Yang-Mills correlation functions from Dyson-Schwinger equations

Markus Q. Huber

in collaboration with Anton K. Cyrol, Lorenz von Smekal

Institute of Physics, University of Graz

7th International Conference on the Exact Renormalization Group, Lefkada

HIC for FAIR
Helmholtz International Center

MARIE CURIE ACTIONS

Correlation functions of QCD

Investigate:

- Confinement
- Dynamical symmetry breaking
- Bound states

(Non-perturbative) Determination of correlation functions:

- Lattice
- Effective theories, e.g., (refined) Gribov-Zwanziger, massive Yang-Mills
- Functional equations
- . . .

Functional equations and truncations

Functional equations...

- ... are exact.
- ... form an infinite system of equations.

Truncating the system: How close/far from exact solution?

- Calculate physical quantities.
- Compare to other methods.
- Modify the truncation.

Functional equations and truncations

Functional equations...

- ... are exact.
- ... form an infinite system of equations.

Truncating the system: How close/far from exact solution?

- Calculate physical quantities.
- Compare to other methods. ☺
- Modify the truncation.

Functional equations and truncations

Functional equations. . .

- . . . are exact.
- . . . form an infinite system of equations.

Truncating the system: How close/far from exact solution?

- Calculate physical quantities.
- Compare to other methods. ☺ If available. ☹
- Modify the truncation.

Functional equations and truncations

Functional equations...

- ... are exact.
- ... form an infinite system of equations.

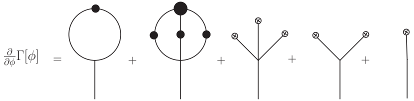
Truncating the system: How close/far from exact solution?

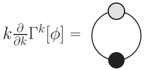
- Calculate physical quantities.
- Compare to other methods. ☺ If available. ☹
- Modify the truncation.

Recent results indicate that primitively divergent correlation functions of Landau gauge Yang-Mills theory provide reasonable truncation for a quantitative, self-consistent and self-contained description.

Comparison: DSEs and flow equations

Dyson-Schwinger equations (DSEs)	Functional RG equations (FRGEs)
'integrated flow equations'	'differential DSEs'
effective action $\Gamma[\phi]$	effective average action $\Gamma^k[\phi]$
–	regulator
n-loop structure (n *const*.)	1-loop structure
exactly only bare vertex per diagram	no bare vertices





- Both systems of equations are exact
- Both contain infinitely many equations.

Outline

- Vacuum
  - Propagators: Introduction
  - Four-gluon vertex
- Non-vanishing temperature
  - Ghost-gluon vertex
  - Three-gluon vertex

Landau Gauge Yang-Mills theory

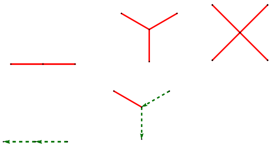
Gluonic sector of quantum chromodynamics: Yang-Mills theory

$$\mathcal{L} = \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$

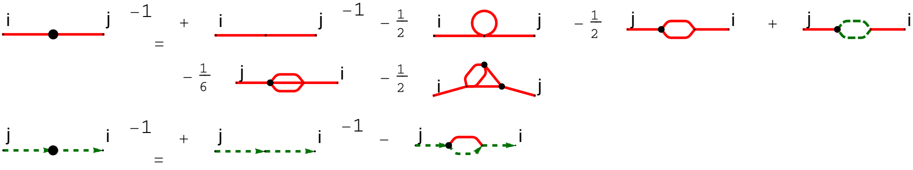
$$F_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + i\,g\,[\mathbf{A}_\mu, \mathbf{A}_\nu]$$

Landau gauge

- simplest one for functional equations
- $\partial_\mu \mathbf{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \mathbf{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\mathbf{c}}\,(-\Box + g\,\mathbf{A}\,\times)\,\mathbf{c}$

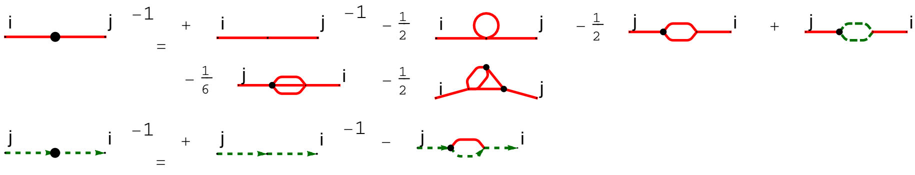


Propagators



Models **or** results for vertices required.

