



FRG-QCD: Status and Prospects

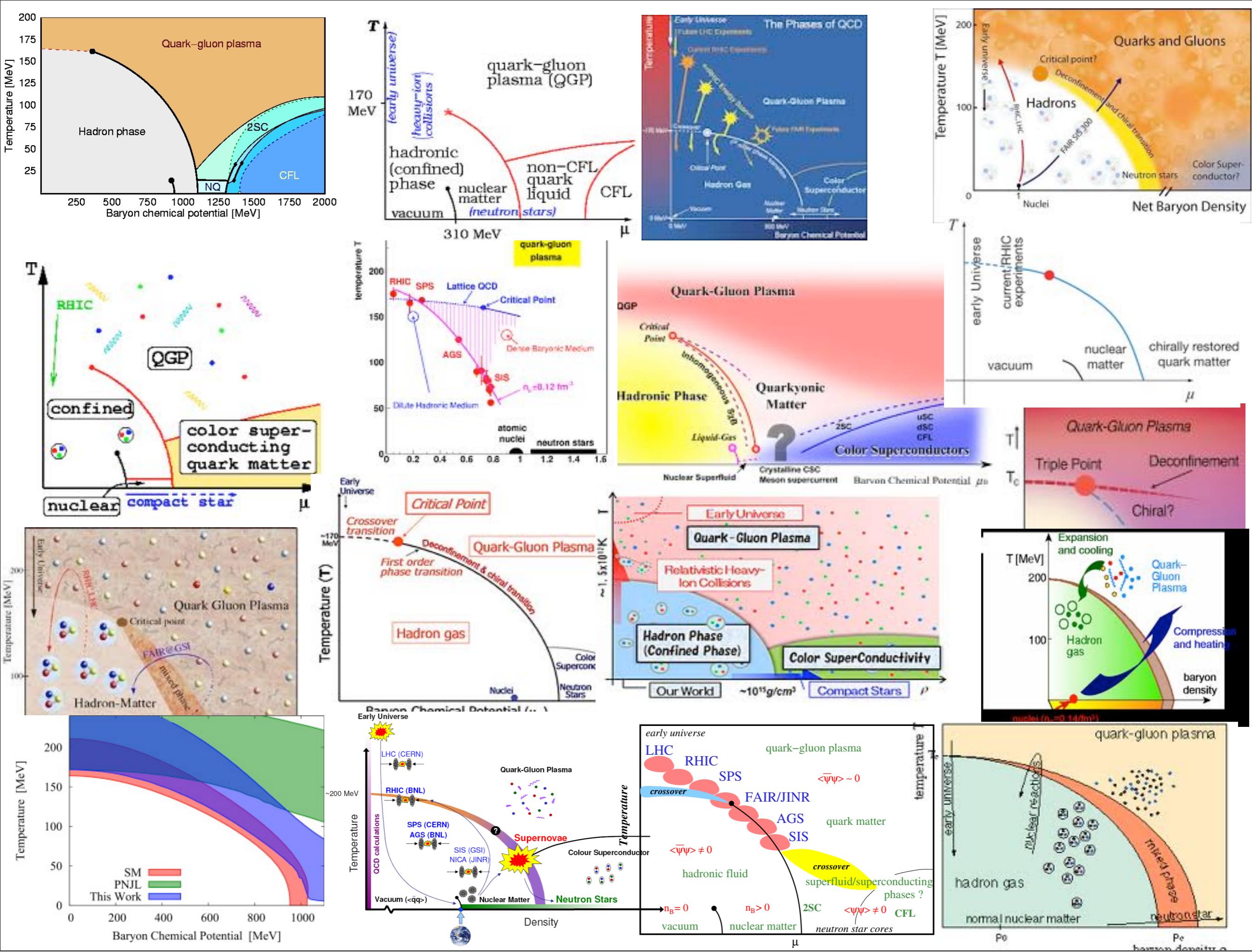


Jan M. Pawłowski

Universität Heidelberg & ExtreMe Matter Institute

Lefkada, September 23rd 2014





FRG-QCD Talks

- L. Fister** '**Correlation functions in Yang-Mills theory**' **cancelled by Air France**
- M. Mitter** '**QCD and dynamical hadronisation**'
- N. Strodthoff** '**Spectral functions**'
- F. Rennecke** '**The chiral phase transition of QCD**'

FRG-QCD collaboration: **J. Braun, L. Fister, T.K. Herbst, M. Mitter, JMP, F. Rennecke, N. Strodthoff**

TARDIS, ERGE

DoFun

Braun, Huber, Comput.Phys.Commun. 183 (2012) 1290-1320

FRG-QCD Talks

| | |
|---------------|---|
| L. Fister | 'Correlation functions in Yang-Mills theory' cancelled by Air France |
| M. Mitter | ' <u>QCD and dynamical hadronisation</u> ' |
| N. Strodthoff | ' <u>Spectral functions</u> ' |
| F. Rennecke | ' <u>The chiral phase transition of QCD</u> ' |

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Related Talks

| | |
|-----------------|--|
| M. Huber | ' <u>Yang-Mills correlation functions from Dyson-Schwinger equations</u> ' |
| A. Juricic | ' <u>Probing the QCD phase diagram with generalised quark susceptibilities</u> ' |
| K. Kamikado | ' <u>Magnetic susceptibility of the strongly interacting matter</u> ' |
| N. Khan | ' <u>The role of fluctuations in the QCD phase diagram</u> ' |
| J. Luecker | ' <u>Polyakov loop potential from functional methods</u> ' |
| S. Mao | ' <u>The Deconfinement phase transition in the Friedberg-Lee model</u> ' |
| N. Mueller | ' <u>QCD in magnetic fields</u> ' |
| S. Rechenberger | ' <u>Chiral dynamics in external magnetic fields</u> ' |
| B.J. Schaefer | ' <u>Phase structure, thermodynamics and fluctuations in QCD</u> ' |
| P. Springer | ' <u>Dynamical locking of chiral and deconfinement phase transitions</u> ' |
| W. Weise | ' <u>Chiral nuclear thermodynamics</u> ' |
| M. Yamada | ' <u>Beyond LPA for dynamical chiral symmetry breaking</u> ' |

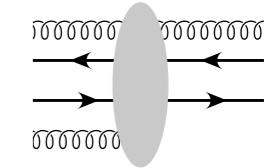
Outline

- **Functional Methods for QCD**
- **Confinement & Chiral Symmetry Breaking**
- **Phase Structure and Transport**
- **Outlook**

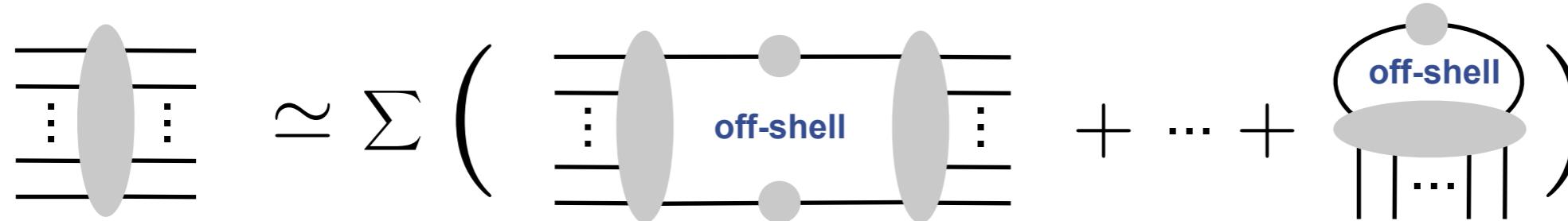
Functional Methods for QCD

quark-gluon correlations

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) \rangle$$



functional relations

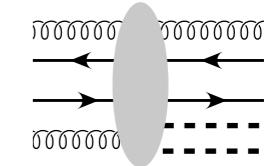


scattering amplitude/
vertex functions

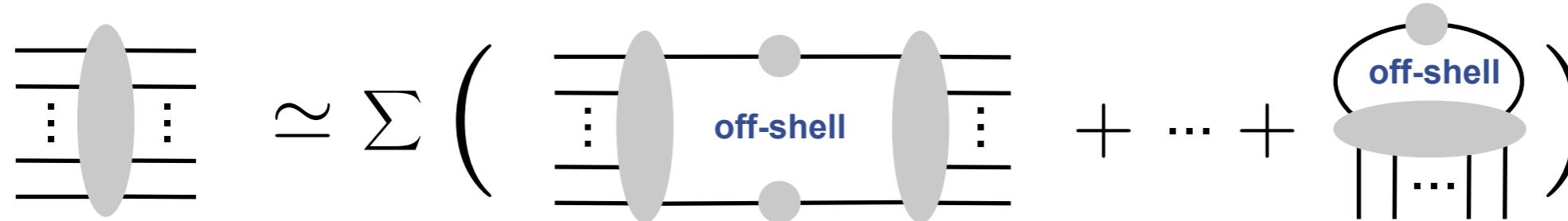
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quark-gluon-hadron correlations

$$\langle q(x_1) \cdots \bar{q}(x_{2n}) A_\mu(y_1) \cdots A_\mu(y_m) h(z_1) \cdots h(z_l) \rangle$$



functional relations



scattering amplitude/
vertex functions

Functional renormalisation group equations

Dyson-Schwinger equations

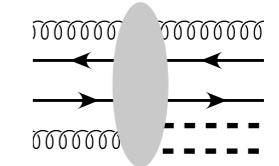
2PI/nPI hierarchies

⋮

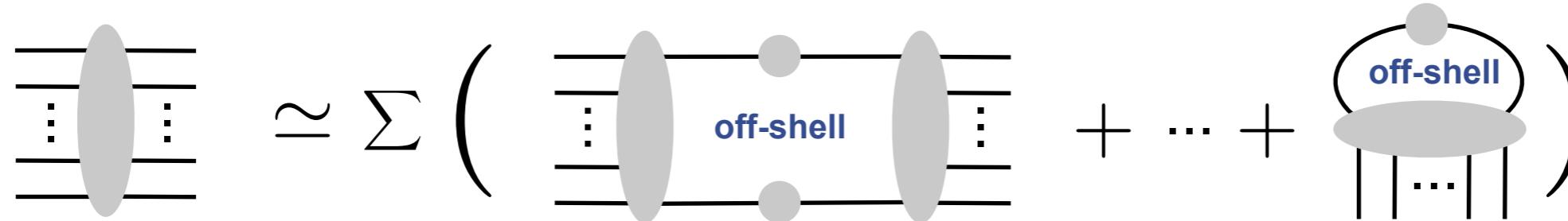
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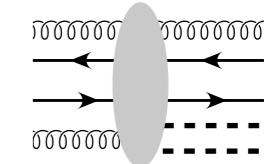
properties

- access to physics mechanisms 
- numerically tractable
no sign problem
systematic error control via closed form
- low energy models naturally incorporated

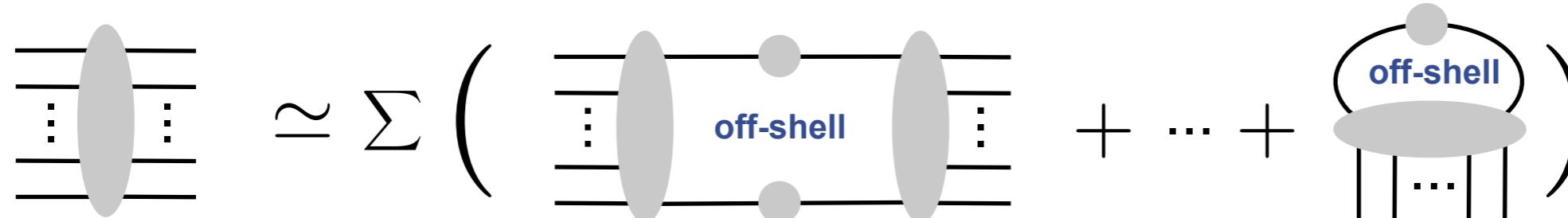
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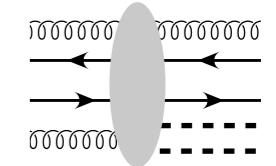
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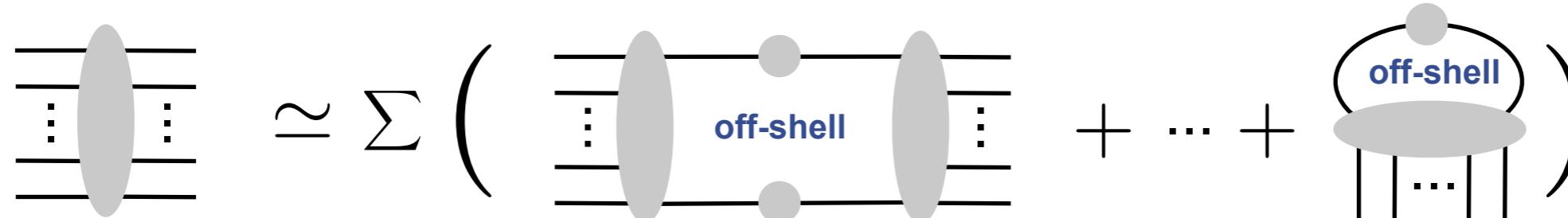
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functional relations



scattering amplitude/
vertex functions

e.g. lattice input on rhs

e.g. volume flucs., finite density,
dynamics, ...

properties

- access to physics mechanisms
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FunMethods complementary to lattice



Functional Methods for QCD

Scales

- **intrinsic scale of QCD:**

$$\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$$



Functional Methods for QCD

Scales

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- **current quark masses:**
$$\frac{m_{q,\text{current}}}{\Lambda_{\text{QCD}}} \Big|_{\text{light quarks}} \approx 10^{-2}$$
 $m_\pi \approx 140 \text{ MeV}$



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 - see talk of F. Rennecke
- meson masses**
-
- QCD** — **low energy models**

Functional Methods for QCD

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- **nucleon binding energy** $\approx 16 \text{ MeV}$

see talk of W. Weise



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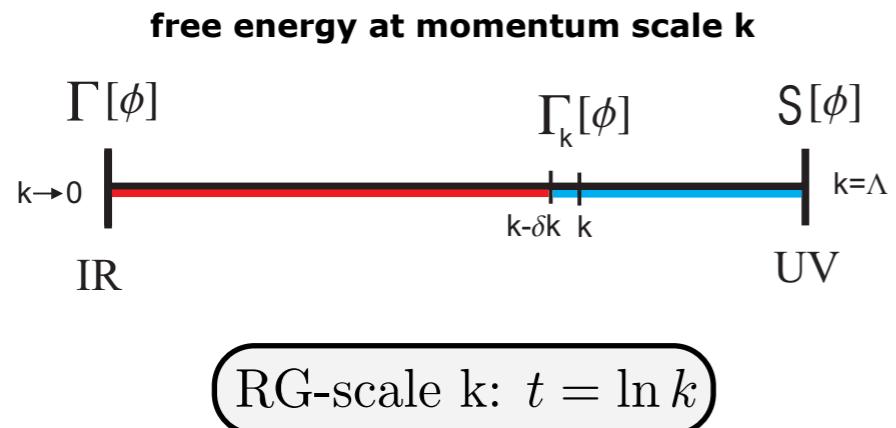
best done with a combination of imaginary and real time flows



Functional Methods for QCD

Functional RG

JMP, AIP Conf.Proc. 1343 (2011)

