Higgs mass bounds from the functional renormalization group

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Research Training Group
Quantum and Gravitational Fields

- first Higgs mass bounds were derived and discussed in perturbation theory [e.g.: Sher '89, Ford et al. '93, Casas et al. '96, Isidori et al. '01, ...]

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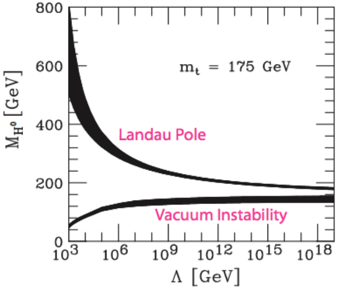

[Hagiwara et al. '02]

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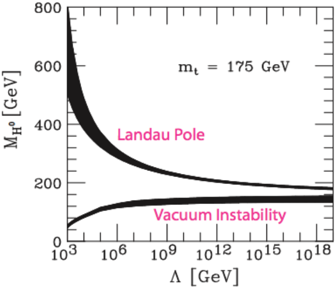

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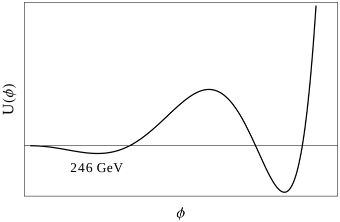

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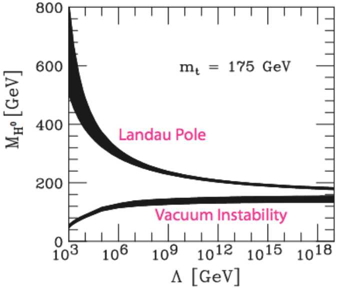

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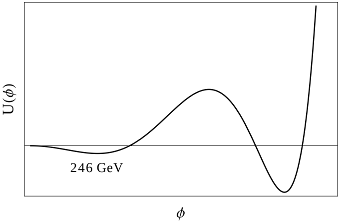

- vacuum stability?
  - Second minimum occurs at a trans-Planckian scale?
  - Convexity properties?
  - discrepancy to lattice simulations

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  $\Rightarrow$ Higgs-Yukawa toy model with $\mathbb{Z}_2$ symmetry

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- interaction part of the fermion determinant is strictly positive
  $\Rightarrow$ cannot induce instability for any finite Λ

$$U_F(\phi^2) = -\frac{\Lambda^2}{8\pi^2} h_t^2 \phi^2 + \frac{1}{16\pi^2}\left[h_t^4 \phi^4 \ln\left(1 + \frac{\Lambda^2}{h_t^2 \phi^2}\right) + h_t^2 \phi^2 \Lambda^2 - \Lambda^4 \ln\left(1 + \frac{h_t^2 \phi^2}{\Lambda^2}\right)\right]$$

[Holger Gies, RS: arXiv:1407.8124]

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Systematic derivative expansion:

$$\Gamma_k = \int d^d x \left(\frac{Z_{\phi k}}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi^2) + Z_{\psi k} \bar{\psi} i \partial\!\!\!/ \psi + i h_k \phi \bar{\psi} \psi \right)$$

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β functions:

$$\partial_t U_k = \beta_{U_k} \qquad \eta_\phi := -\partial_t \ln Z_{\phi k} = \beta_{\eta_\phi}$$

$$\partial_t h_k^2 = \beta_{h_k^2} \qquad \eta_\psi := -\partial_t \ln Z_{\psi k} = \beta_{\eta_\psi}$$

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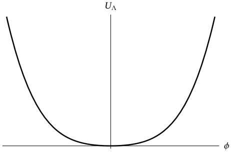
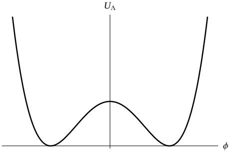
Initial conditions and fine tuning:

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 \qquad \text{or} \qquad U_\Lambda = \frac{\lambda_{2\Lambda}}{8}(\phi^2 - v_\Lambda^2)^2$$





