Magnetic susceptibility of strongly interacting matter

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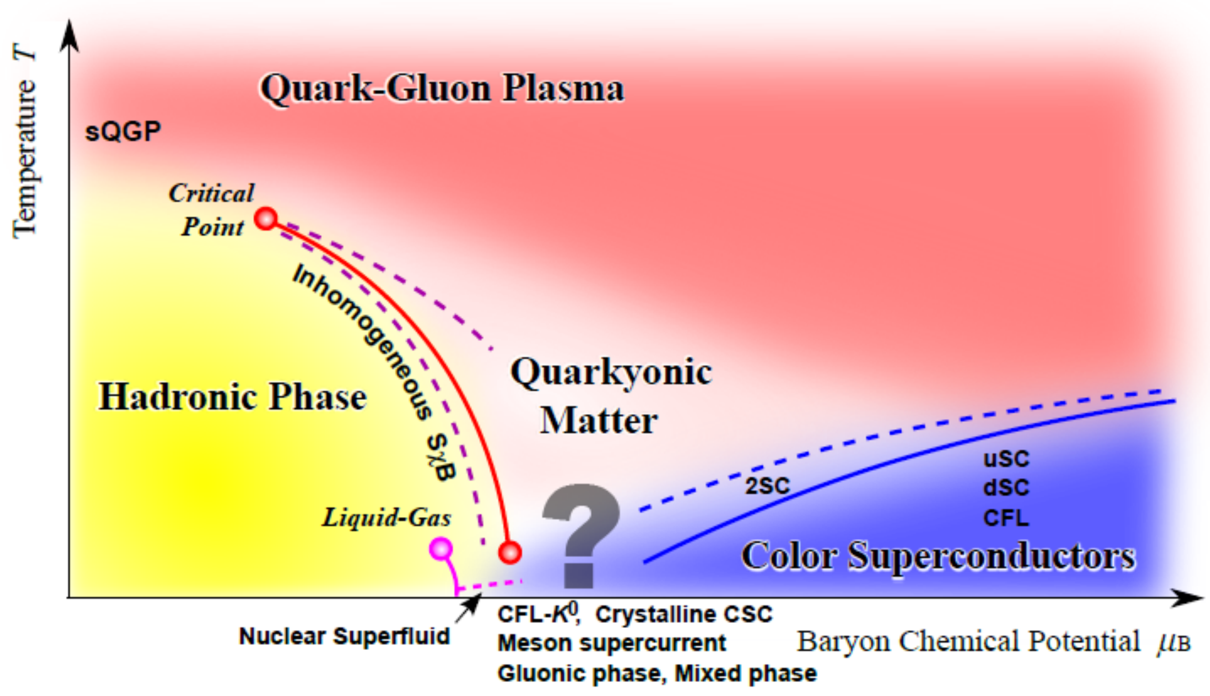
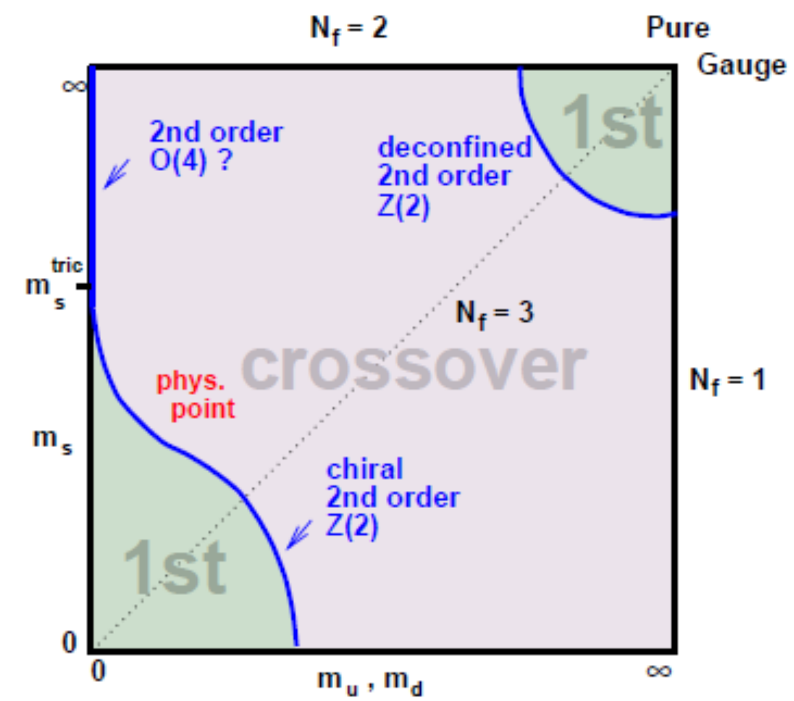
QCD phase diagram



Fukushima et al., 1005.4814



Bonati et al., 1201.2769

- QCD shows rich phenomena when internal/external parameters are changed
  T, μ, N_c, N_f, m_u, m_s,…

- External magnetic field is one of important parameters.
  Magnetars : 10^{10} [T] = 10^{6} [G]
  Non central heavy ion collision :
  10^{11} [G] ~ $eB = 0.1$ [GeV^2] ~ $5\, m_\pi^2$

Aim of this talk

- In this talk, we discuss properties of the strongly interacting matter under static magnetic field.
  - Magnetism of the QCD vacuum
  - Chiral phase transition under strong magnetic field

Free energy and χ

$$B^{ind} = B^{ext} + M = (1+\chi)B^{ext}$$

Free energy: $\Omega = -P$

Magnetisation: $M = -\dfrac{\partial\Omega}{\partial(eB)} \sim \chi(eB)$

Magnetic susceptibility: χ

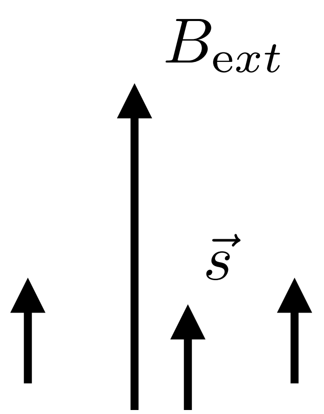
$$\Omega \sim \Omega_0 - \frac{\chi}{2}(eB)^2 + O(eB)^4$$ or $$P \sim P_0 + \frac{\chi}{2}(eB)^2 + O(eB^4)$$

- Magnetic susceptibility is the second order coefficient of the free energy.

Electron gas (Landau vs Pauli)

$$B_{net} = (1+\chi)B_{ext} \qquad \chi_{\text{tot}} = \chi_{\text{spin}} + \chi_{\text{orbit}} > 0$$

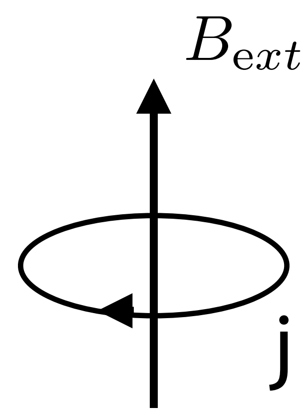
Pauli paramagnetism (Spin)



$$\chi_{\text{spin}} = 2\mu_B^2\rho(\epsilon_f) = \frac{\mu_B^2 m k_f}{3\hbar^2\pi^2}$$

Landau diamagnetism (Orbit)

Landau, 1930



$$\chi_{\text{orbit}} = -\frac{1}{3}\chi_{\text{spin}}$$

- Competition of spin and orbital angular momentum
- Totally, electron gas is paramagnetic.
- Magnetism of QCD vacuum will be determined by the nature of charged quarks and mesons.

