On IR Fixed Points of Quantum Gravity.

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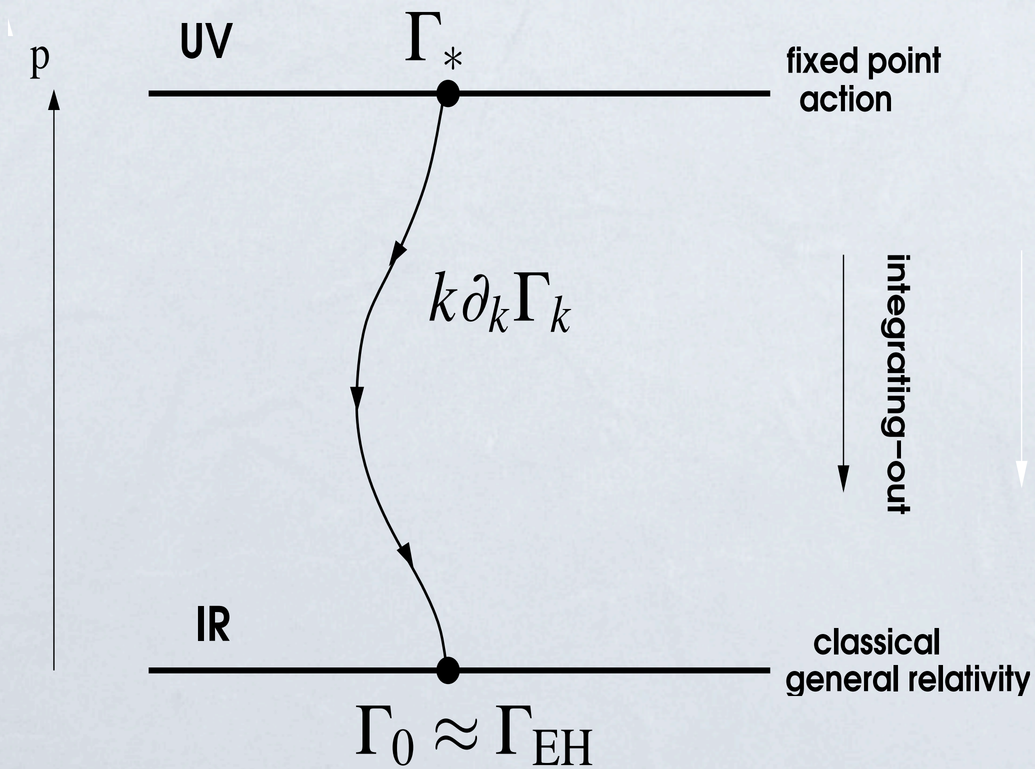
Renormalisation Group for Gravity.

RG for Gravity.



Wilson’s Renormalisation Group.

Explore theories with different energy scales.

$$k\partial_k\Gamma_k = \frac{1}{2}Tr\left(\Gamma^{(2)} + R_k\right)^{-1} k\partial_k R_k,$$

Example: Einstein-Hilbert Action.

$$\Gamma_k = \int d^4x\sqrt{g}\left[\frac{1}{16\pi G_k}(-R + 2\Lambda_k)\right]$$

$$g = G(k)k^2 \qquad \lambda = \Lambda(k)k^{-2}$$

Anomalous Dimension.

$$\eta = k\partial_k \log G_k$$

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Schematic Trajectory.

Gravity Weakening.