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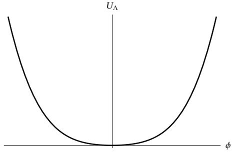
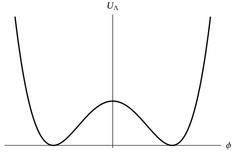
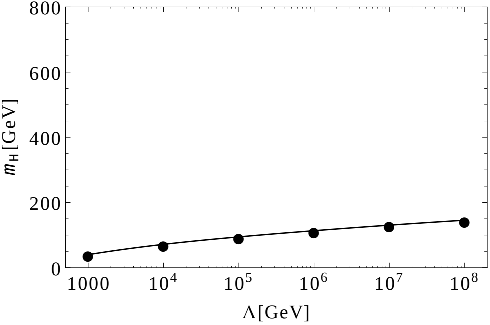
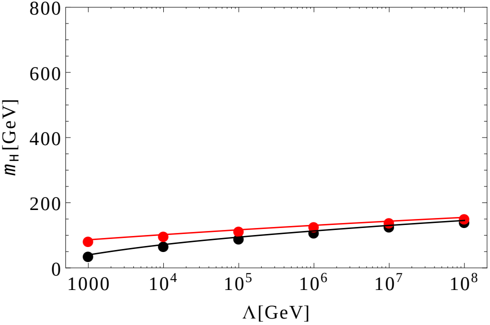




$$\lambda_{1\Lambda} \text{ (or } v_\Lambda) \to v_0 = 246\,\text{GeV}$$
$$h_\Lambda \to m_{\text{top}} = 173\,\text{GeV}$$

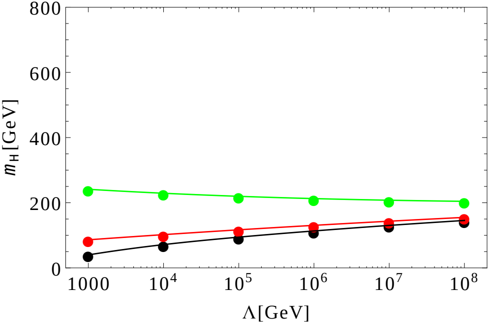


$\lambda_{2\Lambda} = 0$

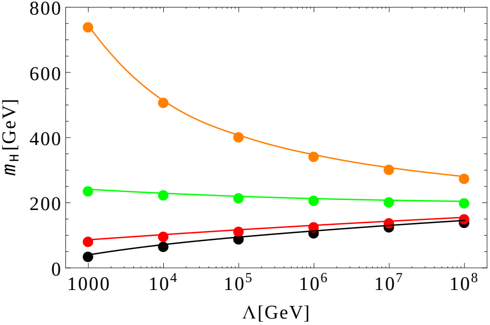


$\lambda_{2\Lambda} = 0$

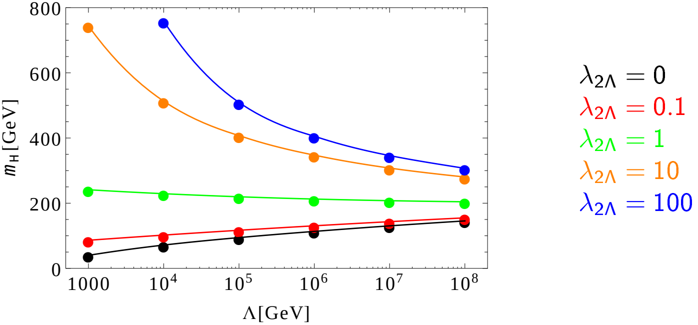
$\lambda_{2\Lambda} = 0.1$



$\lambda_{2\Lambda} = 0$
$\lambda_{2\Lambda} = 0.1$
$\lambda_{2\Lambda} = 1$



$\lambda_{2\Lambda} = 0$
$\lambda_{2\Lambda} = 0.1$
$\lambda_{2\Lambda} = 1$
$\lambda_{2\Lambda} = 10$


800
600
400
200
0
m_H[GeV]
1000
10^4
10^5
10^6
10^7
10^8
Λ[GeV]
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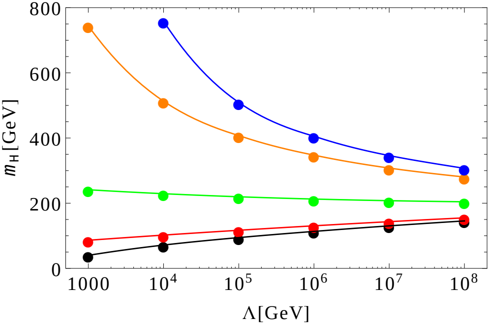

