2$$

$\sim m_\psi \bar{\psi}\psi$

Quark-Meson-Diquark Model

- Order parameter Potential: $O(6)$ + explixit breaking terms

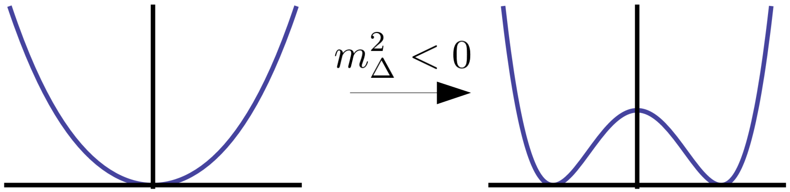
$$U = V\left(Z_\varphi(\vec{\pi}^2 + \sigma^2) + Z_\Delta(\Delta_1^2 + \Delta_2^2)\right) - cZ_\varphi\sigma - \mu^2 Z_\Delta|\Delta|^2$$

- Global minimum determines condensates: $\langle\sigma\rangle, \langle|\Delta|\rangle$

Normal phase: $m^2 - \mu^2 > 0\,, \quad \langle\sigma\rangle = \frac{c}{m^2}\,, \quad \langle|\Delta|\rangle = 0$

Superfluid phase: $\underbrace{m^2 - \mu^2}_{m_\Delta^2} < 0\,, \quad \langle\sigma\rangle = \frac{c}{\mu^2}\,, \quad \langle|\Delta|\rangle \neq 0$

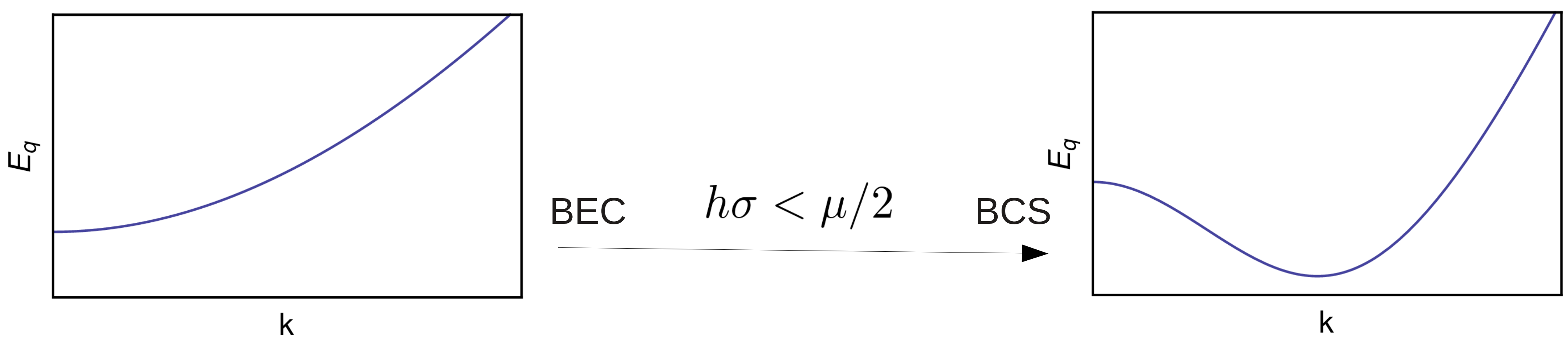
m: mass parameter in U



BEC-BCS Crossover

- Energy dispersion of quarks:

$$E_q = \sqrt{\left(\sqrt{k^2 + h^2\sigma^2} - \mu/2\right)^2 + h^2\Delta^2}$$

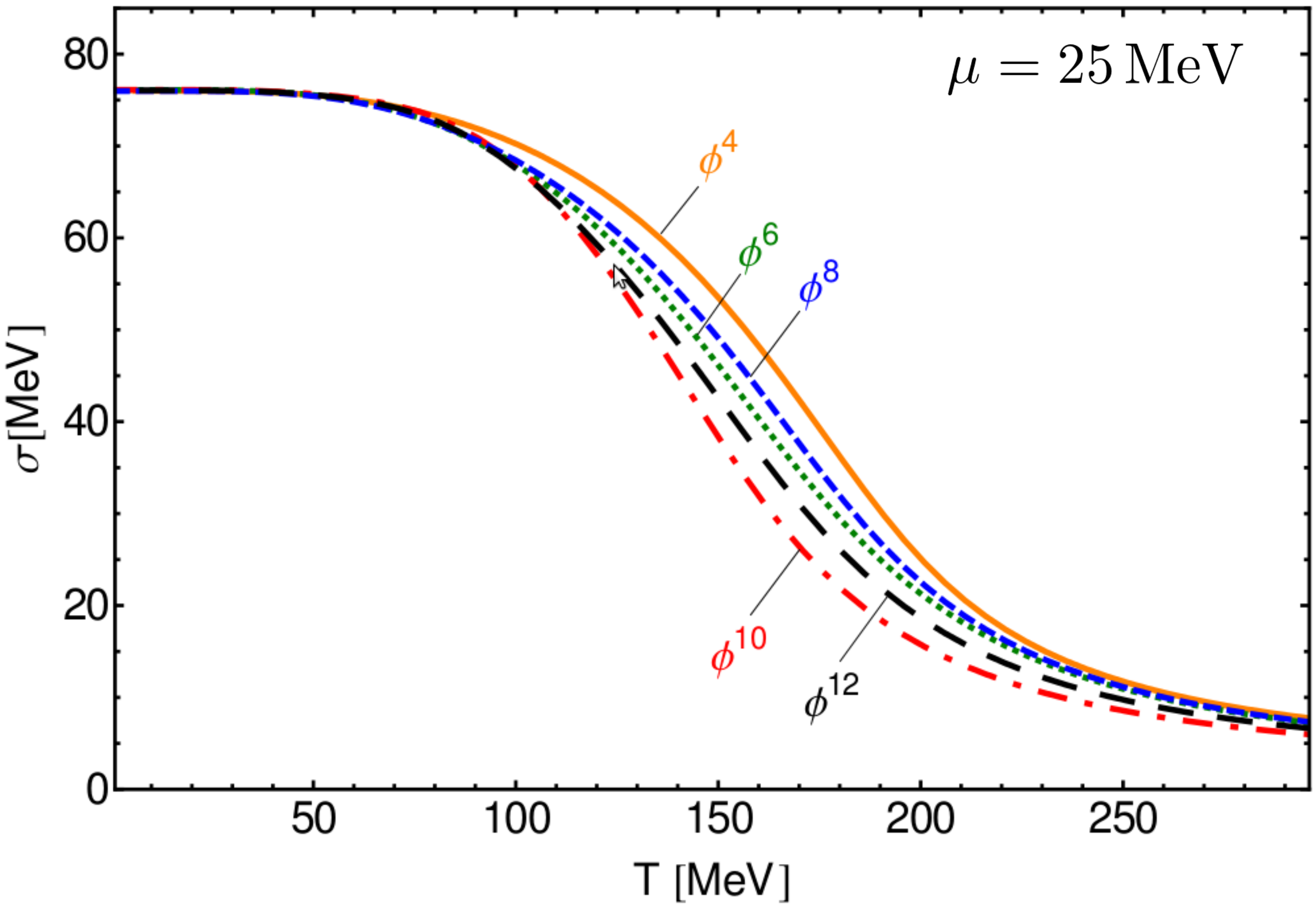


- Rule of thumb: Minimum of E_q goes from zero to nonzero

[L. He ’10]