Free particle (vacuum)

$$\Omega_q = \mathrm{Tr}\log[i\not{D} + m_q]$$

Andersen and Khan (2011)

$$x_f = \frac{m_q^2}{q_f B}$$

$$= \frac{N_c}{16\pi^2}\sum_f \left(\frac{\Lambda^2}{2|q_f B|}\right)^{\epsilon}\left[\left(\frac{2(q_f B)^2}{3} + m_q^4\right)\left(\frac{1}{\epsilon}+1\right) - 8(q_f B)^2\zeta^{(1,0)}(-1, x_f) - 2|q_f B| m_q^2 \log x_f + \mathcal{O}(\epsilon)\right]$$

$$\sim \#\frac{1}{\epsilon}(q_f B)^2 + \text{regular terms}$$

- The B square term has a divergence (ε->0).
- χ must be renormalised by the renormalisation of QED.
- To avoid the cutoff dependence, the following renormalisation condition is usually imposed in non-perturbative methods.

$$\chi(T=0) = 0$$

Normalised pressure

Bonati et.al 2013

$$\Delta P = (P(T,B) - P(T,0)) - (P(0,B) - P(0,0))$$

$$\sim \frac{\hat{\chi}(T)}{2}(eB)^2 = \frac{\chi(T) - \chi(0)}{2}(eB)^2$$

Free particle (thermal)

Quark (s = 1/2)

$$P_q^f = N_c \frac{|e_f B| T}{\pi^2} \sum_{n=0}^{\infty} \alpha_n \int_0^{\infty} dp \, \log\left(1 + \mathrm{e}^{-\beta\sqrt{p^2+m_q^2+2|e_f B|n}}\right)$$

$$\sim P_q^f(T) + \frac{\chi_q}{2}(T)(eB)^2$$

$$\begin{aligned} \hat{\chi}\big|_q &= -\frac{1}{3}\left(\frac{e_f}{e}\right)^2 \mathcal{G}_{\mathrm{q}}{}^{(1)}(0) \\ &= \frac{N_c}{3\pi^2}\left(\frac{e_f}{e}\right)^2 \int_0^{\infty} dp \frac{1}{\sqrt{p^2+m_q^2}} \frac{1}{\mathrm{e}^{\beta\sqrt{p^2+m_q^2}}+1} \quad > 0 \end{aligned}$$

Meson (s = 0)

$$P_{\pi^\pm} = -\frac{|eB| T}{\pi^2} \sum_{n=0}^{\infty} \int_0^{\infty} dp \, \log\left(1 - \mathrm{e}^{-\beta\sqrt{p^2+m_\pi^2+(2n+1)|eB|}}\right)$$

$$\sim P_{\pi^\pm}(T) + \frac{\chi_\pi}{2}(T)(eB)^2$$

$$\begin{aligned} \hat{\chi}\big|_{\pi^\pm} &= \frac{1}{12} \mathcal{G}_{\pi^\pm}{}^{(1)}(0) \\ &= -\frac{1}{12\pi^2} \int_0^{\infty} dp \frac{1}{\sqrt{p^2+m_\pi^2}} \frac{1}{\mathrm{e}^{\beta\sqrt{p^2+m_\pi^2}}-1} \quad < 0 \end{aligned}$$

- Quarks show paramagnetism ($\chi > 0$), while pions show the diamagnetism ($\chi < 0$).
- This may be understood as a competition of the orbital and spin magnetisation.
  (cf. Pauli paramagnetism and Landau diamagnetism)
- Let's see effects of interaction and phase transition.

⟶ Analysis on the quark meson model

3-flavor Quark meson model

$$\mathcal{L} = \overline{\psi}\left[\partial\!\!\!/ + g\sum_{a=0}^{8} T_a(\sigma_a + i\gamma_5\pi_a)\right]\psi + \mathrm{tr}[\partial_\mu\Sigma\partial_\mu\Sigma^\dagger] + U(\rho_1,\rho_2) - h_i\sigma_i - c_a\xi$$

$$\xi = \det\Sigma + \det\Sigma^\dagger\,, \qquad \Sigma = \sum_{a=0}^{8} T_a(\sigma_a + i\pi_a)$$

$$U(\rho_1,\rho_2) = a^{(1,0)}\rho_1 + \frac{a^{(2,0)}}{2}\rho_1^2 + a^{(0,1)}\rho_2$$

$$\rho_1 = \mathrm{tr}\left[\Sigma\Sigma^\dagger\right]$$

$$\rho_2 = \mathrm{tr}\left[\Sigma\Sigma^\dagger - \frac{1}{3}\rho_1\right]^2$$

- Σ is 3x3 complex matrix i.e., composed of 8 scalar and 8 pseudo scalar mesons.

- ρ_1 and ρ_2 are invariants under the U(3) x U(3) flavour-chiral rotation.

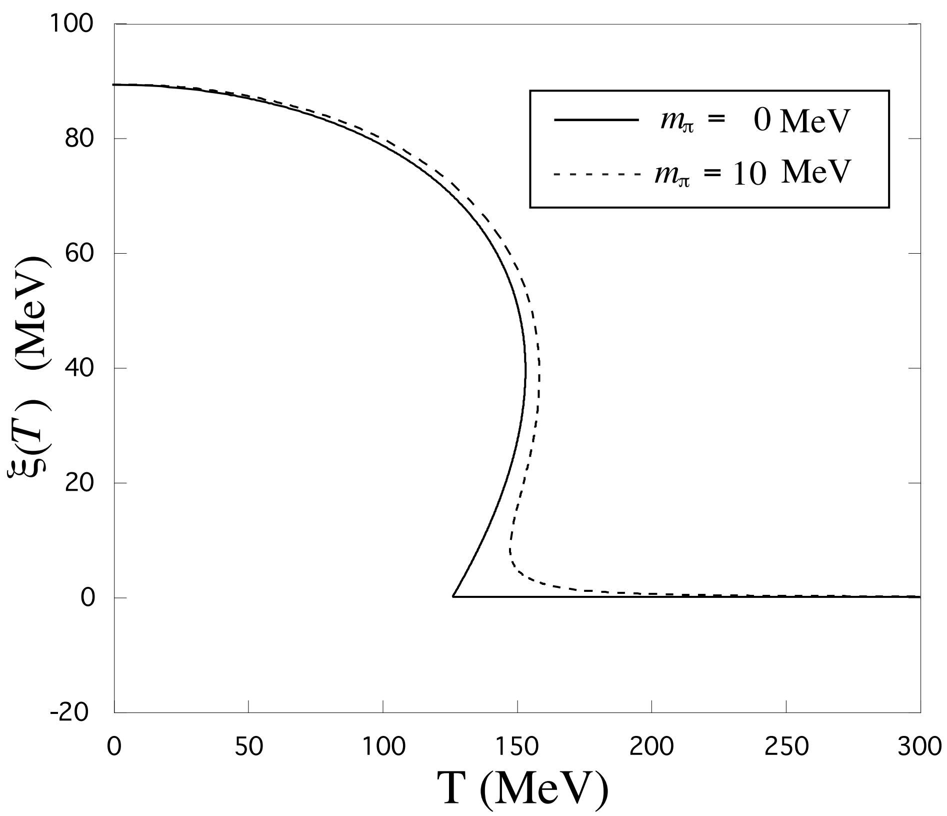
- C_a is Kobayashi-Masukawa term which represents the effects of $U_A(1)$ anomaly.