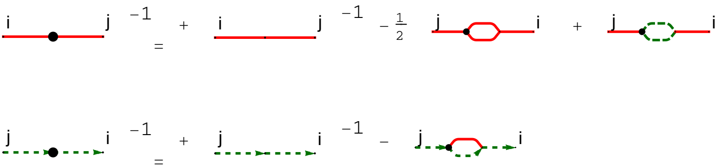
Propagators



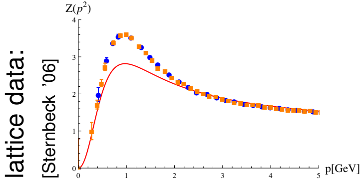
Models **or** results for vertices required.

(Tadpole vanishes perturbatively, but can contribute non-perturbatively [MQH, von Smekal '14].)

Propagators



Typical truncation:
no four-gluon vertex, bare ghost-gluon vertex, model for three-gluon vertex

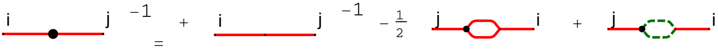


one-loop truncation

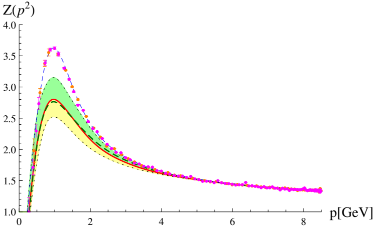
Comparison with lattice results
→ missing strength in mid-momentum regime; attributed to

- neglected two-loop diagrams?
- vertices?

Propagators



Typical truncation:
no four-gluon vertex, bare ghost-gluon vertex, model for three-gluon vertex

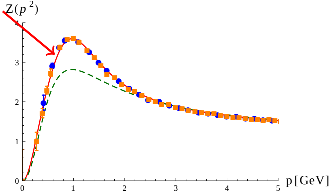


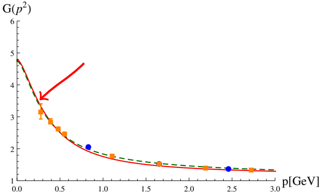
→ Using results from a three-gluon vertex calculation, importance of two-loop diagrams shown.

[Blum, MQH, Mitter, von Smekal '13]

Propagator results

Dynamic ghost-gluon vertex, opt. eff.
three-gluon vertex [MQH, von Smekal '13]

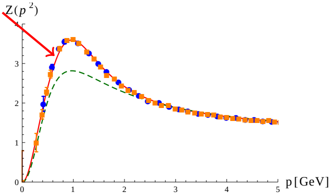


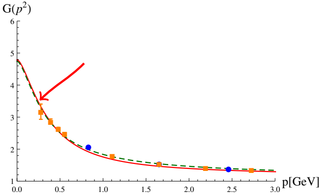


Good quantitative agreement for ghost *and* gluon dressings. ⇒ Input for further calculations.

Propagator results

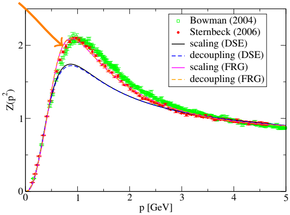
Dynamic ghost-gluon vertex, opt. eff. three-gluon vertex [MQH, von Smekal '13]

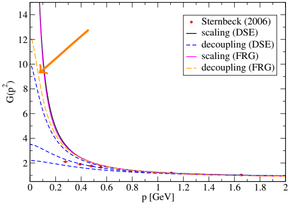




FRG results

[Fischer, Maas, Pawlowski '08]