$\lambda_{2\Lambda} = 0$
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$\lambda_{2\Lambda} = 100$

Higgs mass is a monotonically increasing function of $\lambda_{2\Lambda}$!
$\Rightarrow$ natural lower bound for $\lambda_{2\Lambda} = 0$ for a quartic UV potential
cf. lattice [Holland and Kuti '04], [Jansen et al. '12]

- generalised bare potentials, e.g.:

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{48\Lambda^2}\phi^6$$

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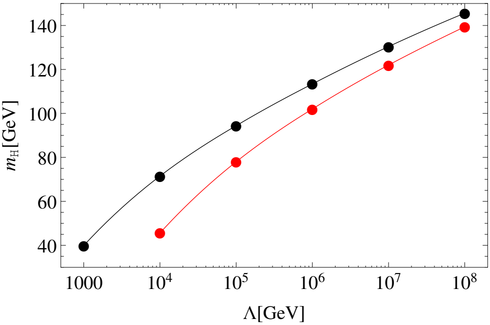

$\lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

$\lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.08$

- Extension of the simple toy model to a chiral Higgs–top-bottom model:

$$S = \int \Big[\partial_\mu \phi^\dagger \partial^\mu \phi + U(\phi^\dagger \phi) + \bar{t} i \partial\!\!\!/ t + \bar{b} i \partial\!\!\!/ b$$
$$+ i h_b (\bar{\psi}_L \phi b_R + \bar{b}_R \phi^\dagger \psi_L) + i h_t (\bar{\psi}_L \phi_C t_R + \bar{t}_R \phi_C^\dagger \psi_L) \Big]$$

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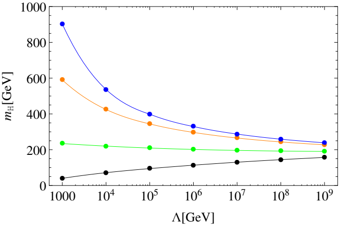
- ϕ^4 type bare potentials



$\lambda_{2\Lambda} = 0$, $\lambda_{2\Lambda} = 1$, $\lambda_{2\Lambda} = 10$,
$\lambda_{2\Lambda} = 100$

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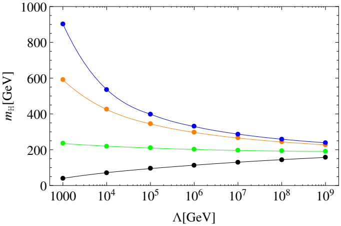

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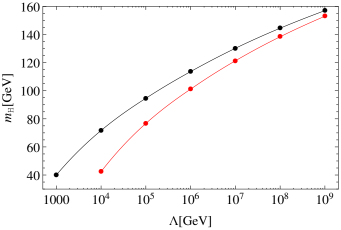
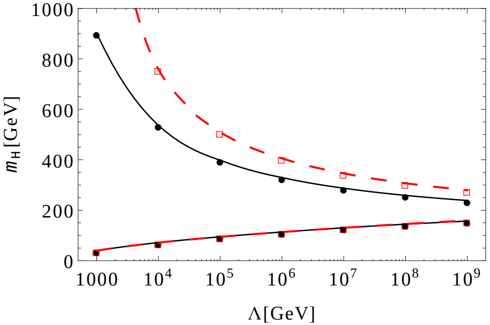
- extended bare potentials



$\lambda_{2\Lambda} = 0$ and $\lambda_{3\Lambda} = 0$,
$\lambda_{2\Lambda} = -0.1$ and $\lambda_{3\Lambda} = 3$

- simple Higgs-top Yukawa-model (red, dashed) vs chiral Higgs-top-bottom model (black, solid)



- running of the Yukawa couplings mainly influenced by the gauge sectors

$$\partial_t h = \frac{1}{16\pi^2}\left[\frac{9}{2}h^3 - 8g_s^2 h - \frac{9}{4}g^2 h - \frac{17}{12}g'^2 h\right]$$

