Probing phase transitions between discrete and continuum quantum spacetime

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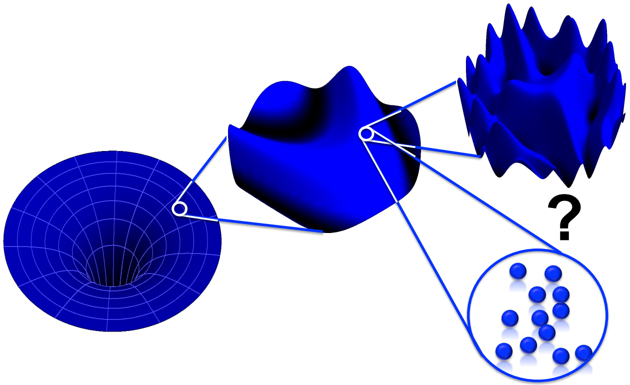
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Outline

- Matrix models for 2-d quantum gravity
- Continuum limit as RG fixed point
- New FRG tools and results [with T. Koslowski, 2013/14]
- Relation to asymptotic safety?

What is the fundamental nature of spacetime?

Quantum gravity: spacetime fluctuations at the Planck scale



Goal: $\int_{\text{spacetimes}} e^{i\,S} \quad \to \int_{\text{spacetimes}} e^{-\,S}$

Path-integral for quantum gravity

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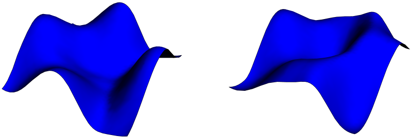

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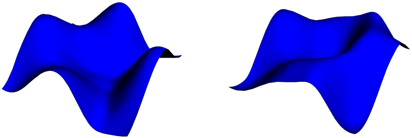

dimensionality

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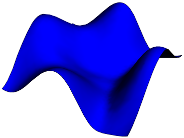
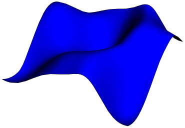
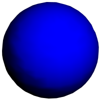
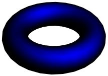




dimensionality

topology





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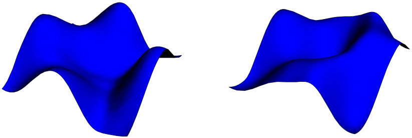

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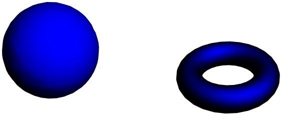
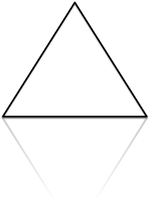


matrix/tensor models [Weingarten, Ambjorn, Durhuus, Fröhlich, Kazakov, Migdal, Boulatov, 1980's]
(group field theories/ tensor track [Freidel, Gurau, Oriti, Rivasseau, 2000's]):
metric & topology fluctuate

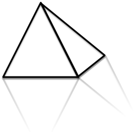
evaluate path integral by discretization

Building quantum spacetime

building blocks of spacetime:



2d



3d

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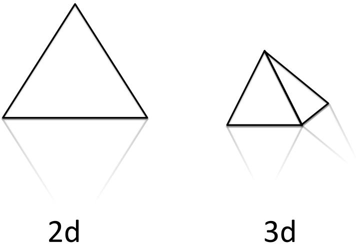

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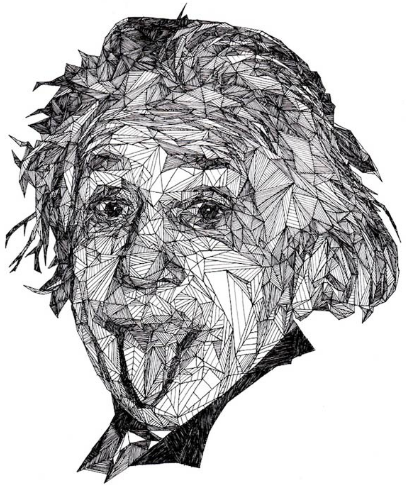
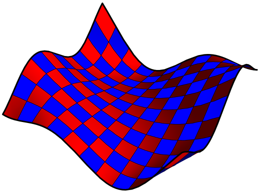


[by Josh Bryan]

Building quantum spacetime

$$\sum_{\text{top.}} \int \mathcal{D} g_{\mu\nu} e^{-S[g_{\mu\nu}]} \longrightarrow \sum_{\mathcal{T}} e^{-S[\mathcal{T}]}$$

superposition of quantum spacetimes → sum over simplicial complexes



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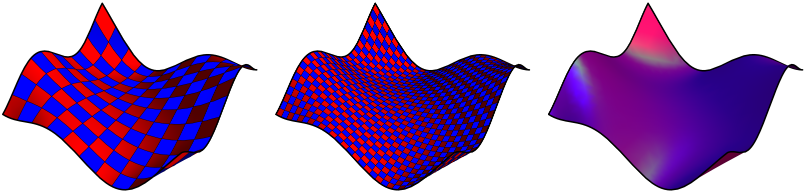
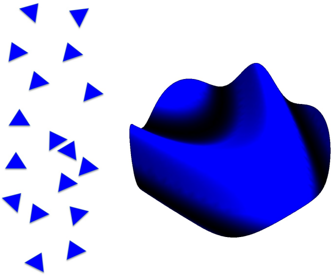


Main challenge: What is the continuum limit?