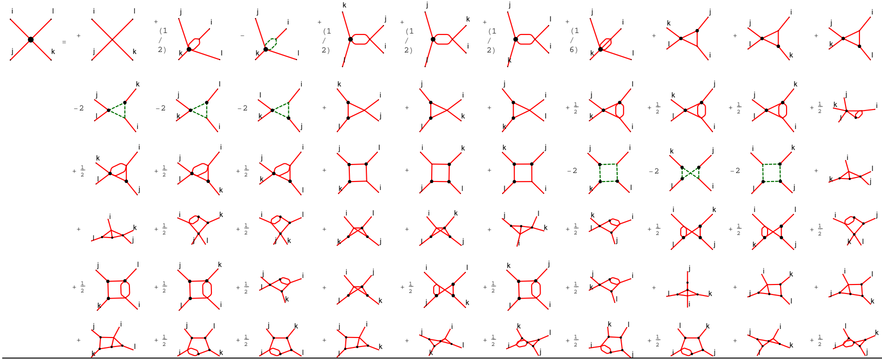




Good quantitative agreement for ghost *and* gluon dressings. ⇒ Input for further calculations.

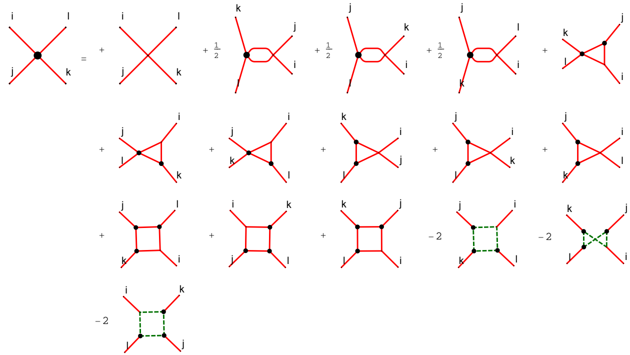
Four-gluon vertex

- 20 one-loop, 39 two-loop diagrams



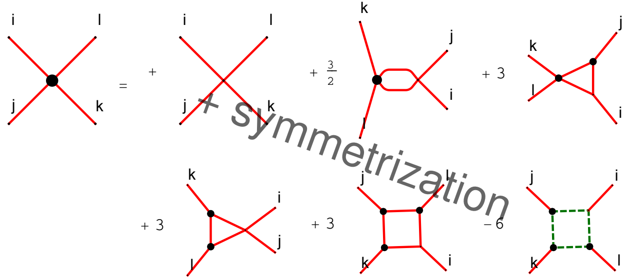
Four-gluon vertex

- 20 one-loop, 39 two-loop diagrams
- Keep UV leading diagrams $\rightarrow$ 16 diagrams



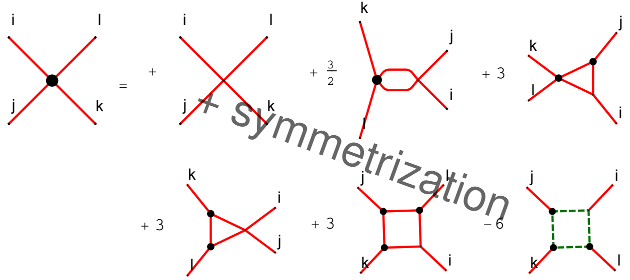
Four-gluon vertex

- 20 one-loop, 39 two-loop diagrams
- Keep UV leading diagrams $\rightarrow$ 16 diagrams
- Calculate **full** momentum dependence.
  $\rightarrow$ Access to all permutations of this diagram. $\rightarrow$ 6 diagrams



Four-gluon vertex

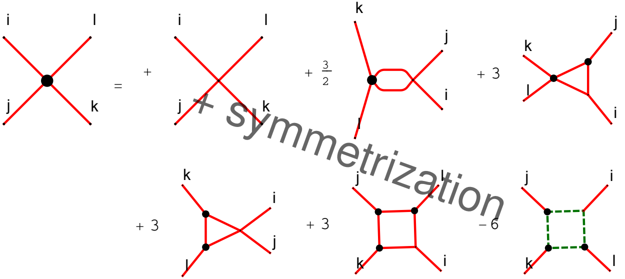
- 20 one-loop, 39 two-loop diagrams
- Keep UV leading diagrams $\rightarrow$ 16 diagrams
- Calculate **full** momentum dependence.
  $\rightarrow$ Access to all permutations of this diagram. $\rightarrow$ 6 diagrams



No dependence on unknown Green functions! $\rightarrow$ 'Truncation closes.'