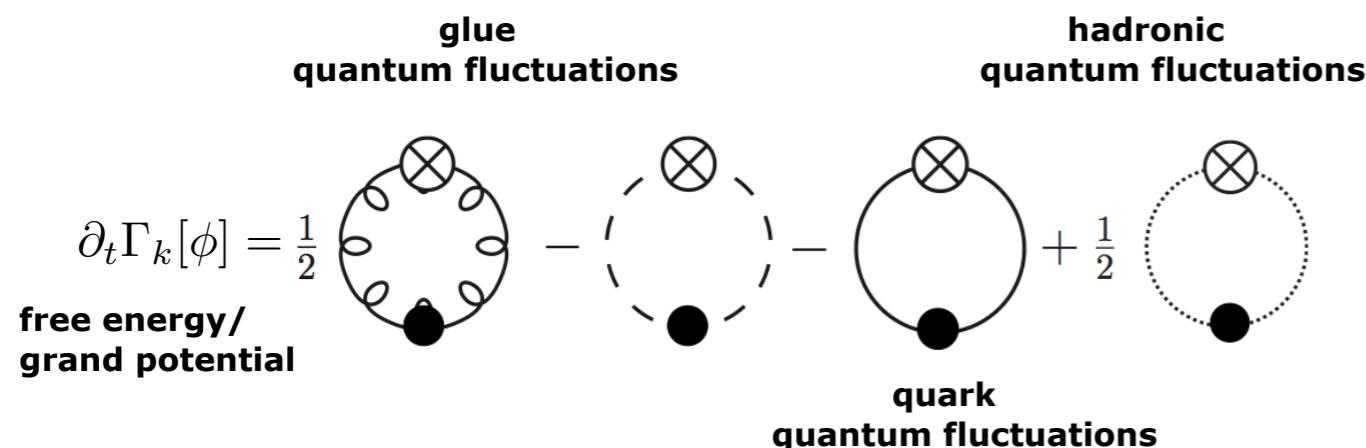


ERG12 QCD survey

JMP, Aussois '12

ERG14 QCD survey

B.J. Schaefer, Lefkada '14



Phase diagram survey

JMP, Schladming '13

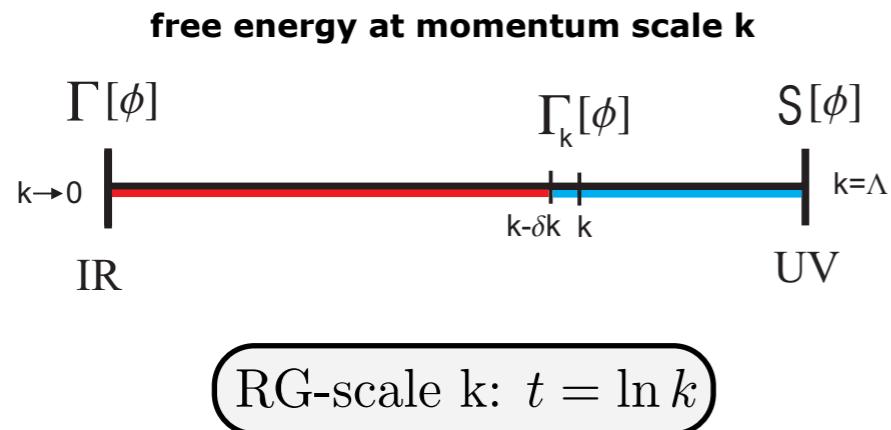
EoS & phase diagram

JMP, Quark Matter2014

Functional Methods for QCD

Functional RG

JMP, AIP Conf. Proc. 1343 (2011)

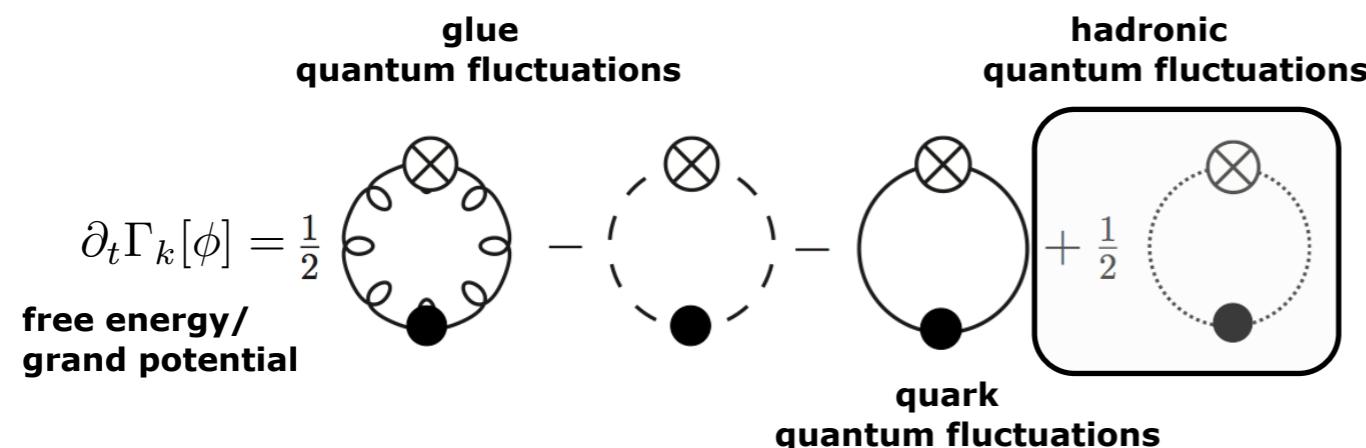


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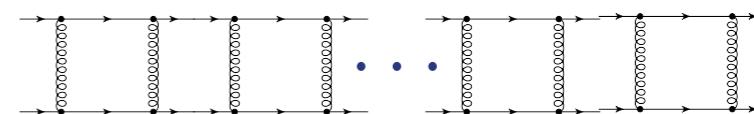


Phase diagram survey

JMP, Schladming '13

EoS & phase diagram

JMP, Quark Matter2014



Dynamical hadronisation

dynamical

Gies, Wetterich '01

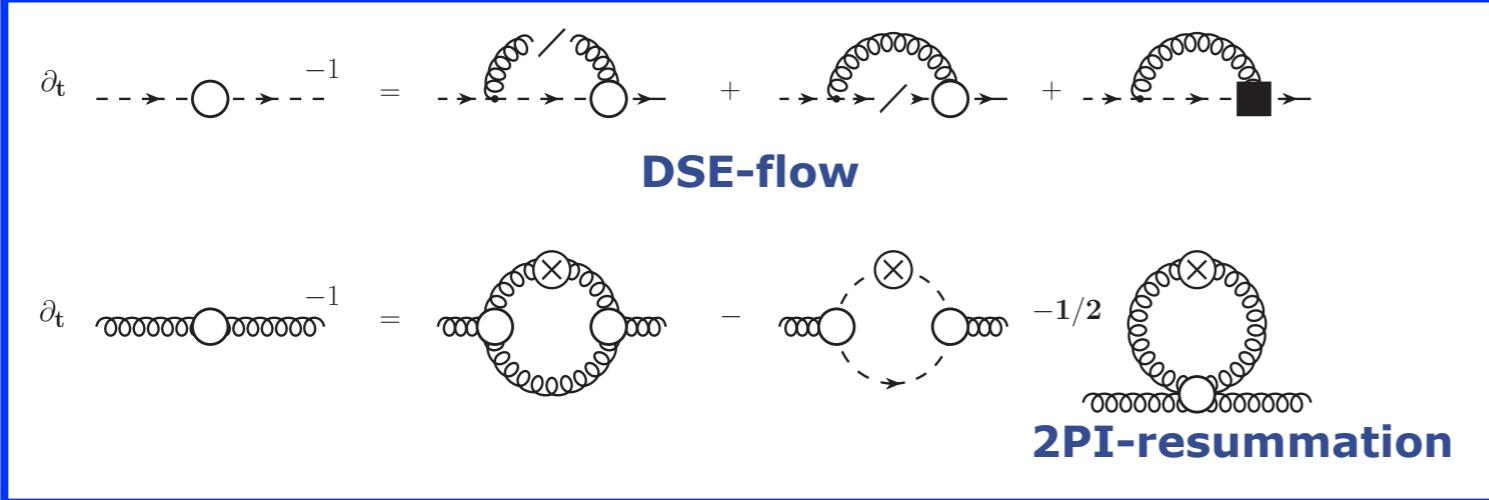
JMP '05

Flörchinger, Wetterich '09

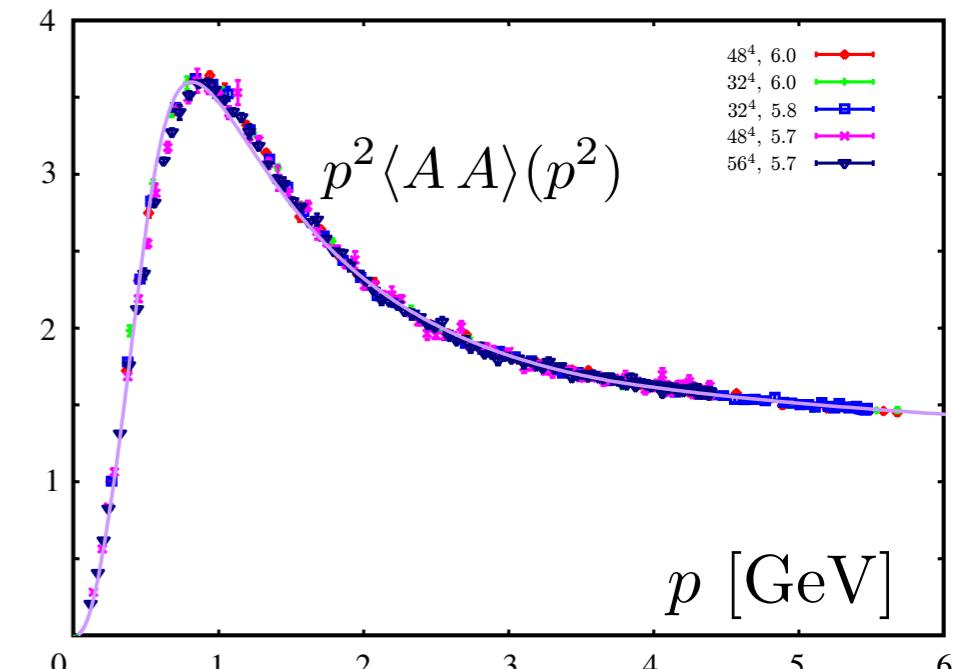
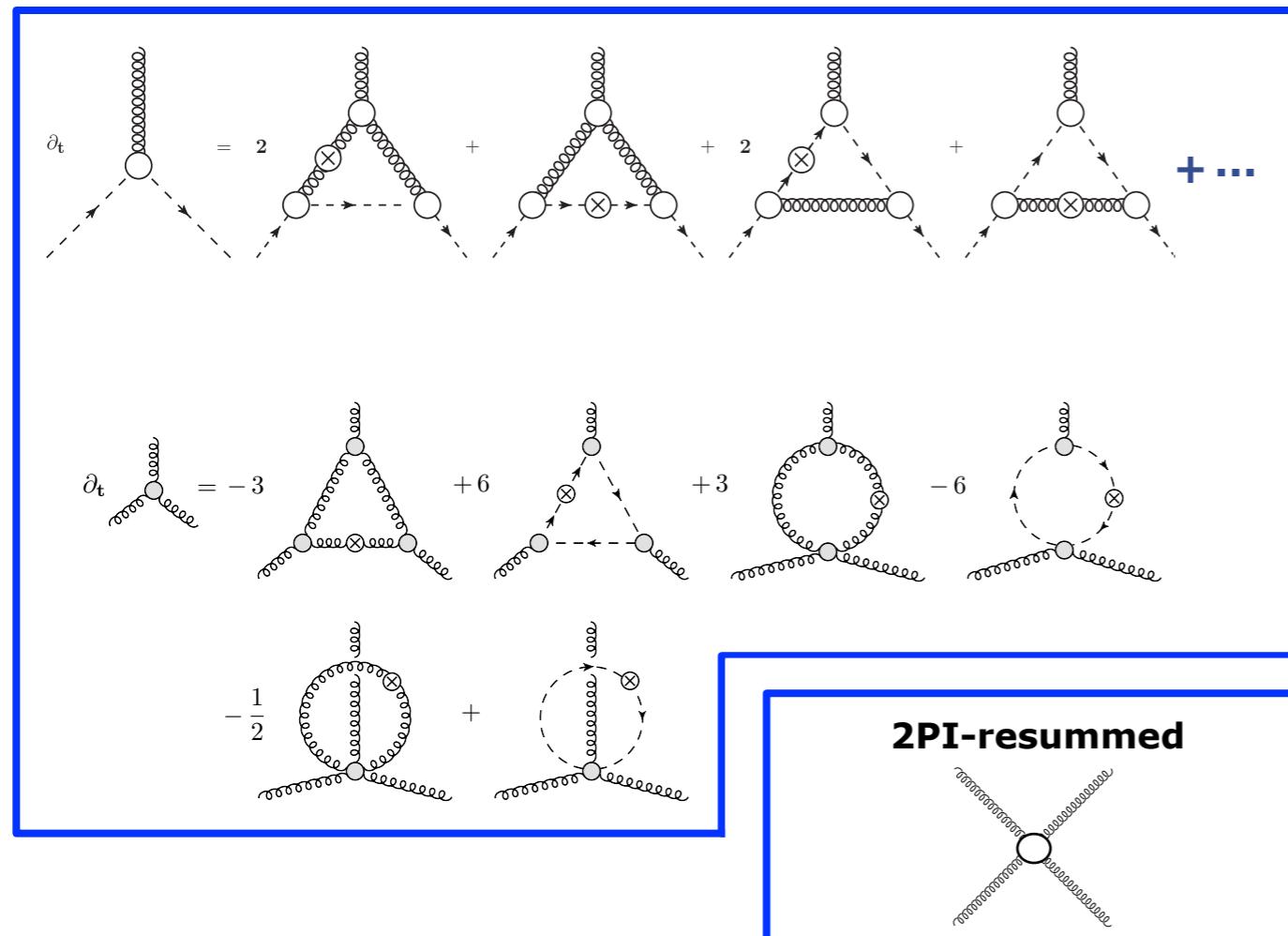
Functional Methods for QCD

Yang-Mills theory

Yang-Mills



Yang-Mills propagators



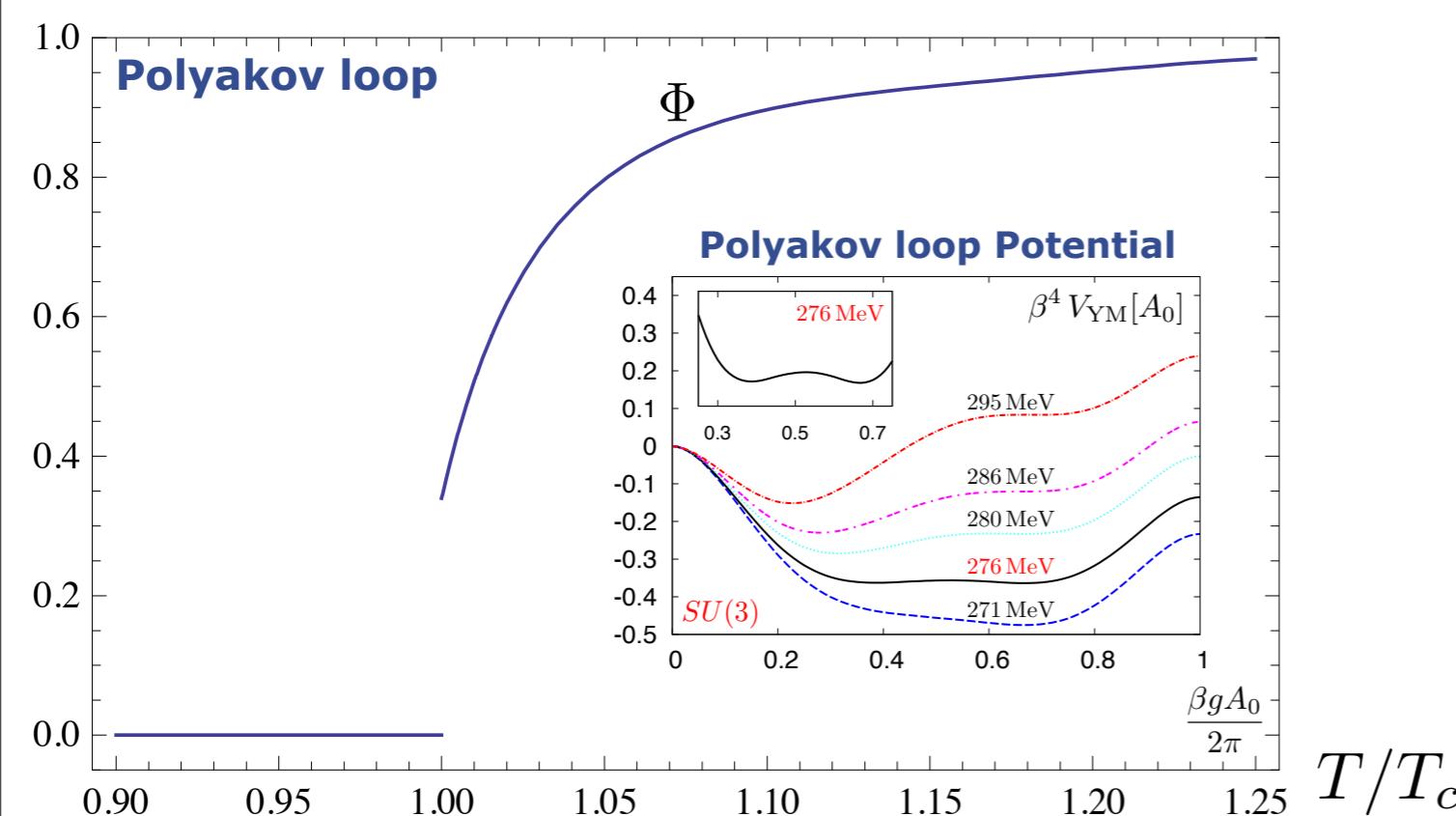
FRG: Fischer, Maas, JMP, Annals Phys. 324 (2009) 2408

