RG flow of the Higgs potential

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Helmholtz-Institut Jena

& C. Gneiting, R. Sondenheimer, PRD 89 (2014) 045012, [arXiv:1308.5075],
& S. Rechenberger, M.M. Scherer, L. Zambelli, EPJC 73 (2013) 2652 [arXiv:1306.6508]
& R. Sondenheimer, arXiv:1407.8124; L. Zambelli, arxiv:14XX.YYYY

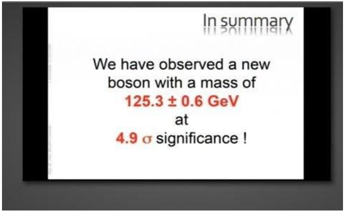
ERG2014 23.09.2014

Search for the Higgs boson

▷ 4 Jul. 2012

ATLAS & CMS

@CERN



▷ 14 Mar 2013, CERN press release:

"... the new particle is looking more and more like a Higgs boson ..."

$$\text{CMS'12}: \quad 125.3 \pm 0.4(stat) \pm 0.5(sys) GeV,$$

$$\text{ATLAS'12}: \quad 126.0 \pm 0.4(stat) \pm 0.4(sys) GeV$$

▷ Success of Experiment & Theory

(ANDERSON'62; BROUT,ENGLERT'64; HIGGS'64; GURALNIK,HAGEN,KIBBLE'64)


JOGHURT MILD
Ehrmann
Bioghurt
PUR
JOGHURT MILD
Ehrmann
Bioghurt
PUR

Numbers matter

standard model

best before:

$$\Lambda = M_{\text{Planck}} \; ?$$

Validity range of the standard model

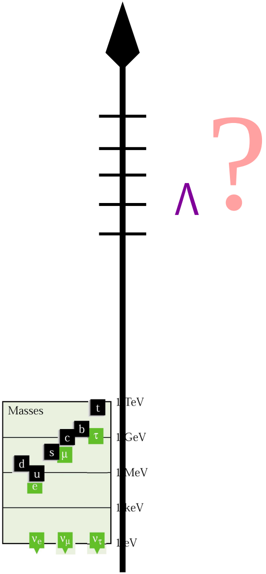
▷ Λ:

- UV cutoff

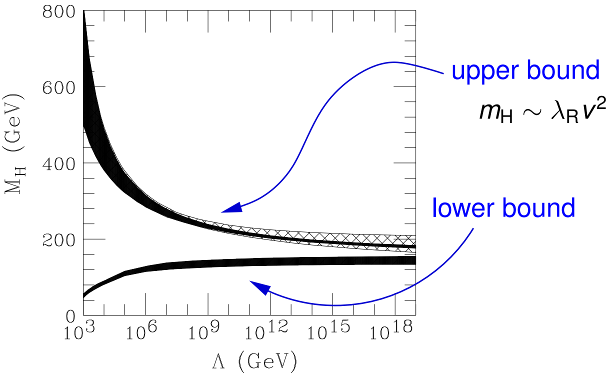
  SM as effective theory

- scale of maximum UV extension

- scale of new physics:
  $\Lambda_{NP} \leq \Lambda$



Higgs boson mass and maximum validity scale

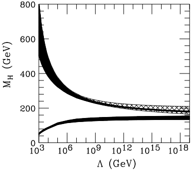


(Hambye,Riesselmann'97)

SM: e.g. (Krive,Linde'76; Maiani,Parisi,Petronzio'78; Krasnikov'78; Politzer, Wolfram'78; Hung'79; Lindner'85; Wetterich'87; Sher'88; Fort,Jones,Stephenson, Einhorn'93; Altarelli,Isidori'94; Schrempp,Wimmer'96; . . .)

BSM: e.g., (Cabbibo, Maiani,Parisi,Petronzio'79; Espinosa,Quiros'91; . . .)

Lower bound of Higgs boson mass



▷ vacuum stability / meta-stability bound

▷ effective potential á la Coleman Weinberg:

$$U_{\text{eff}}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{2}\lambda(\phi)\phi^4$$

▷ e.g., $\lambda(\phi)$ from "RG-improved" perturbation theory:

$$\partial_t\lambda = \frac{3}{4\pi^2}\left(-h_t^4 + h_t^2\lambda + \frac{1}{16}\left[2g^4 + (g^2 + g'^2)^2\right] - \frac{1}{4}\lambda(3g^2 + g'^2) + \lambda^2\right)$$

Lower bound of Higgs boson mass

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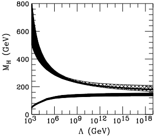

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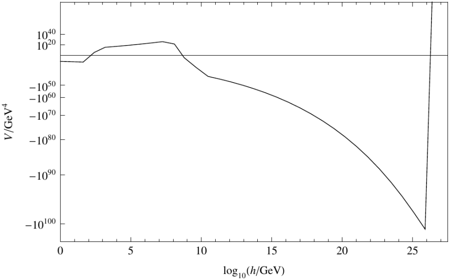
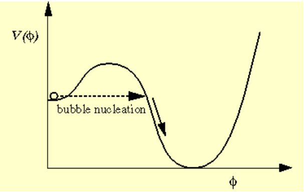


(GABRIELLI ET AL.'13)

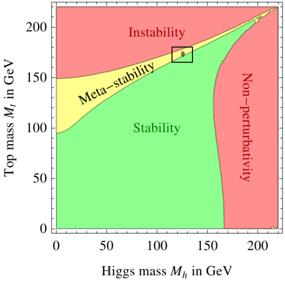
Lower Bound of Higgs boson mass

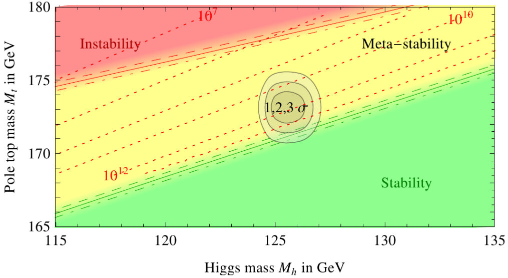
▷ meta-stability:

tunneling time > age of universe



▷ "Near critical" standard model: (BUTTAZZO ET AL.'13)





NNLO calculation (DEGRASSI ET AL.'12)

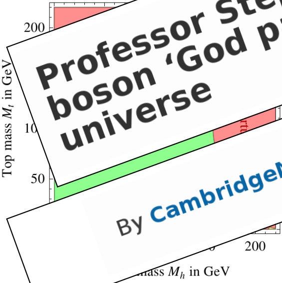
earlier calculations, e.g., (ISIDORI,RIDOLFI,STRUMIA'01)

Lower Bound of Higgs

▷ meta-stability:

tunneling

▷ "Near



Professor Stephen Hawking says the Higgs boson 'God particle' could destroy the universe

