Non-Fermi liquid vs (topological) Mott insulator in electronic systems with quadratic band touching in three dimensions

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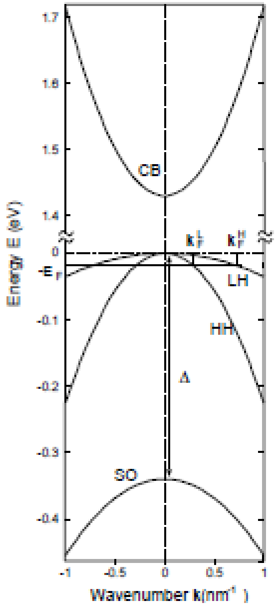


IH and Lukas Janssen,
Phys. Rev. Lett. 113, 106401 (2014)

ERG2014, Lefkada Island

Quadratic band crossing in 2D: (e. g. bilayer graphene)

Irreducible Hamiltonian: ($c = \cos(2\alpha), \; s = \sin(2\alpha)$)

$$H_0 = -\frac{p^2}{4m'}\mathbb{I} - \frac{p^2 c}{4m}\sigma_3 - \frac{s}{4m}\left[\sigma_1(p_x^2 - p_y^2) + \sigma_2 2p_x p_y\right]$$

With short-range interaction:

$$H = \int d\mathbf{r}\left[\mathbf{\Psi}^+(\mathbf{r})H\mathbf{\Psi}(\mathbf{r}) + U\delta n_1(\mathbf{r})\delta n_2(\mathbf{r})\right]$$

has an instability at weak coupling:

$$\frac{dU}{d\ln s} = U^2\rho_0 + O(U^3)$$

towards QAH (gapped) ($|\sin(2\alpha)| > \sqrt{2/3}$) or nematic (gapless) phase. (Sun et al, 2010, Dora, IH, Moessner, 2014)

Three dimensions: gapless semiconductors (gray tin, HgTe,…)

Luttinger spin-orbit Hamiltonian (J= 3/2) (Luttinger, PR 1956)

$$H = \frac{1}{2m}\left((\gamma_1 + \frac{5}{2}\gamma_2)k^2 - 2\gamma_2(\mathbf{k}\cdot\mathbf{S})^2\right)$$

with (twice degenerate) eigenvalues: (with full rotational symmetry)

$$E_L(k) = \frac{\gamma_1 + 2\gamma_2}{2m}k^2 \quad , \quad E_H(k) = \frac{\gamma_1 - 2\gamma_2}{2m}k^2$$

Density of states now vanishes at the QTP: short-range interactions are irrelevant, but there is no screening.

What is the effect of long-range Coulomb interaction?

Without the hole band empty, at ``zero” (low) density:

Wigner crystal !

With the hole band filled and particle band empty: the system is

``critical”

In the RG language, changing the cutoff causes the charge to ``flow”

$$\frac{de^2}{d\ln b} = (z+2-d)e^2 - 4e^4$$

with the dynamical critical exponent: $z = 2 - \frac{16}{15}e^2$

(Coulomb interaction $\sim 1/p^2$.) (Abrikosov, JETP 1974)

Below and near the upper critical dimension, $d_{up} = 4$, the system is in the non-Fermi liquid interacting phase, with the charge at the fixed point value:

$$e_*^2 = 15\epsilon/76 + \mathcal{O}(\epsilon^2)$$

with the small parameter

$$\epsilon = 4 - d$$

and the dynamical critical exponent $z < 2$.

This implies power-laws in various responses, such as specific heat:

$$c_v \sim T^{d/z} \approx T^{1.7}$$

(Abrikosov, JETP 1974, Moon, Xu, Kim, Balents, PRL 2014)

Easy way to get a NFL phase in 3D!

Or not?

The picture must somehow break down before the dimension reaches d = 2; a short range coupling flows like

$$\frac{dg_1}{d\ln b} = (z-d)g_1 \text{ + high. ord. term.}$$

and becomes marginal in d=2.

What can happen to the NFL stable fixed point?

