Generalized Susceptibilities at Finite Volume

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Der Wissenschaftsfonds.

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Locating the critical end-point

- Experiments are built up in a finite volume and finite time
- Correlation length does not diverge!
- Higher order generalized susceptibilities

$$\chi_{n_i,n_j,n_k} = -\frac{\partial^{n_i}}{\partial(\mu_i/T)^{n_i}}\frac{\partial^{n_j}}{\partial(\mu_j/T)^{n_j}}\frac{\partial^{n_k}}{\partial(\mu_k/T)^{n_k}}\left(\frac{U}{T^4}\right)$$

- Higher order cumulants

$$R_{n,2} = \frac{\chi_n}{\chi_2}$$

- Hadron resonance gas model yields positive $R_{n,2}$
- Negative values of $R_{n,2}$ indicate criticality
- $N_f = 2$ quark-meson model

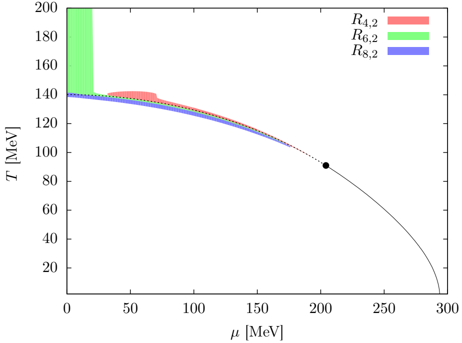
Including fluctuations

- Mean-Field grand potential

$$U_{MFA} = U_B(\sigma, \vec{0}) + \nu T \int \frac{d^3p}{(2\pi)^3} \left(\ln[1 - n_q(T,\mu)] + \ln[1 - n_{\bar{q}}(T,\mu)] \right)$$
$$- \nu \int_0^\Lambda \frac{d^3p}{(2\pi)^3} E_q$$

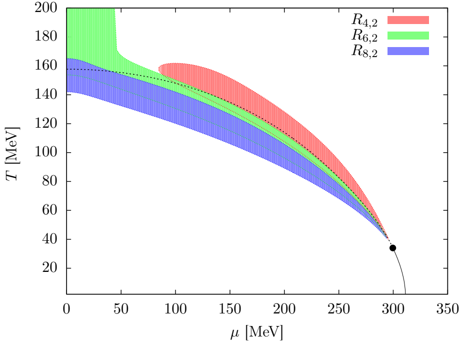
- $\nu = 2N_c N_f$
- standard MFA: only thermal quark fluctuations ($\Lambda = 0$)
- extended MFA: finite Λ
- renormalized MFA: $\Lambda \to \infty$
- Flow equation enables inclution of meson fluctuations
- Wetterich equation with Litim regulator
- Leading order of derivative expansion
- Wave-function renormalizations $Z_\psi = Z_\phi = 1$

Phase diagram in infinite volume



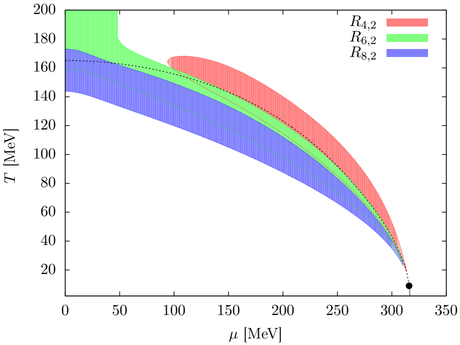
- standard MFA
- Vacuum meson masses $m_\pi = 138$ MeV and $m_\sigma = 550$ MeV

Phase diagram in infinite volume



- extended MFA with cutoff $\Lambda = 0.5$ GeV
- Vacuum meson masses $m_\pi = 138$ MeV and $m_\sigma = 550$ MeV
- CEP moves to higher μ and lower T

Phase diagram in infinite volume



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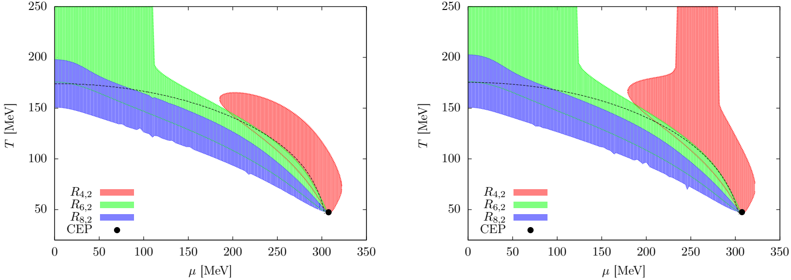
Phase diagram in infinite volume



- RG result
- Two UV cutofffs, $\Lambda_1 = 1.2$ GeV (left) and $\Lambda = 0.95$ GeV (right)