- Inclusion of external magnetic field is achieve by

$$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$$

Why Functional RG?

Perturbation theory at one-loop (O(4) scalar model)



S. Chiku and T. Hatsuda (1998)

$$\partial_\sigma U\Big|_{\sigma=\xi} = 0$$

- Meson fluctuation (at one-loop) makes phase transition first order.
  (contrary to the universality argument [O(n) in 3d])
- We need Functional-RG as a machinery to include meson loops effectively.

Functional RG

$$k\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left[\frac{k\partial_k R_{kB}}{R_{kB} + \Gamma_k^{(0,2)}[\varphi]}\right] - \mathrm{Tr}\left[\frac{k\partial_k R_{kF}}{R_{kF} + \Gamma_k^{(2,0)}[\varphi]}\right]$$

<u>*C. Wetterich (1993)*</u>

Γ_k : scale dependent effective action

$$\Gamma_{k=\Lambda}[\phi] = S[\phi] \longrightarrow \Gamma_{k=0}[\phi] = \Gamma[\phi]$$

UV: classical → IR: quantum

- Local potential approximation

$$\Gamma_k[\psi,\sigma,\pi] = \int_0^\beta dx_4 \int d^3x \left[\bar{\psi}\left[\gamma_\mu D_\mu + g(\Sigma + i\gamma_5 \Pi)\right]\psi + U_k(\rho_1,\rho_2) - h_i\sigma_i + \left((D_\mu\Sigma)^2 + (D_\mu\Pi)^2\right)\right]$$

- Cut off choice (3d sharp cutoff)

$$R_{k,B}(q) = (k^2 - \vec{q}^2)\theta(k^2 - \vec{q}^2)$$

$$R_{k,F}(q) = -i\vec{\not{p}}\left(\frac{k}{|\vec{p}|} - 1\right)\theta(k^2 - \vec{p}^2)$$

RG equation for U_k

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left\{\sum_{\pi,k,a,\phi,\sigma,\sigma',\eta,\eta'}\alpha_{e_b}(k)\frac{\coth\left[\frac{E_b}{2T}\right]}{E_b} - \sum_{u,d,s}\alpha_{e_f}(k)\frac{\tanh\left[\frac{E_f}{2T}\right]}{E_f}\right\}$$

Skokov 2012

$$\alpha_{e_f}(k) = 6N_c\frac{e_fB}{k^2}\left(1+2\sum_{n=1}^{\infty}\sqrt{1-2\frac{e_fB}{k^2}n}\;\theta\left[1-2\frac{e_fB}{k^2}n\right]\right) \to 4N_c\;(e_fB=0)$$

$$\alpha_{e_b}(k) = 3\frac{e_bB}{k^2}\sum_{n=0}^{\infty}\sqrt{1-\frac{e_bB}{k^2}(2n+1)}\;\theta\left[1-\frac{e_bB}{k^2}(2n+1)\right] \to 1\;(e_bB=0);.$$

Scale dependent Energies M.Mitter and B. J .Shaefer 2013

$$E_{u,d}^2 = k^2 + \frac{g^2}{4}\sqrt{\frac{4\rho_1-\sqrt{24\rho_2}}{3}} \qquad E_{k,\pi}^2 = k^2 + \partial_{\rho_1}U_k + \sqrt{24\rho_2}\partial_{\rho_2}U_k - \frac{c_a}{2}\sqrt{\frac{4\rho_1+2\sqrt{24\rho_2}}{3}}$$, etc

- Taylor expansion up to 3rd order (i,j≦3)

$$U_k(\rho_1,\rho_2) = \sum_{i,j}\frac{a_{i,j}}{i!j!}(\rho_1-\rho_{10})^i(\rho_2-\rho_{20})^j$$