The mechanism : collision of UV and IR fixed points (Kaveh, IH, 2005, Gies, Jaeckel 2006, Kaplan, Lee, Son, Stephanov, 2009). First we rewrite the Luttinger Hamiltonian as :

$$H(k) = \epsilon(\mathbf{k}) + \frac{\gamma_2}{m} d_a \Gamma^a$$

where, (Abrikosov, JETP 1974, Murakami et al, PRB 2004)

$$\begin{aligned}
&\epsilon(\mathbf{k}) = \frac{\gamma_1}{2m} k^2, \; d_a(\mathbf{k}) = -3\xi_a^{ij} k_i k_j, \\
&d_1 = -\sqrt{3} k_y k_z, \; d_2 = -\sqrt{3} k_x k_z, \; d_3 = -\sqrt{3} k_x k_y \\
&d_4 = -\frac{\sqrt{3}}{2}(k_x^2 - k_y^2), \\
&d_5 = -\frac{1}{2}(2k_z^2 - k_x^2 - k_y^2).
\end{aligned}$$

and the Dirac matrices satisfy:

$$\{\Gamma^a, \Gamma^b\} = 2\delta_{ab}$$

The full interacting theory, with long-range and short-range interactions is then: (IH and Lukas Janssen, PRL 2014)

$$L = \Psi^\dagger \left(\partial_\tau + ia + d_i(-i\nabla)\gamma_i\right)\Psi + g_1(\Psi^\dagger\Psi)^2 + g_2(\Psi^\dagger\gamma_i\Psi)^2 + \frac{1}{2e^2}(\nabla a)^2$$

and appears (but actually is not!) O(5) symmetric. Change of the cutoff now amounts to (to one loop)

$$\frac{dg_1}{d\ln b} = (z-d)g_1 - \frac{1}{2}g_1 g_2 - \frac{5}{2}g_2^2 - 4e^2 g_2$$

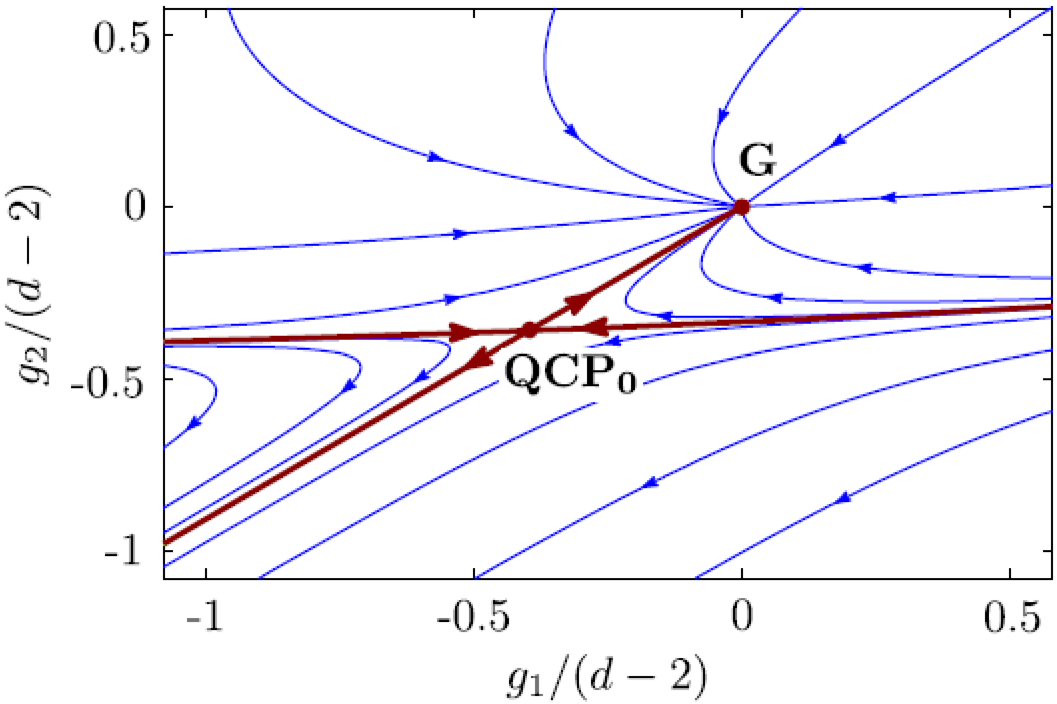
$$\frac{dg_2}{d\ln b} = (z-d)g_2 + \frac{2}{5}g_1 g_2 - \frac{1}{20}g_1^2 - \frac{63}{20}g_2^2 - \frac{4}{5}e^2 g_1 + \frac{16}{5}e^2 g_2 - \frac{16}{5}e^4$$