Lattice: Sternbeck et al, PoS LAT2006 (2006) 076

see talks of L. Fister,
M. Huber

Functional Methods for QCD

Confinement



$$\Phi[A_0] = \frac{1}{3}(1 + 2 \cos \frac{1}{2} \beta g A_0)$$

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

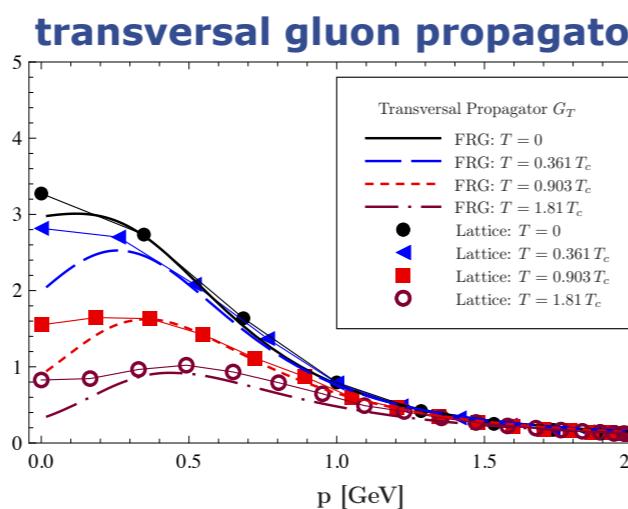
FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

$$\text{lattice : } T_c/\sqrt{\sigma} = 0.646$$

see talk of L. Fister

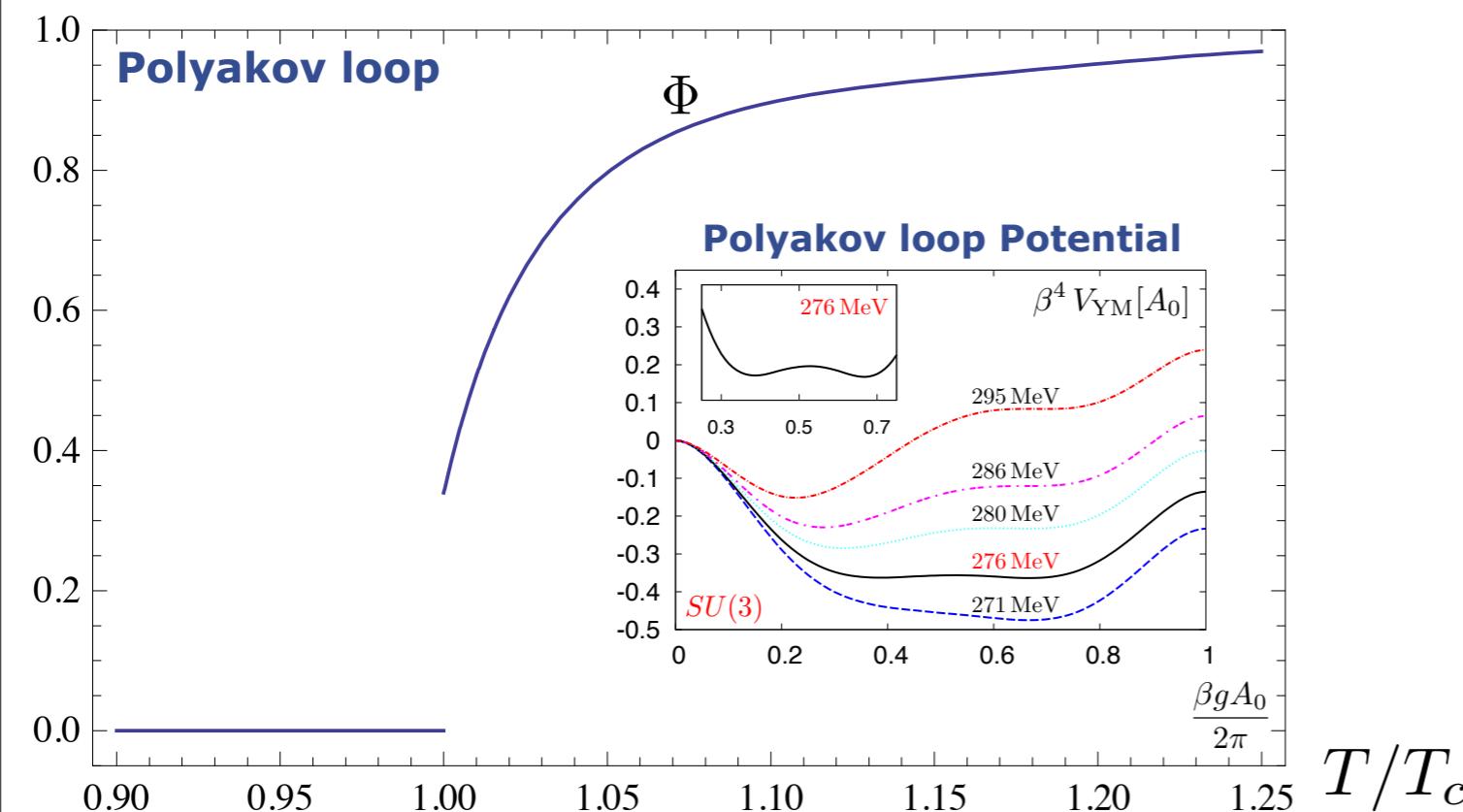
- from the full propagators
- gauge independence
- confinement criteria



Fister, JMP, arXiv:1112.5440

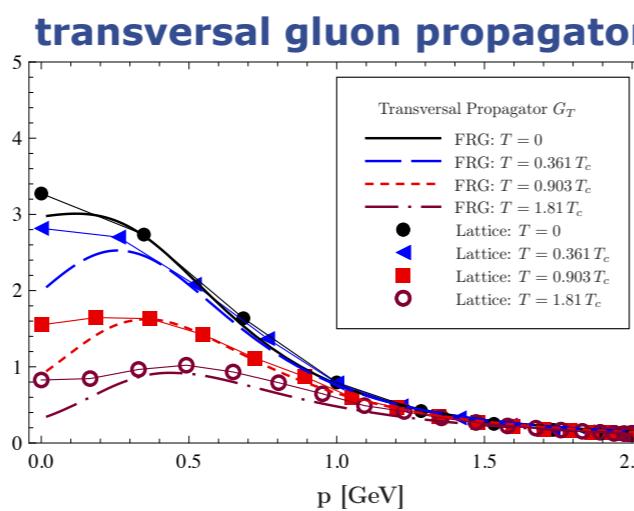
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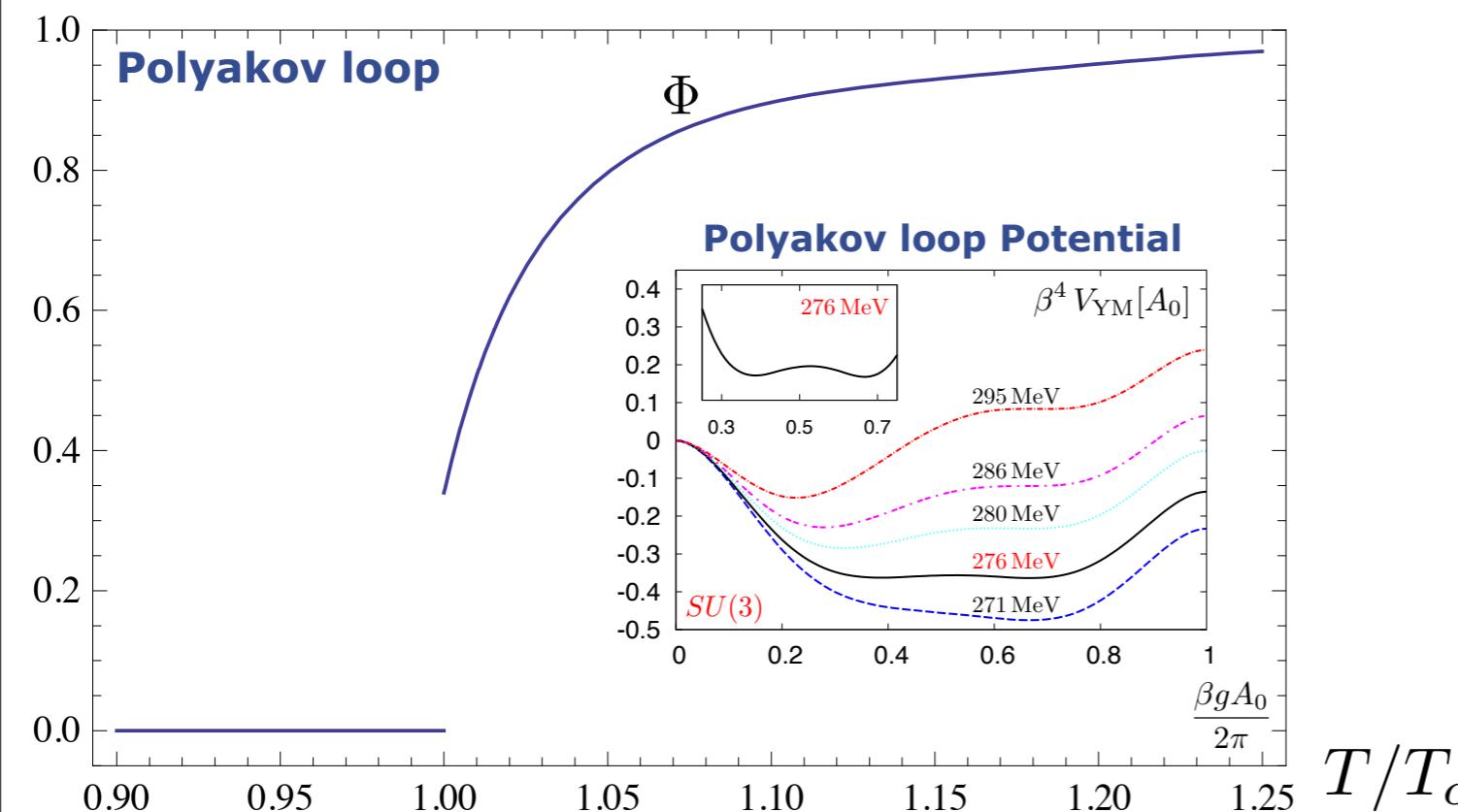
confinement

gluon propagator
gapped relative to
ghost propagator

Braun, Gies, JMP '07
 Marhauser, JMP '08
 Fister, JMP '13

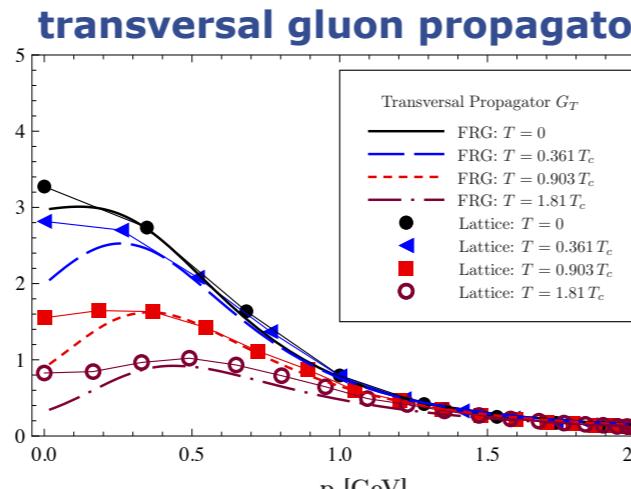
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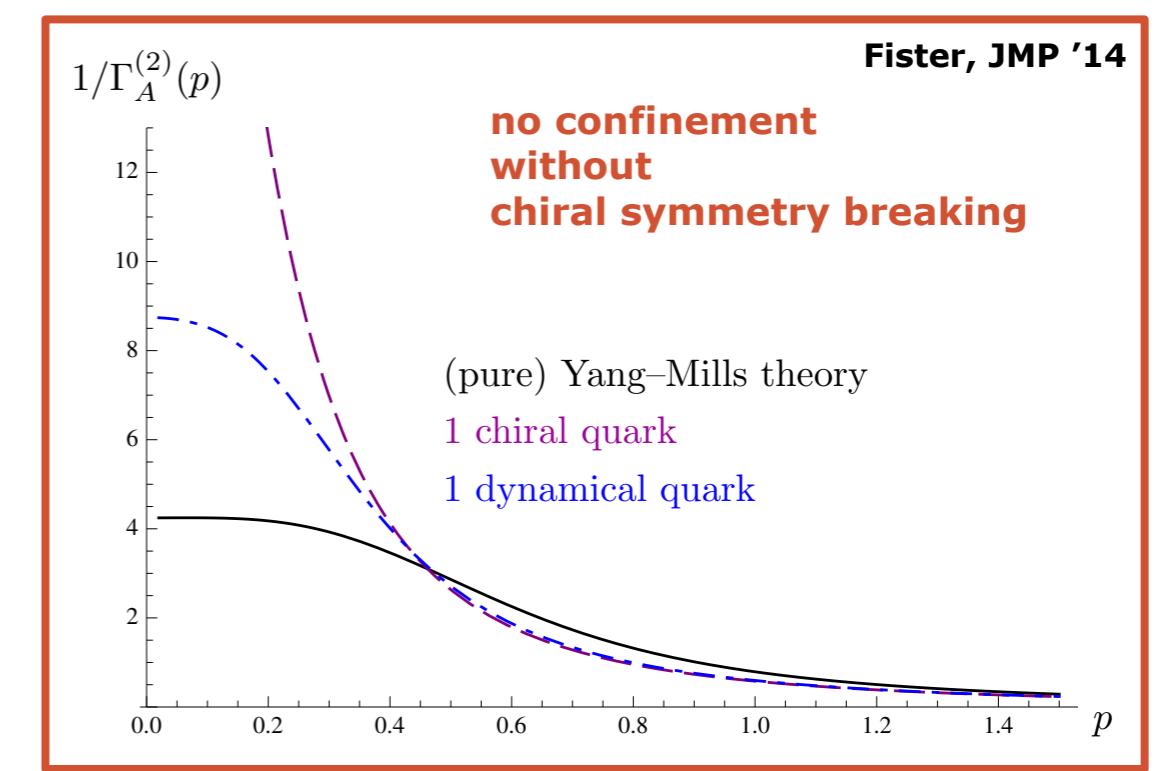
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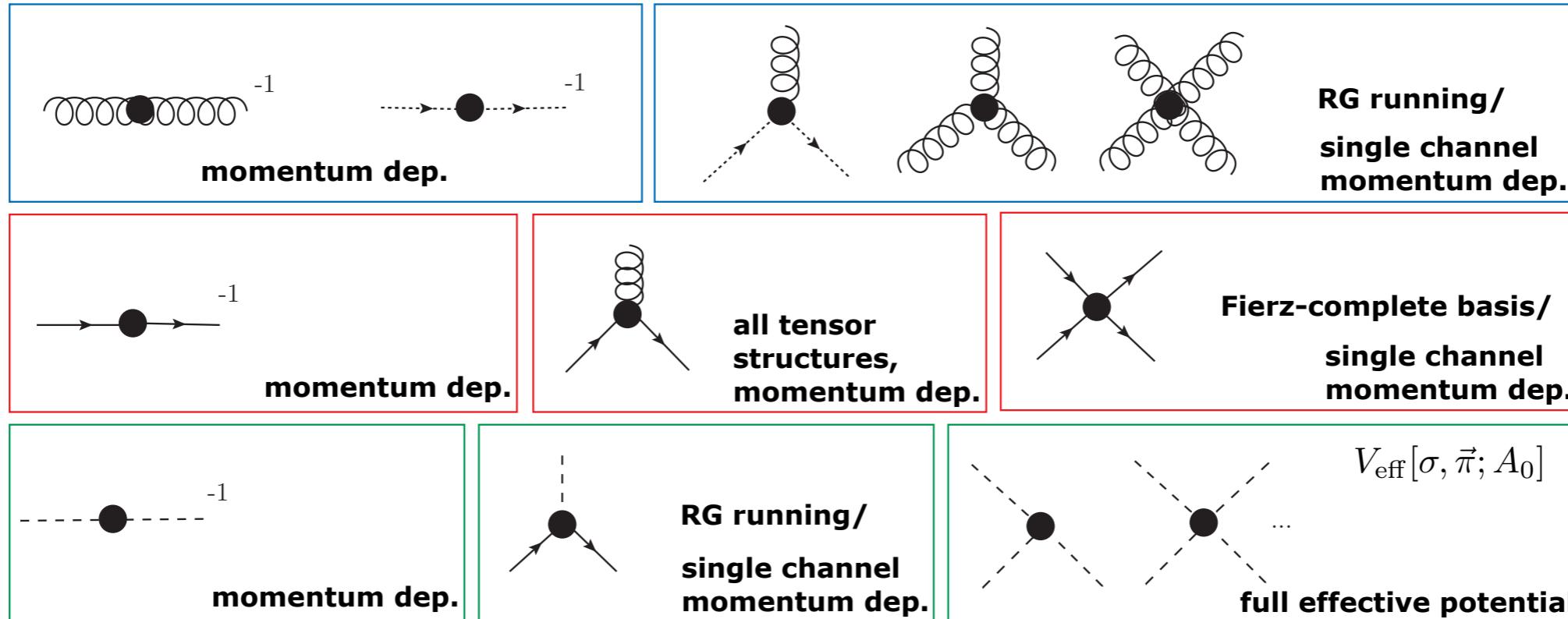
$$\text{lattice : } T_c/\sqrt{\sigma} = 0.646$$

