Convergence of Derivative Expansion in Supersymmetric QM

M. Heilmann, T. Hellwig, B.Knorr, A. Wipf, M. Ansorg
TPI Jena

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Introduction

- if SUSY realized in nature ⇒ FRG appropriate nonperturbative tool (strong coupling phenomena, coll. condensation phenomena)
- completion of lattice studies (formulation manifest supersymmetric)
- advantage in studying QM: exact results known (E_0, E_{ex})
- single-well potential ⇒ easily treated over whole range of couplings
- double-well potential ⇒ allows for tunneling, described in terms of instantons ⇒ challenging

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1 verify convergence of supersymmetric „derivative" expansion within FRG framework

2 analysis of non-convex classical potentials, tunneling processes ↪ Synatschke et al., '09, '10

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4 generalization to more advanced theories

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$\mathcal{N}=1$ SUSY QM

[1]

- **superfield**:

  $$\Phi = \phi + \bar{\theta}\psi + \bar{\psi}\theta + \bar{\theta}\theta F$$

- **superpotential**:

  $$W(\Phi) = W(\phi) + (\bar{\theta}\psi + \bar{\psi}\theta)W'(\phi) + \bar{\theta}\theta(FW'(\phi) - W''(\phi)\bar{\psi}\psi)$$

- **supercharges**:

  $$Q = i\partial_{\bar{\theta}} + \theta\partial_\tau; \quad \bar{Q} = i\partial_\theta + \bar{\theta}\partial_\tau; \quad \{Q, \bar{Q}\} = 2i\partial_\tau$$

  $\Rightarrow$ generate SUSY variations: $\delta_\epsilon = \bar{\epsilon}Q - \epsilon\bar{Q}$

- supercovariant **derivatives**:

  $$D = i\partial_{\bar{\theta}} - \theta\partial_\tau; \quad \bar{D} = i\partial_\theta - \bar{\theta}\partial_\tau; \quad \{D, \bar{D}\} = -2i\partial_\tau$$

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- **bare action**:

$$S[\Phi] = \int d\tau d\theta d\bar{\theta} \left[-\tfrac{1}{2} \Phi K \Phi + iW(\Phi) \right]; \quad K = \tfrac{1}{2}(\bar{D}D - D\bar{D})$$

$$S[\phi,F,\psi,\bar{\psi}] = \int d\tau \left[\tfrac{1}{2}\dot{\phi}^2 - i\bar{\psi}\dot{\psi} + \tfrac{1}{2}F^2 + iFW'(\phi) - iW''(\phi)\bar{\psi}\psi \right]$$

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bosonic potential Yukawa-interaction

- SUSY preserved: $E_0 = 0$, highest power of $W(\phi)$ even
- SUSY spont. broken: $E_0 > 0$, highest power of $W(\phi)$ odd

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I Formalism

microscopic action
$\Gamma_{k\to\Lambda} \equiv S$

transition:

determined by
flow equation
Wetterich, '93

effective action
$\Gamma_{k\to 0} \equiv \Gamma$

$$\partial_k \Gamma_k = \frac{1}{2}\mathbf{S}\mathrm{Tr}\left[\partial_k R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]$$

Horikoshi et al., '98
Kapoyannis, Tetradis, '00
Zapalla, '01
Weyrauch, '06
Synatschke, Bergner, Gies, Wipf, '09

II Truncation Scheme

- **derivative expansion** in $(K)^n$, **off-shell** formulation
- truncation of Γ_k in NNLO:

  $\Gamma_k[\Phi] = \int d\tau d\theta d\bar{\theta} \left[iW_k - \frac{1}{2} Z_k K Z_k + \frac{i}{4} \Omega_k K^2 \Phi + \frac{i}{4} Y_k (K\Phi)^2 \right]$
- general form of **regulator functional**:

  $\Delta S_k[\Phi] = \frac{1}{2} \int d\tau d\theta d\bar{\theta}\, \Phi R_k(D, \bar{D}) \Phi$

  with $R_k(D, \bar{D}) = ir_1(-\partial_\tau^2, k) + Z_k'(\bar{\Phi})^2 r_2(-\partial_\tau^2, k) K$

$\Rightarrow$ manifestly supersymmetric cutoff action ΔS_k;
SUSY links regulators, e.g. $(p^2 r_2, r_2, p r_2) \leftrightarrow (\phi, F, \psi/\bar{\psi})$

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III Projection Scheme

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  $= \int d\tau \frac{i}{2} \left[\partial_k X_k \right]$

- regulator functions: $r_1 = k, \quad r_2 = 0$

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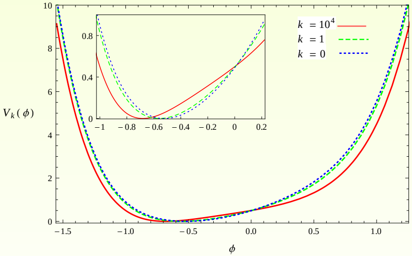
Effective Potential

- unbroken SUSY: UV initial conditions:

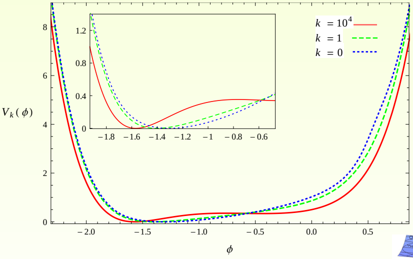
$$W'_\Lambda = e + m\phi + g\phi^2 + a\phi^3, \quad Z'_\Lambda(\phi) = 1, \quad Y_\Lambda = X_\Lambda = 0$$

- running scalar potential $V_k(\phi)$: EOM of F into $\Gamma_{k,\mathrm{off}}$

$$V_k(\phi) = \frac{2}{27Y_k^2}\left(\sqrt{3W'_kY_k + Z'^4_k} - Z'^2_k\right)\left(6W'_kY_k + Z'^4_k - Z'^2_k\sqrt{3W'_kY_k + Z'^4_k}\right)$$



$(e,m,g,a) = (1,1,0,1)$



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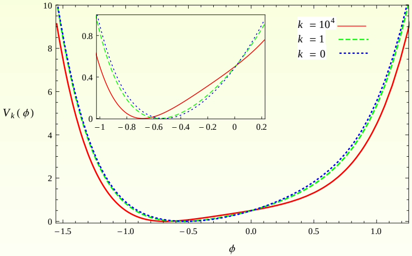

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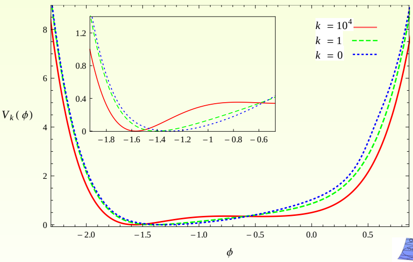

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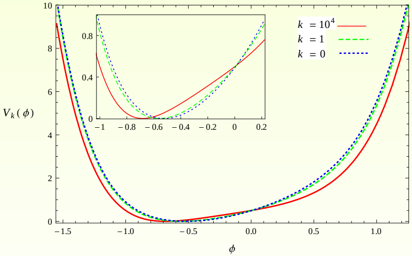

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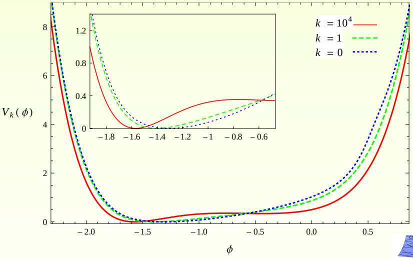