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- Higgs-top-QCD model

$$S = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + U(\phi^2) + \bar{\psi} i \not{D} \psi + i h \phi \bar{\psi} \psi + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu\, i}\right] + S_{\mathrm{gf}} + S_{\mathrm{gh}}$$

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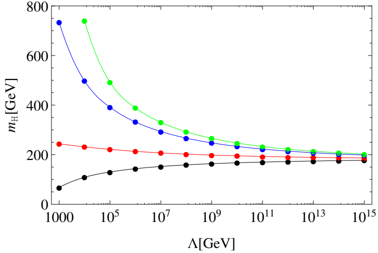
- flow equations

$$\partial_t U_k = \beta^{\text{non-pert}}_{U_k}, \quad \partial_t h_k^2 = \beta^{\text{non-pert}}_{h_k^2}, \quad \partial_t g_k^2 = \beta^{\text{pert}}_{g_k^2}$$

PRELEMINARY results @ next-to-leading order in the derivative expansion

- ϕ^4 type bare potentials



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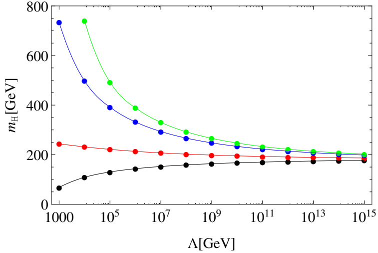
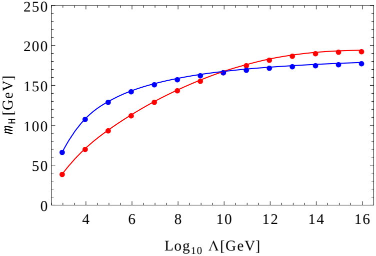
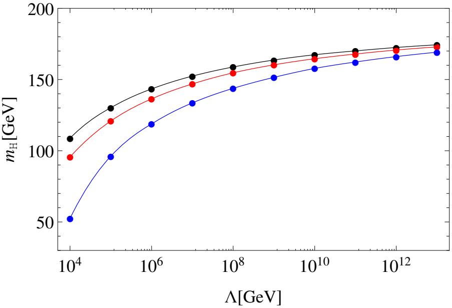


- gauged (blue) vs ungauged model (red)



PRELEMINARY results @ next-to-leading order in the derivative expansion for the class of generalized bare potentials

$$U_\Lambda = \frac{\lambda_{1\Lambda}}{2}\phi^2 + \frac{\lambda_{2\Lambda}}{8}\phi^4 + \frac{\lambda_{3\Lambda}}{48\Lambda^2}\phi^6 + \frac{\lambda_{4\Lambda}}{384\Lambda^4}\phi^8$$



$\lambda_{4\Lambda} = 0,\ \lambda_{3\Lambda} = 0,\ \lambda_{2\Lambda} = 0$

$\lambda_{4\Lambda} = 0,\ \lambda_{3\Lambda} = 3,\ \lambda_{2\Lambda} = -0.1$

$\lambda_{4\Lambda} = 9,\ \lambda_{3\Lambda} = 2,\ \lambda_{2\Lambda} = -0.2$

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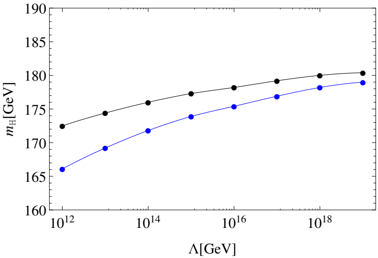

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- $\mathcal{O}(1)$ variations of bare $\lambda_{n,\Lambda}$: $\Delta m_{\mathrm{H}} \simeq \begin{cases} 10\mathrm{GeV} & \text{at} \quad \Lambda \simeq 10^{11}\mathrm{GeV} \\ 5\mathrm{GeV} & \text{at} \quad \Lambda \simeq 10^{15}\mathrm{GeV} \\ 2\mathrm{GeV} & \text{at} \quad \Lambda \simeq 10^{19}\mathrm{GeV} \end{cases}$

Conclusion and Outlook

- We found natural bounds for the Higgs mass in the framework of the functional RG for quartic UV potentials.
- The form of the UV potential can exert a significant influence on the mass bounds.

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- The form of the UV potential can exert a significant influence on the mass bounds.
- Extension of the toy models to the standard model.