in addition to:

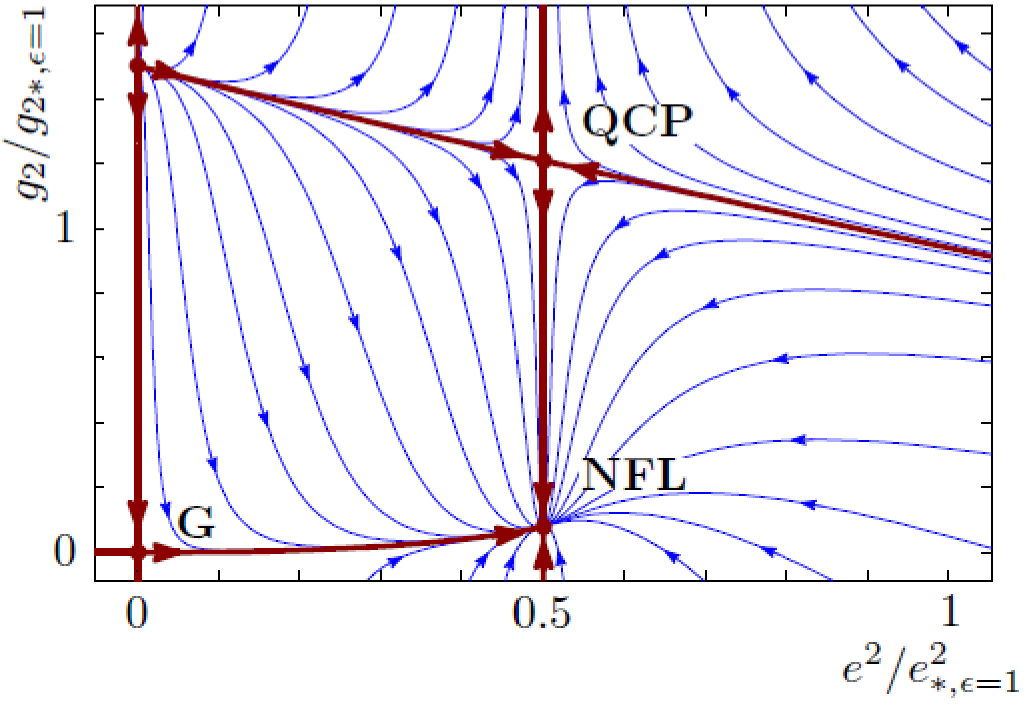
$$\frac{de^2}{d\ln b} = (z+2-d)e^2 - 4e^4$$

(Here, the dimensionless charge is defined as: $e^2 = 2me_{\mathrm{el}}^2/(4\pi\hbar^2\varepsilon)$)

Without the long-range interaction (e=0), the theory possesses a quantum critical point (QCP_0); weakly coupled close to d=2:

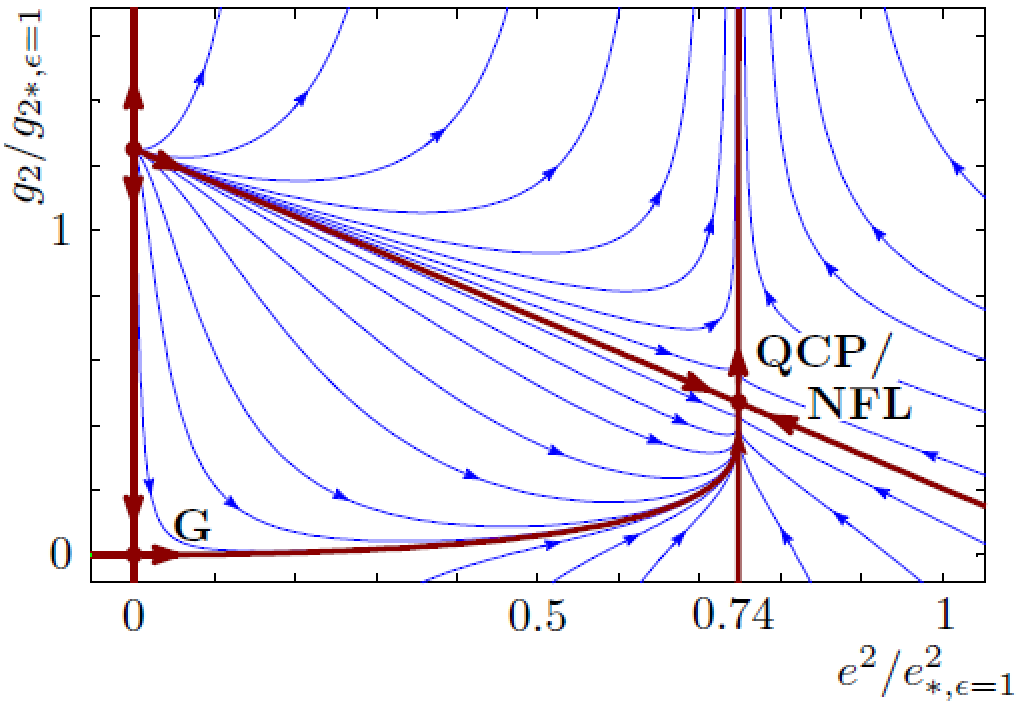


Close to and below d=4 there is a (IR stable) NFL fixed point, but also a (UV stable) quantum critical point at strong interaction: (d=3.5)



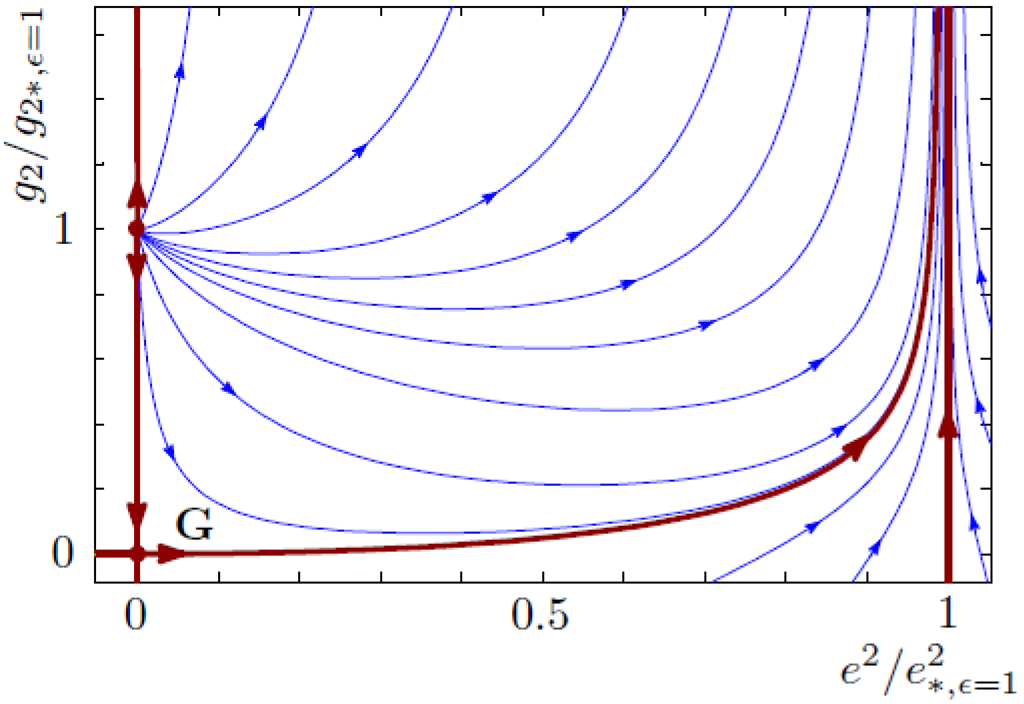
They get closer, but remain separated in the coupling space!