By CambridgeNews | Posted: September 09, 2014

> "The Higgs potential has the worrisome feature that it might become metastable at energies above 100bn gig-electron-volts,"

NNLO calculation (DEGRASSI ET AL.'12)

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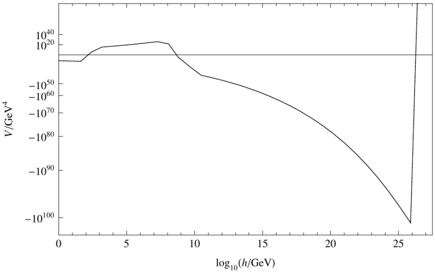
2nd thoughts on the lower bound

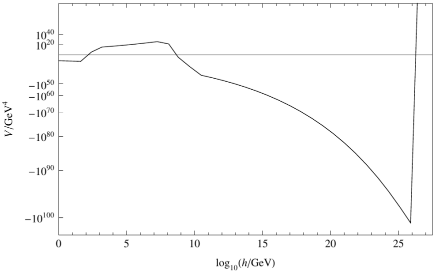


- True minimum of $U_{\text{eff}}(\phi)$ at

  $\phi \sim 10^{25}\text{GeV} > M_{\text{Pl}}$?

- UV$\rightarrow$IR RG flow?

2nd thoughts on the lower bound



▷ True minimum of $U_{\rm eff}(\phi)$ at

$\phi \sim 10^{25}\mathrm{GeV} > M_{\rm Pl}$?

▷ UV→IR RG flow?

▷ simple top-Higgs Yukawa model:

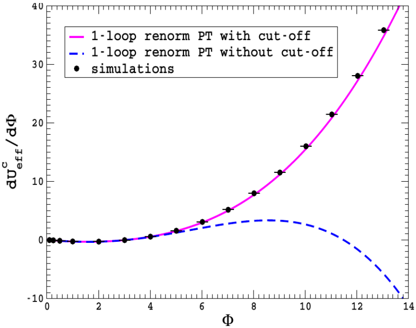
lattice simulation

vs. 1-loop PT with cutoff

vs. 1-loop "Λ-removed" PT

(HOLLAND,KUTI'03; HOLLAND'04)

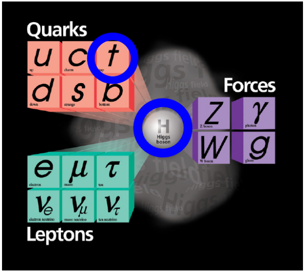
Criticism: to few scales? (EINHORN,JONES'07)



Top-Higgs Yukawa models

Z_2 symmetric model

$$S_{int} = \int ih\phi\bar{\psi}\psi$$



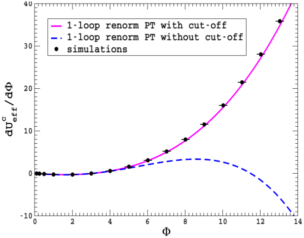
chiral model

$$S_{int} = \int i\bar{h}_t(\bar{\psi}_L\phi_C t_R + \bar{t}_R\phi_C^\dagger\psi_L)$$

- includes relevant top quark + Higgs field (+ largest Yukawa coupling)
- discrete model $\rightarrow$ no Goldstone bosons (as in SM)
- gauged models: (ERG2014: R. SONDENHEIMER)

Top-Higgs Yukawa models

(HOLLAND, KUTI'03; HOLLAND'04)



▷ generating functional:

$$
\begin{aligned}
Z[J] &= \int_\Lambda \mathcal{D}\phi\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-S[\phi,\bar{\psi},\psi]+\int J\phi} \\
&= \int_\Lambda \mathcal{D}\phi \; e^{-S[\phi,\bar{\psi},\psi]-S_{F,\Lambda}[\phi]+\int J\phi}
\end{aligned}
$$

▷ top-induced effective potential

$$
U_F(\phi) = -\frac{1}{2\Omega} \ln \frac{\det_\Lambda(-\partial^2 + h_t^2 \phi^\dagger \phi)}{\det_\Lambda(-\partial^2)},
$$

▷ CAVE: cutoff Λ / regularization dependent

Top-induced effective potential

▷ exact results for fermion determinants for homogeneous ϕ

▷ e.g., sharp cutoff:

$$U_{\mathrm{F},t}(\phi) = \underbrace{-\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2}_{<0 \text{ (mass-like term)}} + \frac{1}{16\pi^2} \underbrace{\left[h_t^4 |\phi|^4 \ln\left(1 + \frac{\Lambda^2}{h_t^2|\phi|^2}\right) + h_t^2|\phi|^2\Lambda^2 - \Lambda^4 \ln\left(1 + \frac{h_t^2|\phi|^2}{\Lambda^2}\right)\right]}_{>0 \text{ (interaction part)}}$$

▷ mass-like term: contributes to χSB $\Longrightarrow$ $v \simeq 246$GeV

▷ interaction part: strictly positive

$\Longrightarrow$ cannot induce instability for any finite Λ

(HG,Sondenheimer'14)

“Rederiving” the instability

- try to send $\Lambda \to \infty$:

$$U_{\mathrm{F},t}(\phi) = -\frac{\Lambda^2}{8\pi^2} h_t^2 |\phi|^2 + \frac{1}{16\pi^2} h_t^4 |\phi|^4 \left[\ln \frac{\Lambda^2}{h_t^2 |\phi|^2} + \text{const.} + \mathcal{O}\left(\frac{h_t^2 |\phi|^2}{\Lambda^2} \right) \right]$$

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▷ renormalization: trade $(\Lambda, m_\Lambda, \lambda_\Lambda)$ for (μ, v, λ_v)

$$U_{\mathrm{F},t}(\phi) \overset{?}{\to} -\frac{1}{16\pi^2} h_t^4 |\phi|^4 \left(\ln \frac{h_t^2 |\phi|^2}{\mu^2} + \text{const.} \right)$$

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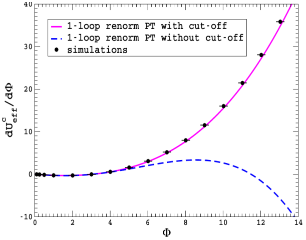


(Holland, Kuti'03; Holland'04)

▷ "instability" occurs beyond $\frac{h_t^2 |\phi|^2}{\Lambda^2} > 1$

(HG,Sondenheimer'14)

▷ similar problems for other reg's

▷ implicit renormalization conditions would violate unitarity

(Branchina,Faivre'05; Gneiting'05)

Summary, Part I

- no in-/meta-stability from top (fermion) fluctuations

  . . . if cutoff Λ is kept finite but arbitrary
- no in-/meta-stability at all ?

$$U_{\text{eff}}(\phi) = U_{\Lambda}(\phi) + U_{\text{B}}(\phi) + U_{\text{F}}(\phi)$$

Summary, Part I

- no in-/meta-stability from top (fermion) fluctuations

  …if cutoff Λ is kept finite but arbitrary

- no in-/meta-stability at all ?

$$U_{\text{eff}}(\phi) = \underbrace{U_\Lambda(\phi)}_{\text{arbitrary}} + \underbrace{U_{\text{B}}(\phi) + U_{\text{F}}(\phi)}_{\text{generically stable}}$$

…in-/meta-stabilities from the bare action/UV completion

- lower Higgs mass bounds?