(Tensor models/ group field theories [Ben Geloun, Freidel, Gurau, Oriti, Rivasseau...])

Spacetime as a “condensate” of building blocks



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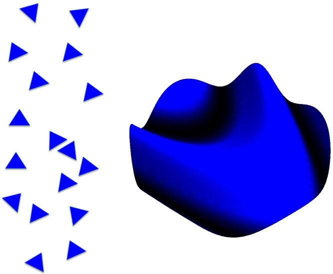
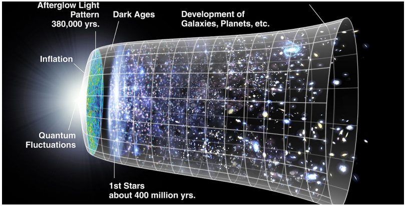


physical phase transition from pregeometric phase (Big Bang?)



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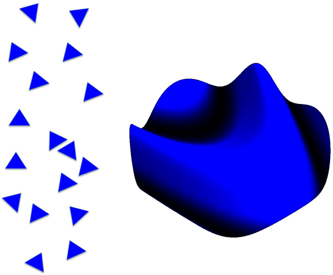

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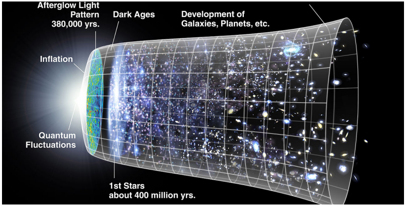

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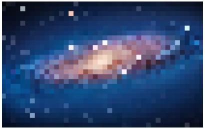
mathematical transition to the only physical phase



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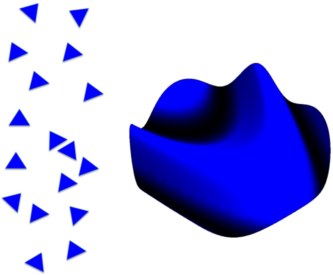

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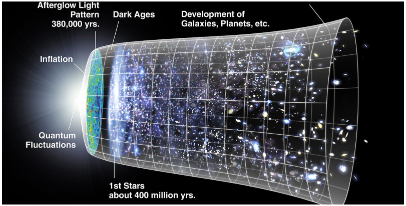

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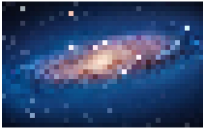

technically: Renormalization Group fixed point

two-dimensional quantum gravity

Matrix model for two-dimensional quantum gravity

2 d: $\int d^2x \sqrt{g} R = 4\pi\chi$

$\chi = 2 - 2h$:

$\chi = 2$

$\chi = 0$

$\chi = -2$

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$S = \frac{1}{16\pi G} \int d^2x \, \sqrt{g} \, (R - 2\Lambda) = \frac{1}{4G}\chi - 2\Lambda A$

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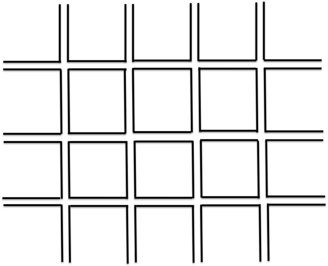

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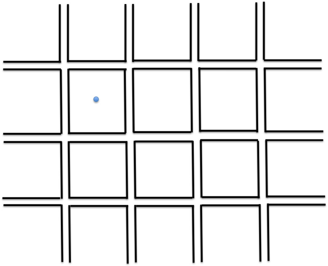

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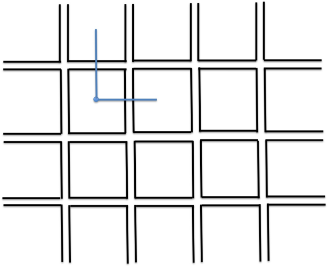

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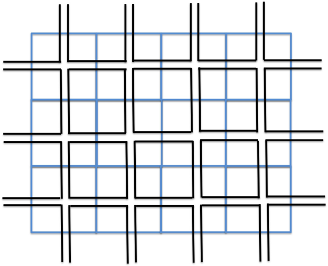

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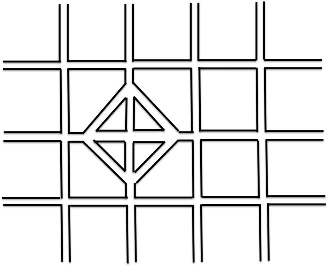

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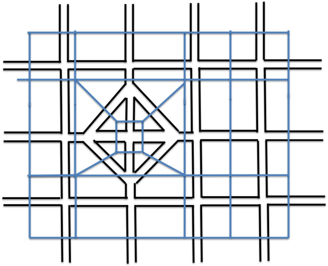

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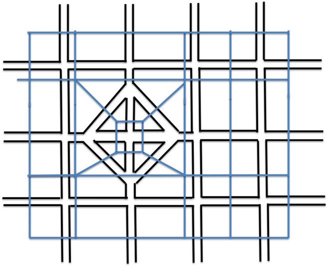
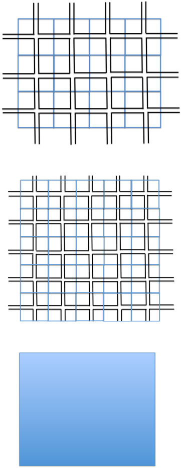


similar: d-dimensional gravity $\leftrightarrow$ rank-d tensor model

Matrix model for two-dimensional quantum gravity

continuum limit: $Z_{grav} \sim \ln \left(\int d\phi e^{-\mathrm{tr}\phi^2 - g\mathrm{tr}\phi^4} \right)$

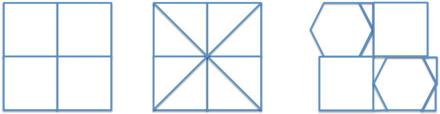


$g \to g_c$

$\langle A \rangle \to \infty$

(cf. phase transition)

$\to$ universal behavior!