At some “lower critical dimension” NFL and QCP collide:



In one loop calculation, this occurs at $d_l = 3.26240$, and thus above, but close to three dimensions.

Finally, below dₗ the NFL and QCP become complex, and there is only a runaway flow left:



The system is unstable.

The instability, and the nature of the QCP:

At $d=d_l$ the NFL and QCP merge at (0.002, -0.153). Neglecting g_1, the flow of g_2 in the large-N theory is :

$$\frac{dg_2}{d\ln b} = -g_2 - \frac{4N}{5} g_2^2$$

For N >>1, introducing the order parameter $\chi_i = 2g_2 \langle \Psi^\dagger \gamma_i \Psi \rangle$ the saddle point at

$$\chi_i = -4g_2 N \int \frac{d\vec{p}}{(2\pi)^3} \frac{d_i(\vec{p}) + \chi_i}{\sqrt{(d_j(\vec{p}) + \chi_j)^2}}$$

is an exact solution. The O(5) and the rotational symmetry are therefore broken at

$$g_c^{-1} = 4N \int \frac{d\vec{p}}{(2\pi)^3} \frac{d_i^2(\vec{p}) - p^4}{p^6} = -\frac{4N}{5} \frac{2\Lambda}{\pi^2}$$

which is precisely the RG fixed point.

At large negative g_2 the system should develop anisotropic gap and,

$$\chi_5 > 0$$

The gap is minimal at the equator (in momentum space) at

$$p^2 = \chi_5 / 2$$

and the system looks as if under strain. The resulting ground state:

(topological) Mott insulator

(IH and Janssen, PRL 2014)

The state is equivalent in symmetry to ``uniaxial nematic”.

The fate of NFL: if dl is above but close to d=3, the flow becomes slow close to (complex!) NFL fixed point. The RG ``escape time" is long:

$$b_0 = e^{\frac{C}{\sqrt{d_{\rm low}-d}} - B + \mathcal{O}(d_{\rm low}-d)}$$

with non-universal constants C and B. There is wide crossover region of the NFL behavior within the temperature window

$$(T_{\rm c}, T_*)$$

with the critical temperature,

$$T_{\rm c} \approx T_* b_0^{-z}$$

And the characteristic energy scale for interaction effects as

$$k_{\rm B} T_* \sim \frac{e_{\rm el}^2}{\varepsilon L_*} = \frac{\hbar^2}{2mL_*^2} = \frac{4m}{m_{\rm el}\varepsilon^2} E_0$$

Assuming a small band mass

$$m/m_{\mathrm{el}} \approx 1/50$$

and a high dielectric constant

$$\varepsilon \approx 30$$

still gives a reasonable

$$T_* \sim 10\,\mathrm{K} - 100\,\mathrm{K}$$

and a detectable

$$T_{\mathrm{c}} \approx T_*/100$$

Conclusion:

1) Abrikosov's non-Fermi liquid phase at T=0 exists only in dimensions d:

$$d_{low} < d < d_{up} = 4$$

with lower critical dimension $d_{low} > 2$, and probably close to three.

2) Below d_{low} the system develops a gap, and most likely becomes a (topological) Mott insulator. (The other possibility is a s-wave superconductor, with an isotropic gap.)

3) NFL shows up in a possibly wide crossover regime of energy scales.

4) Gray tin or mercury telluride should be a (topological) Mott insulator at T=0, and at zero doping!