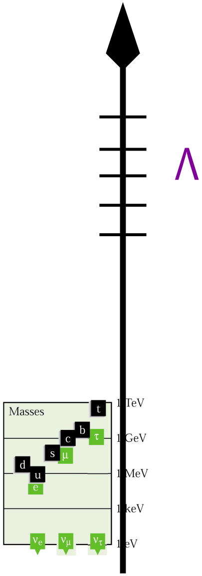
$\Longrightarrow$ nonperturbative methods recommended if not needed

▷ extensive lattice simulations: (Fodor,Holland,Kuti,Nogradi,Schroeder'07)
(Gerhold,Jansen'07'09'10)

▷ constraining 4th generations: (Gerhold,Jansen,Kallarackal'10; Bulava,Jansen,Nagy'13)

▷ implications for dark matter models: (Eichhorn, Scherer'14) (Poster@ERG2014)

Higgs boson mass bounds as a UV to IR mapping



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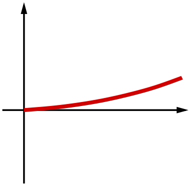

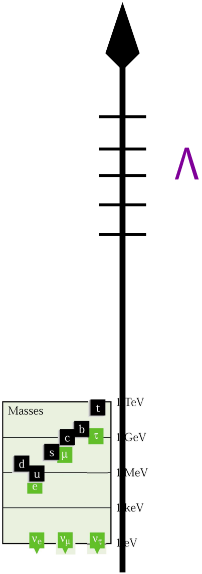

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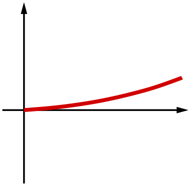

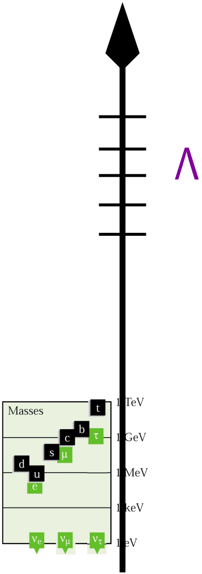

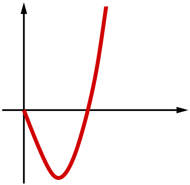

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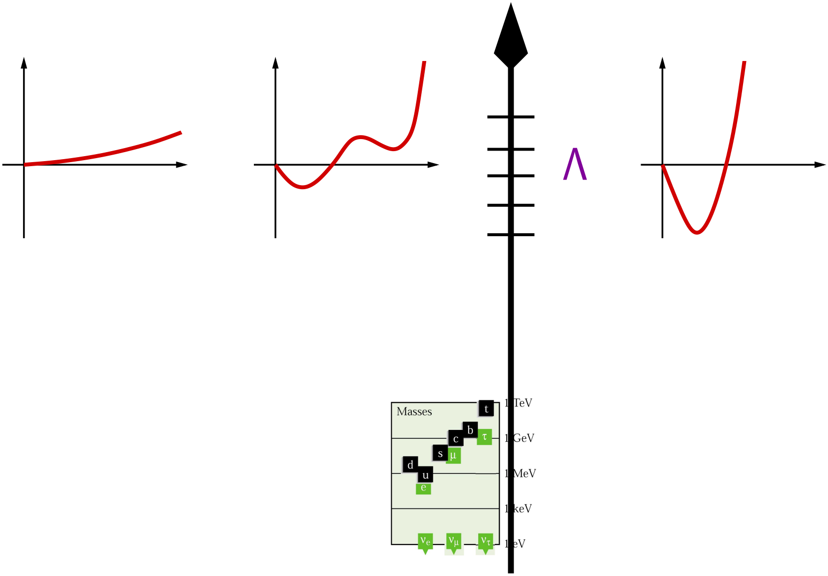

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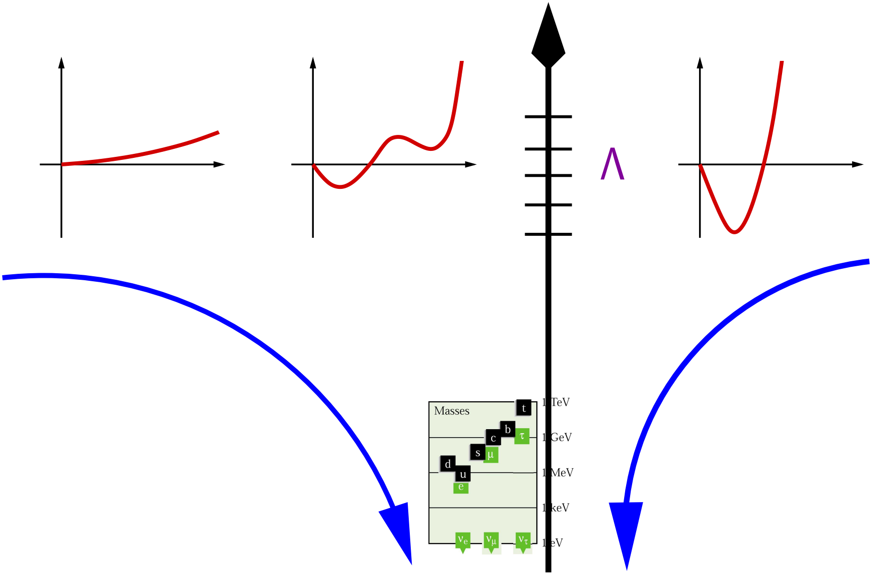