Large N limit

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Double-scaling limit: [Brézin& Kazakov, Gross & Migdal, Douglas & Shenker, 1990]

$N \to \infty$ and $g \to g_c$ such that N^{-2h} is compensated

$$N(g - g_c)^{1-\gamma_s/2} = \text{const.}, \qquad \gamma_s = -1/2.$$

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$\to$ looks like RG fixed-point behavior: $g(k) = g_* + c\left(\frac{k}{k_0}\right)^{-\theta}$

Double-scaling (continuum) limit $\leftrightarrow$ Renormalization Group fixed point

$$g(N) = g_c + \left(\frac{N}{c}\right)^{-2/(2-\gamma_s)} \leftrightarrow g(k) = g_* + c\left(\frac{k}{k_0}\right)^{-\theta}$$

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perturbative approach [Brézin, Zinn-Justin, 1992]

FRG approach [A.E., Tim Koslowski, 2013]

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main advantage:
extension to higher-dimensional models conceptually clear
(technically: larger theory space [Rivasseau, 2014])

Renormalization group scale

basic RG requirement: distinguish "low-momentum" from "high-momentum" quantum fluctuations

quantum gravity: metric fluctuates

discrete approaches: based on "pre-geometric" building blocks
$\Rightarrow$ No notion of momentum

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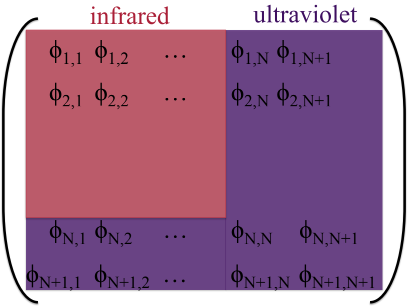
use N to set a scale [Zinn-Justin, Brezin, 1993; Oriti, Rivasseau, Gurau..., since 2010]



$N \to \infty$ no degrees of freedom integrated out (ultraviolet)

$N \to 0$ all degrees of freedom integrated out (infrared)

FRG for matrix models

regulator $\phi_{a,b} R_N(a,b)_{abcd} \phi_{cd}$ with $R_N(a,b) = \left(\frac{2N}{a+b} - 1\right) \theta\left(1 - \frac{a+b}{2N}\right)$

inspired by "Litim-cutoff"

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effective action Γ_N from $Z_N = \int d\phi \, e^{-S[\phi] - \frac{1}{2}\mathrm{tr}\phi R_N \phi}$

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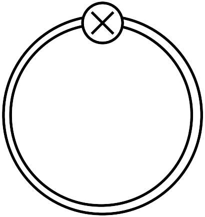
$N\partial_N \Gamma_N = \frac{1}{2}\mathrm{tr}\left(\Gamma_N^2 + R_N\right)^{-1} N\partial_N R_N$



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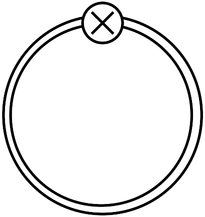

theory space: $\Gamma_N = \sum_i \frac{g_{2i}}{2}\mathrm{tr}\phi^{2i} + \sum_{i,j;\, i+j\,\mathrm{even}} \frac{g_{i,j}}{2}\mathrm{tr}\phi^i\,\mathrm{tr}\phi^j + \ldots$

assignment of canonical dimensionality: $\bar{g}_i = g_i N^{-(i-2)/2}$

SU(N) symmetry breaking and tadpole approximation

RG approach: cutoff at $N \Rightarrow$ symmetry-breaking

cutoff function: $R_N(a,b) = \left(\frac{2N}{a+b} - 1\right)\theta\left(1 - \frac{a+b}{2N}\right) \Rightarrow$ completely broken symmetry $\Rightarrow \phi_{ab} f(a,b) \phi_{ba}$ etc are generated

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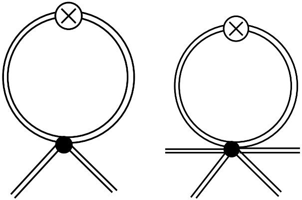

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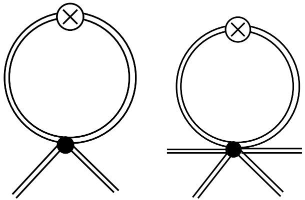

✓ $SU(N)$ symmetry unbroken

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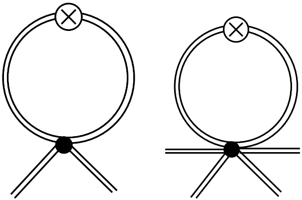