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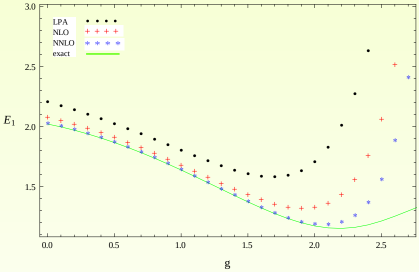
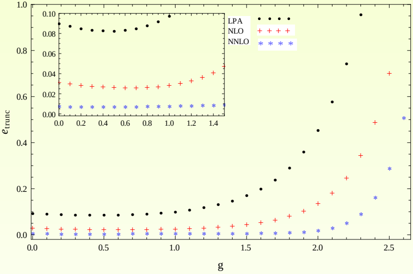
Energy Gap

- energy gap $E_1 - E_0$: from pole of propagator at ϕ_{min}:

$$E_1^2 = \lim_{k\to 0} \frac{2}{X_k^2}\left(Z_k'^4 + X_k W_k'' - Z_k'^2\sqrt{Z_k'^4 + 2X_k W_k''}\right)\bigg|_{\phi=\phi_{\text{min}}}$$





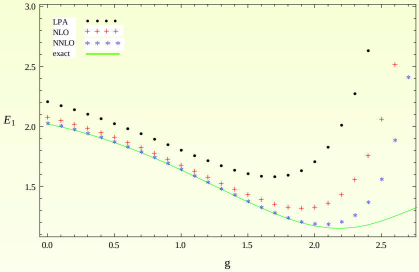
- breakdown of NNLO at $g \approx 2$ ($V_k \in \mathbb{C}$, generation of further (large) masses ↪ Heilmann et al., '12)

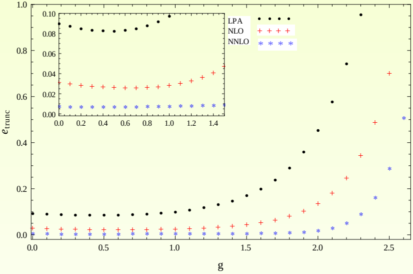
Research Training Group Quantum and Gravitational Fields

Energy Gap

- energy gap $E_1 - E_0$: from pole of propagator at $\phi_{\min}$:

$$E_1^2 = \lim_{k\to 0} \frac{2}{X_k^2}\left(Z_k'^4 + X_k W_k'' - Z_k'^2 \sqrt{Z_k'^4 + 2 X_k W_k''} \right)\Big|_{\phi=\phi_{\min}}$$





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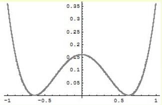
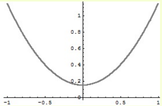
Research Training Group Quantum and Gravitational Fields

$\mathcal{N}=1$ Wess-Zumino model in $d=3$

- Goal: analysis of Wilson-Fisher type fixed-point:

  SUSY/sp. broken $\mathbb{Z}_2 \Leftrightarrow$ sp. broken SUSY/$\mathbb{Z}_2$





- derivation in Minkowski spacetime ($\hookrightarrow$ Synatschke et al, '10) $\Rightarrow$ Wick-rotation $\Rightarrow$ similar flows as in QM
- adequate regulator functional:

  $$\Delta S_k[\Phi] = \frac{1}{2}\int dz \Phi \left[2r_1 - Z'(\bar{\Phi})^2 r_2 K\right]\Phi$$

  $\Rightarrow r_1 = 0,\ r_2 = (k^2/p^2 - 1)\Theta(k^2/p^2 - 1)$
  $\Rightarrow$ *no* BFA, choose $\bar{\Phi} = \bar{\phi} = \phi_0$

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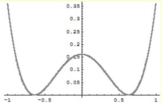

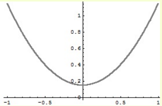

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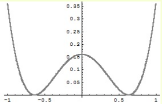

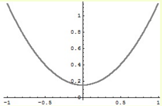

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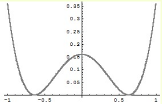