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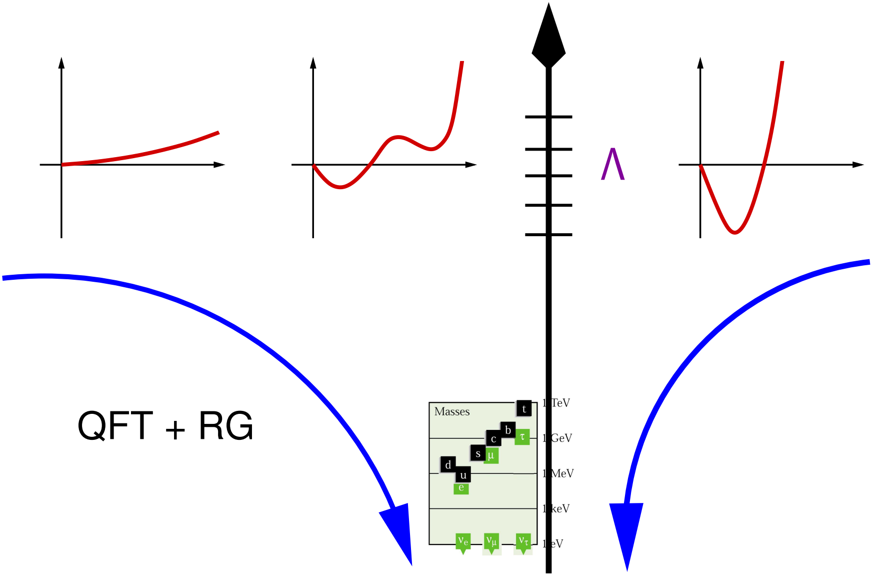

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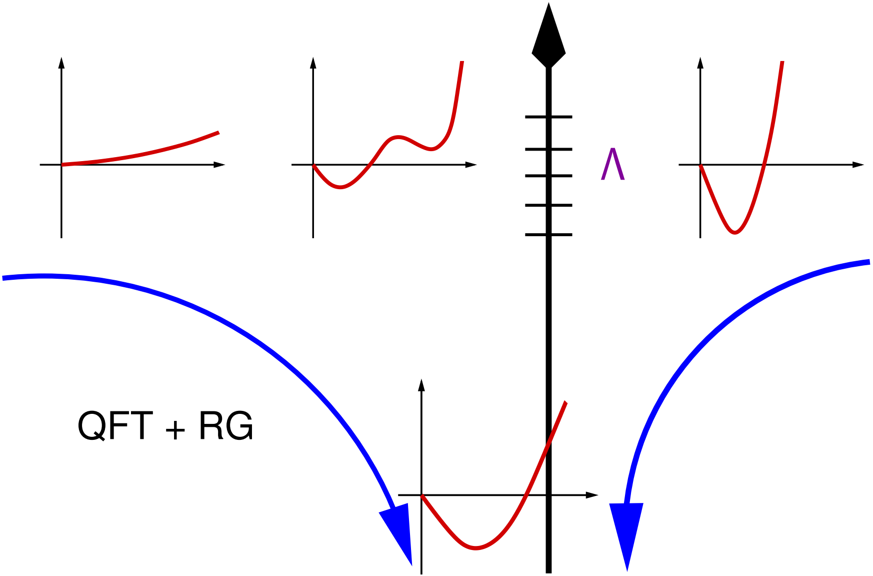
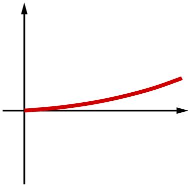


Higgs boson mass bounds as a UV to IR mapping

▷ microscopic action at cutoff Λ:

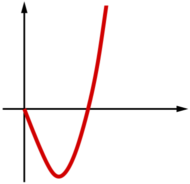
(HG,GNEITING,SONDENHEIMER'13)



$$S_\Lambda = S_\Lambda(m_\Lambda^2, \lambda_\Lambda, \lambda_{6,\Lambda}, \dots, h_\Lambda, \dots)$$

$\Longrightarrow$ RG: mapping to IR observables

$$\overset{\text{RG}}{\Longrightarrow} \; v \simeq 246\text{GeV}, \; m_{\text{top}} \simeq 173\text{GeV}, \; m_{\text{H}} = m_{\text{H}}[S_\Lambda]$$



▷ e.g, "renormalizable" (?) UV bare potential

$$U_\Lambda(\phi) = \frac{1}{2} m_\Lambda^2 \phi^2 + \frac{1}{8} \lambda_\Lambda \phi^4, \quad \lambda_\Lambda \geq 0$$

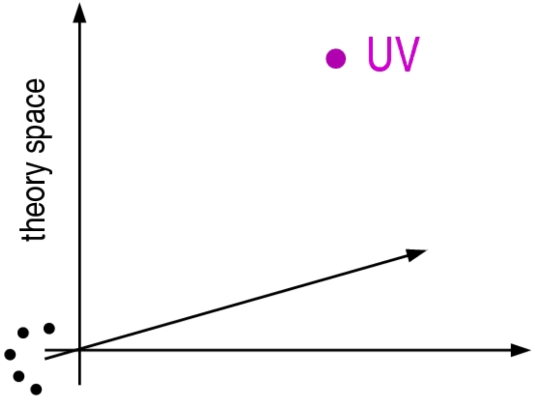
▷ trade: $m_\Lambda, h_\Lambda \quad \Longleftrightarrow \quad v, m_{\text{top}} \qquad \Longrightarrow \quad m_{\text{H}} = m_{\text{H}}(\lambda_\Lambda, \Lambda)$

Nonperturbative tool: functional RG

(WETTERICH'93)

$$\partial_t \Gamma_k = \frac{1}{2} \,\mathrm{Tr}\, \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

▷ RG trajectory: $\Gamma_{k=\Lambda} = S_\Lambda = \int \frac{1}{2}(\partial\phi)^2 + U_\Lambda(\phi) + \dots$



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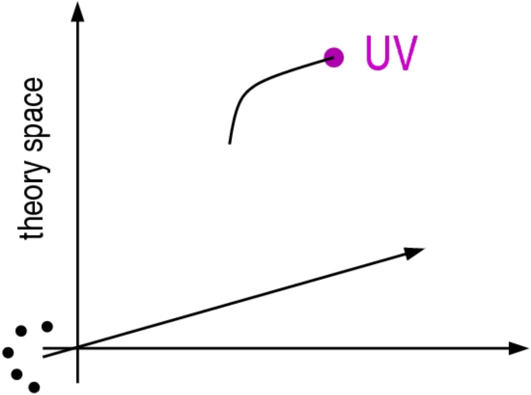

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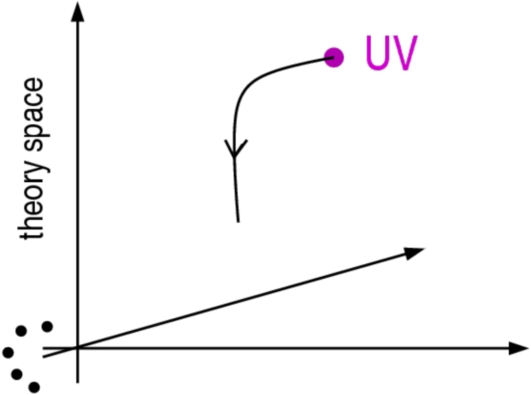