✓ $SU(N)$ symmetry unbroken

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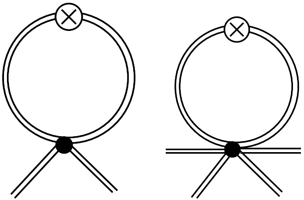

- ✓ $SU(N)$ symmetry unbroken
- ✓ computational simplicity (important at multi-trace level)
- ✓ consistent approximation if fixed-point couplings small

Results: Double-scaling limit

search for: fixed point at $g_4 < 0$ with $\theta_1 = 0.8$ and $\theta_n < 0$ ($n \geq 2$)

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$\eta = -\partial_t \ln Z = 2 g_4 x$

$\beta_{g_{2n}} = ((n-1) + \eta n)\, g_{2n} - 2n\, x\, g_{2(n+1)}$ [A.E., T. Koslowski, 2013, 2014]

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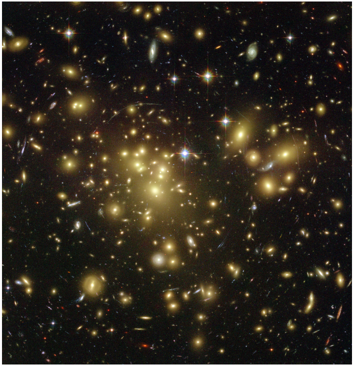
Double-scaling limit recovered with Functional RG (multi-trace truncation confirms result)

Multicritical points

Experimental fact:

Multicritical points

Experimental fact: Universe contains gravity & matter



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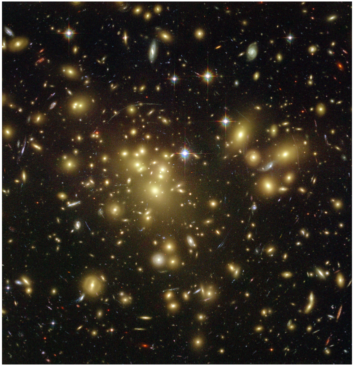

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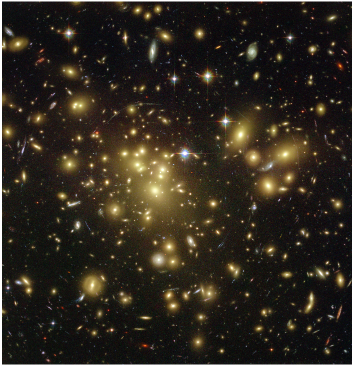

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Here: matter $\leftrightarrow$ collective excitations of matrix model

Multicritical points

fixed point $V[\phi] = \frac{1}{2}\mathrm{tr}\phi^2 + g_4\mathrm{tr}\phi^4$ with $g_4 < 0 \rightarrow$ 2d gravity

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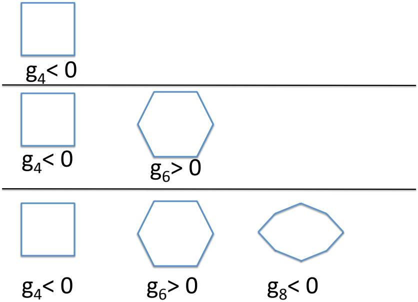

pure-gravity case

multicritical case

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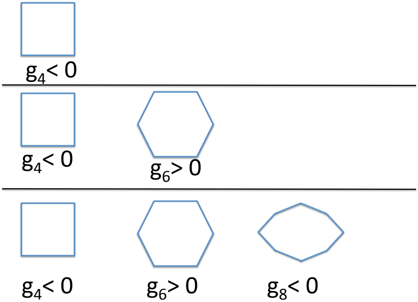

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some geometric d.o.f. contribute with negative weight $\Rightarrow$ extra d.o.f.?

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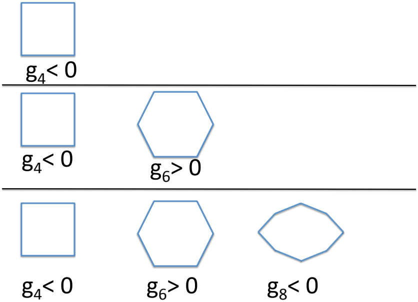

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$\rightarrow$ gravity critical exponent: $\gamma_s = 3/2 - m$ $\quad [\theta = 2/(m + 1/2)]$

for $m = 2$ (pure gravity), $m > 2$ (gravity + conformal matter)

Results: Multicritical points

analytic results for fixed points: [A.E., T. Koslowski, 2014]

g_4	g_6	g_8	g_{10}	g_{12}	g_{14}	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
0	0	0	0	0	0	-1	-2	-3	-4	-5	-6
$\frac{-1}{4x}$	0	0	0	0	0	1	$-\frac{1}{2}$	-1	$-\frac{3}{2}$	-2	-$\frac{5}{2}$
$\frac{-1}{3x}$	$\frac{1}{36x^2}$	0	0	0	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	-1	$-\frac{4}{3}$
$\frac{-3}{8x}$	$\frac{3}{64x^2}$	$-\frac{1}{512x^3}$	0	0	0	1	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{3}{4}$
$\frac{-2}{5x}$	$\frac{3}{50x^2}$	$\frac{-1}{250x^3}$	$\frac{1}{10^4x^4}$	0	0	1	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$-\frac{1}{5}$	$-\frac{2}{5}$
$\frac{-5}{12x}$	$\frac{5}{72x^2}$	$-\frac{5}{864x^3}$	$\frac{5}{20736x^4}$	$-\frac{1}{248832x^5}$	0	1	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$
$\frac{-3}{7x}$	$\frac{15}{196x^2}$	$-\frac{5}{686x^3}$	$\frac{15}{38416x^4}$	$-\frac{3}{268912x^5}$	$\frac{1}{7529537x^6}$	1	$\frac{6}{7}$	$\frac{5}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	$\frac{2}{7}$