see talk of L. Fister



Functional Methods for QCD

present best approximation

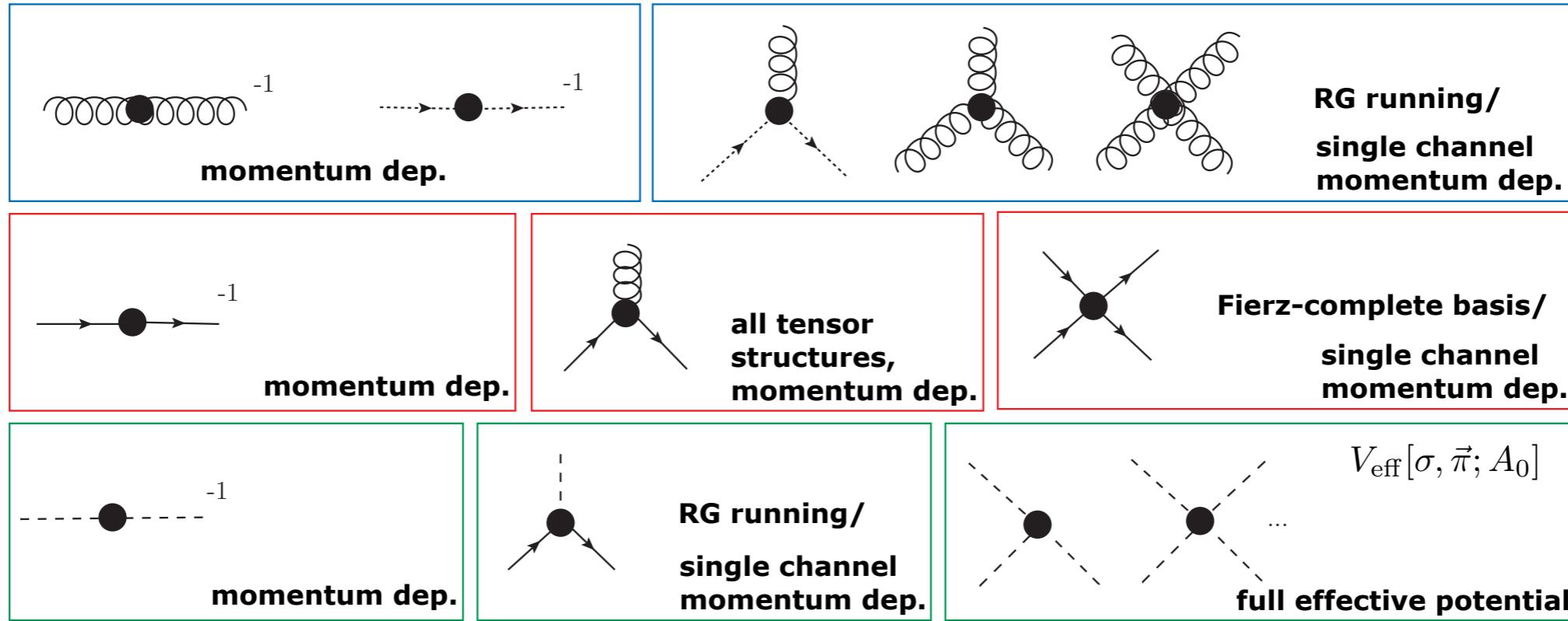


see talk of M. Mitter

FRG-QCD: Braun, Fister, Herbst, Mitter, JMP, Rennecke, Strodthoff

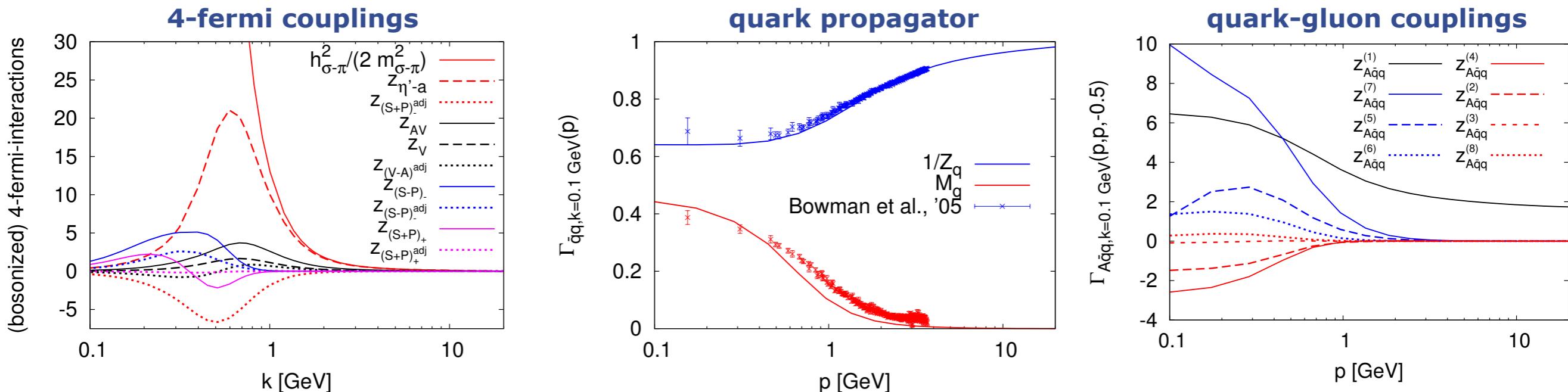
Functional Methods for QCD

chiral symmetry breaking



see talk of M. Mitter

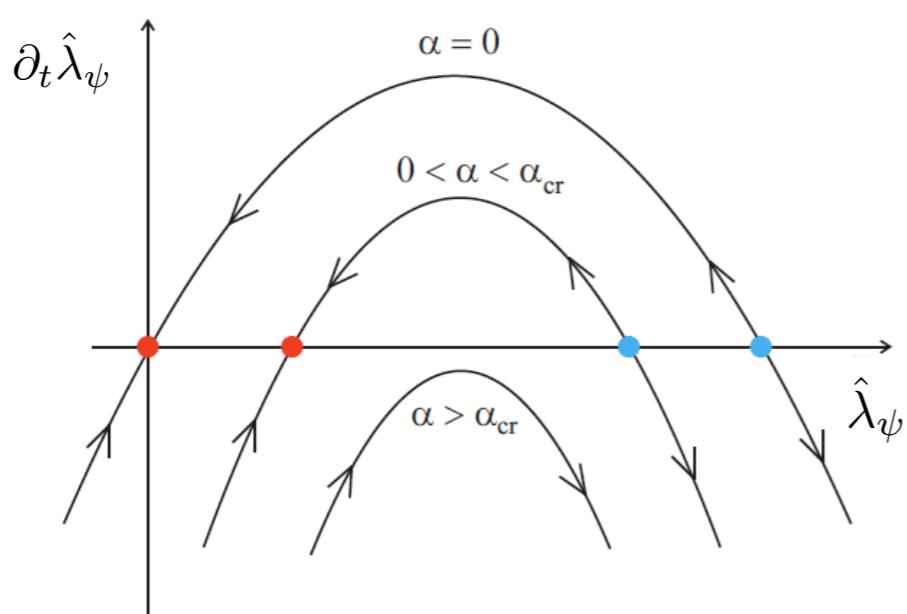
FRG-quenched QCD vs lattice-quenched QCD



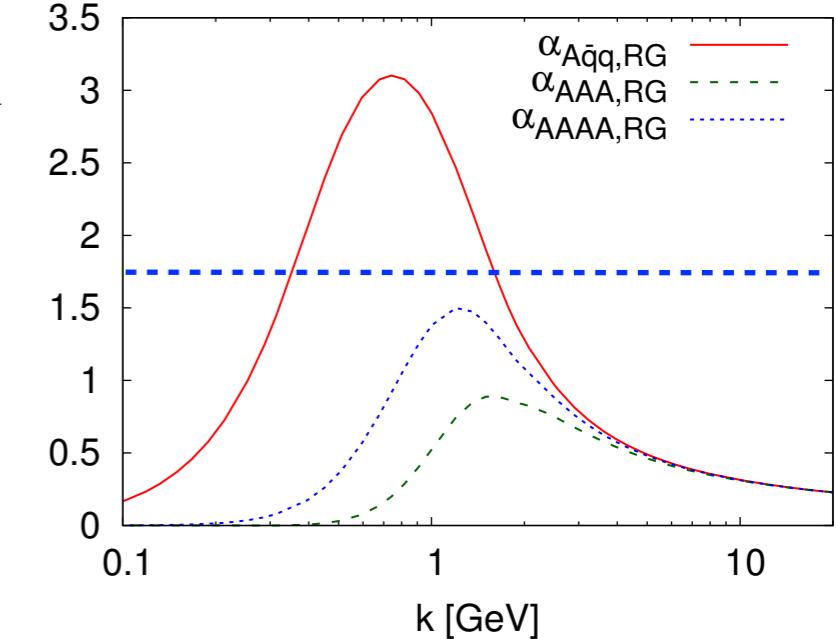
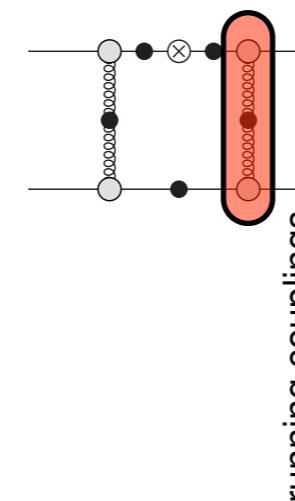
Mitter, JMP, Strodthoff, in preparation

Functional Methods for QCD

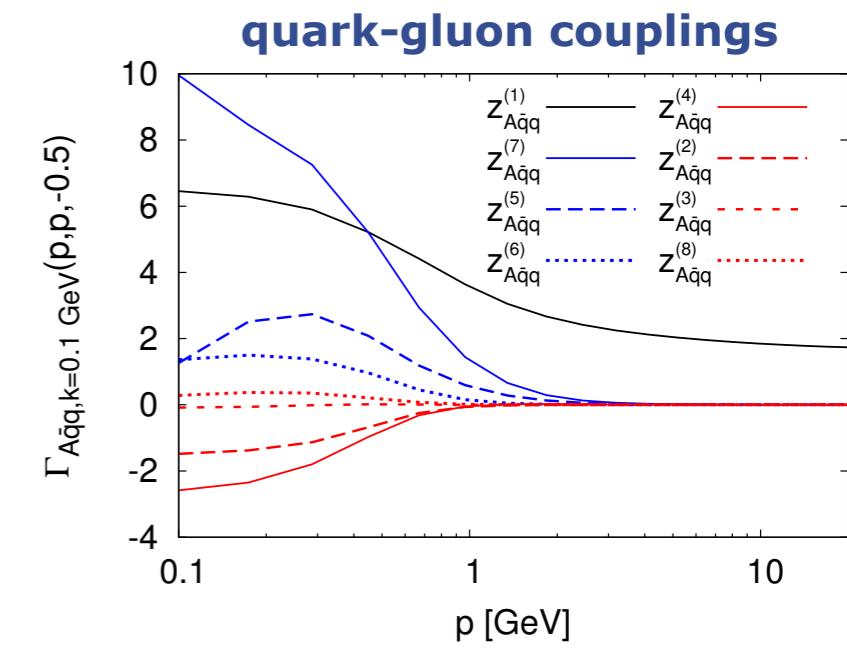
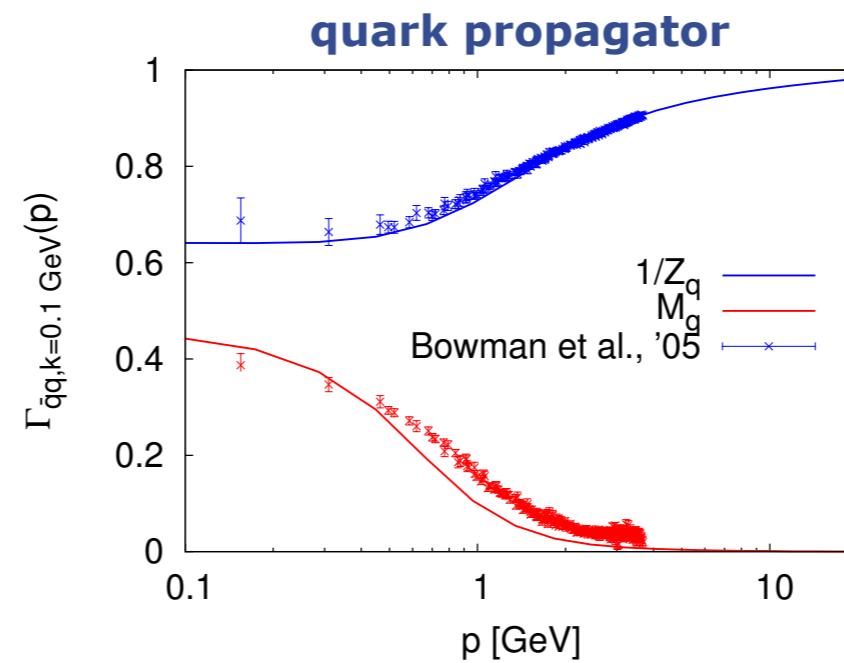
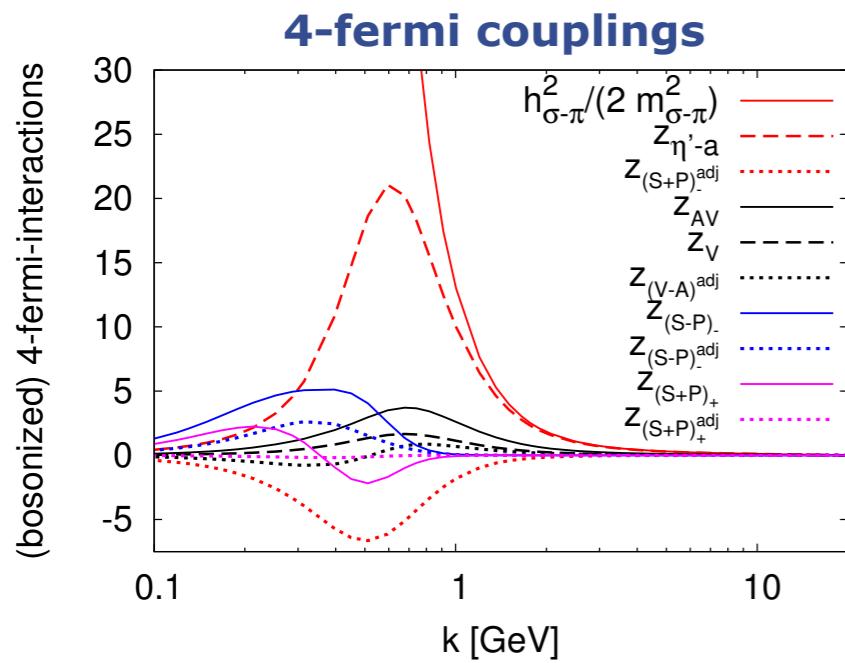
chiral symmetry breaking



mag. catalysis vs inverse mag. catalysis: see talk of S. Rechenberger



FRG-quenched QCD vs lattice-quenched QCD



Mitter, JMP, Strodthoff, in preparation

Functional Methods for QCD

chiral symmetry breaking

**dynamical correlation of confinement
and
chiral symmetry breaking**

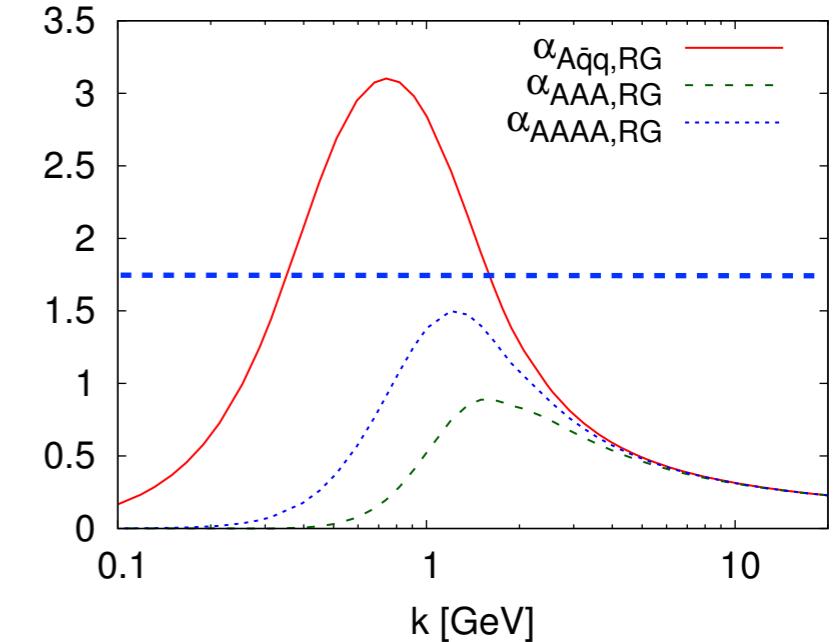
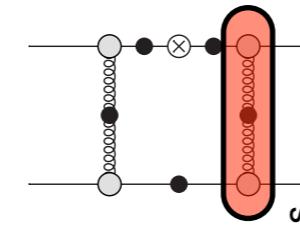
confinement