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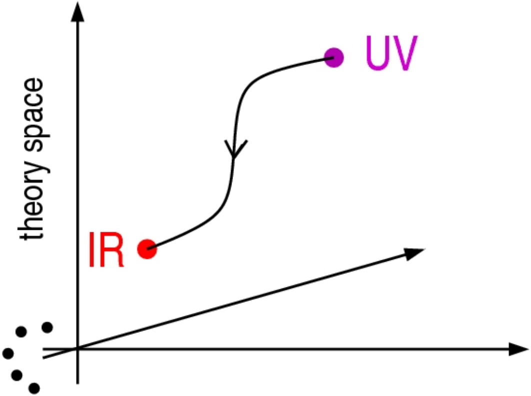
(WETTERICH'93)

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

▷ RG trajectory: $\Gamma_{k\to 0} = \Gamma = \int U_{\text{eff}} + \ldots$

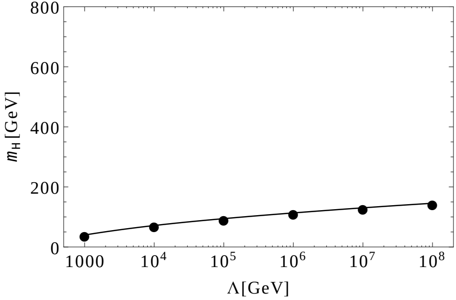


Higgs boson mass bounds from functional RG

▷ Z_2 Yukawa model + ϕ^4 bare potential (HG, Gneiting, Sondenheimer'13)

▷ Systematic derivative expansion:

$$\Gamma_k = \int d^d x \left(\frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \phi \bar{\psi} \psi \right)$$



$\lambda_\Lambda = 0$

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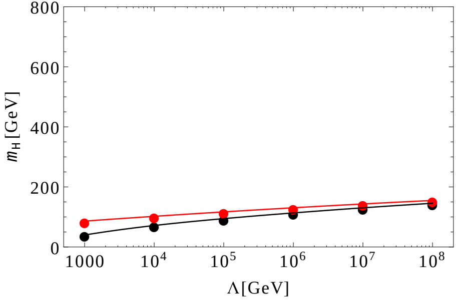

$\lambda_\Lambda = 0$

$\lambda_\Lambda = 0.1$

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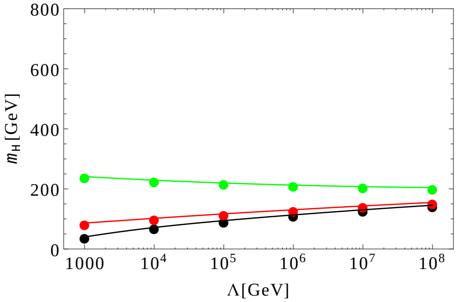

$\lambda_\Lambda = 0$
$\lambda_\Lambda = 0.1$
$\lambda_\Lambda = 1$

Higgs boson mass bounds from functional RG

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- ▷ Systematic derivative expansion:

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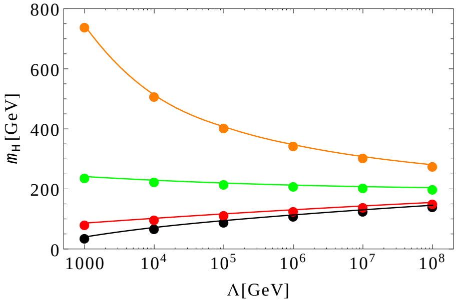


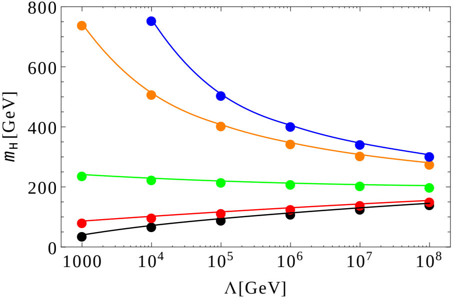
$\lambda_\Lambda = 0$
$\lambda_\Lambda = 0.1$
$\lambda_\Lambda = 1$
$\lambda_\Lambda = 10$

Higgs boson mass bounds from functional RG

▷ Z_2 Yukawa model + ϕ^4 bare potential (HG,Gneiting,Sondenheimer'13)

▷ Systematic derivative expansion:

$$\Gamma_k = \int d^d x \left(\frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \phi \bar{\psi} \psi \right)$$



$\lambda_\Lambda = 0$
$\lambda_\Lambda = 0.1$
$\lambda_\Lambda = 1$
$\lambda_\Lambda = 10$
$\lambda_\Lambda = 100$

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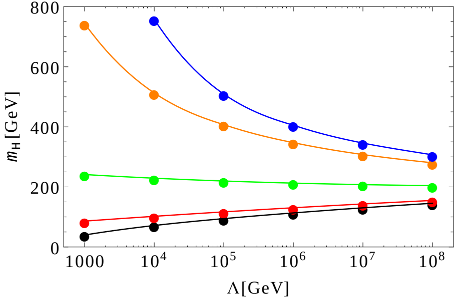


$\lambda_\Lambda = 0$

$\lambda_\Lambda = 0.1$

$\lambda_\Lambda = 1$

$\lambda_\Lambda = 10$

$\lambda_\Lambda = 100$

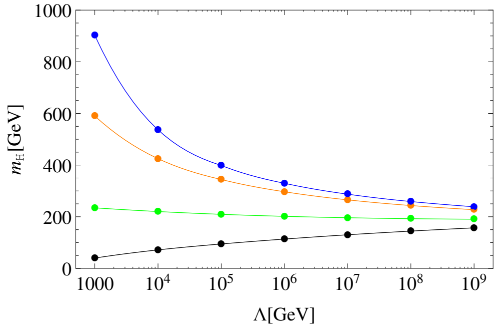
$\Longrightarrow$ lower bound for $\lambda_\Lambda = 0$ ($\simeq$ mean-field result)

agreement with lattice (HOLLAND'04; FODOR,HOLLAND,KUTI,NOGRADI,SCHROEDER'07; GERHOLD,JANSEN'07'09'10)

Conventional lower Higgs boson mass bound

▷ chiral top-bottom-Higgs Yukawa model (+Goldstone decoupling):

(HG,Sondenheimer'14)



FRG:
NLO derivative
expansion

$\lambda_{2\Lambda} = 0$
$\lambda_{2\Lambda} = 1$
$\lambda_{2\Lambda} = 10$
$\lambda_{2\Lambda} = 100$

$\Longrightarrow$ lower bound close to Z_2 model:

...bottom quark has little quantitative influence

General microscopic actions

▷ S_Λ is a priori unconstrained. Consider, e.g., (HG,GNEITING,SONDENHEIMER'13)

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{24}\phi^6$$

▷ for $\lambda_{3\Lambda} > 0$ we can choose $\lambda_{2\Lambda} < 0$:

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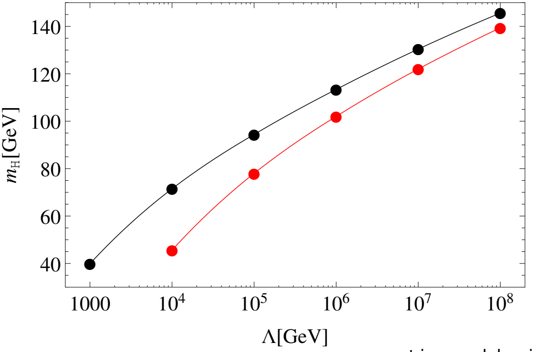


$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.08$

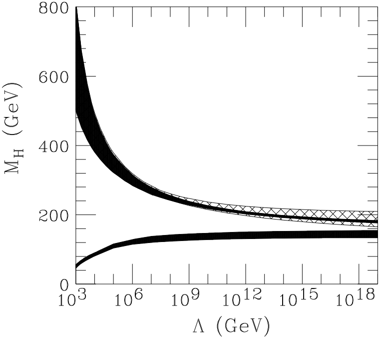
▷ lower bound relaxed

string models with $\lambda < 0$: (HEBECKER,KNOCHEL,WEIGAND'13)

Renormalizable field theories

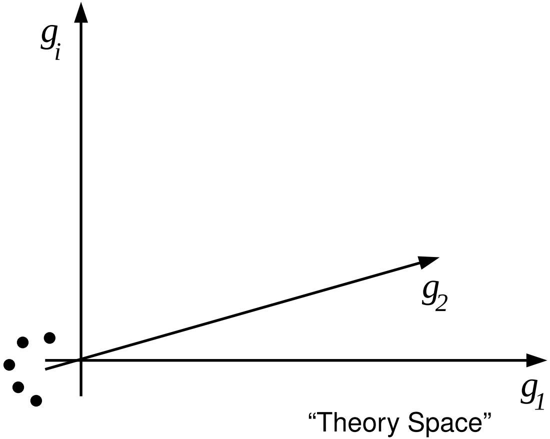
▷ seeming contradiction with common wisdom . . . ?

". . . observables are determined by renormalizable operators . . ."

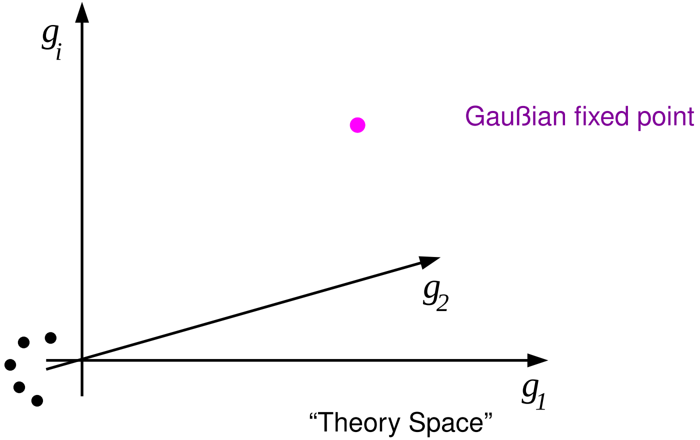


▷ y-axis: m_H observable ✓, x-axis: Λ ?

RG mechanism for “lowering” the lower bound

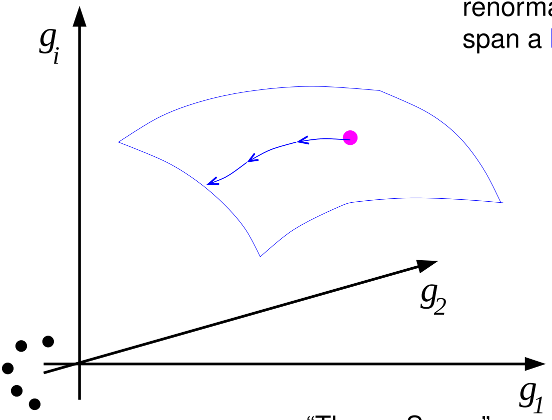


RG mechanism for “lowering” the lower bound



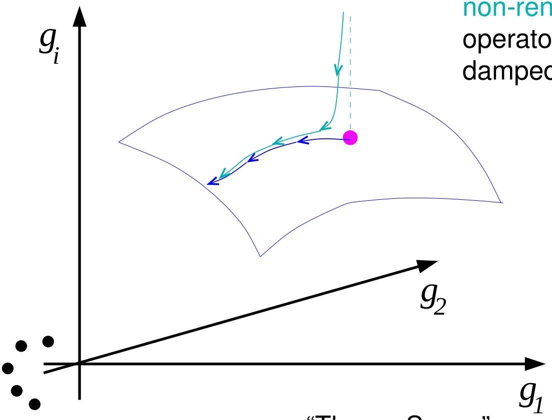
RG mechanism for “lowering” the lower bound

renormalizable operators
span a hyperplane



“Theory Space”

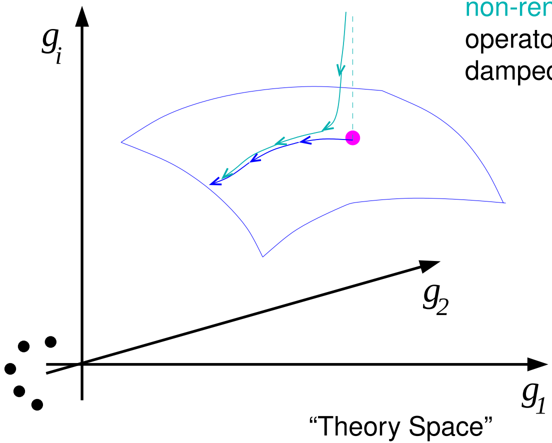
RG mechanism for “lowering” the lower bound



non-renormalizable operators are exponentially damped towards the IR

“Theory Space”

RG mechanism for “lowering” the lower bound



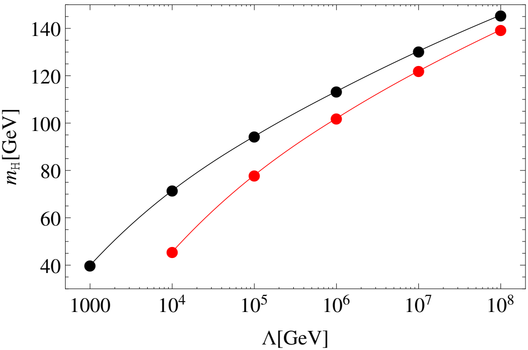
non-renormalizable operators are exponentially damped towards the IR

BUT: RG “time” (scales) is needed for this damping

Lower bounds from generalized bare actions

▷ e.g., (HG, Gneiting, Sondenheimer'13)

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{24}\phi^6$$



$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

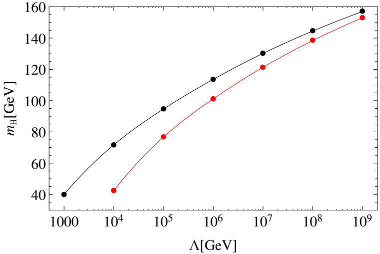
$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.08$