Effects of Fluctuations

- Higher orders in $V(\phi^2)$

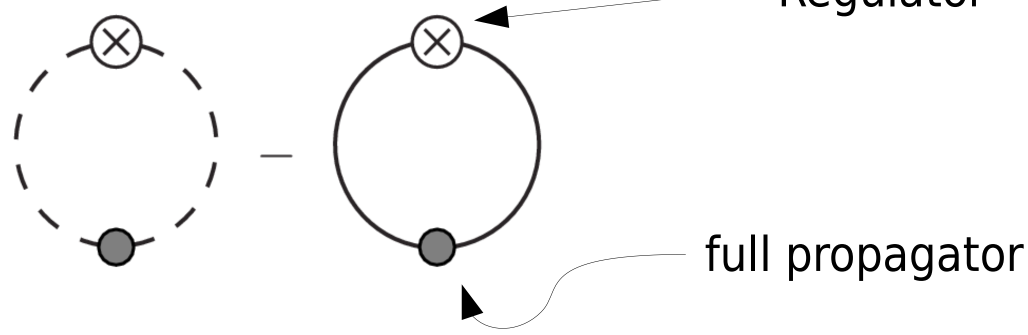


- Convergence not satisfactory
- Convergence of Taylor-expansion in QCD much better

→ **[Rennecke '14]**

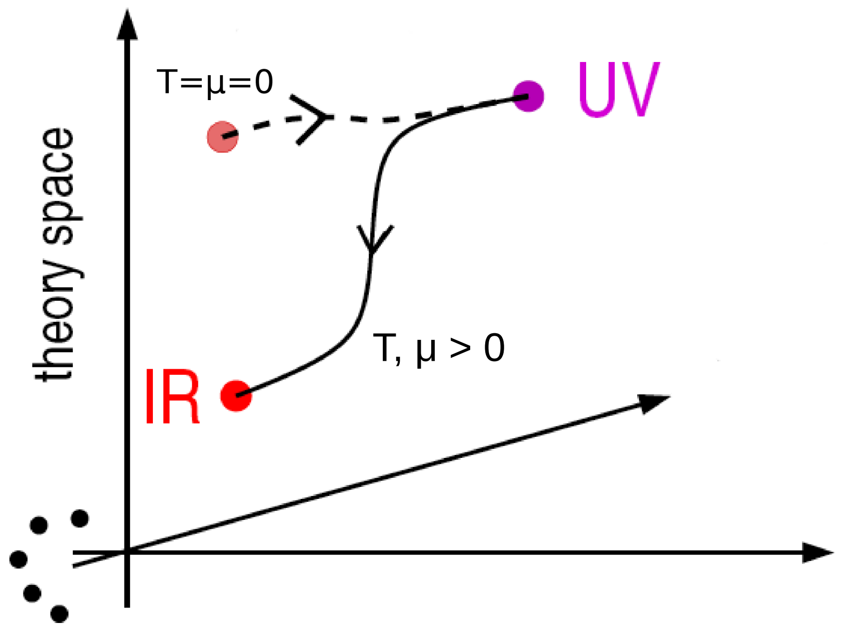
Functional Renormalization Group

