Asymptotic Safety & Background Independence

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ERG2014, Greece

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Outline

Asymptotic Safety & Background Independence

Asymptotic Safety (UV)

Non-perturbative renormalizability $:\Leftrightarrow$

- Existence of a (non-trivial) UV-fixed point of FRGE . . .
  for different fields, gauge groups, constraints, and topologies ✓
- . . . with finite dimensional critical hypersurface $\mathscr{S}_{\mathrm{UV}}$
  Tests: $f(R)$-truncations ✓

Background Independence

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2. Generic background ⇒ EAA

   Choose arbitrary background $\bar{g}_{\mu\nu}$ on which dynamical field $g_{\mu\nu}$ propagate

   Physical sector of $\Gamma_{k=0} \equiv \Gamma$ should be independent of $\bar{g}$!

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   Physical sector of $\Gamma_{k=0} \equiv \Gamma$ should be independent of $\bar{g}$!

   $\Rightarrow$ Split-symmetry condition: $\Gamma^{\text{grav}}_{k=0}[g, \bar{g} + \delta\bar{g}] = \Gamma^{\text{grav}}_{k=0}[g, \bar{g}]$

A Global Requirement

Requirements on the FRGE flow for quantum gravity:

- In the **UV**: Asymptotic Safety (NGFP, $\dim \mathscr{S}_{\text{UV}} < \infty$)
- In the **IR**: Restoration of split-symmetry $\Gamma_{k=0}$

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Question:

Existence of RG trajectories satisfying **both** conditions (global issue!)

Bi-Metric Einstein-Hilbert Truncation

A Bi-Metric Einstein-Hilbert Truncation

$$\Gamma_k^{\text{grav}}[g,\bar{g}] = -\frac{1}{16\pi {G^{\text{B}}}_k}\int\sqrt{\bar{g}}\,\left(\bar{R}-2{\Lambda^{\text{B}}}_k\right) - \frac{1}{16\pi {G^{\text{Dyn}}}_k}\int\sqrt{g}\,\left(R-2{\Lambda^{\text{Dyn}}}_k\right)$$

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Its single-metric approximation:

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Background Independence: $k \to 0$

Single-metric reliability: $\forall\, k \in [0,\infty)$

Structure of the Beta-Functions

$$\partial_t g_k^{\text{Dyn}} = \beta_g^{\text{Dyn}}(g_k^{\text{Dyn}}, \lambda_k^{\text{Dyn}}) \,, \qquad \partial_t \lambda_k^{\text{Dyn}} = \beta_\lambda^{\text{Dyn}}(g_k^{\text{Dyn}}, \lambda_k^{\text{Dyn}})$$

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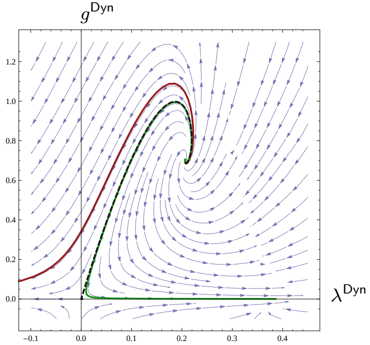

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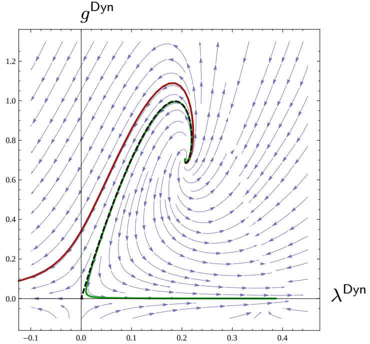

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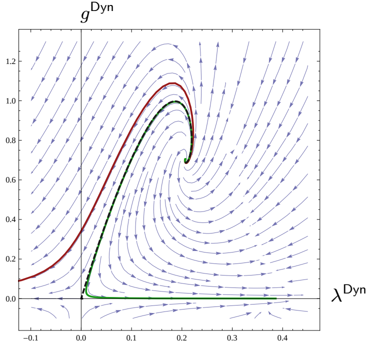

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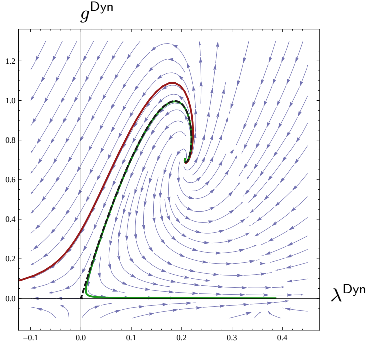