$\Longrightarrow$ lowered bound $\sim$ shifted Λ axis

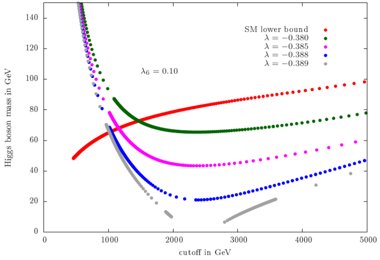
“Lowering” the lower Higgs boson mass bound

▷ chiral model with $\lambda_{6,\Lambda}(\phi^\dagger\phi)^3$ interaction

▷ comparison with lattice data: $\Longrightarrow$ RG mechanism confirmed



(HG, SONDENHEIMER'14)



(HEDGE, JANSEN, LIN, NAGY'13)

▷ $\mathcal{O}(1)$ variations of bare $\lambda_{n,\Lambda}$: $\Delta m_H \simeq \begin{cases} 10\,\text{GeV} & \text{at } \Lambda \simeq 10^{11}\text{GeV} \\ 5\,\text{GeV} & \text{at } \Lambda \simeq 10^{15}\text{GeV} \\ 2\,\text{GeV} & \text{at } \Lambda \simeq 10^{19}\text{GeV} \end{cases}$

(ERG2014: R. SONDENHEIMER)

Summary, Part II

- Bounds on the Higgs boson mass (or any other physical IR observable) arise from a mapping

$$S_{\rm micro} \quad \rightarrow \quad \mathcal{O}_{\rm phys}$$

. . . provided by the RG

- For "effective quantum field theories" (with a cutoff Λ):

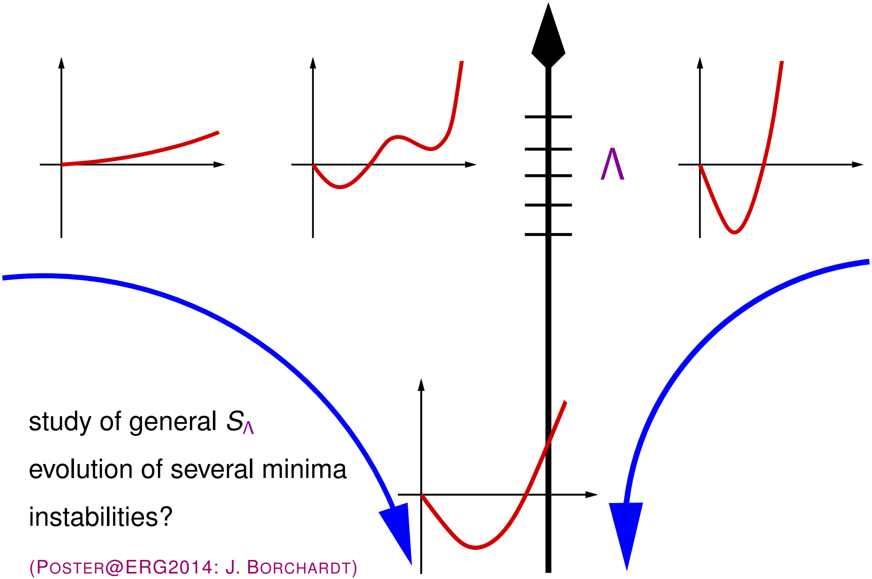
$$\text{bounds on } \mathcal{O}_{\rm phys} = f[S_\Lambda]$$

. . . full S_Λ not just the "renormalizable" operators

- "lowering" the conventional lower Higgs boson mass bound is possible

. . . without in-/meta-stable vacuum

TODO list



study of general S_Λ

evolution of several minima

instabilities?

(POSTER@ERG2014: J. BORCHARDT)

Implications

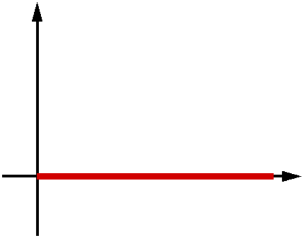
- if $m_H <$ conventional lower bound:
  - new physics at lower scales
  - first constraints on underlying UV completion

- if m_H exactly on the convenional lower bound:

  …"criticality"

  - underlying UV completion has to explain absence of higher dimensional operators

▷ flat interaction potential



Candidates

▷ Asymptotically free YM-Higgs-Yukawa models?

possible, but no pheno-viable model known (CALLAWAY'88)

general perturbative UV prediction:

$$\lambda \sim g^2 \to 0$$

...analysis relies on the deep Euclidean region

▷ standard model + asymptotically safe gravity (WEINBERG'76; REUTER'96)

gravity fluctuations induces a UV fixed point $\lambda_* \simeq 0$

(PERCACCI ET AL'03'09)

$\Longrightarrow$ $m_{\rm H}$ put onto conventional lower bound (WETTERICH,SHAPOSHNIKOV'10)

(BEZRUKOV,KALMYKOV,KNIEHL,SHAPOSHNIKOV'12)

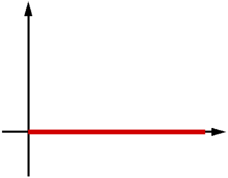
▷ asymptotically safe ($\neq$ free) particle physics models?

$\Longrightarrow$ "guaranteed" in gauged Yukawa models (LITIM,SANNINO'14)

(ERG2014: D. LITIM)

Asymptotically free UV gauge scaling solutions?

(RECHENBERGER,SCHERER,HG,ZAMBELLI'13; HG,ZAMBELLI'14IP)



▷ (Almost) flat potentials:

$\Longrightarrow$ large amplitude fluctuations

▷ If flatness is driven by asymptotically free gauge sector:

gauge rescaling of fields: $$X = g^{2P} \frac{Z_\phi |\phi|^2}{k^2}$$

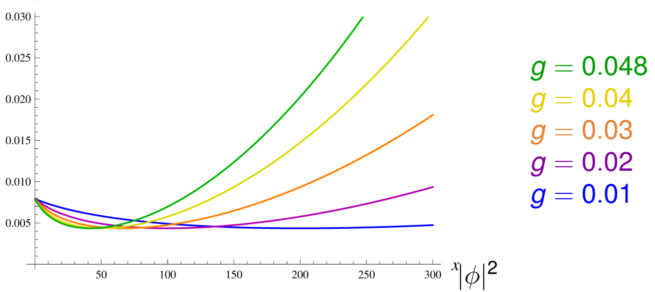
$\Longrightarrow$ solution of fixed-point equation for effective potential ($P = 1$):

SU(2) Yang-Mills-Higgs : $$V(X) = \xi X^2 - \left(\frac{3}{16\pi}\right)^2 \left[2X + X^2 \ln\left(\frac{X}{2+X}\right)\right]$$