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$\frac{-2}{5x}$	$\frac{3}{50x^2}$	$\frac{-1}{250x^3}$	$\frac{1}{10^4x^4}$	0	0	1	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$-\frac{1}{5}$	$-\frac{2}{5}$
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Gaußian fixed point

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g_4	g_6	g_8	g_{10}	g_{12}	g_{14}	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
0	0	0	0	0	0	-1	-2	-3	-4	-5	-6
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$\frac{-1}{3x}$	$\frac{1}{36x^2}$	0	0	0	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	-1	$-\frac{4}{3}$
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$\frac{-2}{5x}$	$\frac{3}{50x^2}$	$\frac{-1}{250x^3}$	$\frac{1}{10^4x^4}$	0	0	1	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$-\frac{1}{5}$	$-\frac{2}{5}$
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Gaußian fixed point

pure gravity fixed point

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$\frac{-2}{5x}$	$\frac{3}{50x^2}$	$\frac{-1}{250x^3}$	$\frac{1}{10^4x^4}$	0	0	1	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{5}$	$-\frac{1}{5}$	$-\frac{2}{5}$
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pure gravity fixed point

multicritical points

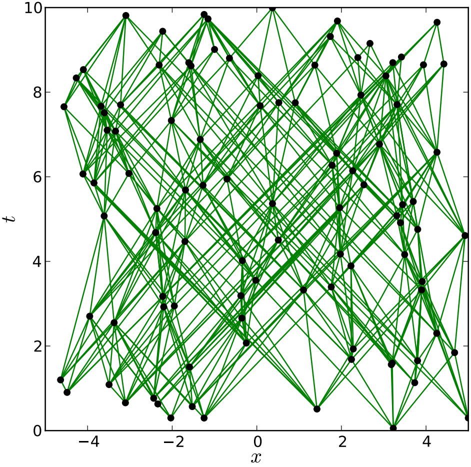
exact result: $\theta = 2/(m + 1/2)$, here: $\theta = 2/m$

Multicritical points recovered with Functional RG

Quantum spacetime is ...

Quantum spacetime is ...

... a causal set



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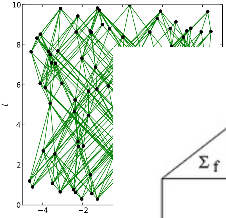
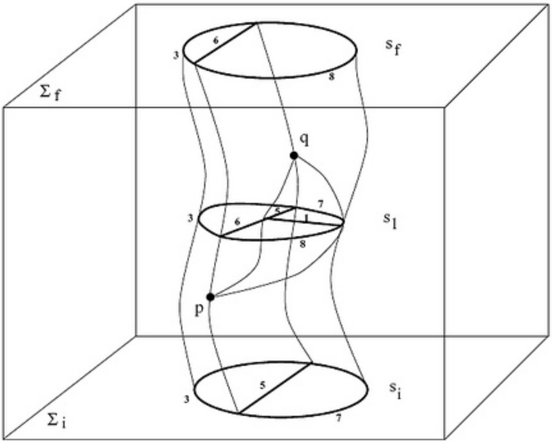


... a spinfoam



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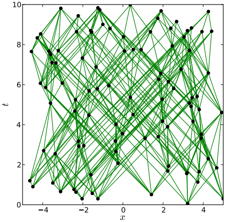

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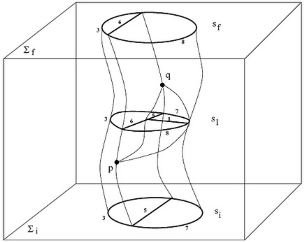
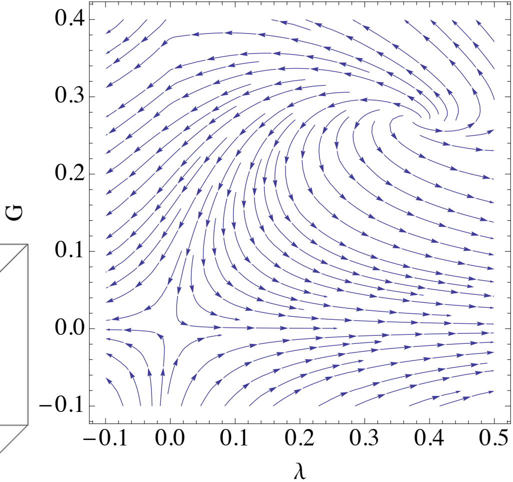


... a QFT described by an RG fixed point



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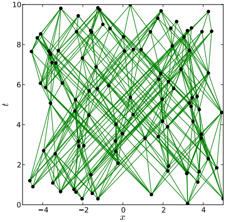
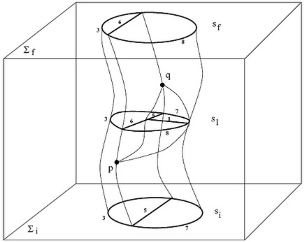


