UV critical behaviour in quantum gravity

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Based on: **Asymptotic safety and the cosmological constant, KF** (arXiv:1408.0276) and work in preparation.

What is the physics of asymptotically safe gravity?

- Classical gravity: GR.
- Physical degrees of freedom: gravitational waves i.e. graviton.
- Conformal degree of freedom constrained.
- Conformal factor fixed by cosmological constant.
- What changes in the quantum theory?
- What is the physical meaning of running couplings e.g. G_k, Λ_k .
- How are they related to physical degrees of freedom.
- FRG approach to quantum gravity:

$$\mathcal{Z} = \int \mathcal{D}\, g_{\mu\nu} e^{iS[g_{\mu\nu}]}$$

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\partial_t \mathcal{R}_k \cdot \frac{1}{\Gamma_k^{(2)} + \mathcal{R}_k} \right]$$

What is the physics of asymptotically safe gravity?

- **Approach** (see : (arXiv:1408.0276), KF)

  - Decompose metric into physical degrees of freedom.
  - Cancel gauge variant and ghosts (Benedetti arXiv:1107.3110)
  - Cancellation of conformal fluctuations with functional measure
  - Take care to ensure convexity of the effective average action.
  - Approximation scheme based on heat kernel expansion.

- Some Physics: Critical behaviour in quantum gravity

  - What are the physical fields and couplings.
  - Asymptotic safety vs phase transitions.
  - Continuum vs discrete spacetime.

Scaling in quantum gravity

- Einstein Hilbert action:

$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-\det g_{\mu\nu}} R(g_{\mu\nu}) \propto g_{\mu\nu}$$

- Can scale out Newtons constant $g_{\mu\nu} \to 16\pi G \, g_{\mu\nu}$
- Spoilt by linear decomposition

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

- Decompose into conformal factor:

$$g_{\mu\nu} = \phi^2 \hat{g}_{\mu\nu} \,, \quad \det \hat{g}_{\mu\nu} = \det \bar{g}_{\mu\nu}$$

- Dressed conformal factor:

$$Z_\phi = \frac{1}{16\pi G} \qquad \omega \equiv Z_\phi^{\frac{1}{2}} \phi \qquad S[g_{\mu\nu}] \sim \omega^2$$

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$$\Delta S_k = \frac{1}{2}\int d^4x \sqrt{-\det \bar{g}_{\mu\nu}}\,\omega\, R_k(\bar{\Delta})\,\omega$$

Cosmological constant

- Dynamics of volume.
- Dimensionless product:

$$\tau = G \cdot \Lambda$$

- measurable quantity $\tau \approx 10^{-122}$.
- Cosmological constant problem.

- $\sim \tau\omega^4$ interaction for the dressed conformal mode.

Conformal fluctuations

- Conformal factor is non-dynamical in general relativity
- Quantum theory: W rong sign in kinetic term
- Mottola and Mazur (1990): Dynamics of conformal factor cancelled on-shell by Jacobian in the functional measure
- Fluctuations Wick rotated: $\delta\phi \to i\delta\phi \quad \text{for} \quad \Delta_0 > 0$

$$S_\omega^{(2)} = 16\pi G S_\phi^{(2)} = \Delta_0 + 2\left(\frac{R}{4} - \Lambda\right) \qquad J_0 = (\det(\Delta_0))^{\frac{1}{2}}$$

where $\quad \Delta_0 = -\nabla^2 - \dfrac{R}{3}$

- Cancellation of the conformal factor can be realised at the level of the flow equation after suitable field redefinitions.

$$\partial_t \Gamma_k \sim \frac{1}{2}\mathrm{Tr}\left[\partial_t \mathcal{R}_k \cdot (\Gamma_k^{(2)} + \mathcal{R}_k)^{-1}\right]_\omega - \frac{1}{2}\mathrm{Tr}\left[\partial_t \mathcal{R}_k \cdot (\Gamma_k^{(2)} + \mathcal{R}_k)^{-1}\right]_J + \ldots$$

Convexity

- Need to ensure convexity when we make approximations
- In presence of a regulator $\Gamma_k^{(2)} + \mathcal{R}_k > 0$
- Consider:

$$S_\omega^{(2)} = 16\pi G S_\phi^{(2)} = \Delta_0 + 2\left(\frac{R}{4} - \Lambda\right)$$