- Integrate out fluctuations: $\Phi = (\phi, \psi, \bar{\psi})$

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}$$ 

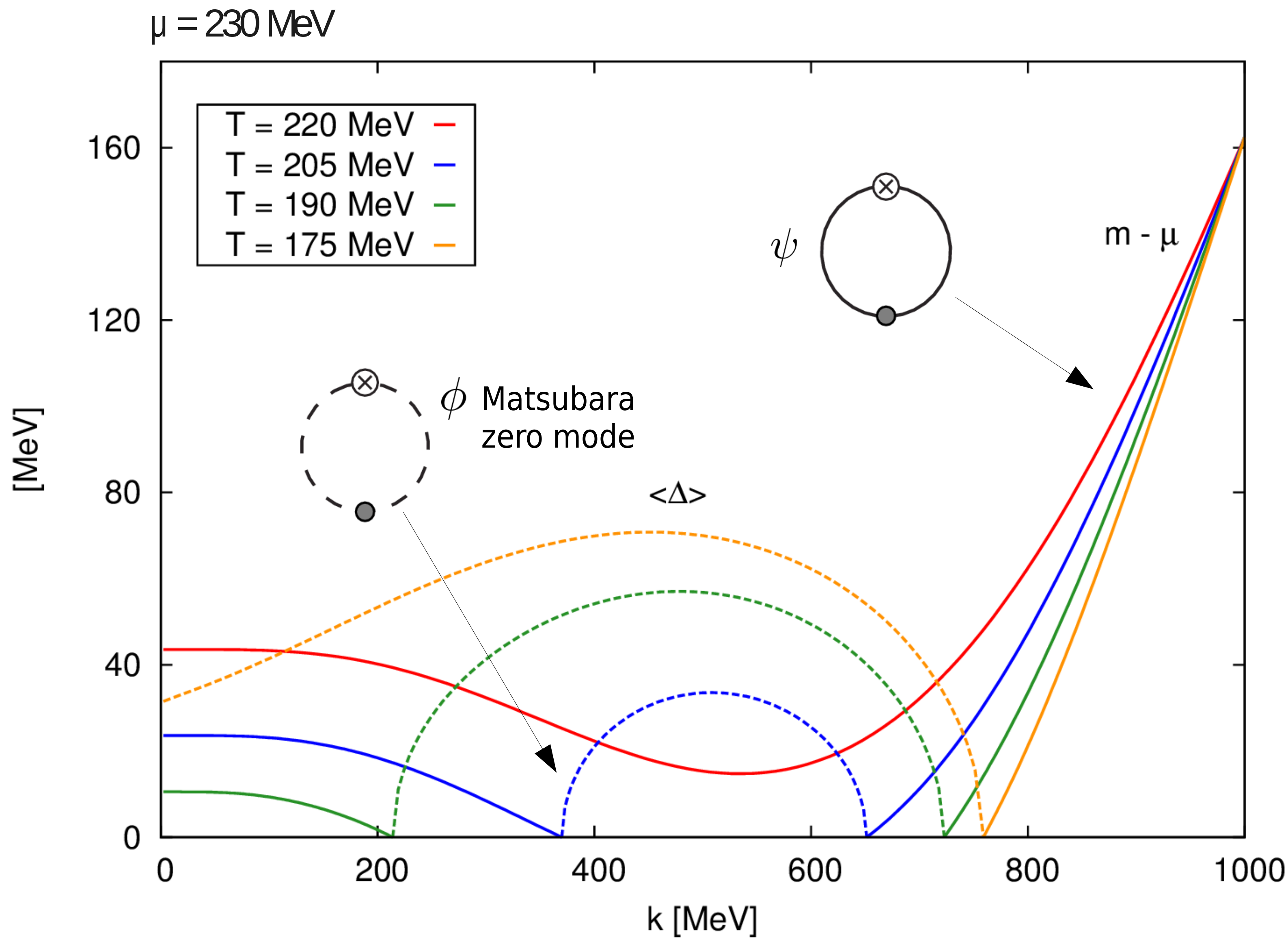
- Fix vacuum conditions:

$$f_\pi,\ m_\pi,\ m_\sigma,\ m_q,$$



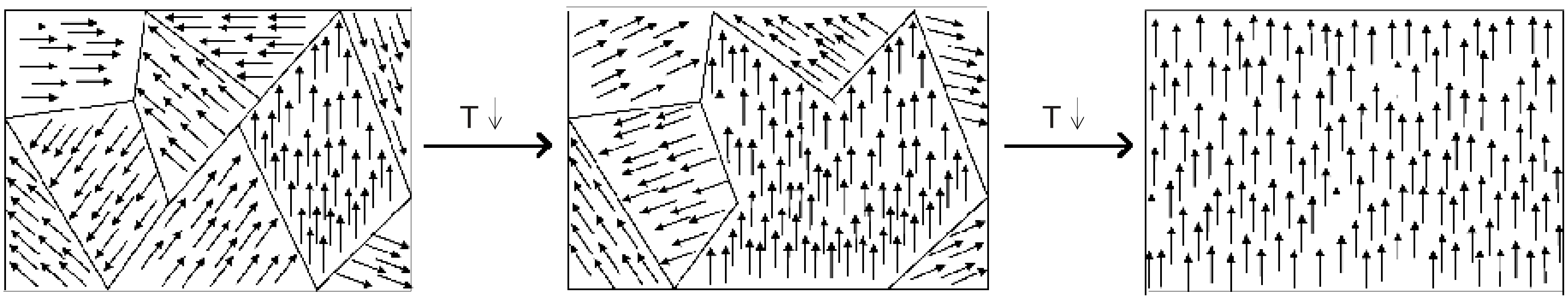
[Diehl *et al* '10]