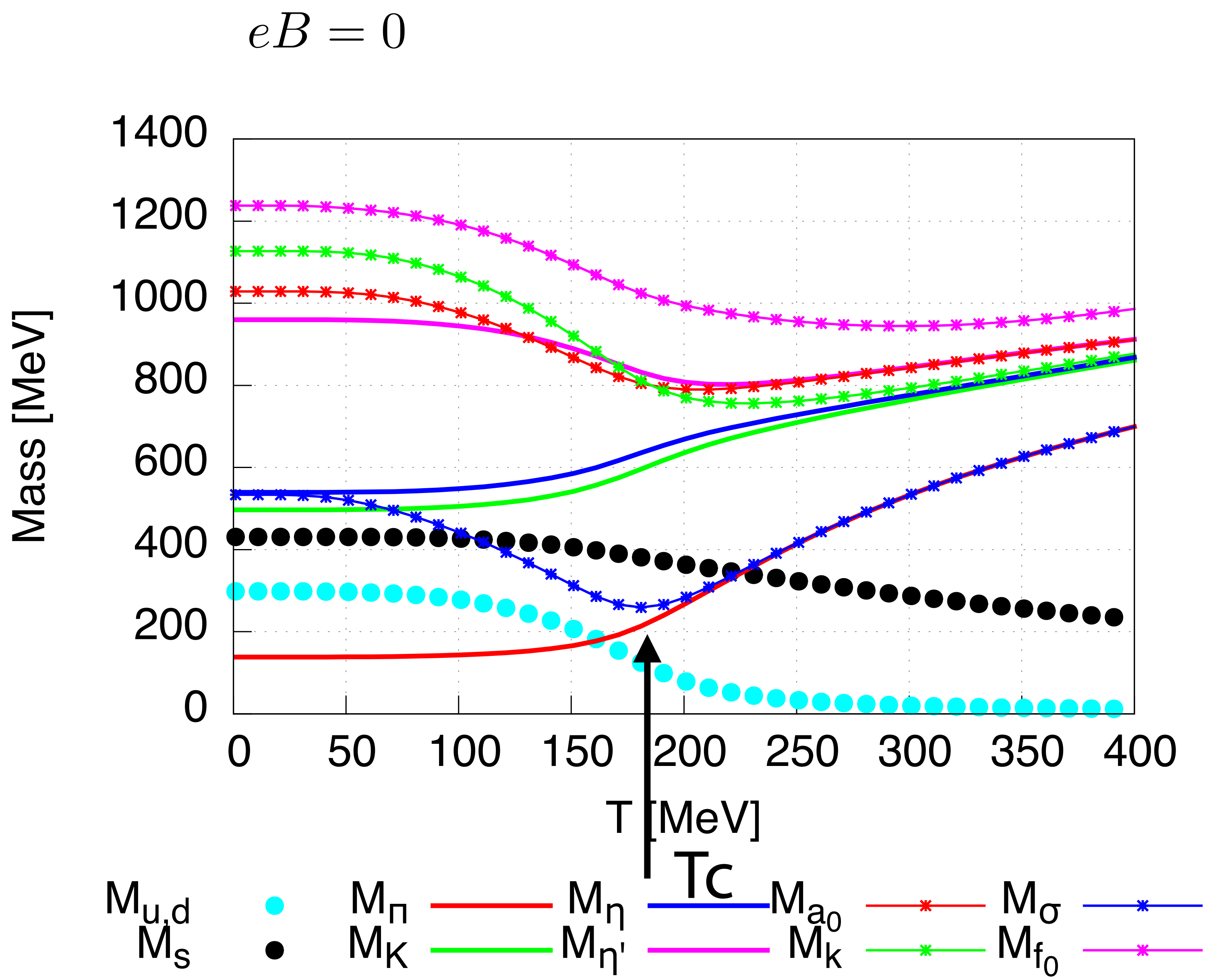
- Pressure is

$$P = -a_{0,0}|_{k=0} + c\sigma_0 + c_a\xi_0 + \int_{\Lambda}^{\infty} dk\partial_k U_k^F|_{\rho_1=\rho_2=0}$$

T. Herbst, et al. (2010)

Numerical Results

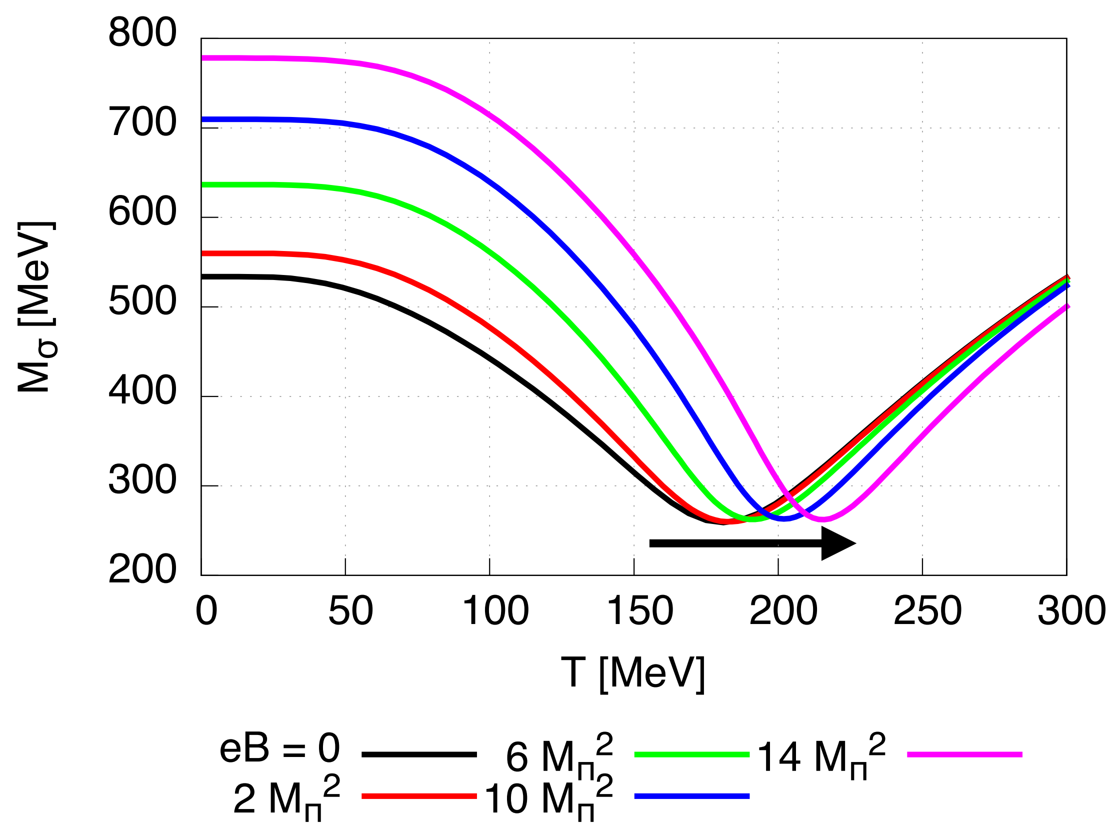
Meson curvature masses



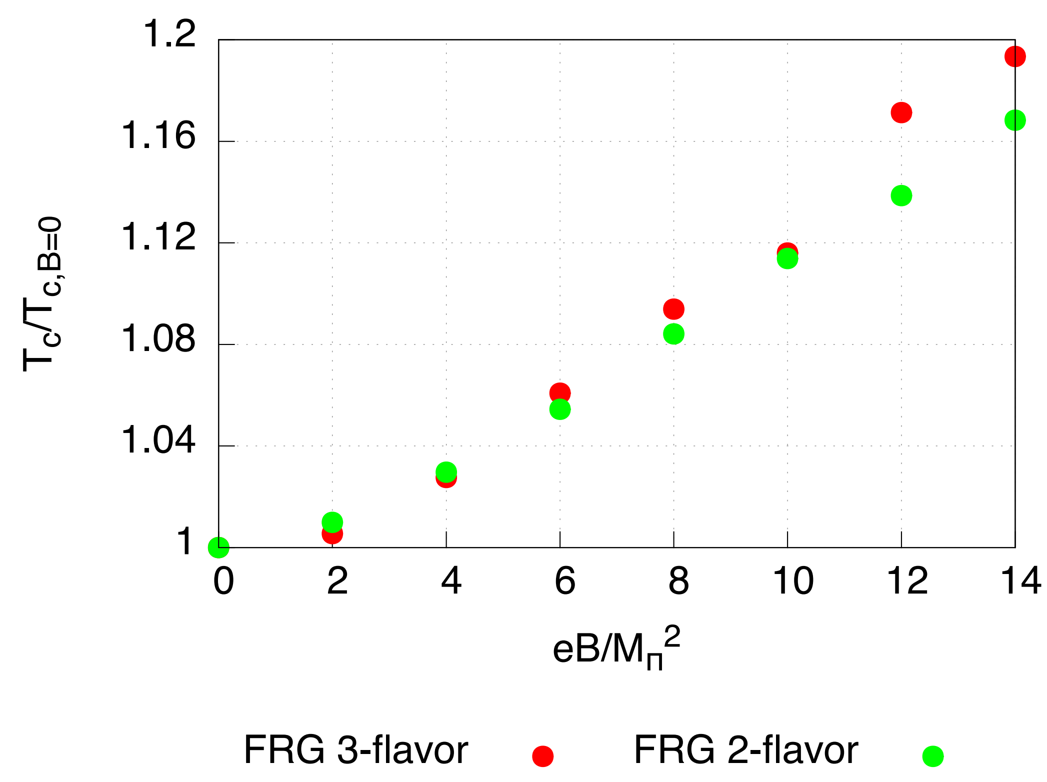
- Chiral phase transition occurs.
- Tc is determined by the minimum of sigma mass.