- Convex for $R > 4\Lambda$
- Expansion around $R = 0$ leads to poles (avoided by IR fixed points)

RG scheme

- Want to keep on-shell cancellations and avoid poles arising at non-convex part of the action
- Regulator

$$\Delta S_k = \frac{1}{2}\int d^4x\sqrt{-\det \bar{g}_{\mu\nu}}\,\omega\, R_k(S^{(2)}_\omega)\,\omega$$

- Regulator vanishes for k=0 provided $R > 4\Lambda_k$

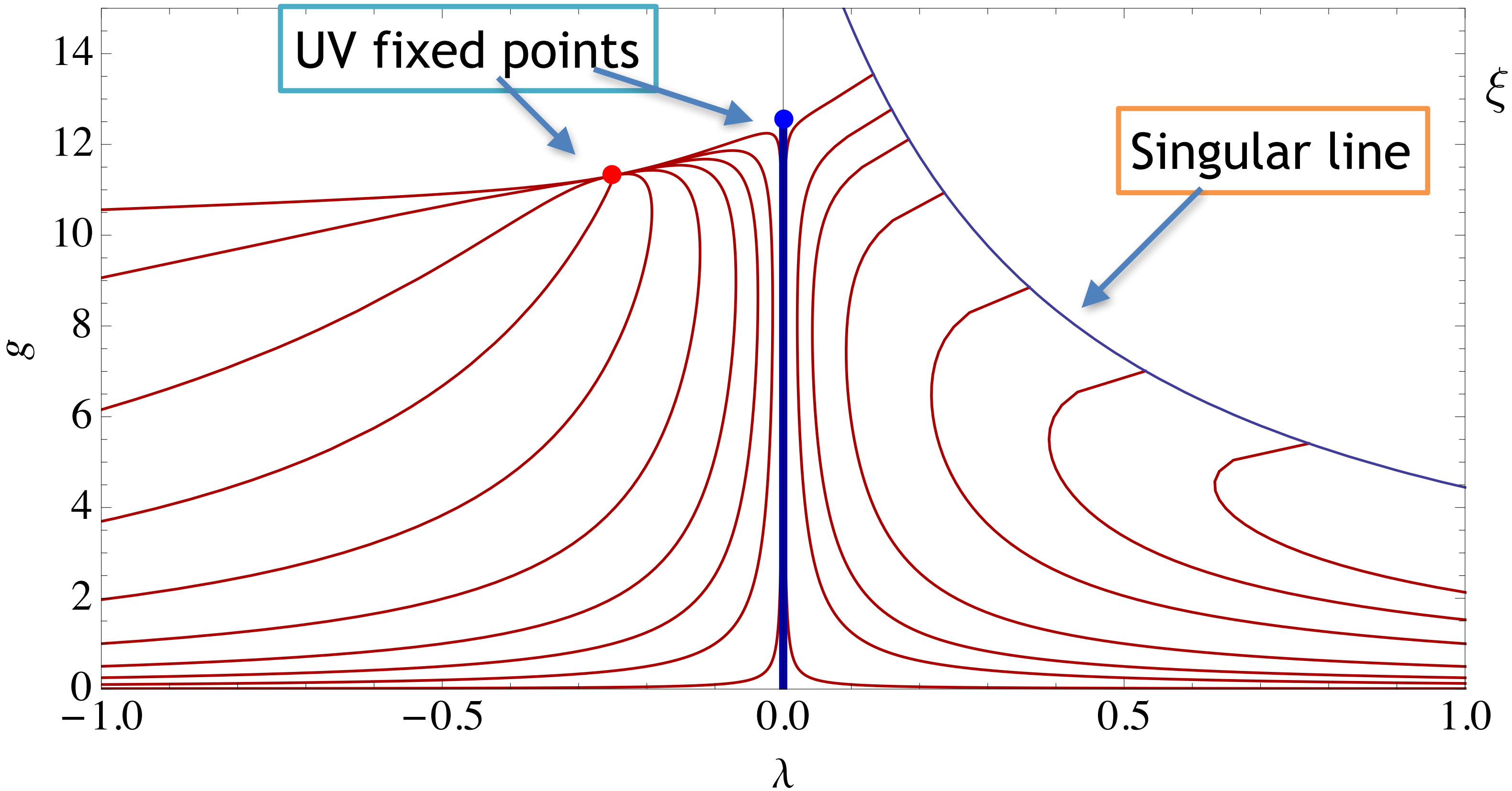
- UV behaviour captured by early heat kernel expansion