**gluon propagator
gapped relative to
ghost propagator**

chiral symmetry breaking

**gluon propagator
not gapped too much**

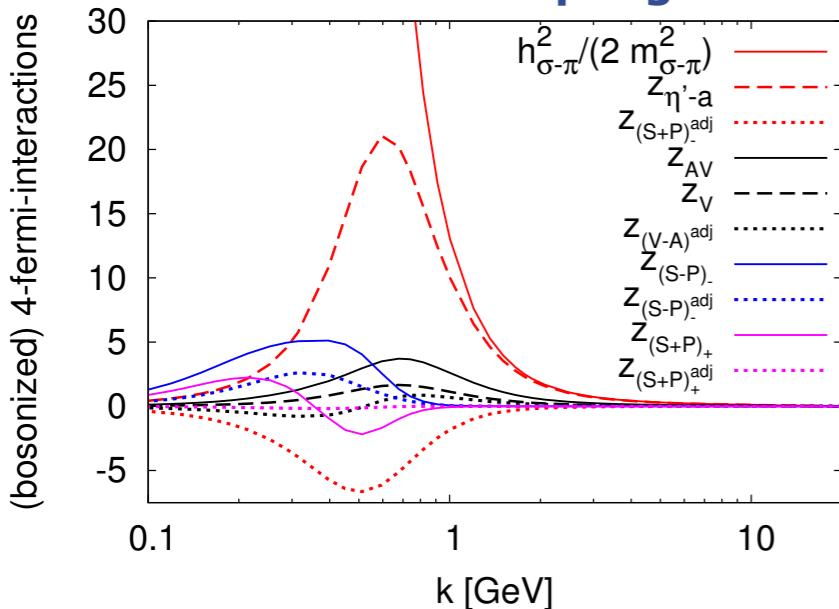
Fister, Mitter, JMP, Strodthoff '14



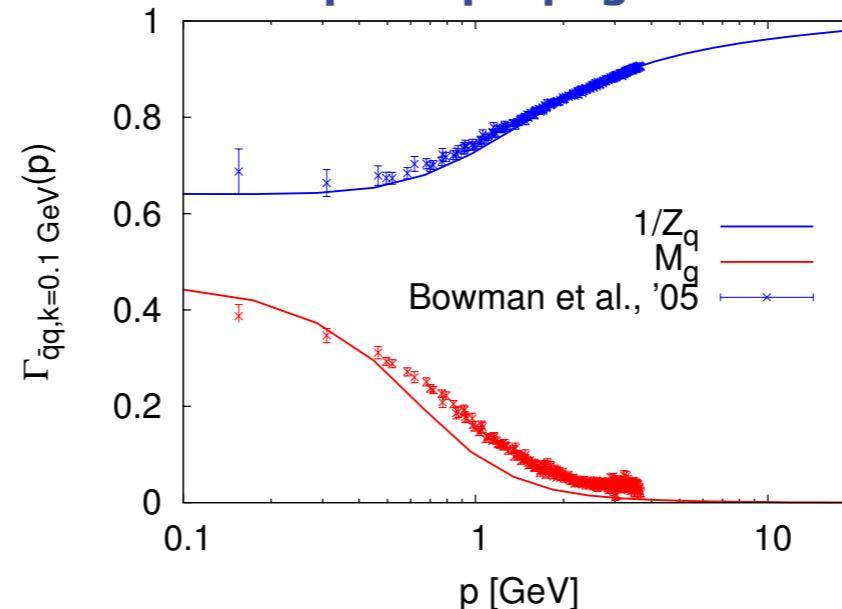
dynamical locking at finite T: see talk of P. Springer

FRG-quenched QCD vs lattice-quenched QCD

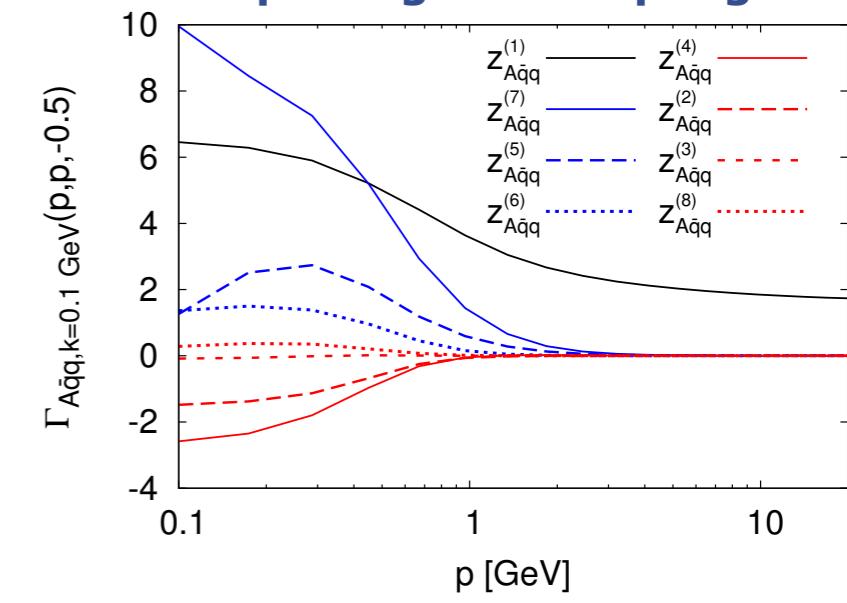
4-fermi couplings



quark propagator



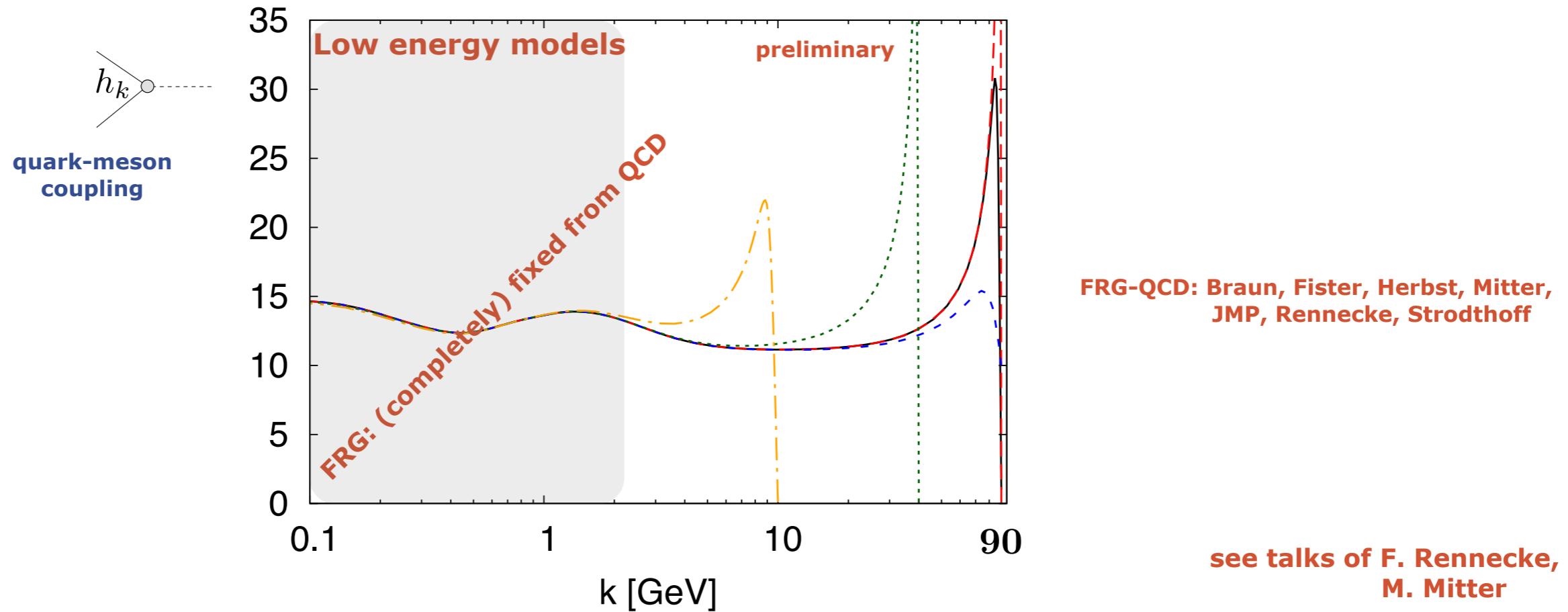
quark-gluon couplings



Mitter, JMP, Strodthoff, in preparation

QCD

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} + \frac{1}{2} \text{Diagram 4} \right)$$



Model results on the phase structure of QCD

PQM-model

$$\frac{1}{2} \text{Diagram 1} - \text{Diagram 2} + \text{Diagram 3} - \text{Diagram 4} + \frac{1}{2} \text{Diagram 5}$$

PNJL-model

$$\frac{1}{2} \text{Diagram 1} - \text{Diagram 2} + \text{Diagram 3}$$

QM-model

$$-\text{Diagram 1} + \frac{1}{2} \text{Diagram 2}$$

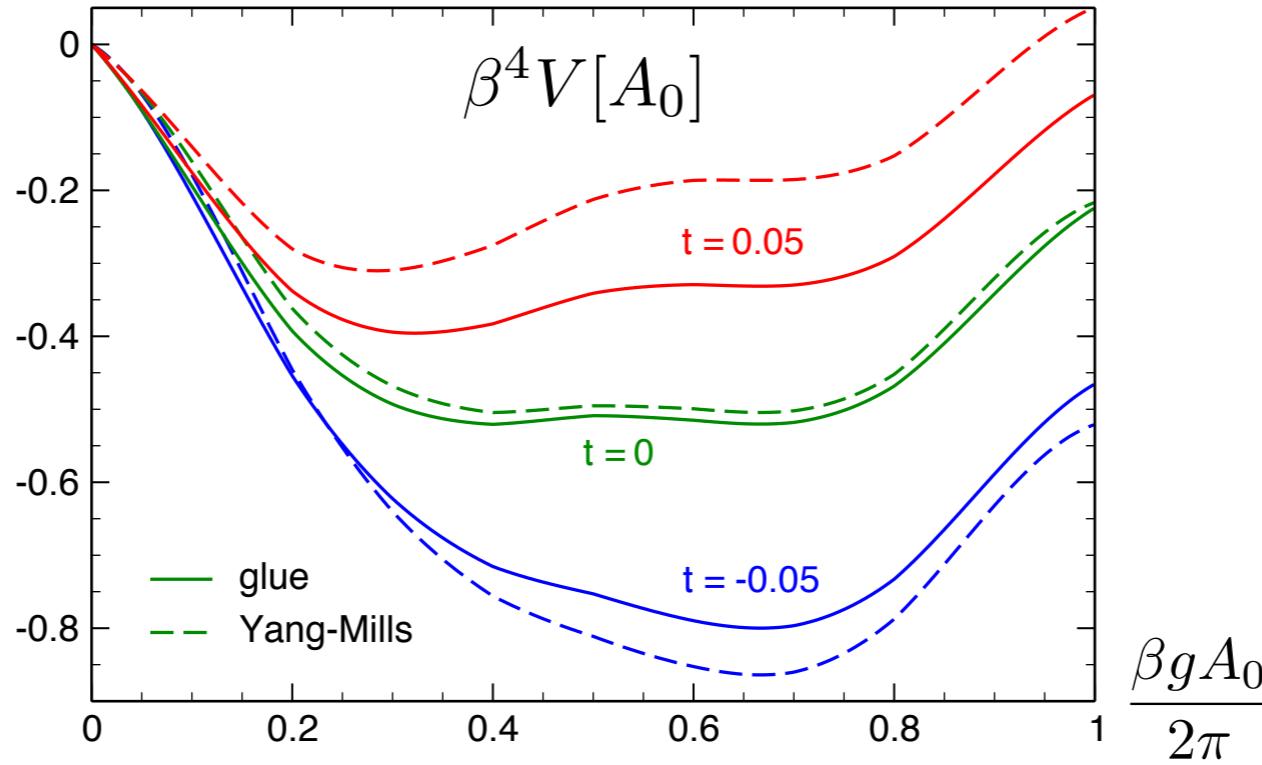
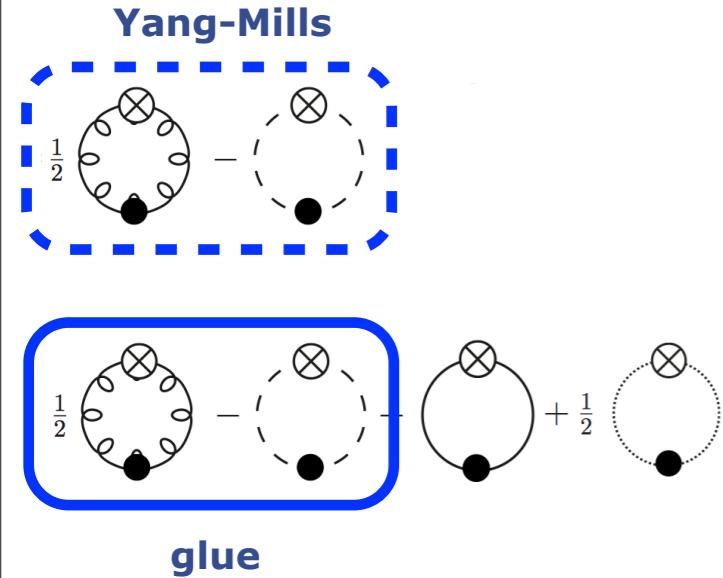
NJL-model

$$-\text{Diagram 1}$$

QCD

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{---} \right) - \left(\text{---} \right) - \left(\text{---} \right) + \frac{1}{2} \left(\text{---} \right)$$

Polyakov loop potential in full QCD



see talk of B.J. Schaefer

JMP, AIP Conf. Proc. 1343 (2011)
Haas, Stiele et al, Phys. Rev. D87 (2013) 076004

Model results on the phase structure of QCD

PQM-model

PNJL-model

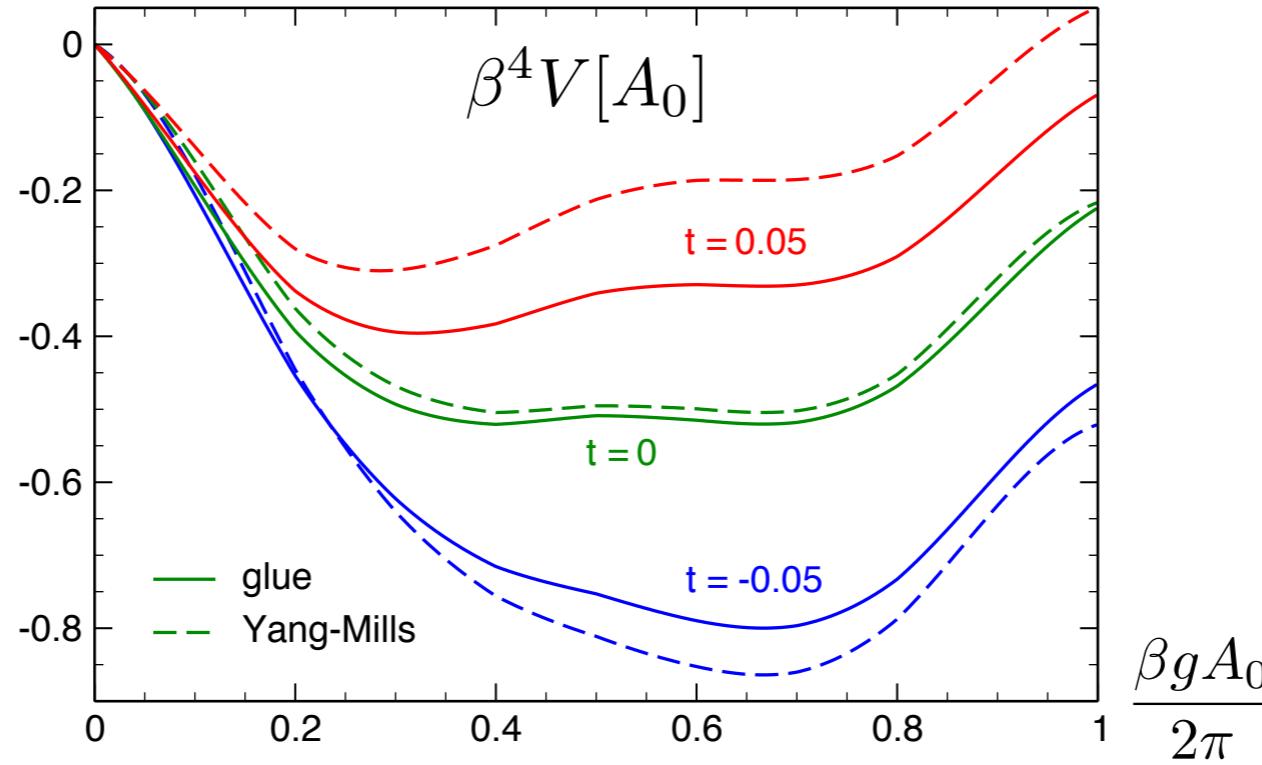
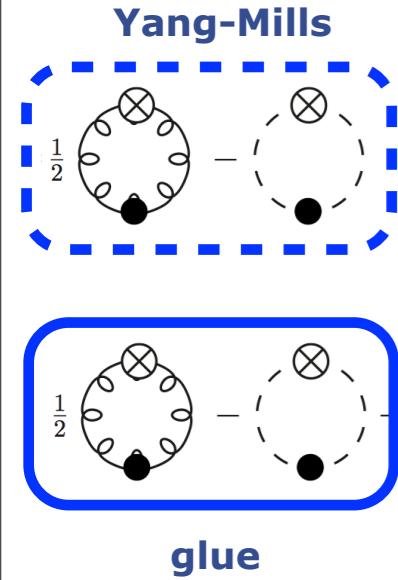
QM-model