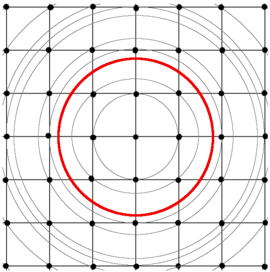
Finite volume

- Meson fields

$$\phi(x+L) = \phi(x)$$

- Periodic momenta

$$\vec{q}_p^{\,2} = \frac{(2\pi)^2}{L^2} \sum_{i=1}^{3} n_i^2$$

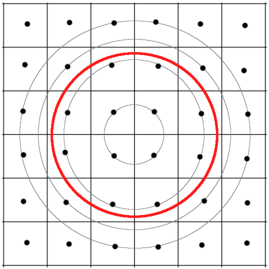


- For quarks

$$\psi(x+L) = \pm\psi(x)$$

- Antiperiodic momenta when $\psi(x+L) = -\psi(x)$

$$\vec{q}_{ap}^{\,2} = \frac{(2\pi)^2}{L^2} \sum_{i=1}^{3} \left(n_i + \frac{1}{2}\right)^2$$



Flow in finite volume

- Mode counting functions due to Litim regulator

$$\mathcal{B}_{p,ap}(kL) = \frac{6\pi^2}{(kL)^3} \sum_{n_i \in \mathbb{Z}} \Theta\left[k^2 - \vec{q}^{\,2}_{p,ap}\right]$$

$$\partial_t U_k(T,\mu) = \frac{k^5}{12\pi^2}\Bigg[\mathcal{B}_p(kL)\left(\frac{1}{E_\sigma}\coth\frac{E_\sigma}{2T} + \frac{3}{E_\pi}\coth\frac{E_\pi}{2T}\right) - \mathcal{B}_{p,ap}(kL)\left(\frac{2N_cN_f}{E_q}\tanh\frac{E_q-\mu}{2T} + \frac{2N_cN_f}{E_q}\tanh\frac{E_q+\mu}{2T}\right)\Bigg]$$

- Infinite volume limit

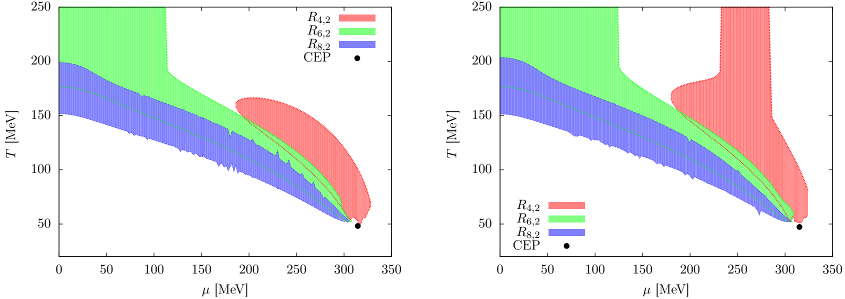
$$\lim_{L\to\infty} \mathcal{B}_{p,ap}(kL) = 1$$

- Small volume limit

$$\lim_{kL\to 0} \mathcal{B}_p(kL) \sim \frac{1}{(kL)^3} \qquad \lim_{kL\to 0} \mathcal{B}_{ap}(kL) = 0$$

Finite volume, periodic boundary conditions

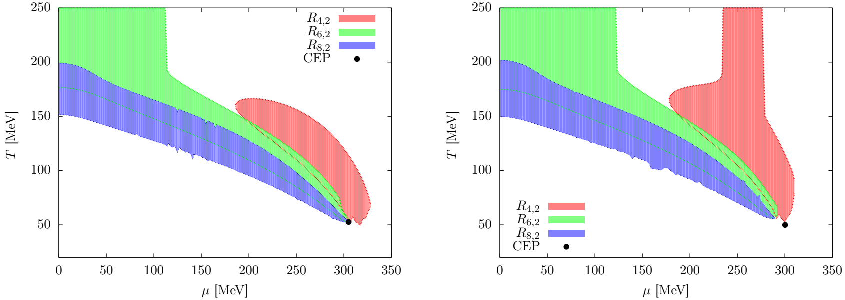
- Preliminary results



- $L = 5$ fm
- UV cutoffs: $\Lambda_1 = 1.2$ GeV (left) and $\Lambda_2 = 0.95$ GeV (right)

Finite volume, antiperiodic boundary conditions

- Preliminary results



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Summary and outlook

- Including the meson fluctuations broadens the negative regions of $R_{n,2}$
- $L = 5$ fm and different boundary conditions
- Cutoff influence on $R_{4,2}$

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Thank you for your attention!