$$\mathrm{Tr}[f(S^{(2)}_\omega)] = \int_0^\infty ds\,\mathrm{Tr}[e^{-S^{(2)}_\omega s}]\,\tilde{f}(s) \approx \frac{1}{(4\pi)^{\frac{d}{2}}}\sum_{n=0}^{\infty} Q_{\frac{d}{2}-n}[f] A_n(R,\Lambda_k)\,,$$

- Truncate expansion no further curvature expansion, closes Einstein-Hilbert approximation.

Linear decomposition

- Linear approximation to conformal mode $g_{\mu\nu} \sim (1+\sigma)\bar{g}_{\mu\nu}$
- Only quantise (linear) conformal modes with non-trivial Jacobian.
- RG flow contains phase with vanishing CC for all scales:



$$\xi k = \frac{1}{(g_* - g)^{\frac{1}{3}}} + \dots$$

$$\nu = 1/3$$

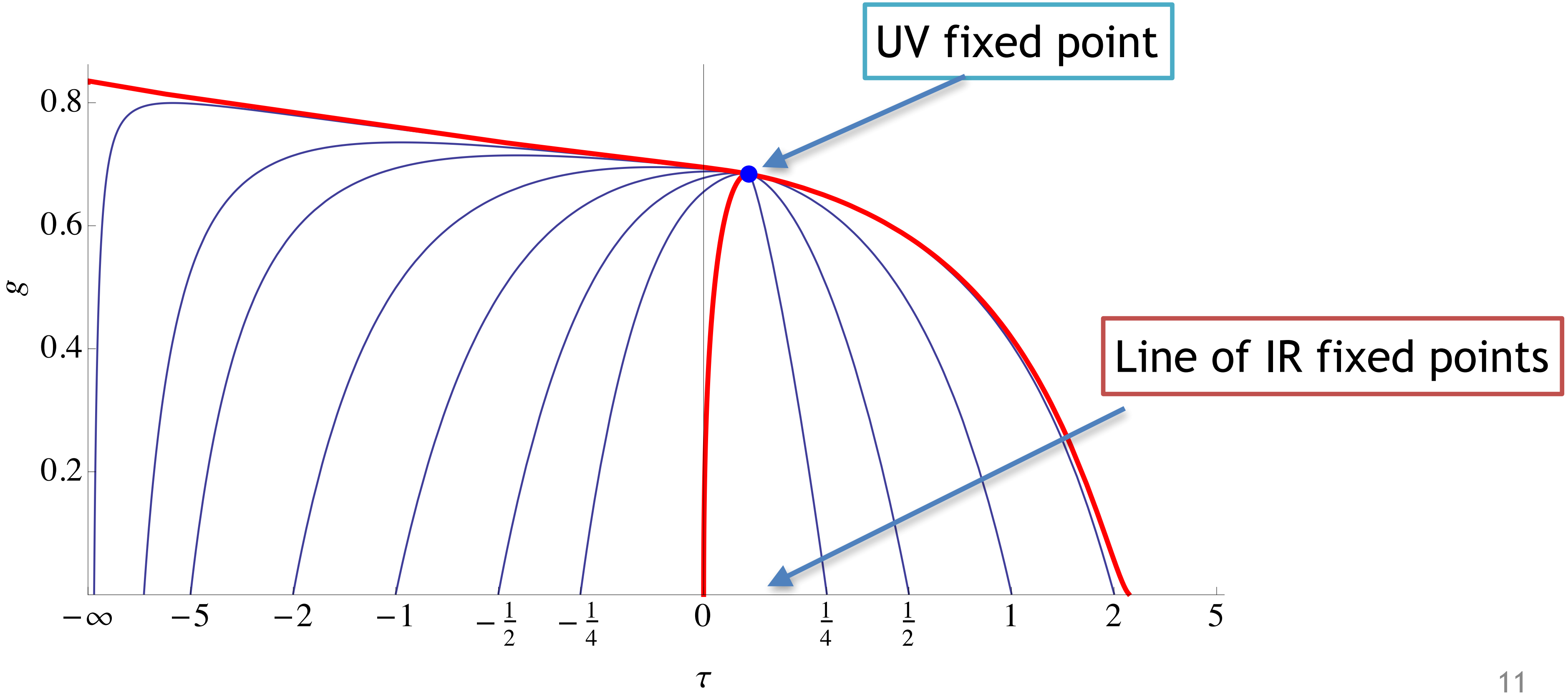
- Critical exponent (obtained with Litim's regulator) in agreement with lattice studies by Hamber $\nu \approx 0.3354.$