... noncommutative

$[x_1, x_2] \sim \theta$

... a spinfoam



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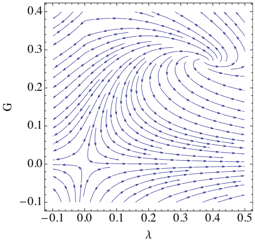

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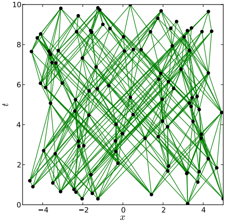

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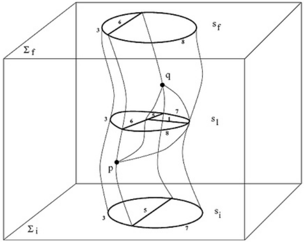
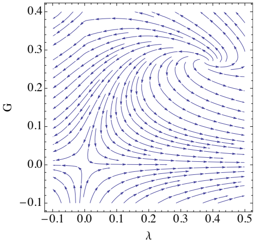


... described by an RG fixed point

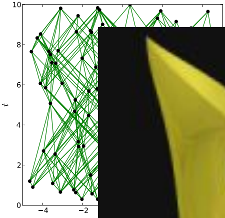


... noncommutative

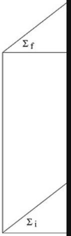
$[x_1,x_2] \sim \theta$

Quantum spacetime is ...

... a causal set



...stringy and braney



py an RG
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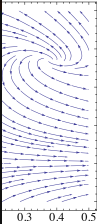

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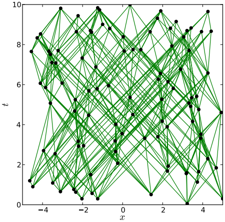

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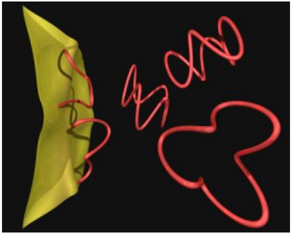

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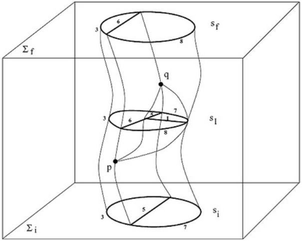

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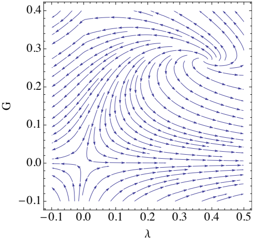

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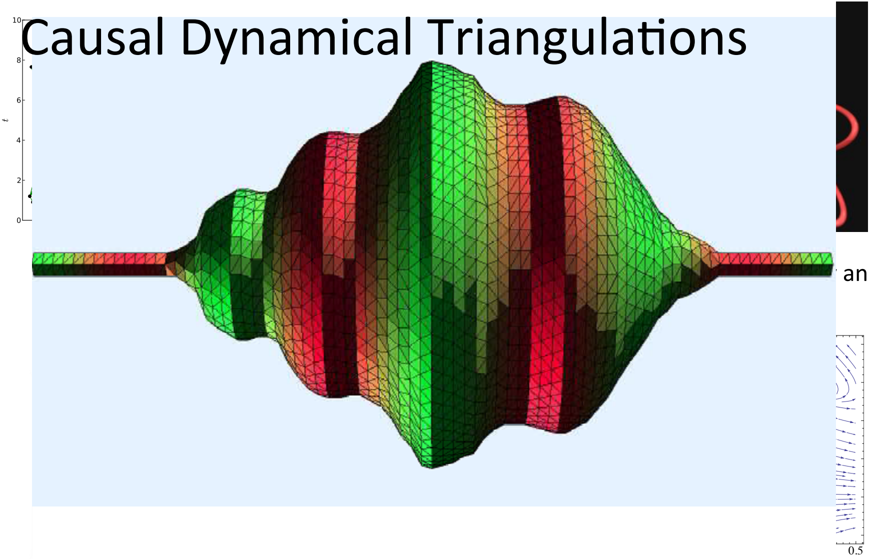
Quantum spacetime is ...

... the continuum limit of Causal Dynamical Triangulations



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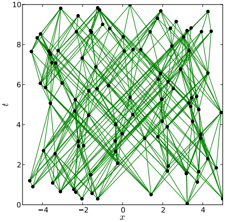

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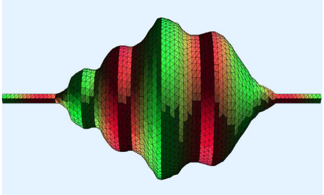

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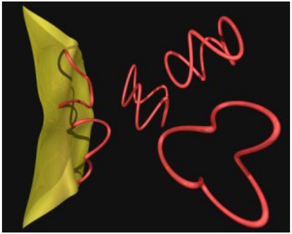

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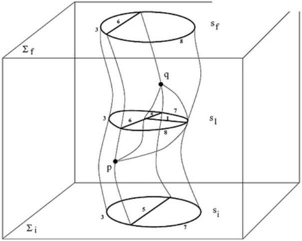