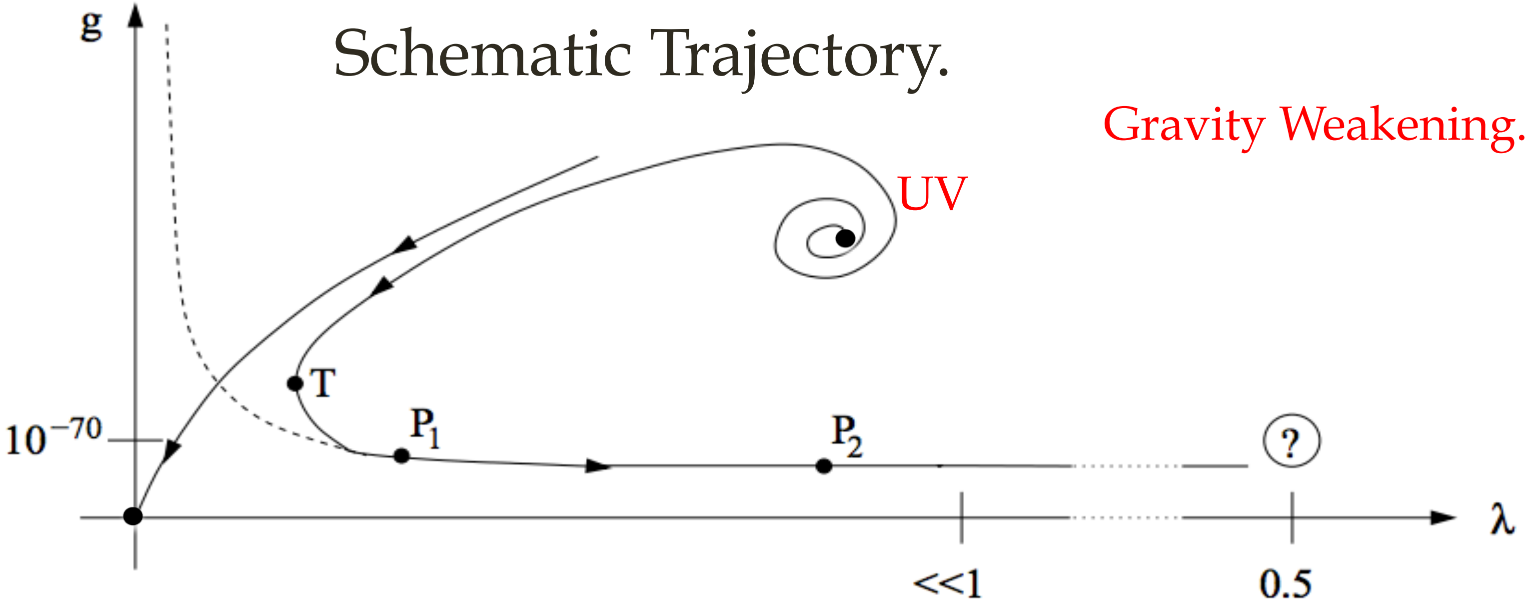

Figure 4: The Type IIIa trajectory realized in Nature and the separatrix. The dashed line is a trajectory of the canonical RG flow.

M. Reuter and H. Weyer, JCAP 0412, 001 (2004)

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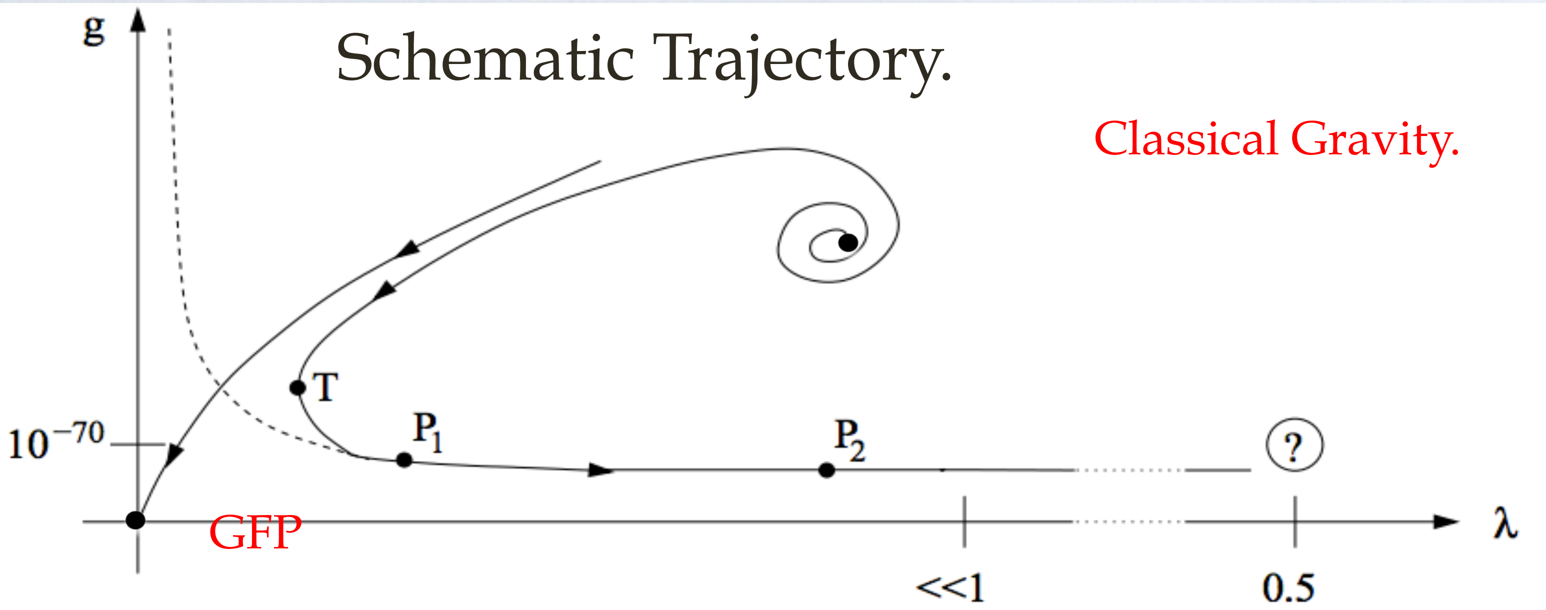