$\Longrightarrow$ Coleman-Weinberg type, one-parameter family ξ

Asymptotically free UV gauge scaling solutions

▷ gauge scaling towards flatness (Rechenberger,Scherer,HG,Zambelli'13; HG,Zambelli'14ip)



▷ approach to UV $k \to \infty$:

$$g^2 \to 0, \quad |\phi_{\mathrm{min}}|^2 \sim \frac{1}{g^2} \to \infty, \quad \underline{\underline{\lambda \sim g^4 \to 0}}, \quad \frac{m_W^2}{k^2} \to \mathrm{const.}$$

$\Longrightarrow$ deep Euclidean region is sidestepped

▷ marginal-relevant direction: self-similar & polynomially bounded

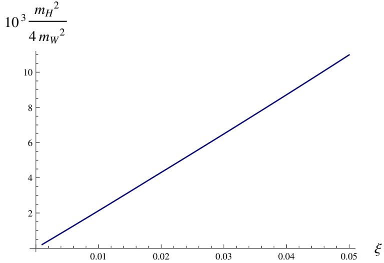
(Morris'98)

Estimates of IR Observables

(HG,ZAMBELLI'14IP)

▷ Higgs to W boson mass ratio:

$$\frac{m_H^2}{m_W^2} \sim \xi$$



▷ UV-IR mapping
of physical parameters:

$$\left.\begin{array}{c} v \\ m_H \\ m_W \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{ll} g^2 & \text{marginal-relevant} \\ \delta m^2 & \text{relevant} \\ \xi & \text{exactly marginal} \end{array}\right.$$

$\Longrightarrow$ pheno-relevant parameter regime is accessible

$\Longrightarrow$ SU(2) non-abelian Higgs model can be UV complete

Summary, Part III

- Numbers matter

  . . . m_{top}, m_{H}

- QFT is more than a collection of recipes

  . . . new insight from new tools

- vacuum stability: no reason for concern

  . . . so far . . .

- UV complete models approaching flat potentials

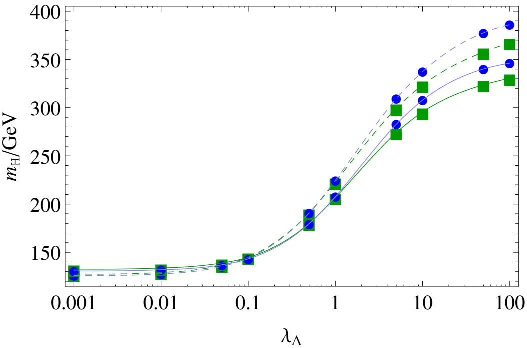
  . . . appealing also in view of current data

The IR window for the Higgs boson mass

▷ $m_{\mathrm{H}} \sim v\lambda_{\mathrm{R}}$ (WETTERICH'87)

mapping: $\lambda_\Lambda \quad \to \lambda_{\mathrm{R}}$ not surjective on $\mathbb{R}_+$

▷ e.g. for ϕ^4 bare potential, fix $\Lambda = 10^7$GeV (HG,GNEITING,SONDENHEIMER'13)



convergence check of

- derivative expansion ΔNLO / LO $\sim$ 10% @ strong coupling
- U_{eff} solver (polynom. exp.)

Towards the standard model

▷ chiral Yukawa model: (HG,Sondenheimer'14)

$$S = \int \Big[\partial_\mu \phi^\dagger \partial^\mu \phi + U(\phi^\dagger \phi) + \bar{t} i \partial\!\!\!/ t + \bar{b} i \partial\!\!\!/ b$$
$$+ i h_b (\bar{\psi}_L \phi b_R + \bar{b}_R \phi^\dagger \psi_L) + i h_t (\bar{\psi}_L \phi_{\mathcal{C}} t_R + \bar{t}_R \phi_{\mathcal{C}}^\dagger \psi_L) \Big]$$

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix} \quad \psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

▷ enforce decoupling of Goldstone bosons ($m_G = 0$)

$$\frac{k^2}{k^2 + m_G^2} \quad \to \quad \frac{k^2}{k^2 + m_G^2 + g v_k^2}$$

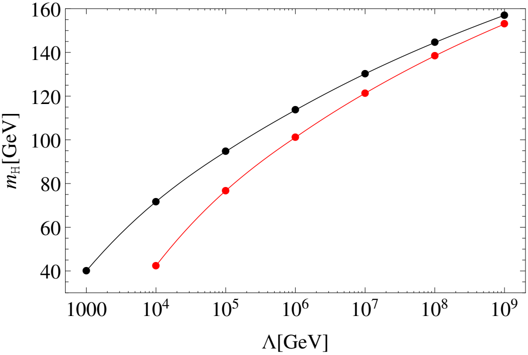
▷ choose "gauge boson" masses $g v_k^2 = (80.4\,\mathrm{GeV})^2$

cf. lattice model (Gerhold,Jansen'07'09'10)

"Lowering" the lower Higgs boson mass bound

▷ generalized bare potential with $\lambda_{6,\Lambda}(\phi^\dagger\phi)^3$ interaction:

(HG,Sondenheimer'14)



$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.1$

$\Longrightarrow$ same RG mechanism at work

Gauge-invariant regulator

▷ ζ function regularization (interpolating reg.: propertime $\leftrightarrow$ dim.reg.)

(HG,SONDENHEIMER'14)

$$U_{\mathrm{F},t}(\phi) = \underbrace{-\frac{4\mu^{4-d}\Lambda^{d-2}}{(d-2)(4\pi)^{d/2}} h_t^2 \, |\phi|^2}_{<0 \text{ (mass-like term)}} + \underbrace{\frac{2\mu^{4-d}}{(4\pi)^{d/2}} \int_{1/\Lambda^2}^{\infty} \frac{dT}{T^{1+(d/2)}} \left(e^{-h_t^2 |\phi|^2 T} + h_t^2 |\phi|^2 T - 1 \right)}_{>0 \text{ (interaction part)}}$$

▷ interaction part: strictly positive

$\Longrightarrow$ cannot induce instability for any finite Λ, μ, d

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(HG,SONDENHEIMER'14)

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▷ BUT: Limit $\Lambda \to \infty$ and expansion about $d = 4 - \epsilon$

$$U_{\mathrm{F},t}(\phi) \overset{?}{\to} \frac{\#}{\epsilon} h_t^4|\phi|^4 - \frac{1}{16\pi^2} h_t^4|\phi|^4 \left(\ln \frac{h_t^2|\phi|^2}{\mu^2} + \text{const.} \right)$$

$\Longrightarrow$ "instability": artifact of dim.reg

use dim.reg. in the presence of large fields: (BROWN'76,LUSCHER'82)