NJL-model

QCD

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{---} \right) - \left(\text{---} \right) - \left(\text{---} \right) + \frac{1}{2} \left(\text{---} \right)$$

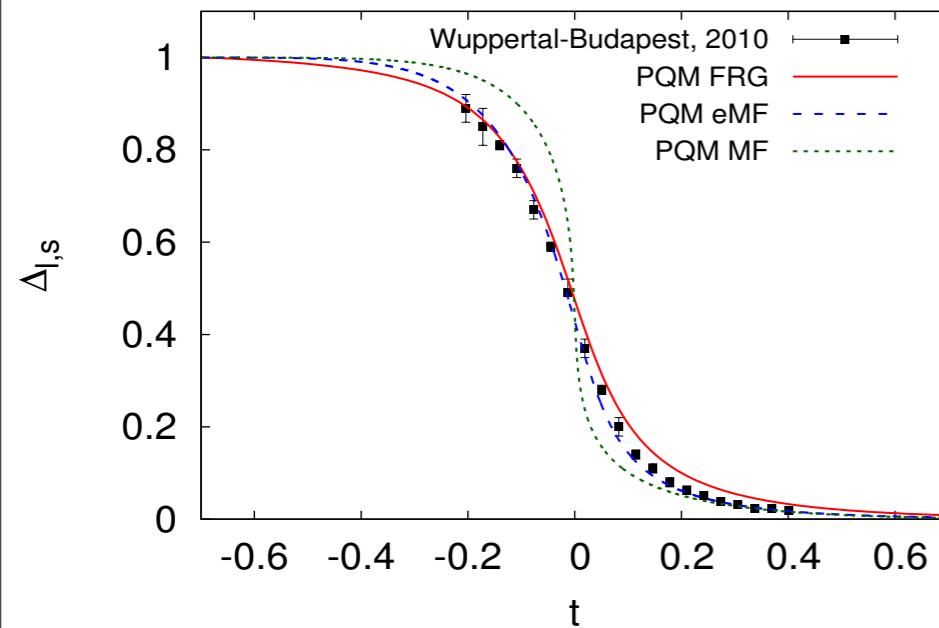
Polyakov loop potential in full QCD



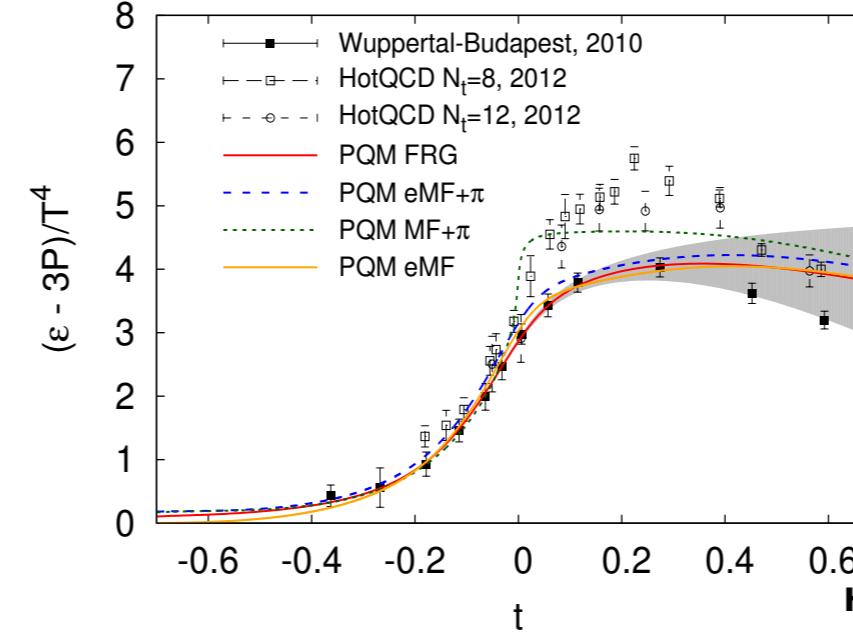
JMP, AIP Conf. Proc. 1343 (2011)
Haas, Stiele et al, Phys. Rev. D87 (2013) 076004

2+1 flavor Polyakov-loop - enhanced QM-model

reduced chiral condensate



Interaction measure



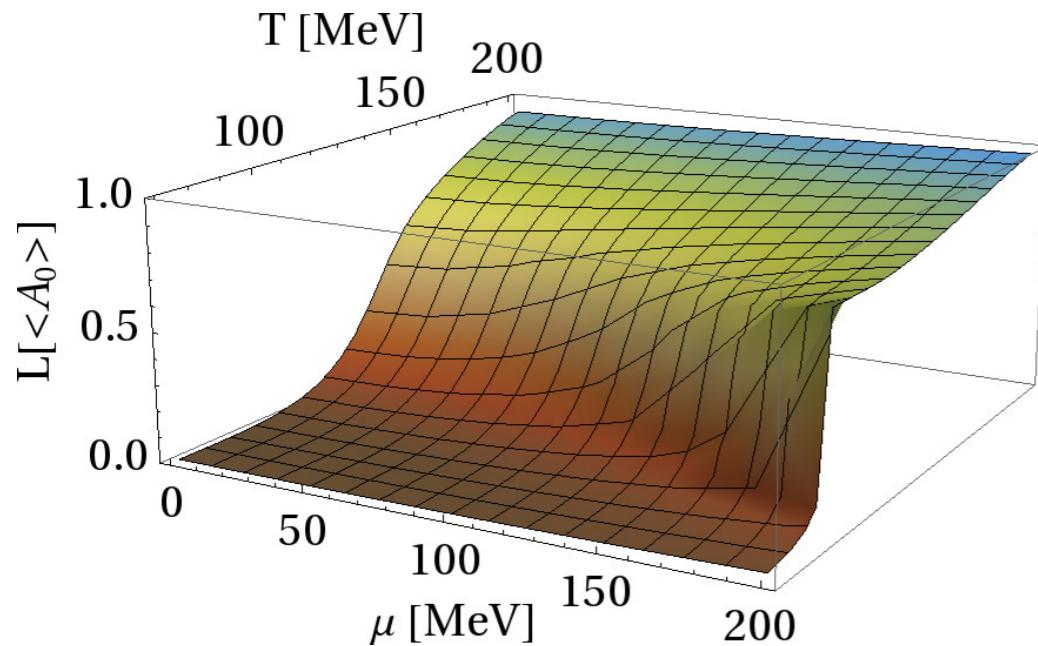
see talk of M. Mitter

Shaded area:
systematic error estimate
due to low initial scale 1 GeV

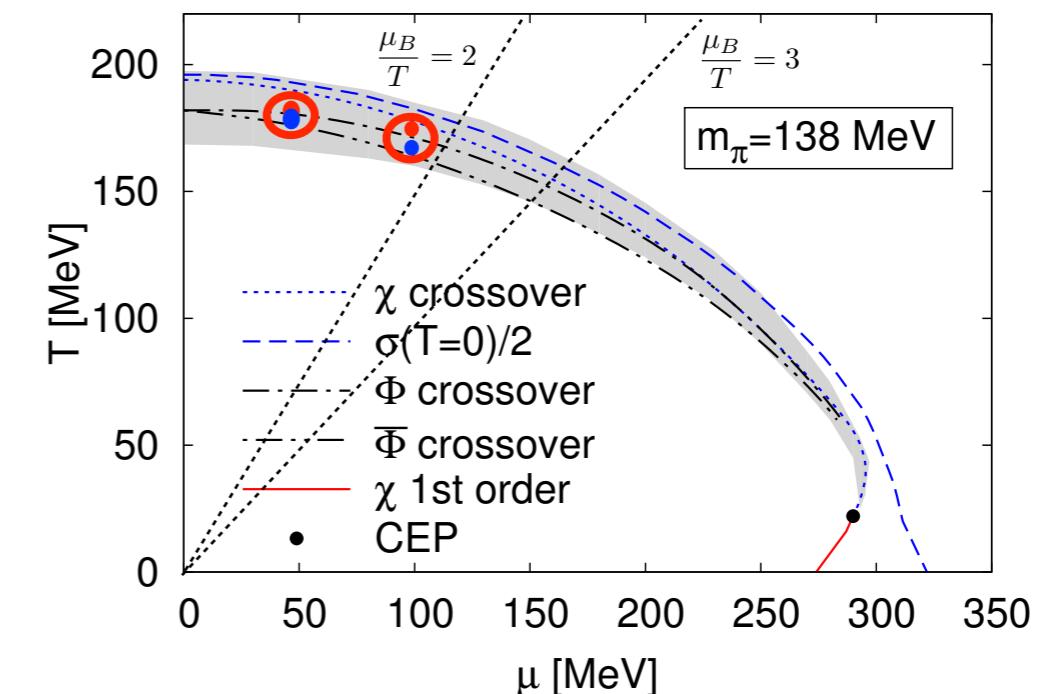
Herbst, Mitter et al, PLB 731 (2014) 248-256

Phase structure at finite density

Polyakov loop at finite density

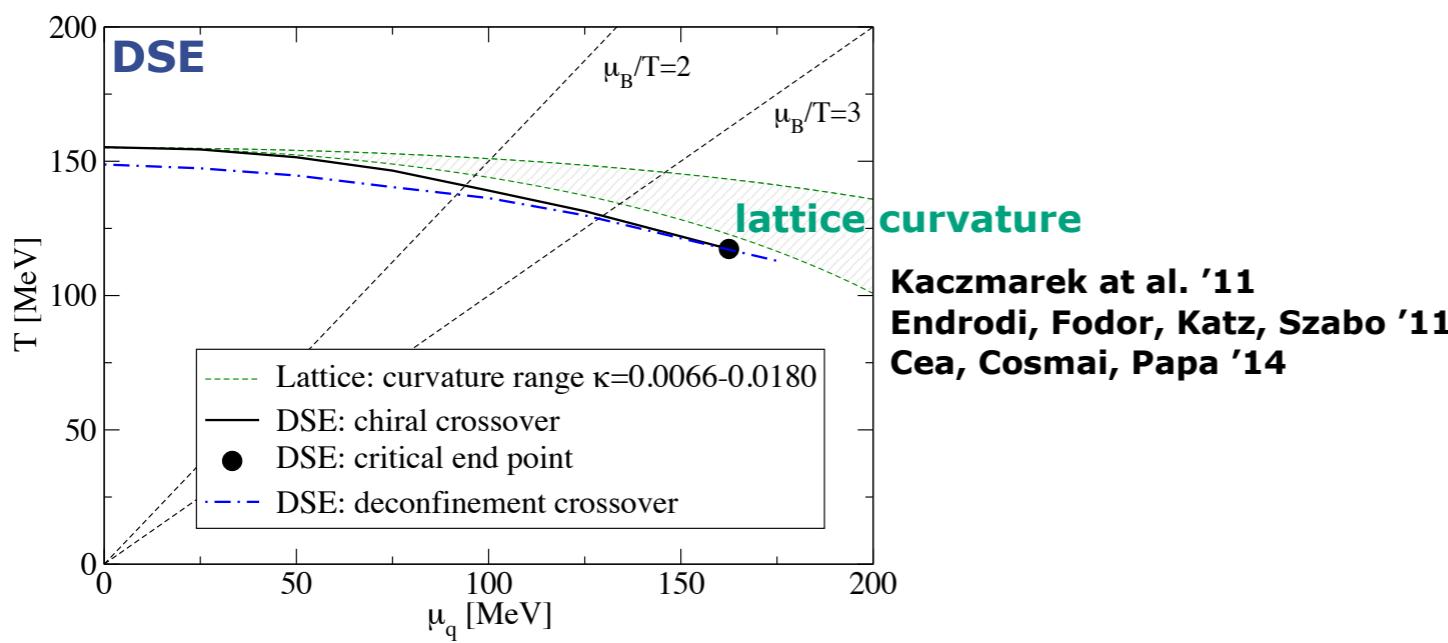


Phase diagram of quantised PQM-model



see talk of J. Luecker

Phase diagram of 2+1 flavor QCD



Fischer, Luecker, PLB 718 (2013) 1036

Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

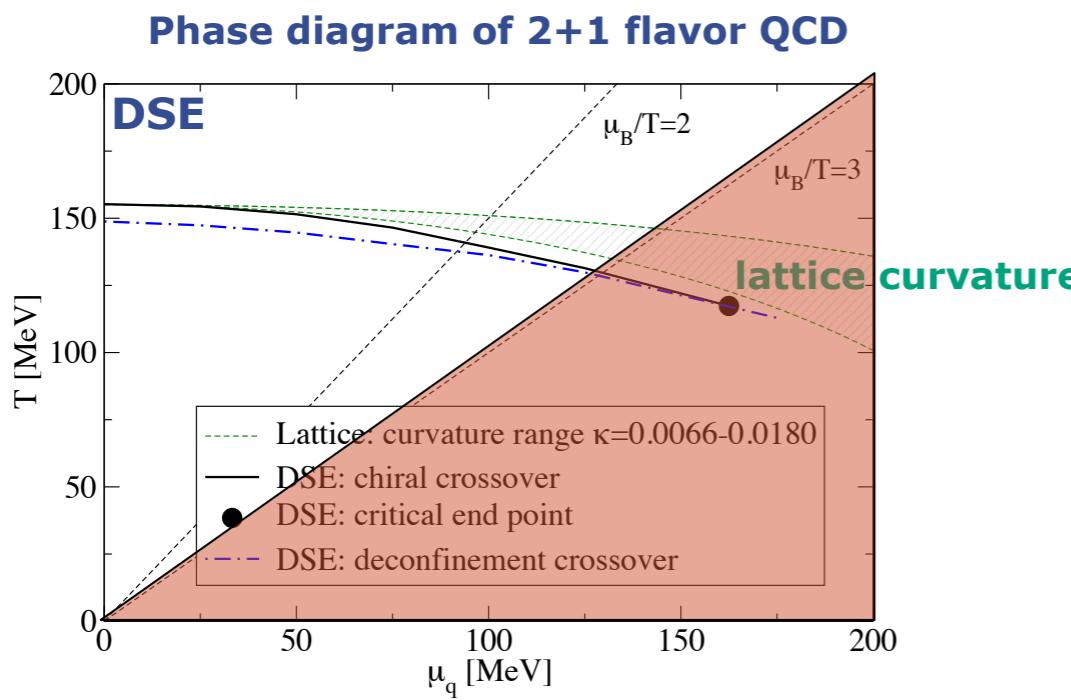
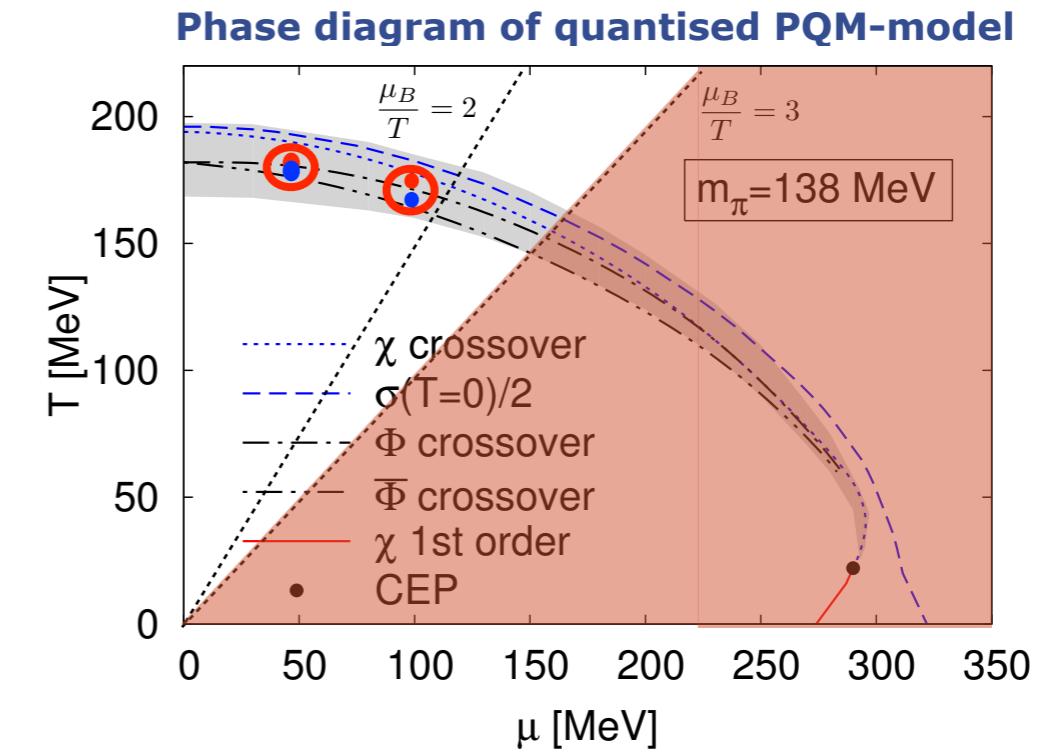
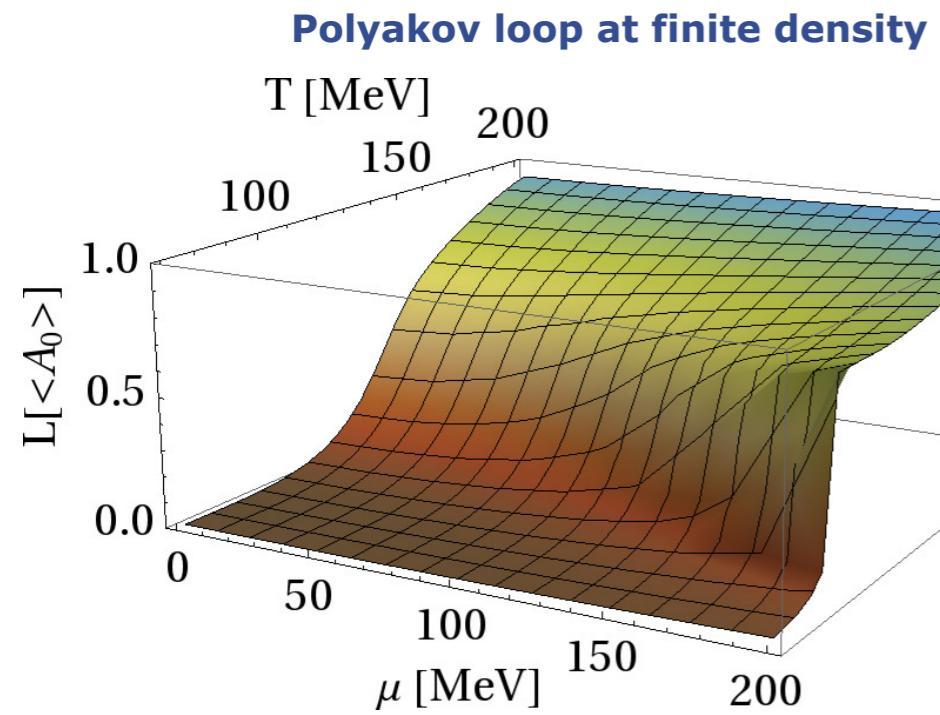
Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007