Four-gluon vertex

- 20 one-loop, 39 two-loop diagrams
- Keep UV leading diagrams $\rightarrow$ 16 diagrams
- Calculate **full** momentum dependence.
  $\rightarrow$ Access to all permutations of this diagram. $\rightarrow$ 6 diagrams



- 6 external variables
- 4 integration variables

No dependence on unknown Green functions! $\rightarrow$ 'Truncation closes.'

Tensor basis

Full calculation: Restriction to tree-level tensor:

$$\Gamma^{abcd}_{\mu\nu\rho\sigma}(p,q,r,s) = \Gamma^{(0),abcd}_{\mu\nu\rho\sigma} D^{4g}(p,q,r,s).$$

Non-perturbative information in $D^{4g}(p,q,r,s)$.

Derivation of DSE using *DoFun* [Braun, MQH '11]:
Mathematica package for derivation of functional equations

Tensor basis

Full calculation: Restriction to tree-level tensor:

$$\Gamma^{abcd}_{\mu\nu\rho\sigma}(p,q,r,s) = \Gamma^{(0),abcd}_{\mu\nu\rho\sigma} D^{4g}(p,q,r,s).$$

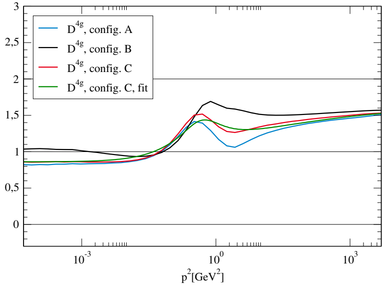
Non-perturbative information in $D^{4g}(p,q,r,s)$.

Derivation of DSE using *DoFun* [Braun, MQH '11]:
Mathematica package for derivation of functional equations

Other DSE results (configuration **A** only):

- Box-only truncation: [Kellermann, Fischer '08]
- Same truncation as here, semi-perturbative approximation (1 iteration): [Binosi, Ibáñez, Papavassiliou '14]

Four-gluon vertex results

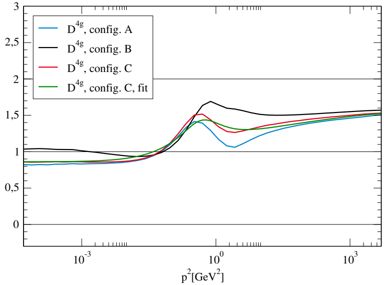


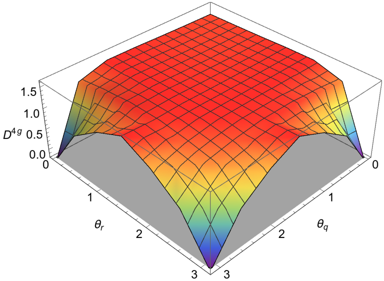
[Cyrol, MQH, von Smekal '14]

2-parameter fit:

$$D^{4g,\,\text{dec}}_{\text{model}}(p, q, r, s) = \left(a \tanh\left(b/\bar{p}^2\right) + 1\right) D^{4g}_{\text{RG}}(p, q, r, s)$$

Four-gluon vertex results





[Cyrol, MQH, von Smekal '14]

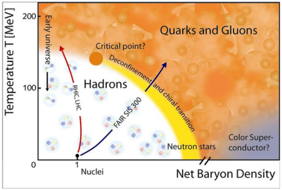
2-parameter fit:

$$D^{4g,\,dec}_{model}(p,\,q,\,r,\,s) = \left(a \tanh\left(b/\bar{p}^2\right) + 1\right) D^{4g}_{RG}(p,\,q,\,r,\,s)$$

Non-vanishing temperature

- Phase diagram: (dual) quark condensates, Polyakov loop potential, . . .
- Can be calculated from correlation functions!

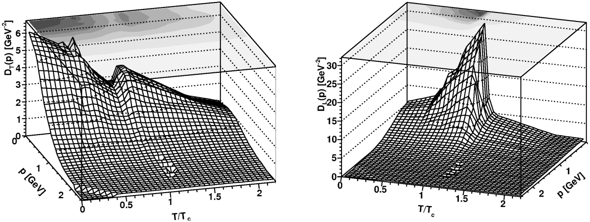
Related talks: Fister, Juricic, Lücker, Kamikado, Khan, Mitter, Müller, Pawlowski, Rechenberger, Rennecke, Schaefer, Springer, Strodthoff, Weise, Yamada, . . .