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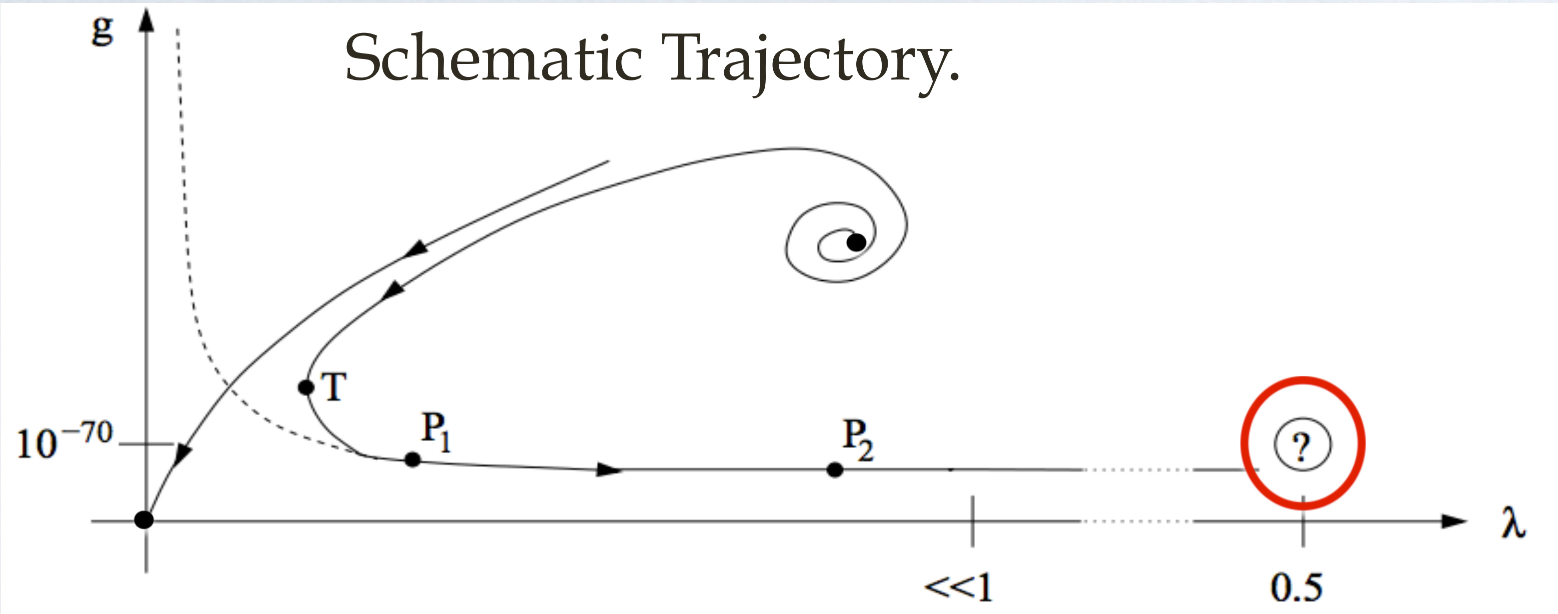