Phase transition

σ mass with varying eB



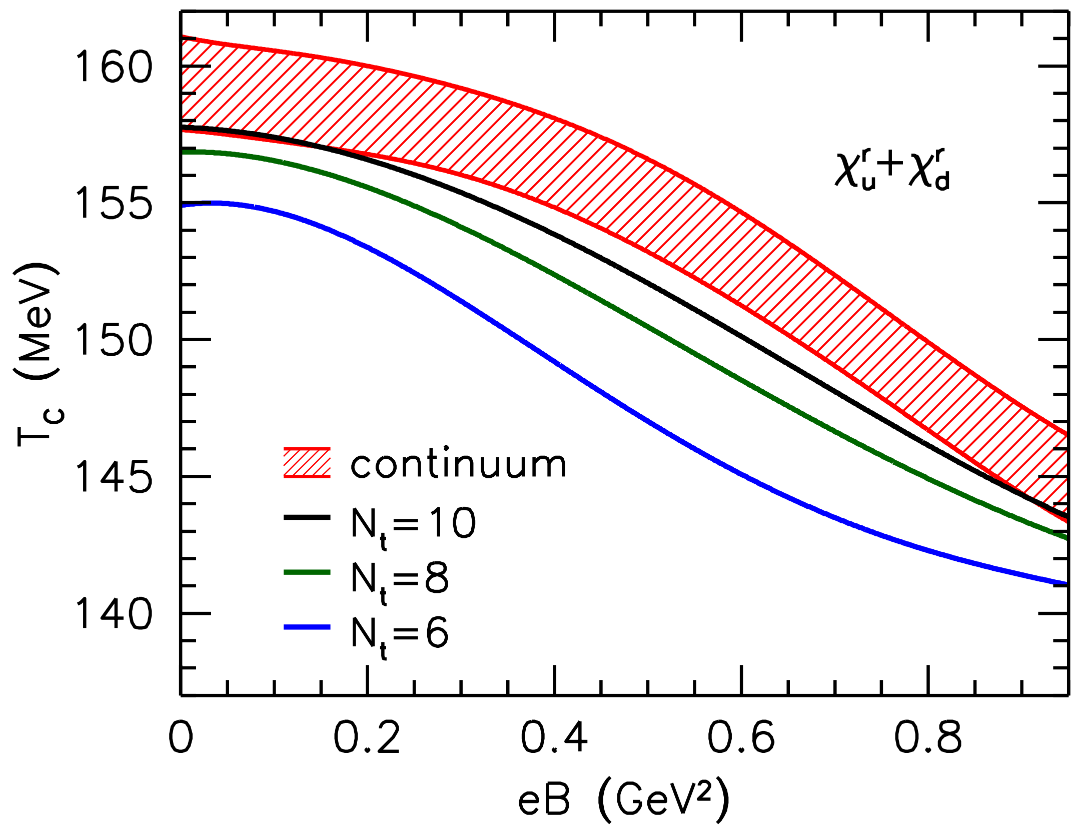
T_c at finite eB



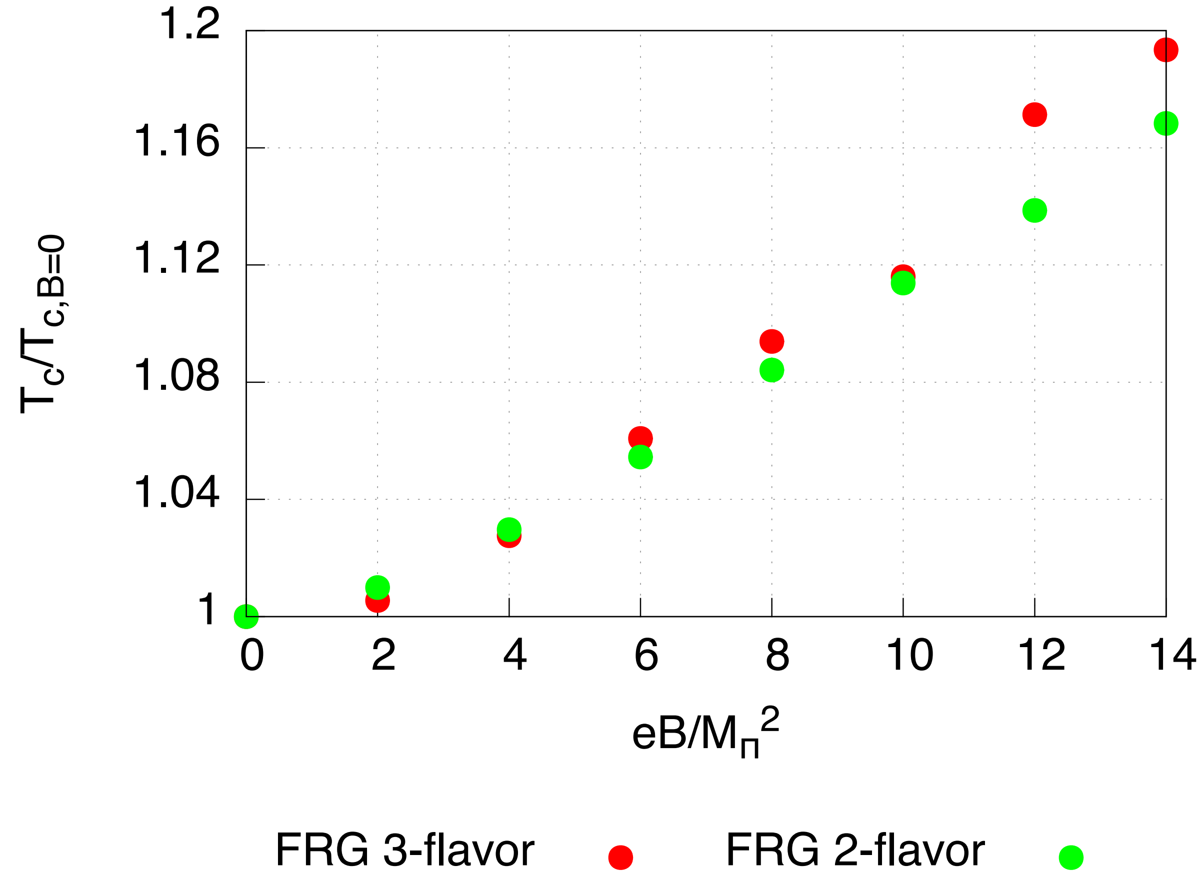
- Tc increases with increasing eB. (Magnetic catalyses)

Anti-Magnetic catalyses

2+1 flavor Lattice [G.S. Bali et al. (2012)]