- For now: Yang-Mills theory
- Goal: Calculate quantities that are difficult to obtain on the lattice.

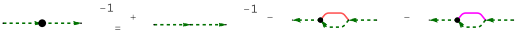
Propagators

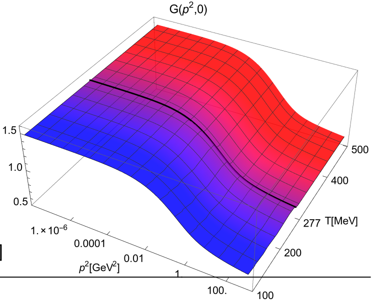
Chromomagnetic and chromoelectric gluons from the lattice [Fischer, Maas, Müller '10]:



→ Input for DSEs

Ghost dressing $G(p^2)$ from DSE [MQH, von Smekal '13]:





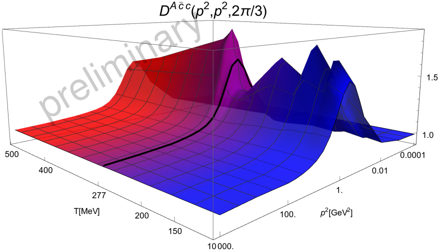
Propagators from

FRG: [Fister, Pawlowski '11]

Massive Yang-Mills: [Reinosa, Serreau, Tissier, Wschebor '13]

Three-point warm-up: Ghost-gluon vertex

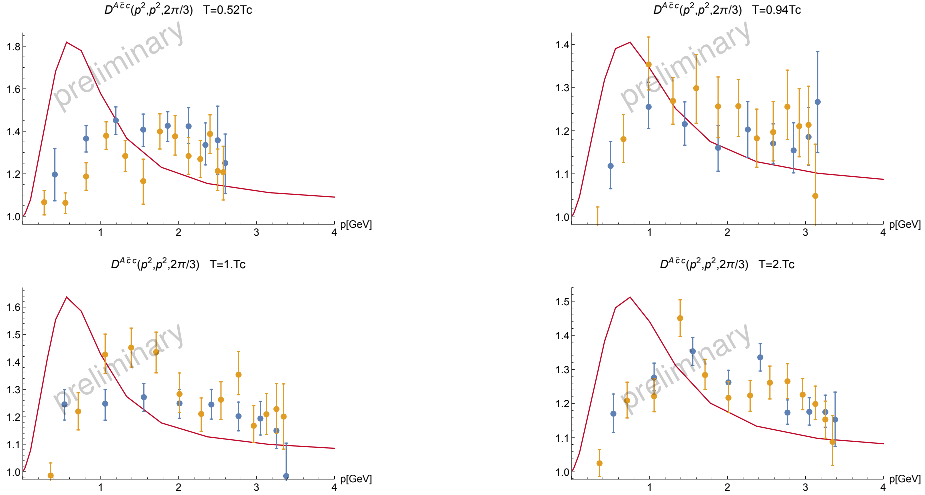
DSE calculation: self-consistent solution of truncated DSE, zeroth Matsubara frequency only



- Vertices quite expensive on lattice.
- Full momentum dependence from functional equations.

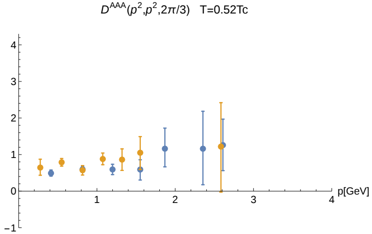
Vertex from FRG: [Fister, Pawlowski '11]

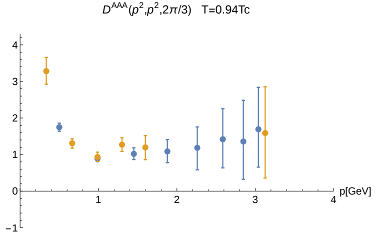
Ghost-gluon vertex: Continuum and lattice