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IR Fixed Point Hypothesis.

$$G_k = \frac{g_*}{k^2}$$

Strong Gravity in the IR.

$$\Lambda_k = \lambda_* k^2$$

High Redshift Type Ia Supernovae.

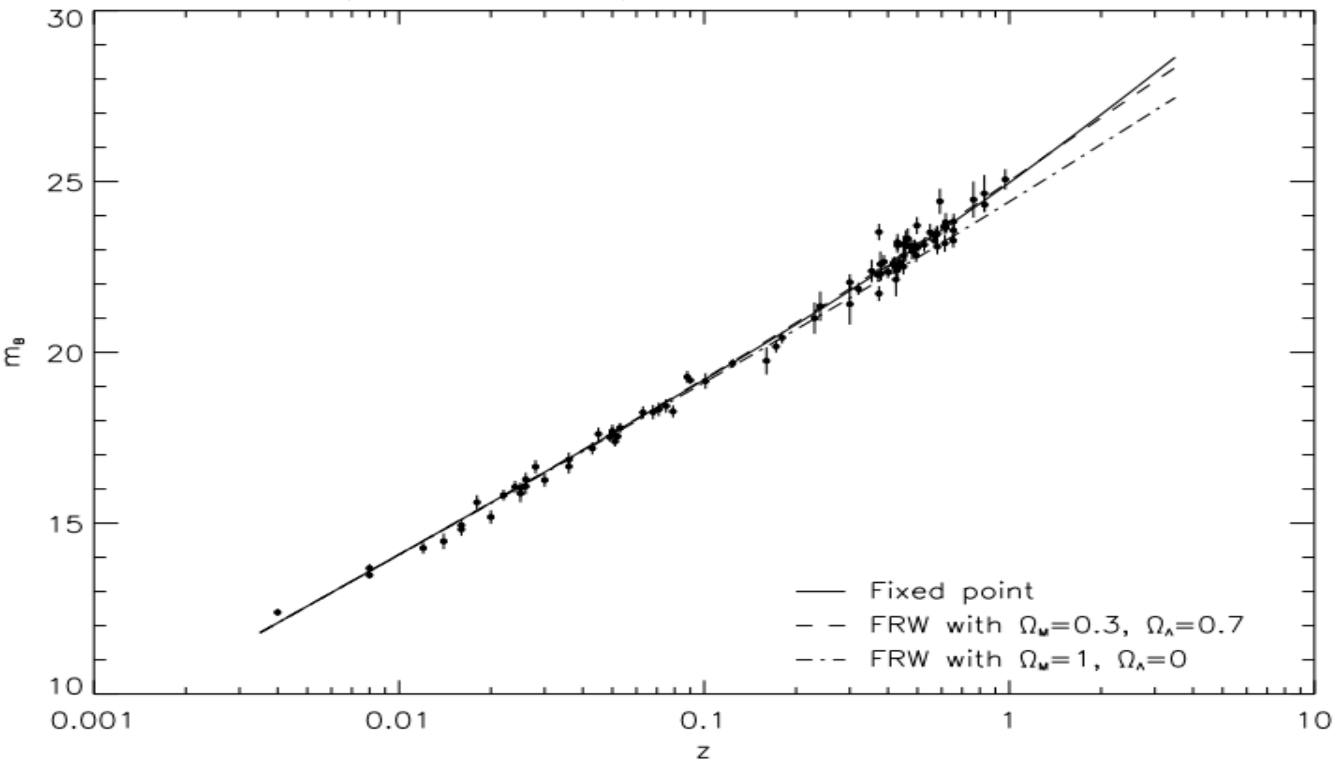


FIG. 1: The measured apparent magnitudes of the supernovae as a function of their redshift. The continuous line represents the prediction of the IRFP cosmology, the dashed one is the best-fit FRW model, and the dot-dashed line is a flat FRW model with zero cosmological constant.