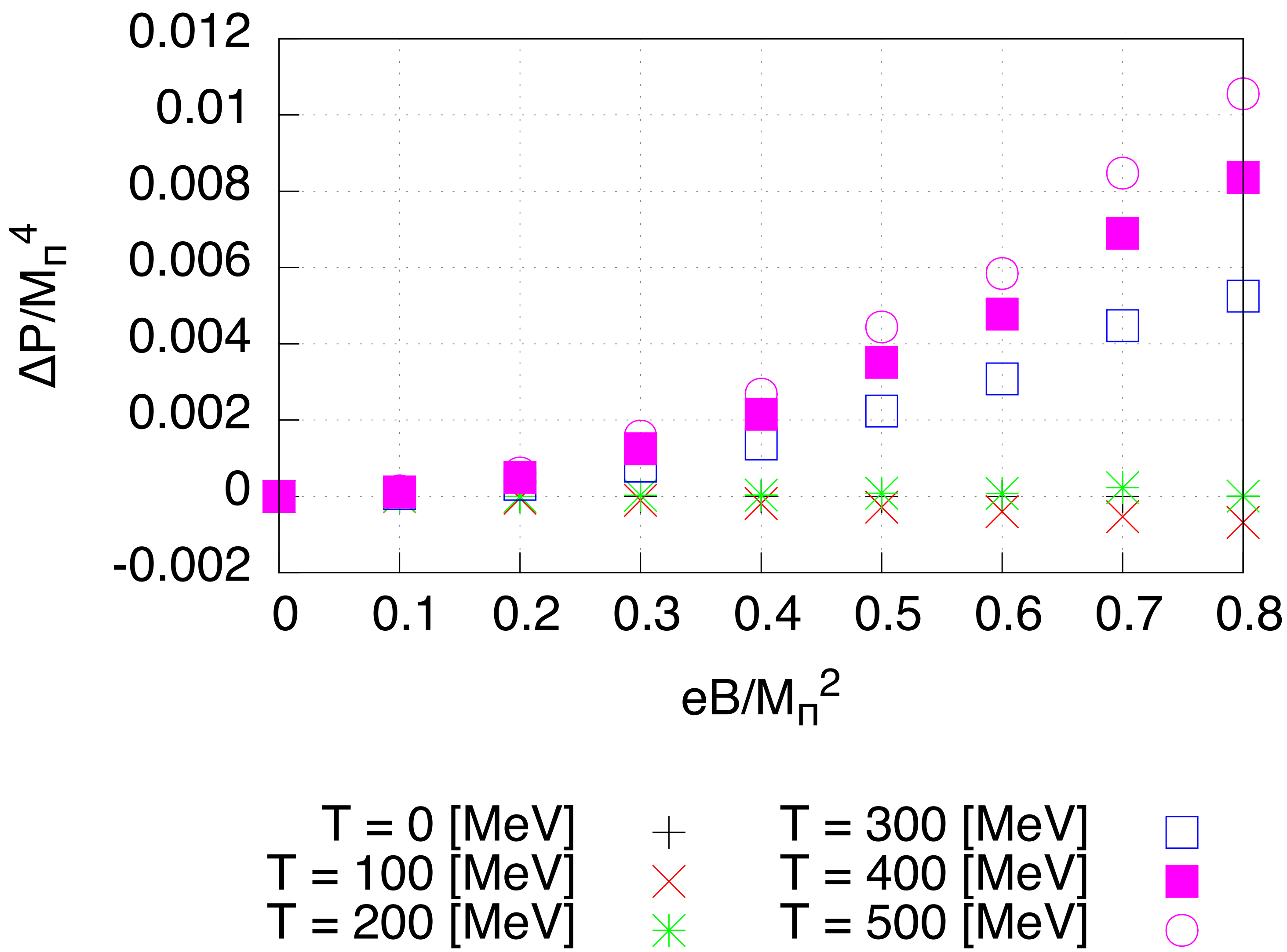


Our results



- Discrepancy between the lattice result.
- As far as we know, no chiral effective model explains the Anti-Magnetic catalyses.

Pressure

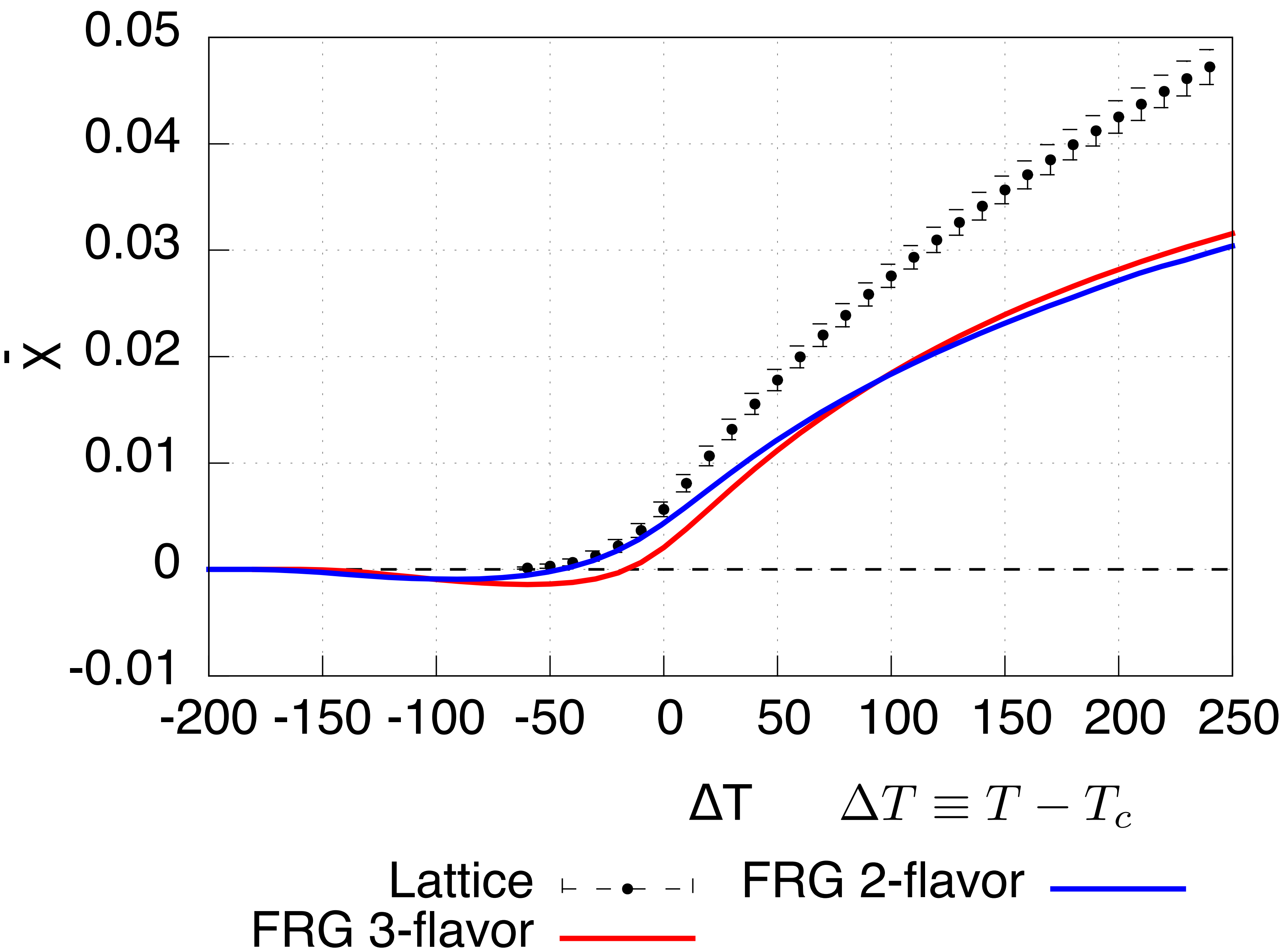


$$\Delta P(eB) \equiv (P(T,B) - P(T,0)) - (P(B,0) - P(0,0))$$

- Pressure vs eB for (0 < T < 500 [MeV] ~ 2.5 T_c)
- We fit the pressure with trial function using Gnuplot