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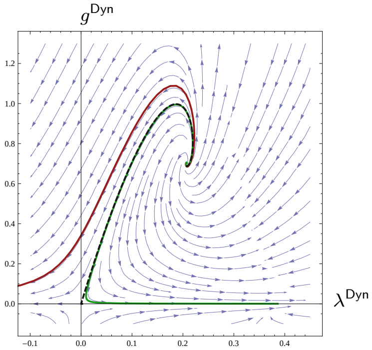
$\Rightarrow$ $\left(g_\bullet^{\text{B}}(k), \lambda_\bullet^{\text{B}}(k)\right)$ defines **running UV-attractor** in B-sector

Asymptotic Safety Condition UV



Non-trivial UV-fixed point

Dynamical sector:

$$(g_*^{\text{Dyn}},\ \lambda_*^{\text{Dyn}}) = (0.7, 0.2)$$

$\lim_{k\to\infty} \left(g_\bullet^{\text{B}}(k), \lambda_\bullet^{\text{B}}(k)\right)$ exists?

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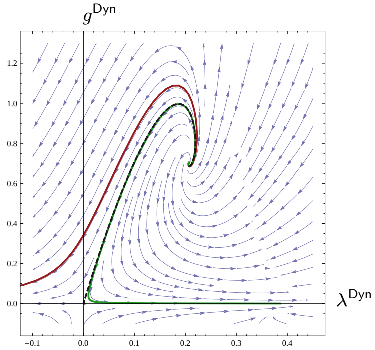

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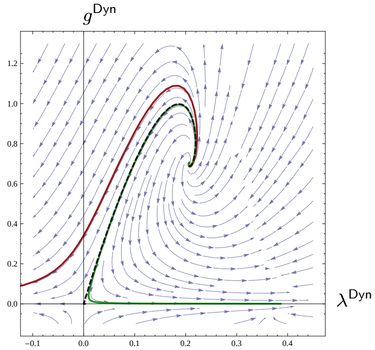

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Split-Symmetry in the IR

For every $k \mapsto (G_k^{\text{Dyn}}, \Lambda_k^{\text{Dyn}})$:

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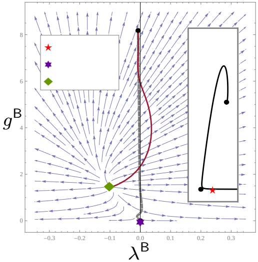
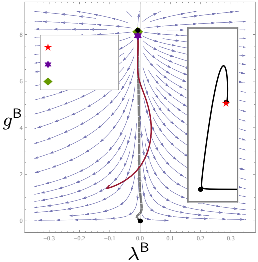
⇒ Asymptotic Safety and Background Independence simultaneously fulfilled! (only 2 free parameters left)

The running UV-attractor

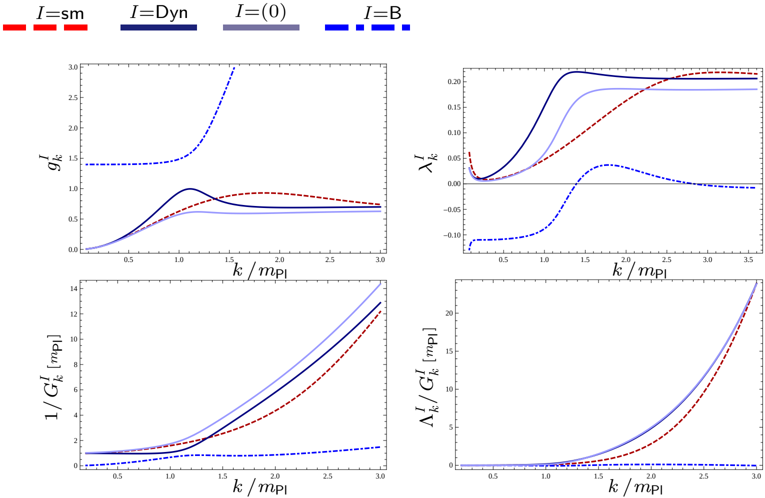
The dual role of the running UV-attractor:

- guarantees non-perturbative renormalizability for $k \to \infty$
- guarantees Background Independence for $k \to 0$





Test of (intermediate) split-symmetry



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Asymptotic Safety & Background Independence

- A *global* requirement that needs bi-metric truncations

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Conclusion

- Background Independence and A. S. can coexist
- Generalize truncations to bi-metric ones (single-metric check)

Thank You!