Linear decomposition

- Linear approximation to conformal mode

$$g_{\mu\nu} \sim (1+\sigma)\bar{g}_{\mu\nu}$$

- Quantise all metric degrees of freedom.
- RG flow UV FP —> line of classical IR fixed points:



Conformal factor decomposition

- Flow of Newton's constant becomes independent of the cosmological constant in full theory.

$$g_{\mu\nu} = \phi^2 \hat{g}_{\mu\nu}\,, \quad \det \hat{g}_{\mu\nu} = \det \bar{g}_{\mu\nu}$$

- Positive Newton's constant UV fixed point.

$$g_* = \frac{8\pi}{19} \approx 1.322$$

- Simple beta function

$$\beta_g = 2g + \frac{4g^2}{g - \frac{24\pi}{19}} \qquad \xi k = \frac{1}{(g_* - g)^{\frac{1}{3}}} + \ldots$$

- Explicit results using Litim's optimised cutoff.
- Universal critical exponent agrees with numerical results of Hamber (hep-th/9912246) from lattice theory (Regge calculus).

Continuum FRG:	Lattice theory:
$\nu = 1/3$	$\nu \approx 0.3354$

Conformal factor decomposition

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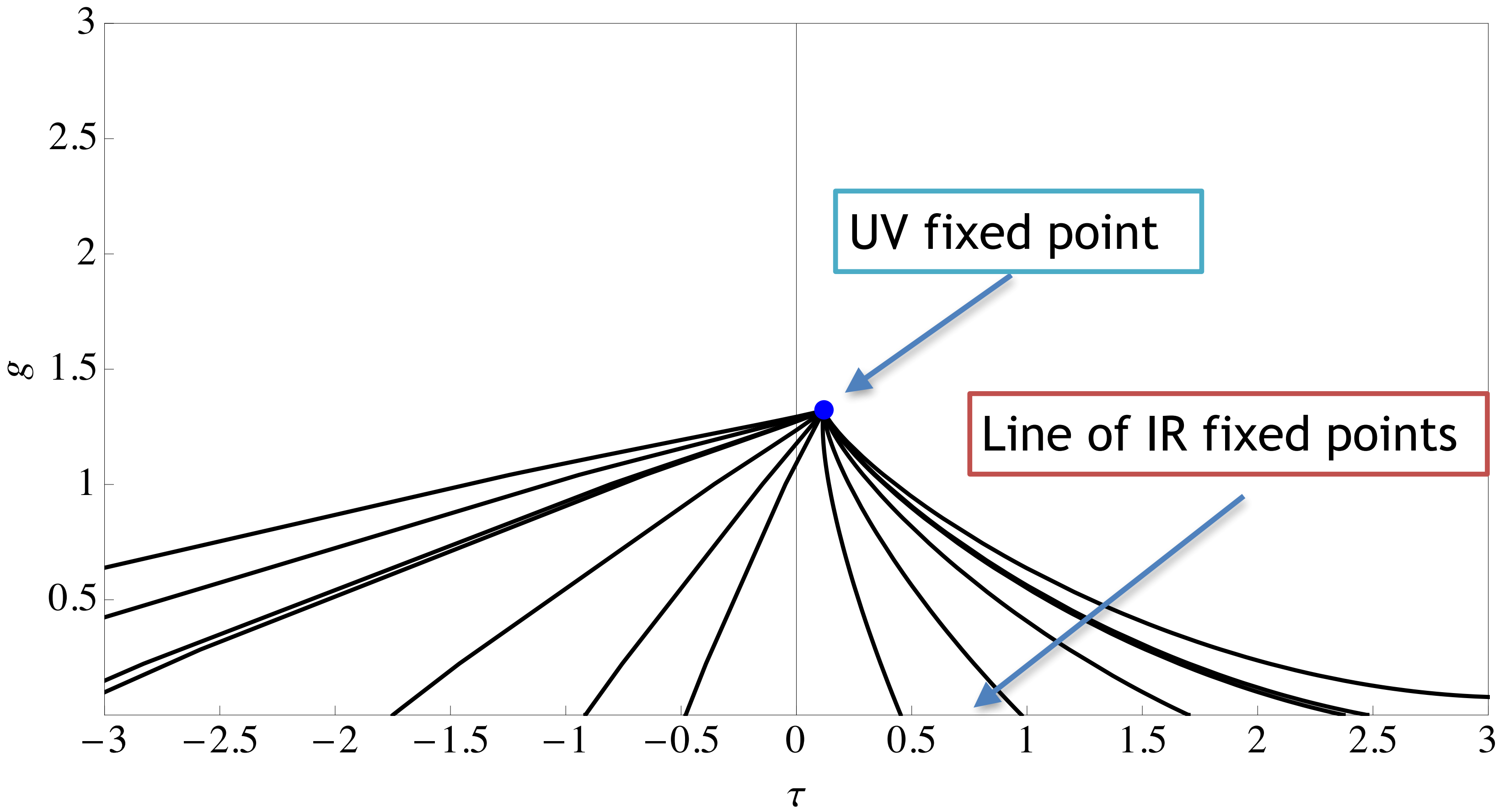