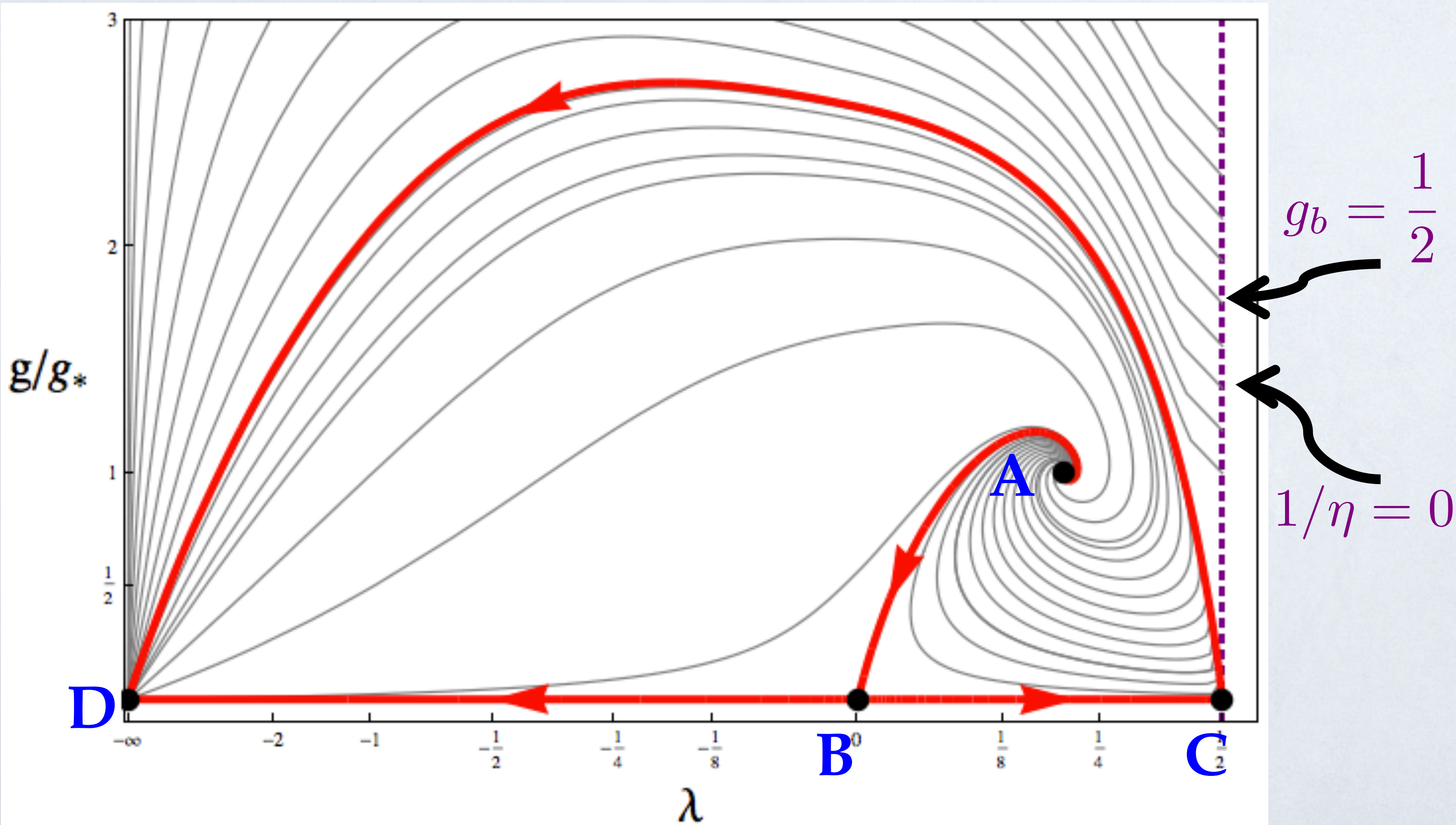
- ***m_B*: *Apparent Magnitude.***
- ***z*: *Red Shift .***

E. Bentivegna, A. Bonanno, M. Reuter, JCAP 0401 (2004) 001

Approximations.