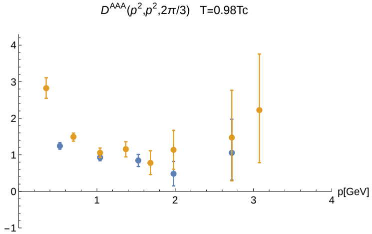


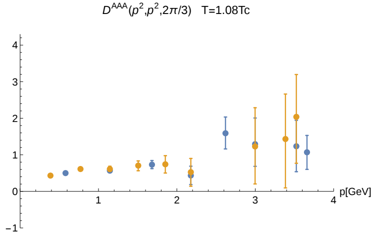
Lattice: [Fister, Maas '14]

Three-gluon vertex: Continuum and lattice



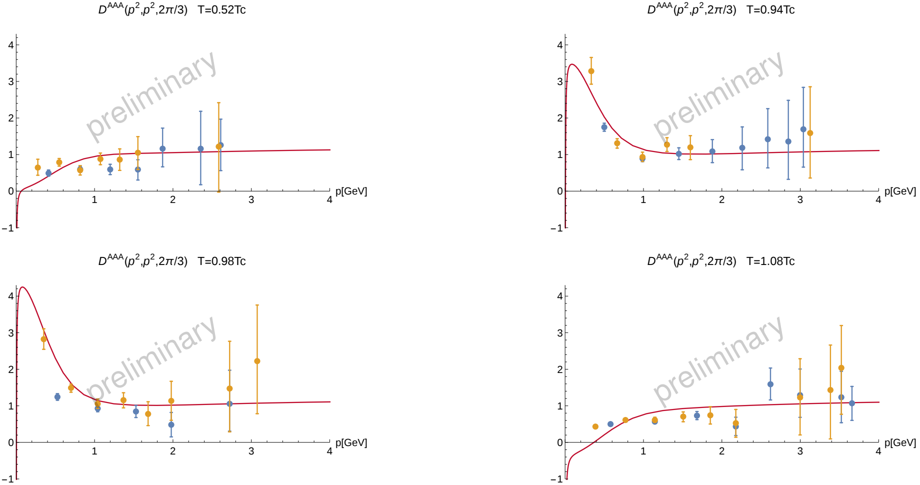






Lattice: [Fister, Maas '14]

Three-gluon vertex: Continuum and lattice

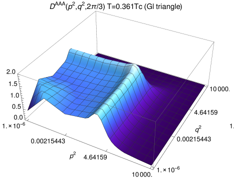


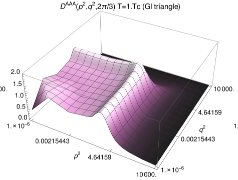
Lattice: [Fister, Maas '14]

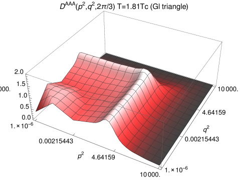
Three-gluon vertex: Diagram contributions

Diagram contributions (T = 0: 6, $T > 0$: 22 diagrams):

- Only chromomagnetic propagators: dominant







Three-gluon vertex: Diagram contributions

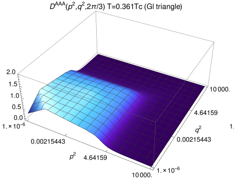
Diagram contributions ($T = 0$: 6, $T > 0$: 22 diagrams):

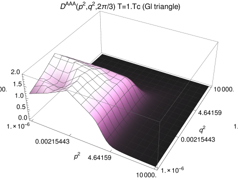
- Only chromomagnetic propagators: dominant
- Both propagators contained: negligible

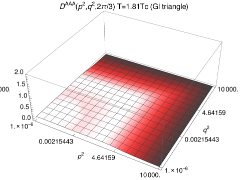
Three-gluon vertex: Diagram contributions

Diagram contributions (T = 0: 6, $T > 0$: 22 diagrams):

- Only chromomagnetic propagators: dominant
- Both propagators contained: negligible
- Only chromoelectric propagators: Importance depends on temperature, especially for $T < T_c$.