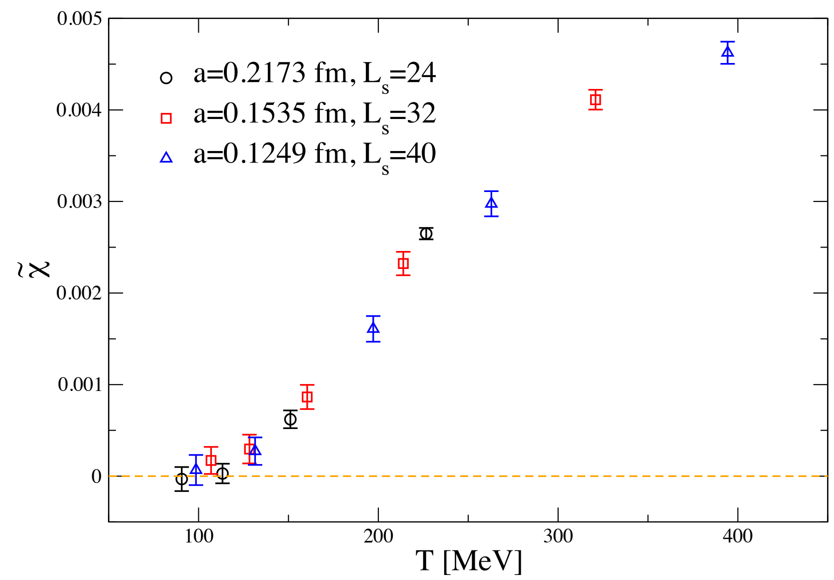
$$f(eB) = \frac{\hat{\chi}}{2}(eB)^2$$

Comparison with Lattice QCD



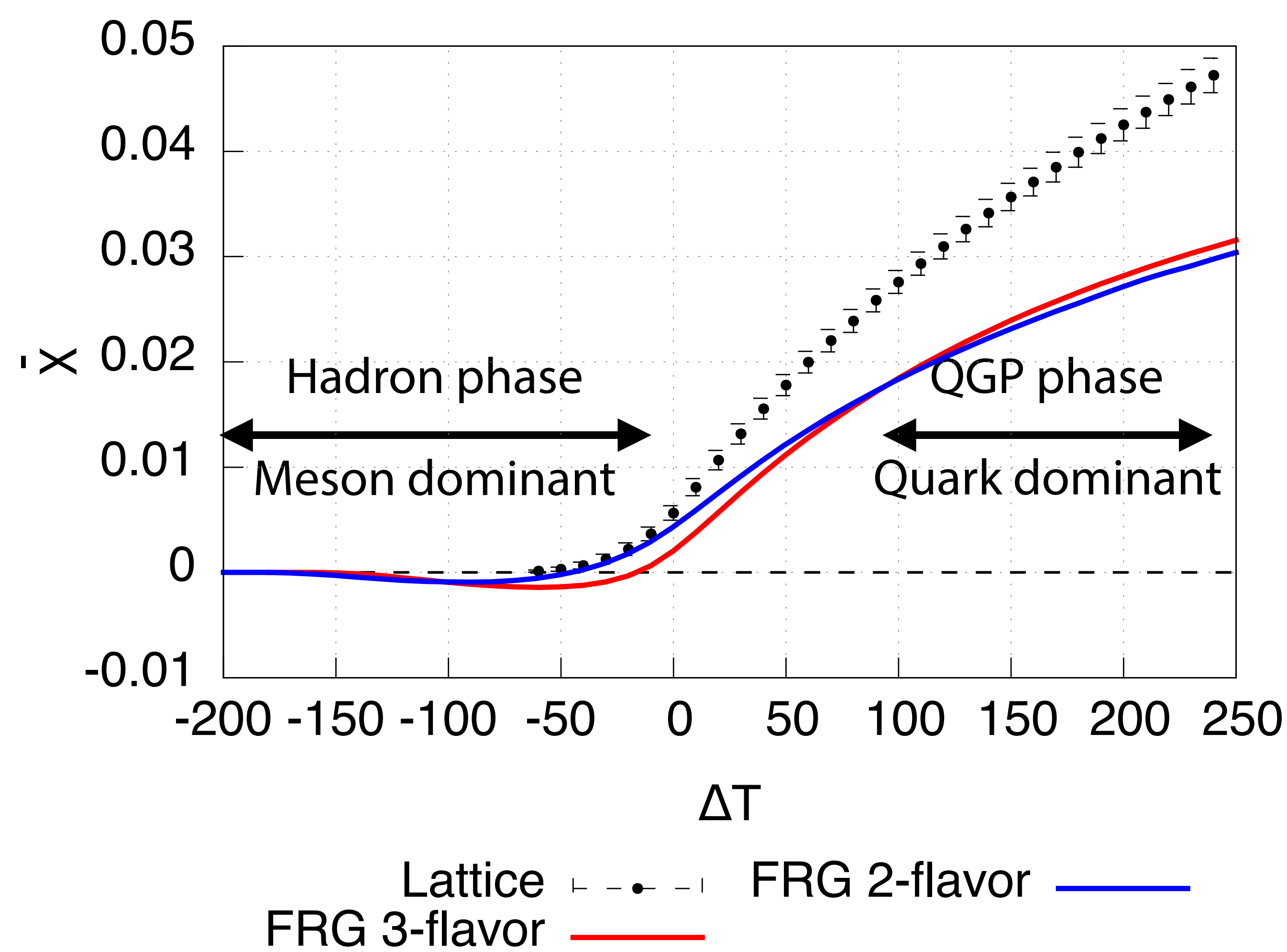
Lattice results on 2+1 flavor

Bonati et.al 2013

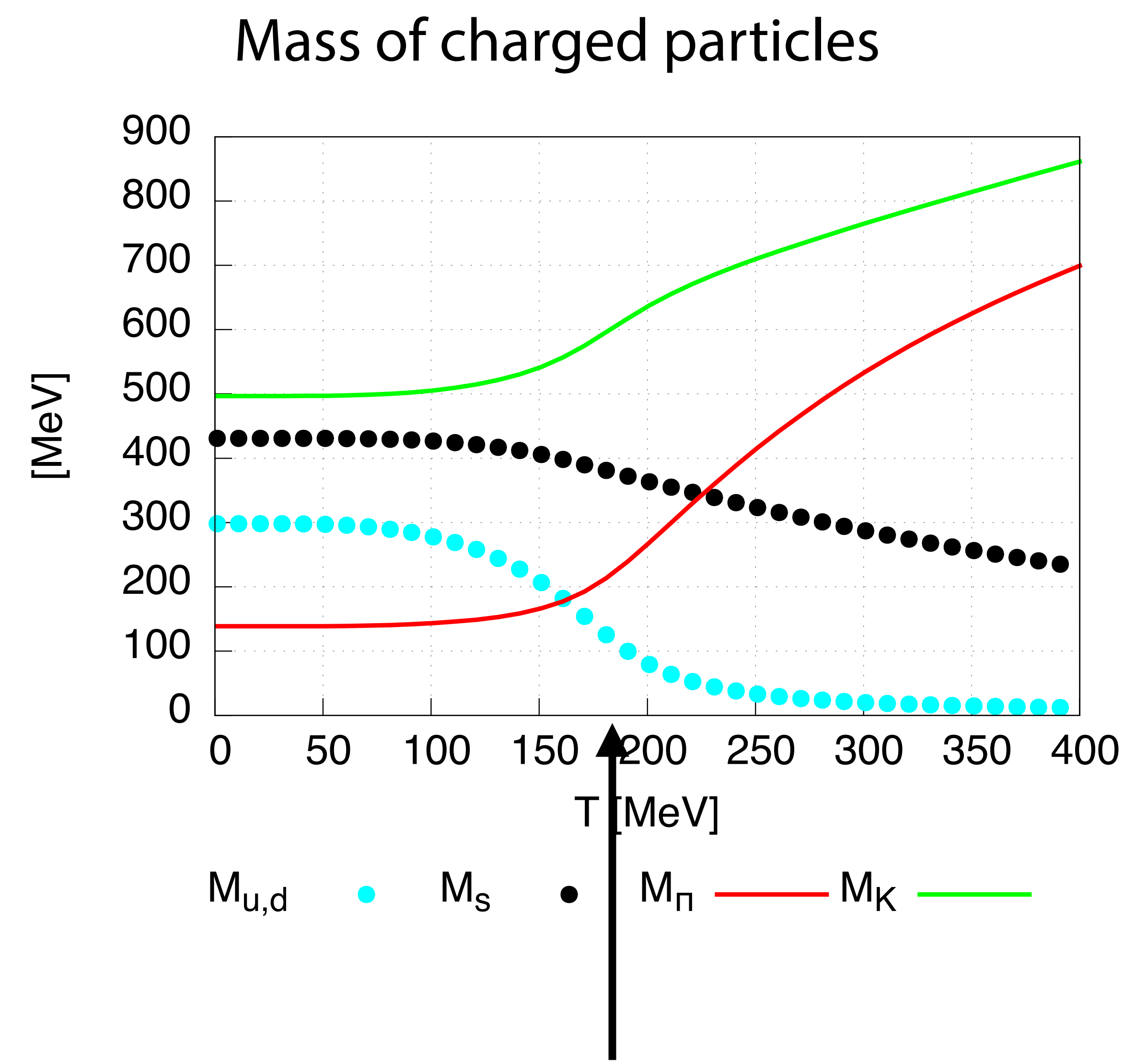


- At low temperature, the matter has diamagnetism.
- Near Tc, χ changes the sign and the vacuum becomes paramagnetic.

Comparison with Lattice QCD



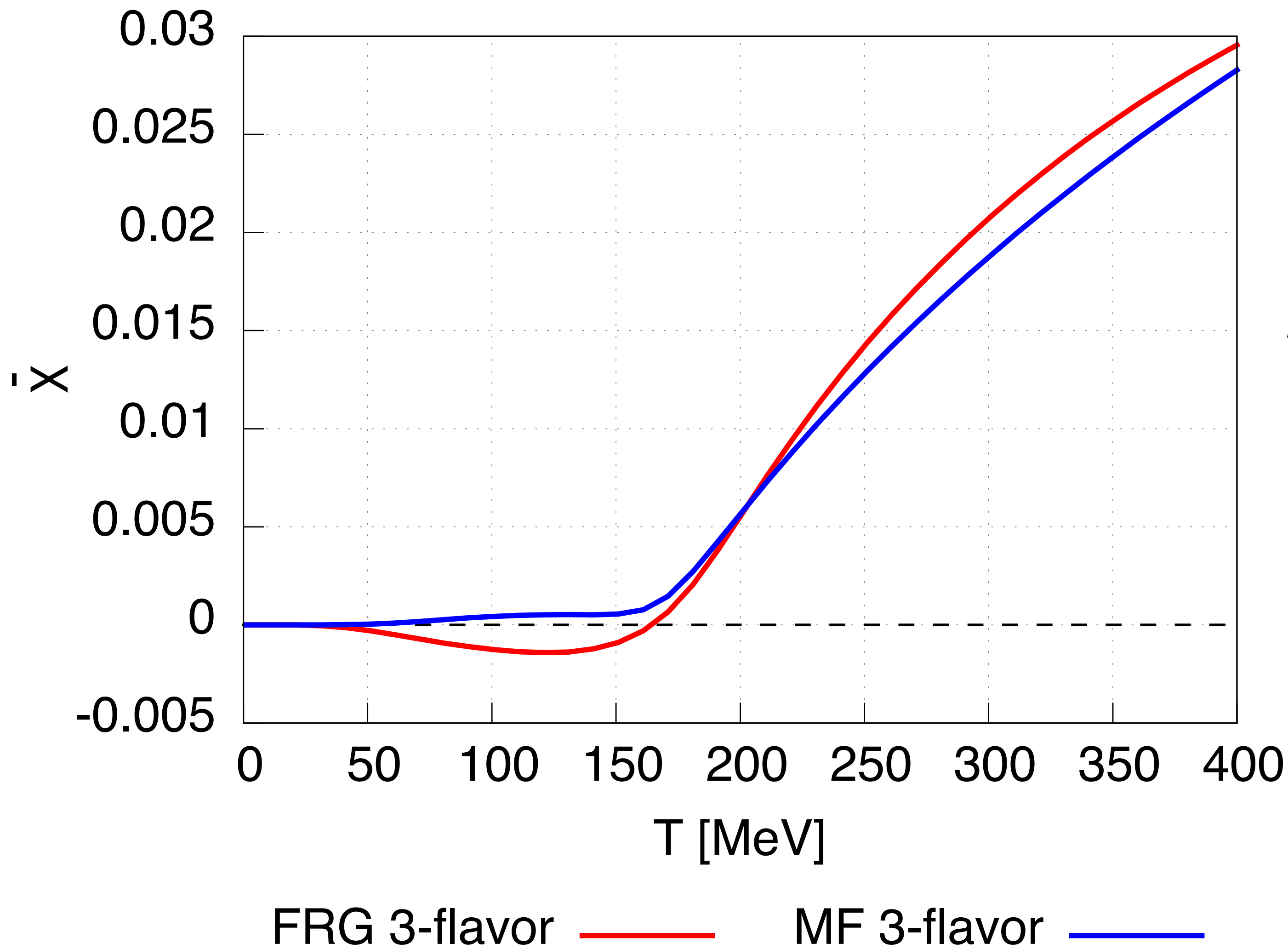
$$\Delta T \equiv T - T_c$$



$$\Delta T = 0$$

- At Hadron phase, charged mesons (especially pion) are dominant
- While QGP phase, u,d quarks are dominant.

Mean field



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MF: no dynamical meson

- If we neglect meson loop contributions (mean field approximation), the matter is paramagnetic at all region.
- The origin of diamagnetism is charged mesons.

Summary

- We solve the 3-flavor quark-meson model under strong magnetic field with Functional-RG.
- We have calculated magnetisation of the QCD vacuum at finite temperature.
- At the hadron phase, the QCD vacuum shows diamagnetism, due to light charged pions.
- At the QGP phase, the matter shows paramagnetism, due to almost bare quarks.