Approximation (Leading order in g).

$$\eta = -ga_1$$



D. Litim, A. Satz, arXiv:1205.4218v1 [hep-th]

Close-up on C.

Approximation - Nullclines.

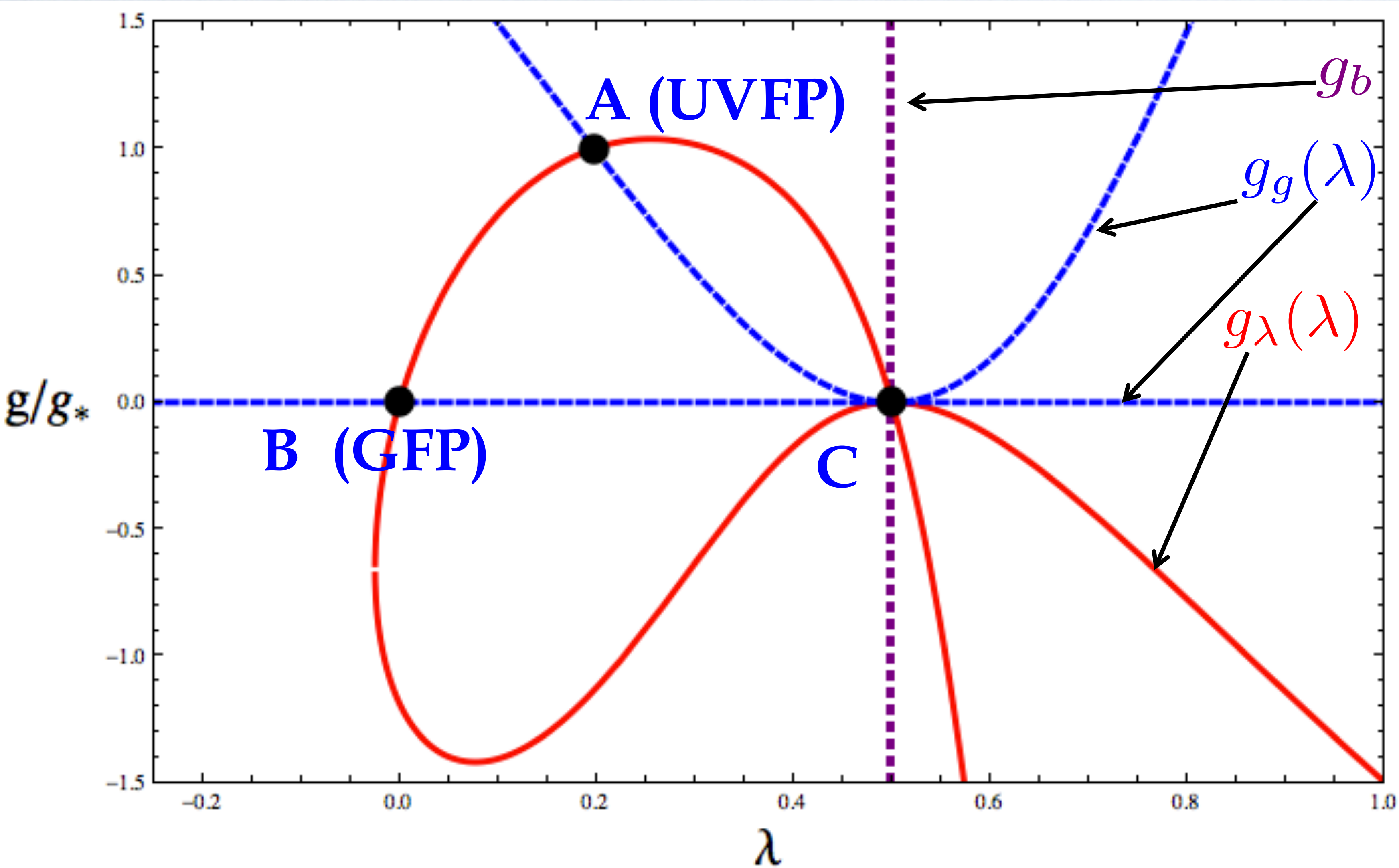
Definition:

NULLCLINES are integral curves where the beta functions vanish.

$$\beta_g(\lambda, g_g(\lambda)) = 0 \qquad \beta_\lambda(\lambda, g_\lambda(\lambda)) = 0$$

- The intersection of two nullclines is a Fixed Point.