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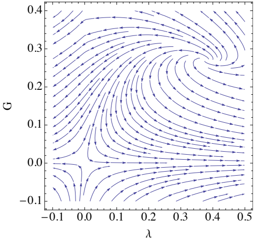

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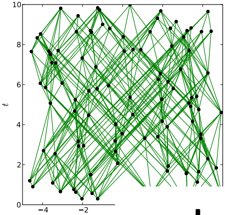

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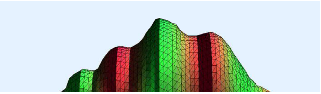

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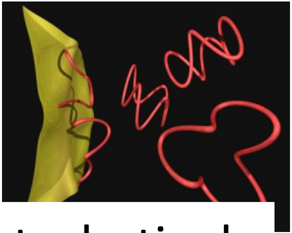
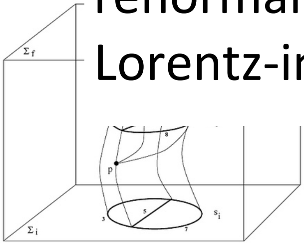


… described by a perturbatively renormalizable theory with Lorentz-invariance violation



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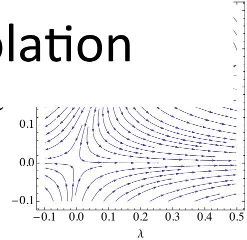

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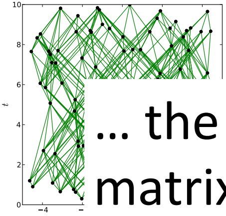

... the continuum limit of
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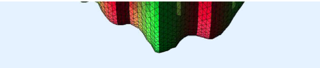
... the continuum limit of a matrix/ tensor model



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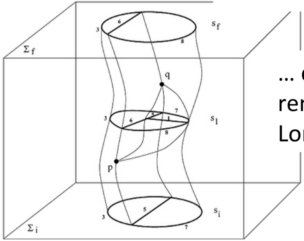

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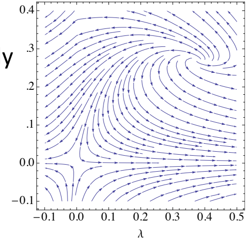

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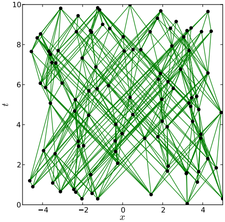

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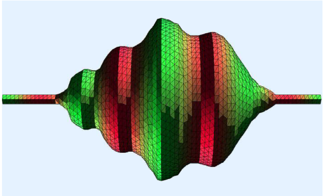

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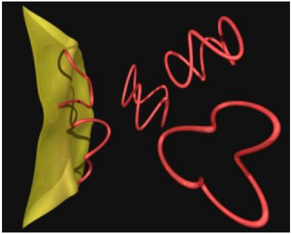

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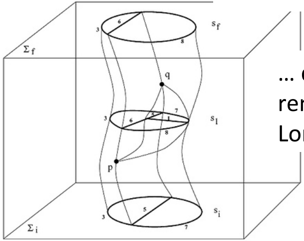

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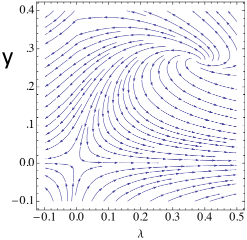

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Which way to quantum gravity?

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Are any of the quantum gravity models related?

Two sides of the same picture?

Two sides of the same picture?

Discrete approaches:
pregeometric phase

phase transition
to continuum spacetime

Asymptotic safety:
geometric phase: $\langle g_{\mu\nu} \rangle$ exists

interacting fixed point
in the ultraviolet

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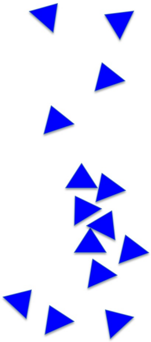

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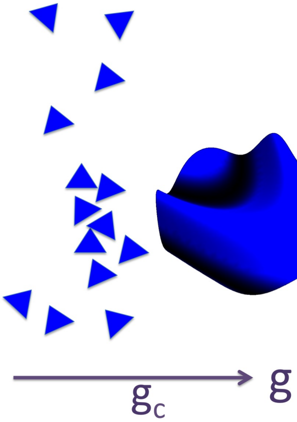

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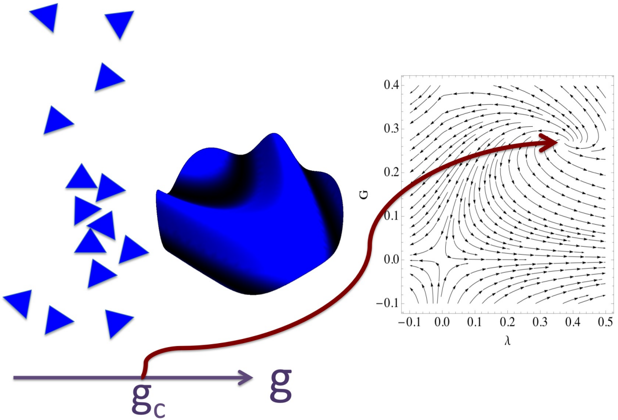
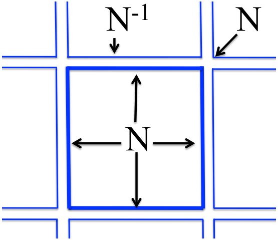