Precondensation

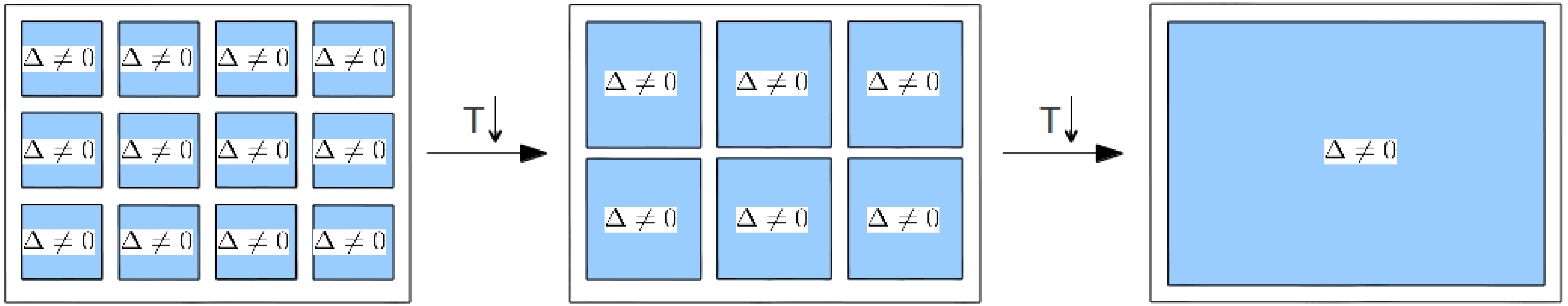


Precondensation

- Magnetic domains:

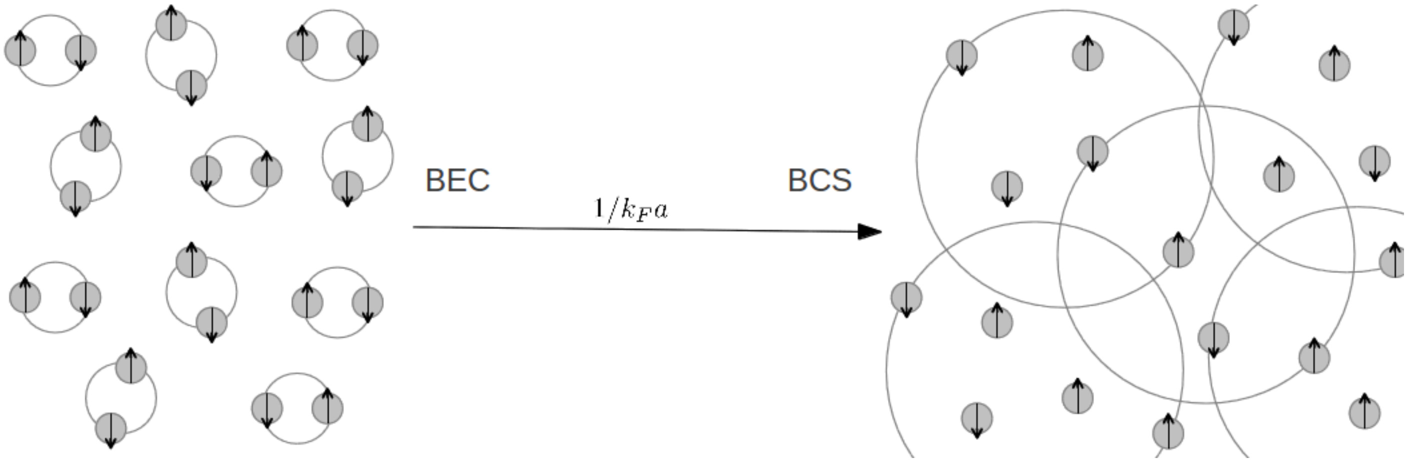


- Domains of BEC:



BEC-BCS Crossover

- In cold atoms



- In QC_2D