Nullclines – Approximation.



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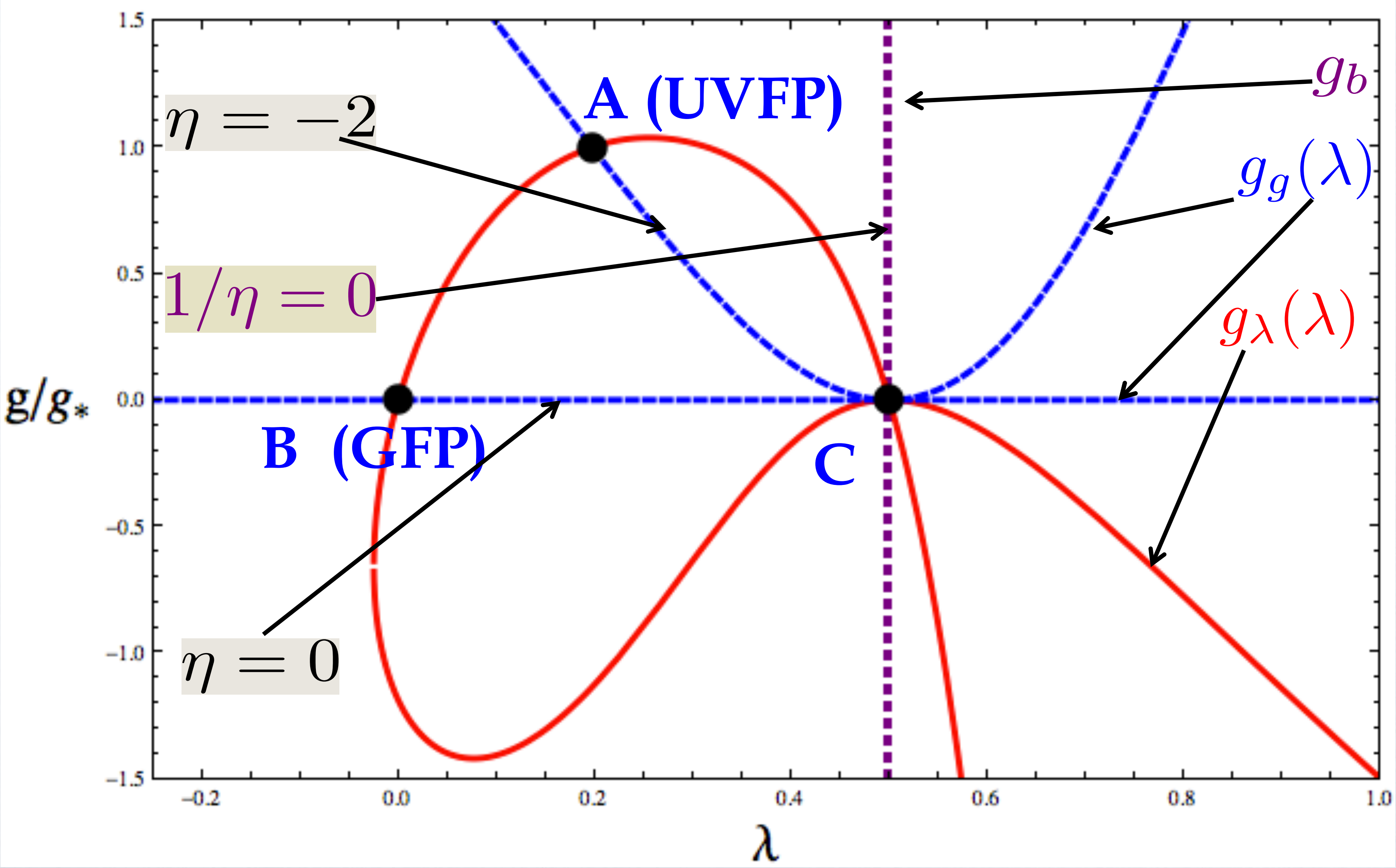