Double-scaling limit and continuum beta function

Double-scaling limit and continuum beta function

matrix model:
configurations scale with

N^{F-E+V}
$= N^{\chi}$



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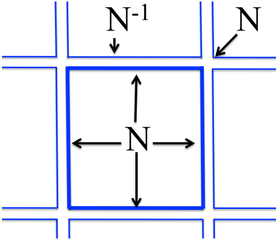

spacetime description:

$e^{-S} = e^{-\Lambda A + \frac{1}{4G_0}\chi}$

configurations scale with

$e^{\frac{1}{4G_0}\chi}$

Double-scaling limit and continuum beta function

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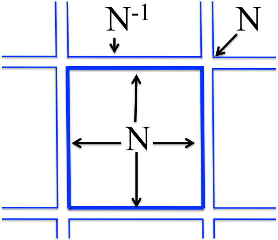

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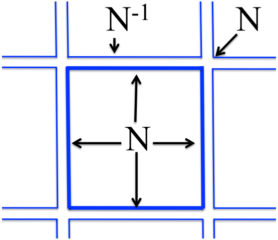

N^{χ}

g_4^n (n: number of building blocks)

spacetime description:

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$e^{-\Lambda A} = e^{-\Lambda a^2 n}$

Double-scaling limit and continuum beta function

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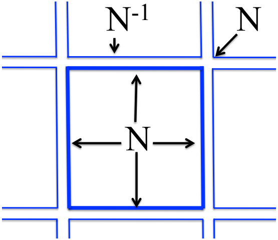

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$e^{\frac{1}{4\mathbf{G_0}}\chi}$

$e^{-\Lambda A} = e^{-\Lambda a^2 n}$

$$(g_4 - g_{4\,c})^{5/4} N = \text{const} \Rightarrow \left(\Lambda_R a^2\right)^{5/4} e^{\frac{1}{4G_0}} = \text{const} \rightarrow G_0(a)$$ [J. Ambjorn]

$$\Rightarrow \beta_G = -a\partial_a G = -10G^2$$

Double-scaling limit and continuum beta function

$\beta_G = -10G^2$

Double-scaling limit and continuum beta function

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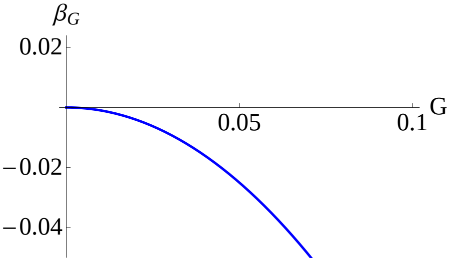

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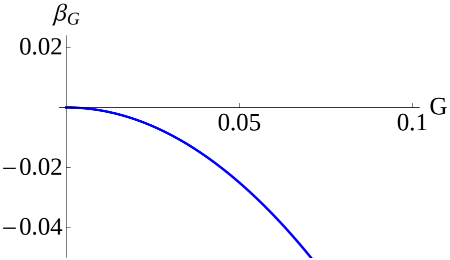
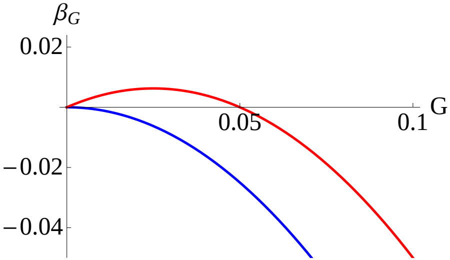


$d = 2 + \epsilon$

[Christensen, Duff, 1978; Gastmanns et al., 1978; Weinberg, 1979]

$\beta_G = \epsilon G - cG^2 \ (c > 0)$

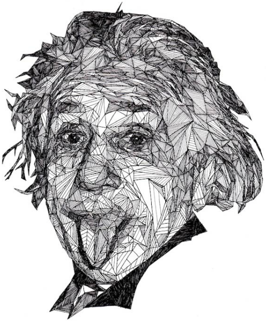


interacting fixed point (asymptotic safety) $\leftrightarrow$ double-scaling limit

Summary

Matrix/tensor models for quantum gravity:
Sum over spacetimes $\leftrightarrow$ Matrix/tensor path integral

Continuum limit:
double scaling limit $\leftrightarrow$ RG fixed point



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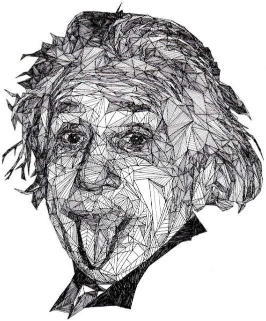

Continuum limit:
double scaling limit $\leftrightarrow$ RG fixed point

new FRG tools: double-scaling limit (pure gravity and with matter) in matrix models

connection between double-scaling limit and $d = 2 + \epsilon$ continuum beta function showing asymptotic safety

Outlook: Towards $d = 4$, quantitative precision, and new connections

- towards quantitative precision: control symmetry-breaking sector
- towards $d = 4$: similar strategy as in $d = 2$, larger theory space
  ongoing work in $d = 3$ tensor models [Ben Geloun, Benedetti, Oriti]
- relation to continuum approaches/ asymptotic safety: compare critical exponents in $d = 4$

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Thank you for your attention!