FRG QCD results at finite density
Haas, Braun, JMP '09, unpublished

Critical point
unlikely for

$$\frac{\mu_B}{T} < 2$$

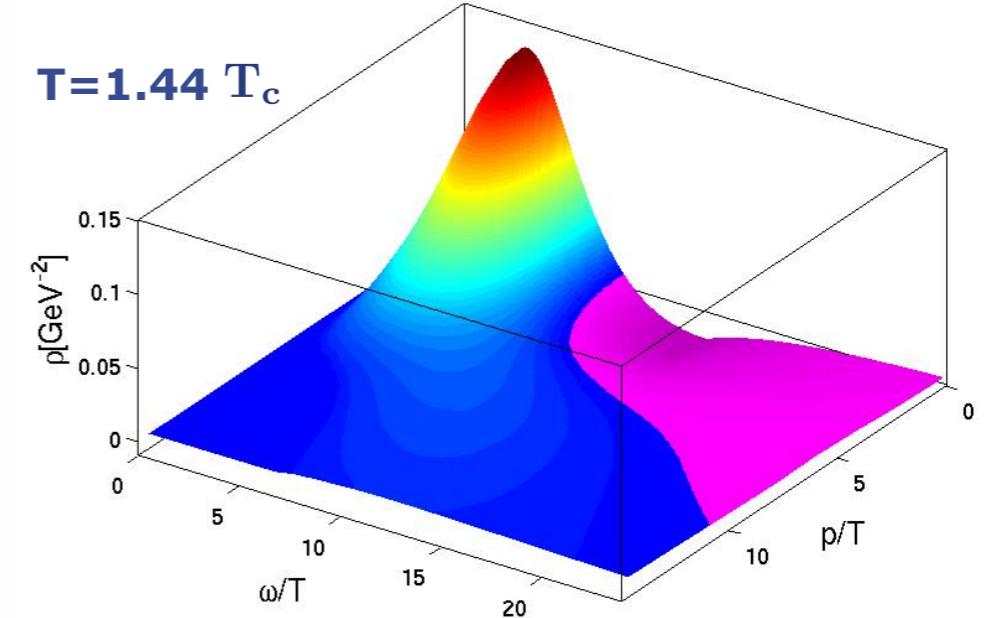
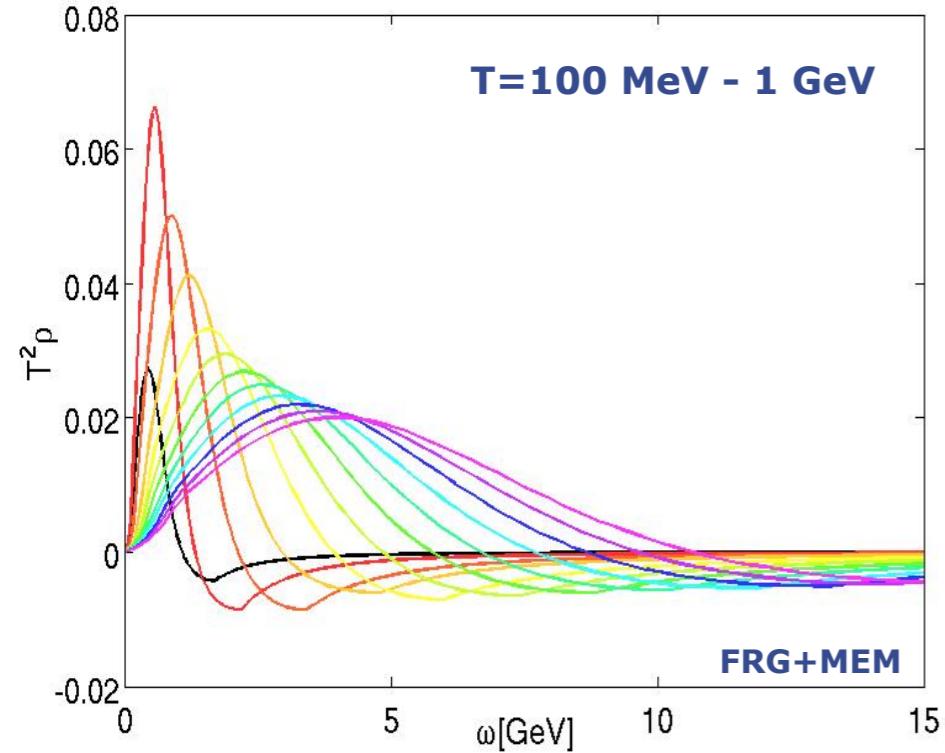
Phase structure at finite density



FRG-QCD with mesons, diquarks & baryons
finite mu and real frequencies

Viscosity in pure glue

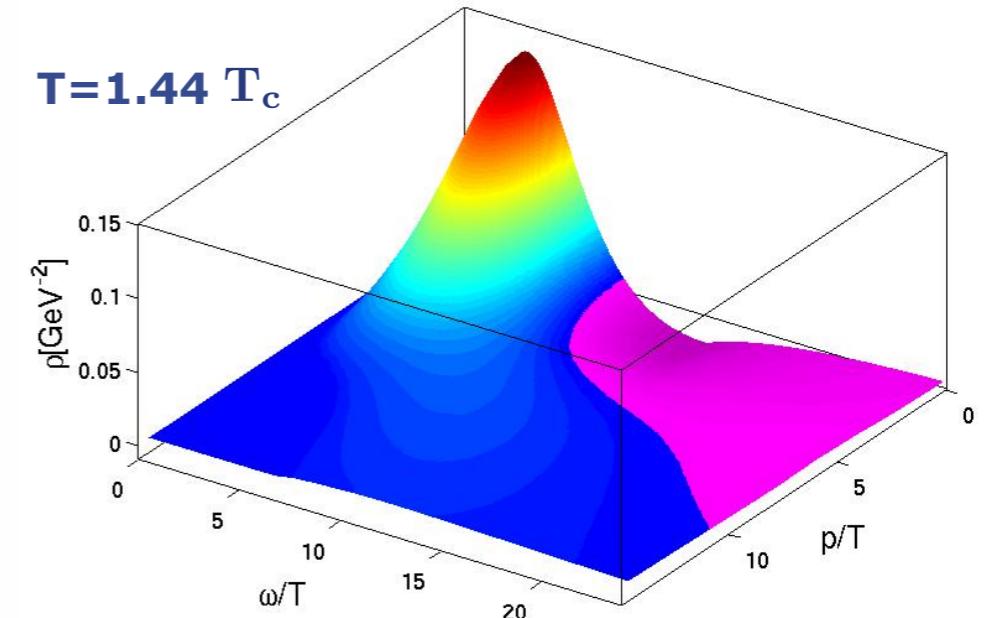
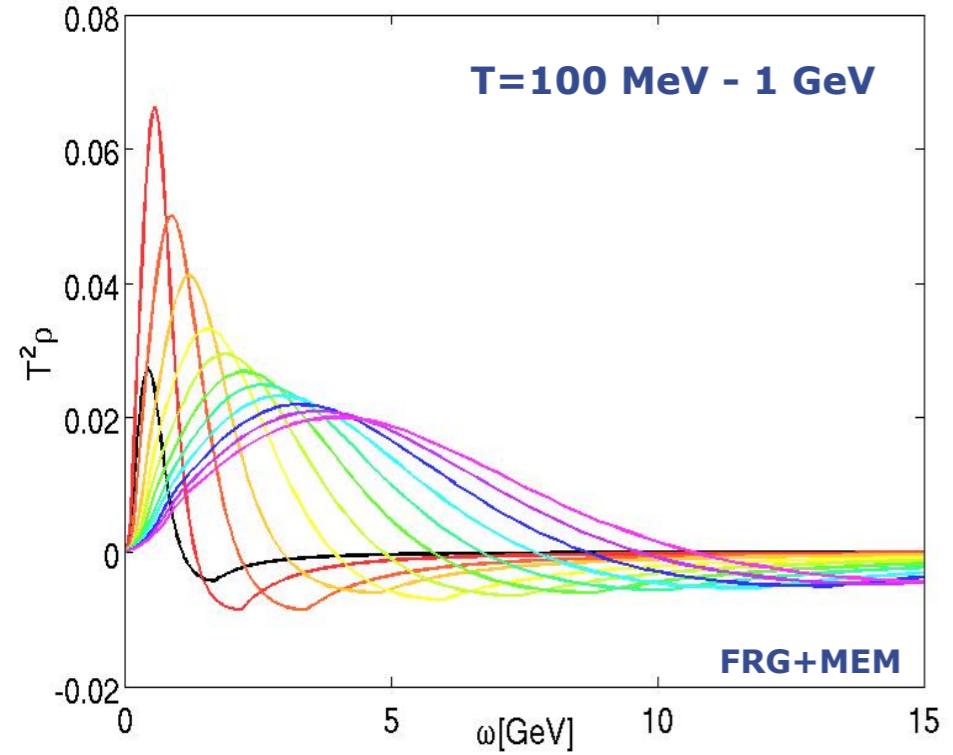
transversal gluon spectral function



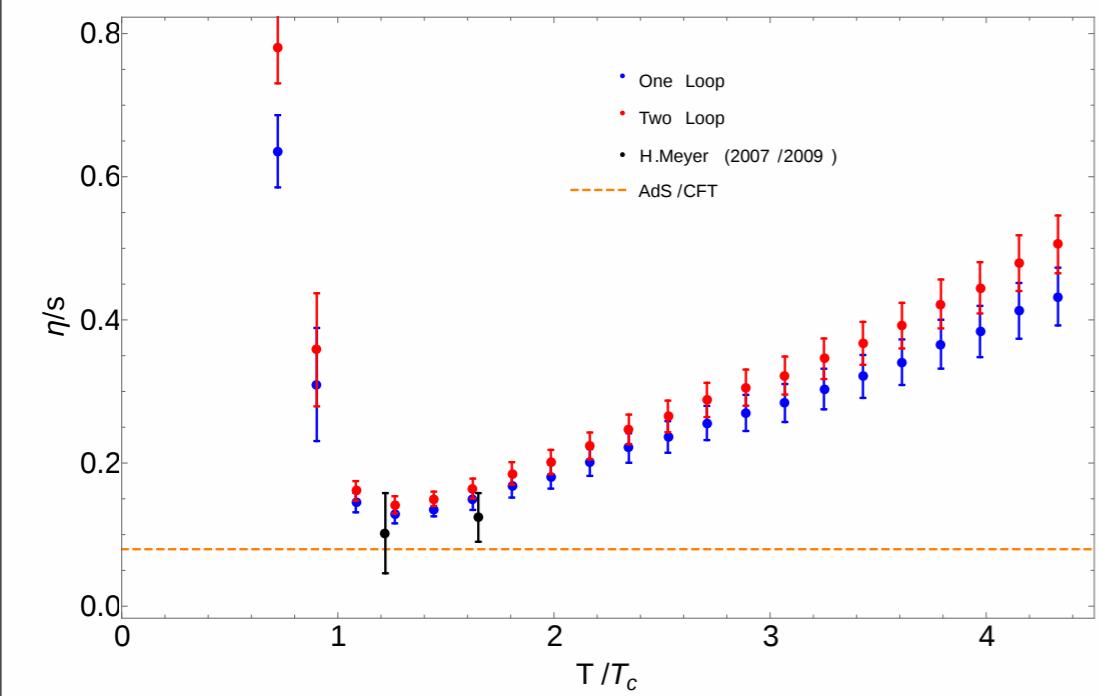
see talk of N. Strodthoff

Viscosity in pure glue

transversal gluon spectral function



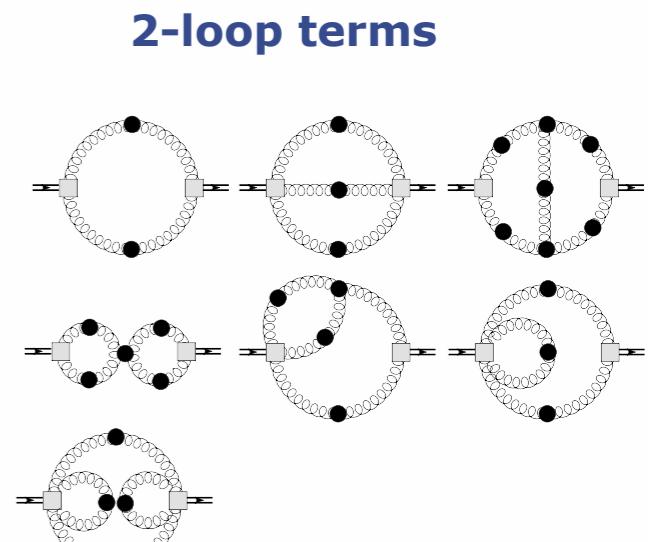
see talk of N. Strodthoff



Kubo relation

$$\eta = \frac{1}{20} \left. \frac{d}{d\omega} \right|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