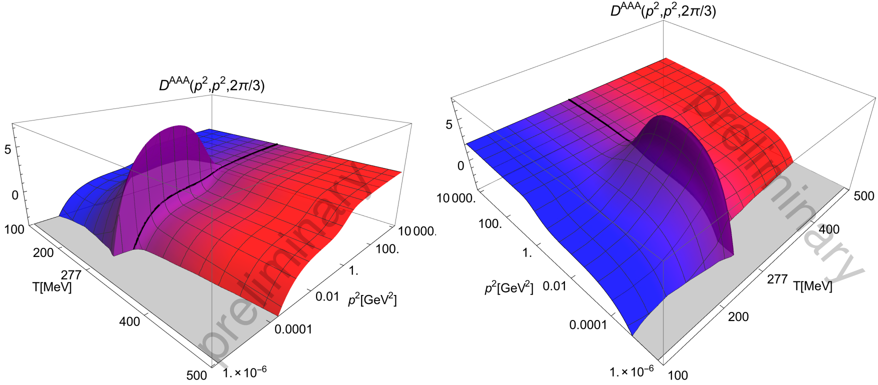






Three-gluon vertex

DSE calculation: semi-perturbative approximation (first iteration only)



Summary

- Conjecture:
  Primitively divergent correlation functions sufficient for a
  quantitative, self-consistent and self-contained description.
- Truncation naturally 'closes' with four-gluon vertex.
- Two-point functions: Quantitative effects understood.
- Three-point functions: Good agreement with lattice.
- Four-point functions: Test four-gluon vertex in DSEs of lower n-point functions.

Summary

- Conjecture:
  Primitively divergent correlation functions sufficient for a quantitative, self-consistent and self-contained description.
- Truncation naturally 'closes' with four-gluon vertex.
- Two-point functions: Quantitative effects understood.
- Three-point functions: Good agreement with lattice.
- Four-point functions: Test four-gluon vertex in DSEs of lower n-point functions.

Vertices at non-vanishing temperature:

- Qualitative agreement with lattice results
- Source of behavior of three-gluon vertex $\sim T_c$ elucidated
- Basis for model building
- Effects of dressed vertices, e.g., in Polyakov loop potential?
- Basis for extension to QCD and $\mu > 0$

Summary

- Conjecture:
  Primitively divergent correlation functions sufficient for a quantitative, self-consistent and self-contained description.
- Truncation naturally 'closes' with four-gluon vertex.
- Two-point functions: Quantitative effects understood.
- Three-point functions: Good agreement with lattice.
- Four-point functions: Test four-gluon vertex in DSEs of lower n-point functions.

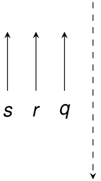
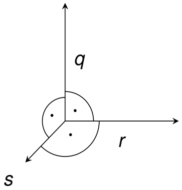
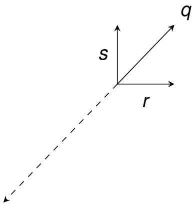
Vertices at non-vanishing temperature:

- Qualitative agreement with lattice results
- Source of behavior of three-gluon vertex $\sim T_c$ elucidated
- Basis for model building
- Effects of dressed vertices, e.g., in Polyakov loop potential?
- Basis for extension to QCD and $\mu > 0$

Thank you for your attention.

One-momentum configurations

6 independent variables: 3 momentum squares, 3 angles

Configuration	**A**	**B**	**C**
Definition	$S^2 = R^2 = Q^2 = p^2$ $\theta_r = \theta_q = \psi_q = 0$	$S^2 = R^2 = Q^2 = p^2$ $\theta_r = \theta_q = \psi_q = \frac{\pi}{2}$	$S^2 = R^2 = p^2, \quad Q^2 = 2p^2$ $\theta_r = \frac{\pi}{2}, \quad \theta_q = \frac{\pi}{4}, \quad \psi_q = 0$
Visualization			