Dressed metric

- We have regulated the conformal factor of the dressed metric $$\Delta S_k = \frac{1}{2}\int d^4x \sqrt{-\det \bar{g}_{\mu\nu}}\, \omega\, R_k(S^{(2)}_\omega)\, \omega$$

$$\chi_{\mu\nu} = \frac{g_{\mu\nu}}{16\pi G_k} \qquad R_k = R_k(g_{\mu\nu})$$

- Interpretation:
  - averaged fluctuations of $\chi_{\mu\nu}$ with respect to length scales on $g_{\mu\nu}$.

$$\ell(g_{\mu\nu}) = \int \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$$

$$k \approx \frac{1}{\ell(g_{\mu\nu})} \qquad \ell(\chi_{\mu\nu}) = \frac{\ell(g_{\mu\nu})}{\sqrt{16\pi G_k}}$$

A smallest length

$$\chi_{\mu\nu} = \frac{g_{\mu\nu}}{16\pi G_k} \quad k \approx \frac{1}{\ell(g_{\mu\nu})} \quad \ell(\chi_{\mu\nu}) = \frac{\ell(g_{\mu\nu})}{\sqrt{16\pi G_k}}$$

- Critical fine grained limit limit:

$$G_k \to k^{-2} g_*,\ k \to \infty,\ \ell(g_{\mu\nu}) \to 0,$$

$$\ell(\chi_{\mu\nu}) \to \text{finite}\,.$$

- RG improvement implies a smallest observable length! (similar observation: arXiv:1008.3621, R. Percacci, G. P. Vacca)

Phase transitions (see talk A. Eichhorn)

- Consider de-Sitter space
- Hubble's constant is the only scale

$$k \propto H(g_{\mu\nu})$$

- Gibbons-Hawking temperature associated to the horizon:

$$T(g_{\mu\nu}) = 2\pi H(g_{\mu\nu}) \,, \quad T(\chi_{\mu\nu}) = 2\pi\sqrt{G_k} H(g_{\mu\nu}) \propto \sqrt{k^2 G_k}$$

- Phase transition on the re-scaled metric.

$$\xi(\chi_{\mu\nu}) \propto (T_c - T(\chi_{\mu\nu}))^{-\nu} \,, \quad T_c \propto \sqrt{g_*}$$

- Hagedorn temperature for quantum gravity?
- Metric description may break down in high temperature phase. Melting point of spacetime.

Conclusions

- Disentangling degrees of freedom disentangles the RG flow of couplings
- Conformally reduced theory consistent with a zero cosmological constant.
- Full theory implies that the CC is a free parameter.
- Critical exponent found in agreement with quantum Regge calculus.
- UV FP —> Phase transition for spacetime.
- Discrete —> continuous.
- Universality in quantum gravity?