3-loop exact functional relation

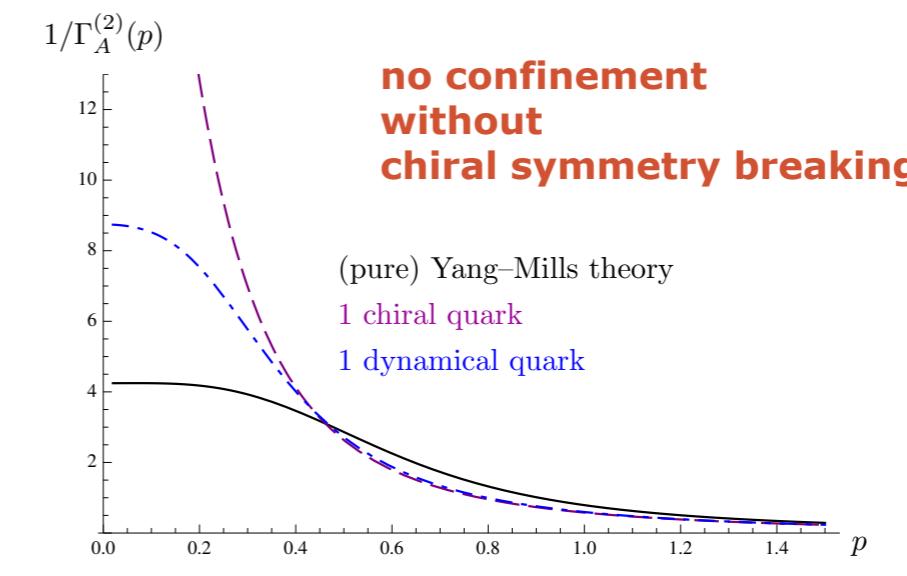
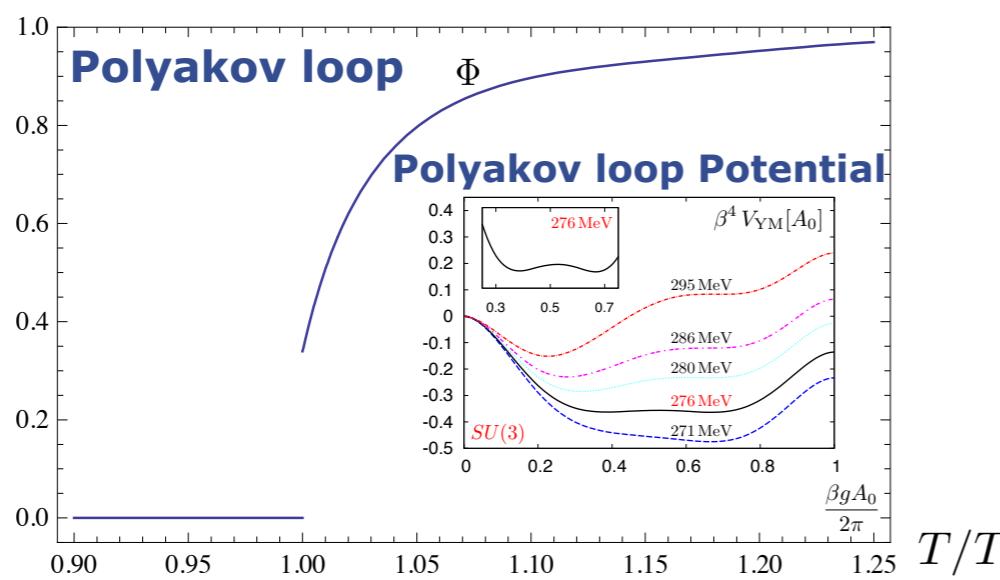
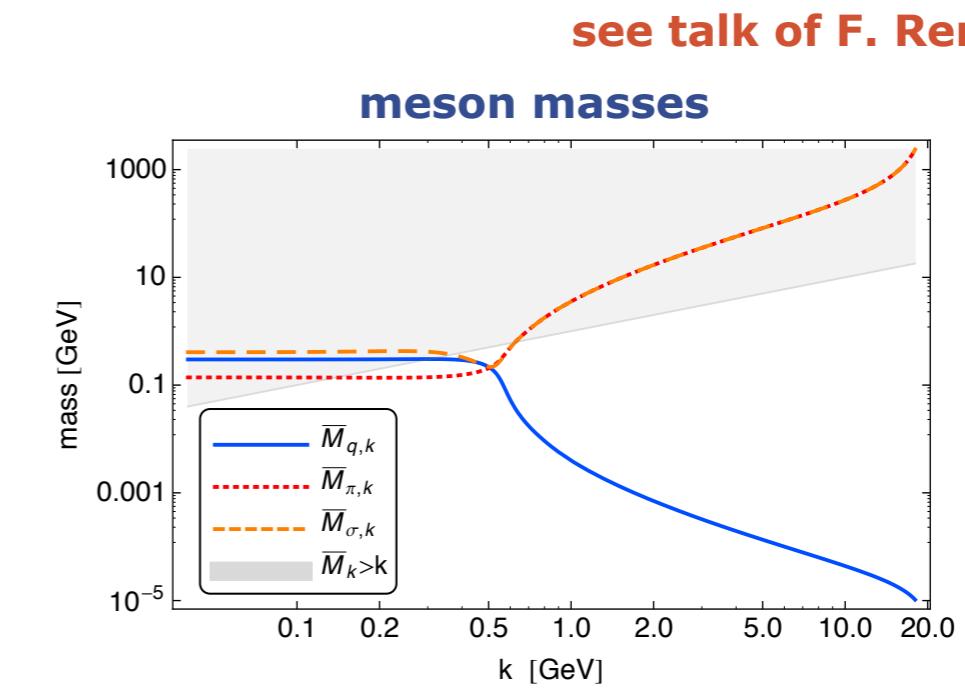
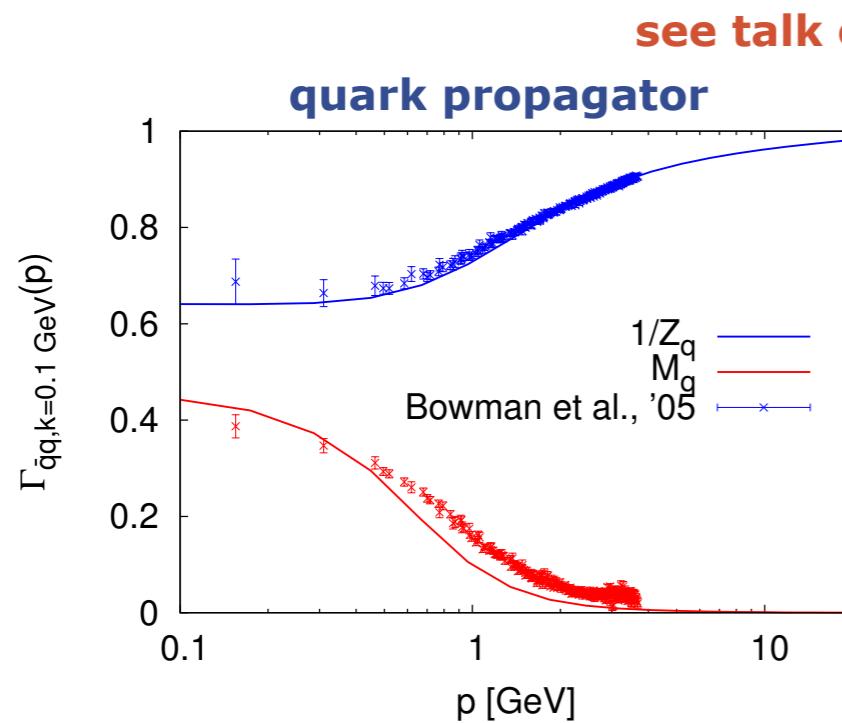


M. Haas, Fister, JMP, arXiv:1308.4960
Christiansen, Haas, JMP, Strodthoff, in preparation

Summary & Outlook

Summary & Outlook

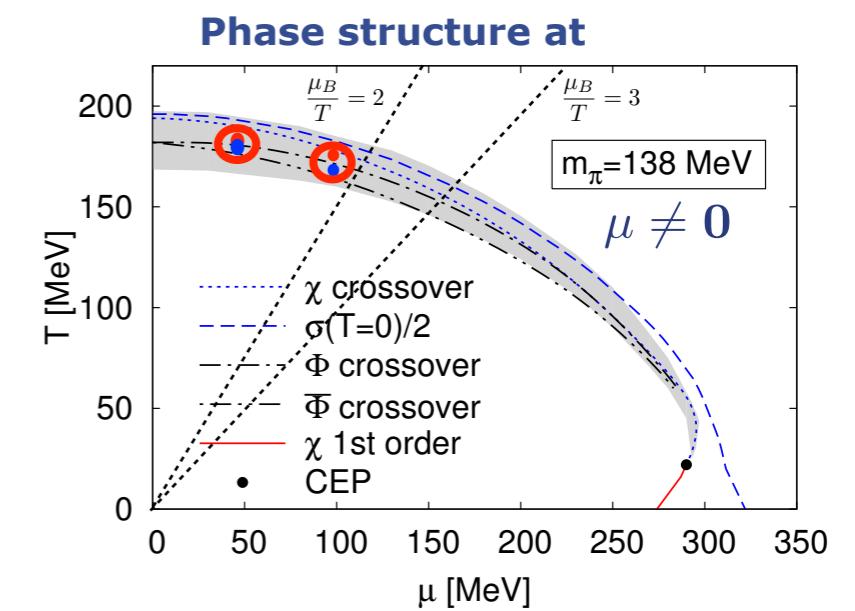
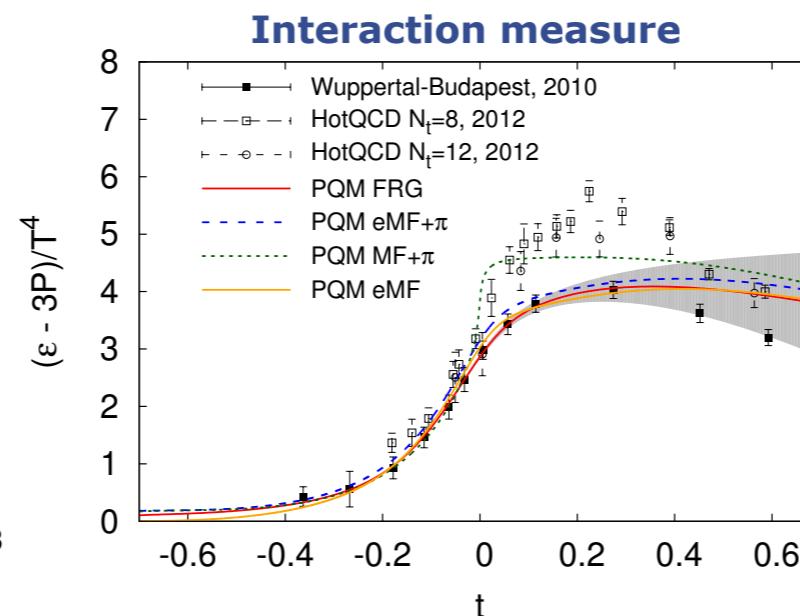
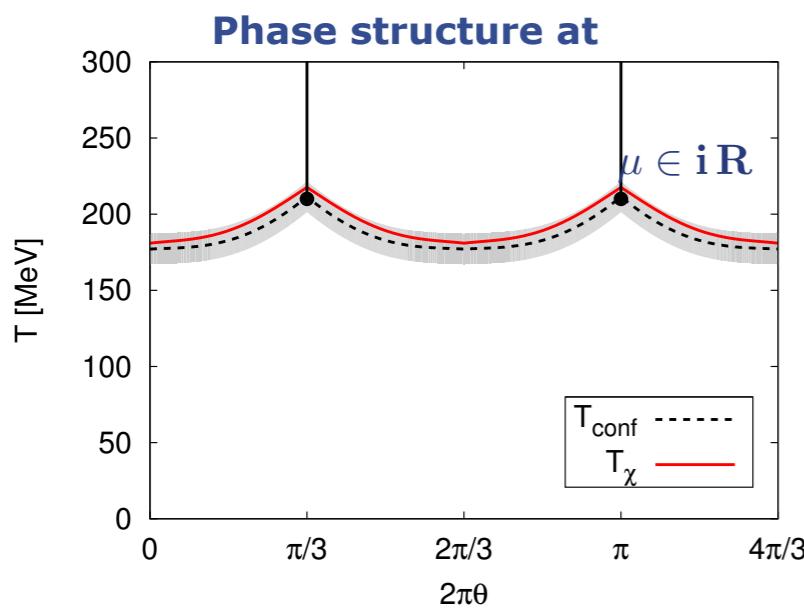
▪ Chiral Symmetry Breaking and Confinement



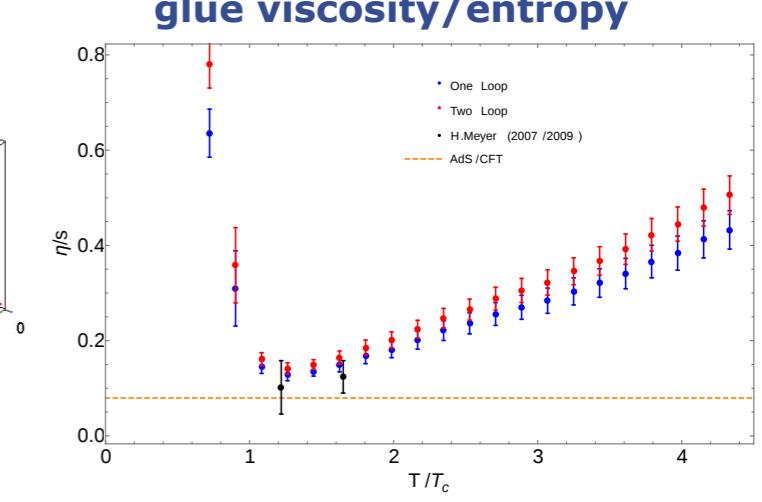
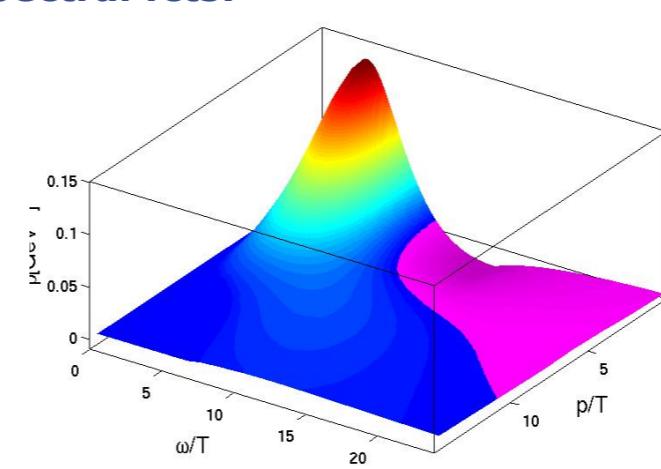
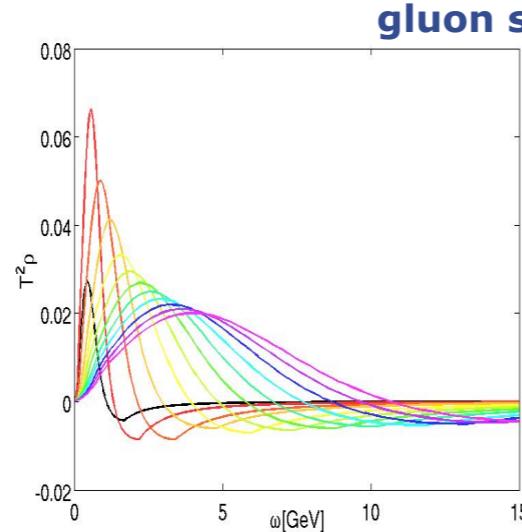
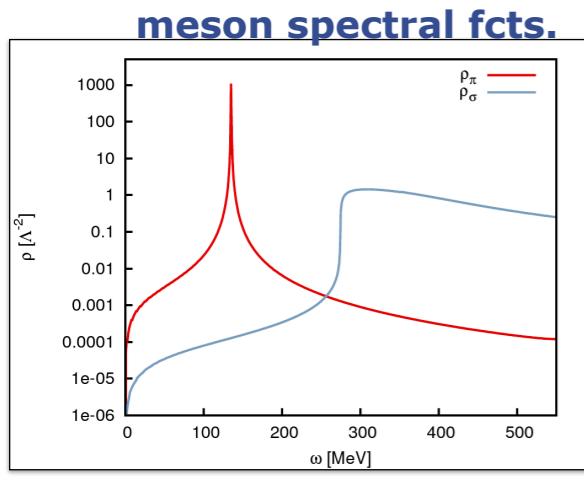
Summary & Outlook

■ Phase structure and Transport

see talks of M. Mitter
B.-J. Schaefer



see talks of L. Fister
N. Strodthoff



Summary & Outlook

- **Chiral Symmetry Breaking and Confinement**
- **Phase Structure and Transport**
- **Towards quantitative precision**
- **Baryons, high density regime, dynamics**
- **Hadronic properties**
 - **hadron spectrum & in medium modifications**
 - **low energy constants**

final word of caution

10 biggest lies in mountaineering =

- o) We are almost there.
- o) We won't need crampons.
- o) I can already see the summit.
- o) Believe me, I know the way.
- o) The hardest part is already behind us.
- ⋮
- ⋮
- ⋮
- ⋮

Thanks for the nice workshop ☺

C.G.

- Original application: sign-problem

- General application: Evaluate systematic error



final word of caution

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:
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Thanx for the nice workshop ☺

C.G.

- Original application: sign-problem

- General application: Evaluate systematic error



Thanx to Holger, Jan, Manuel, Dietrich

...of course we have seen the summit!

...several of them.....

C. Gatteringer, DELTA13-meeting Heidelberg