Other dressing functions

By transverse projection and orthonormalization construct from

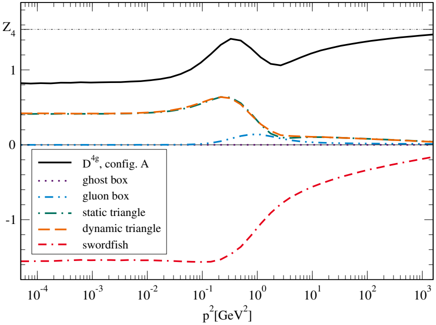
$$\widetilde{V}^{abcd}_{1,\mu\nu\rho\sigma} = \Gamma^{(0),abcd}_{\mu\nu\rho\sigma}$$
$$\widetilde{V}^{abcd}_{2,\mu\nu\rho\sigma} = \delta^{ab}\delta^{cd}\delta_{\mu\nu}\delta_{\rho\sigma} + \delta^{ac}\delta^{bd}\delta_{\mu\rho}\delta_{\nu\sigma} + \delta^{ad}\delta^{bc}\delta_{\mu\sigma}\delta_{\nu\rho}$$
$$\widetilde{V}^{abcd}_{3,\mu\nu\rho\sigma} = (\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})\delta_{\mu\nu}\delta_{\rho\sigma} + (\delta^{ab}\delta^{cd} + \delta^{ad}\delta^{bc})\delta_{\mu\rho}\delta_{\nu\sigma} + (\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd})\delta_{\mu\sigma}\delta_{\nu\rho}$$

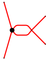
the tensors V_1, V_2, V_3.

Another class (4 momenta):

$$\widetilde{P}^{abcd}_{\mu\nu\rho\sigma} = (\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})\frac{s_\mu r_\nu q_\rho p_\sigma + r_\mu s_\nu p_\rho q_\sigma + q_\mu p_\nu s_\rho r_\sigma}{\sqrt{p^2\, q^2\, r^2\, p^2}}$$

Four-gluon vertex: Individual diagrams



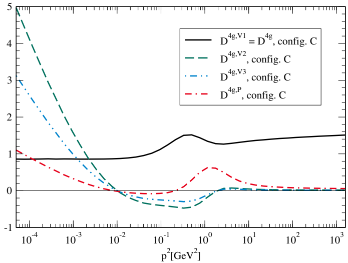
- Swordfish 
- Dynamic triangle 
- Static triangle 
- Gluon box 
- Ghost box 

In general the non-perturbative four-gluon vertex is very close to its tree-level.

Note: Renormalization important to get the correct weights of all diagrams.

Results for other dressing functions

- V_1 close to tree-level.
- Logarithmic IR divergence
- For V_2 and V_3 individual diagrams almost cancel each other.
- P small, but richer structure
- Not calculated self-consistently



[Cyrol, MQH, von Smekal '14]

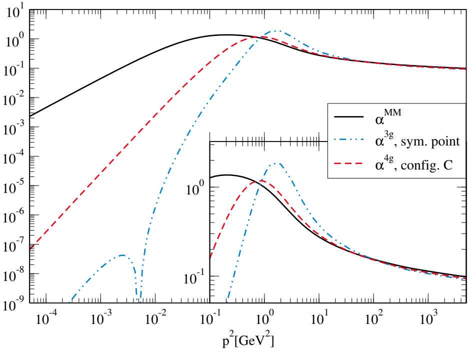
V_1: tree-level
V_2, V_3: only metric tensors, $\sim \delta_{\mu\nu}\delta_{\rho\sigma}$
P: four momenta, $\sim s_\mu r_\nu q_\rho p_\sigma$

Note: Renormalization important to get the correct weights of all diagrams.

Running couplings

Can be determined from

- ghost-gluon vertex $\rightarrow \alpha^{MM}$, cf. MiniMOM scheme
  [von Smekal, Alkofer, Hauck '97, von Smekal, Maltman, Sternbeck '09]
- all other vertices $\rightarrow \alpha^{3g}, \alpha^{4g}, \ldots$ [Alkofer, Llanes-Estrada, Fischer '05]



$$\alpha^{MM}(p^2) = \alpha(\mu^2) G(p^2)^2 Z(p^2)$$

$$\alpha^{3g}(p^2) = \alpha(\mu^2) \frac{\left[D^{4g}(p^2)\right]^2 Z^3(p^2)}{\left[D^{4g}(\mu^2)\right]^2 Z^3(\mu^2)}$$

$$\alpha^{4g}(p^2) = \alpha(\mu^2) \frac{D^{4g}(p^2) Z^2(p^2)}{D^{4g}(\mu^2) Z^2(\mu^2)}$$

[Blum, MQH, Mitter, von Smekal '14,
Cyrol, MQH, von Smekal '14]