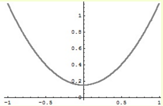

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Wilson-Fisher fixed point

- dimensionless quantities: $t=\ln(k/\Lambda)$, $\eta=-\frac{d}{dt}\ln\left(Z'^2(\phi_0)\right)$

  $[\phi, W, Z', X, Y]=\left(\frac{1}{2}, 2, 0, -1, -\frac{3}{2}\right) \Rightarrow [\chi, w, z', x, y]$
- asymptotics of FP from canonical & anomalous scaling:

  $w'_* \sim \chi^{\left(\frac{3-\eta}{1+\eta}\right)}, z'_* \sim \chi^{-\left(\frac{\eta}{1+\eta}\right)}, x_* \sim \chi^{-2}, y_* \sim \chi^{-3}$ for $|\chi| \to \infty$
- superscaling relation $\forall d \geq 2$:

  $\Theta_0 = 1/\nu_{\mathrm{w}} = \frac{(d-\eta)}{2}$ ("interaction"-flows w'-independent)

	$w'_*(0)$	χ_0	η	Θ_0	Θ_1	Θ_2
LPA	−0.0420	0.147		3/2	−0.702	−3.800
NLO	−0.0292	0.150	0.186	1.407	−0.771	−1.642
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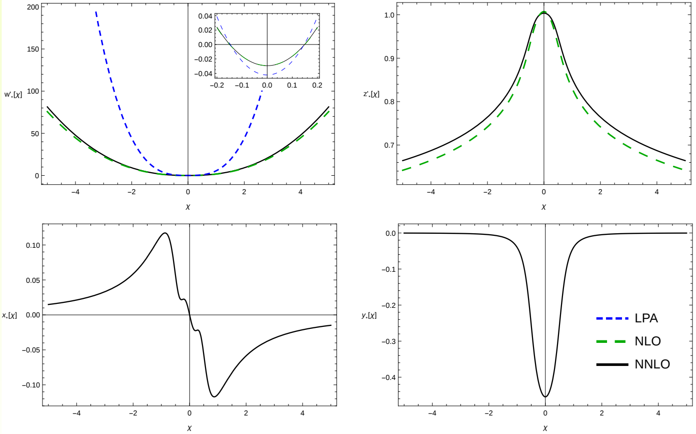

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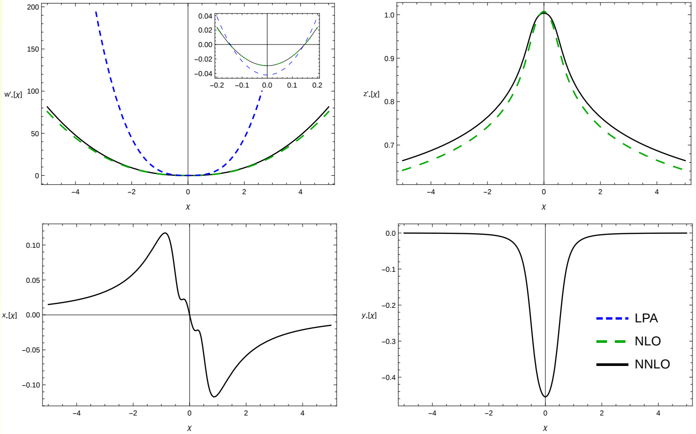

Summary

- Goal: testing supercovariant derivative expansion

(1) unbroken SUSY QM: E_1 rel. error of 1% in NNLO, very good for weak couplings g

(2) spont. broken SUSY QM: E_0, rel error of 4% in NLO, very good for strong couplings g

(3) Wess-Zumino model in $d = 3$: superscaling relation, convergence of Θ_i of Wilson-Fisher fixed point

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Quantum and Gravitational Fields

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References

[1] 10.09.2014:
http://fatalphysics.weebly.com/uploads/2/9/7/8/29780301/1297348orig.jpg