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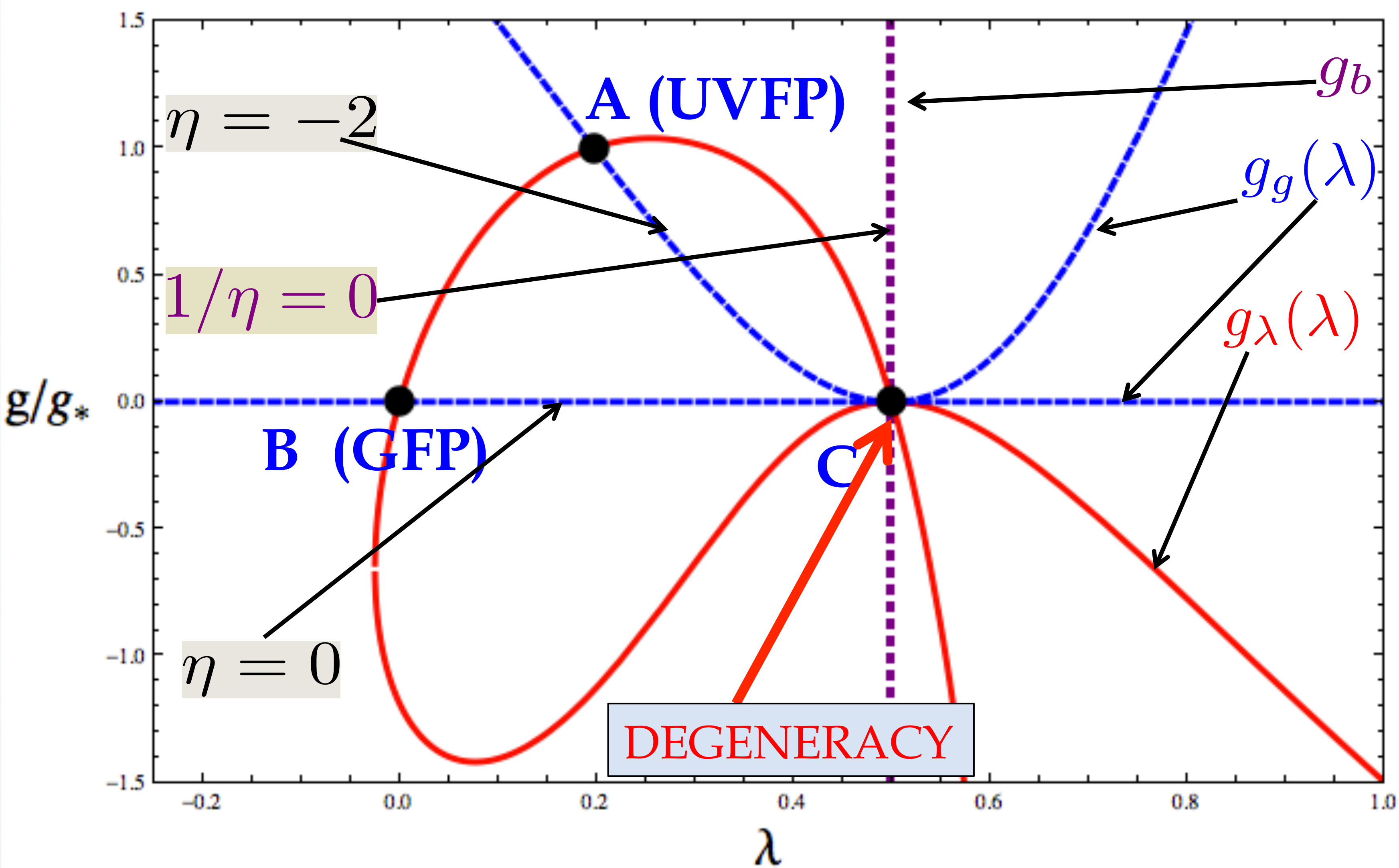

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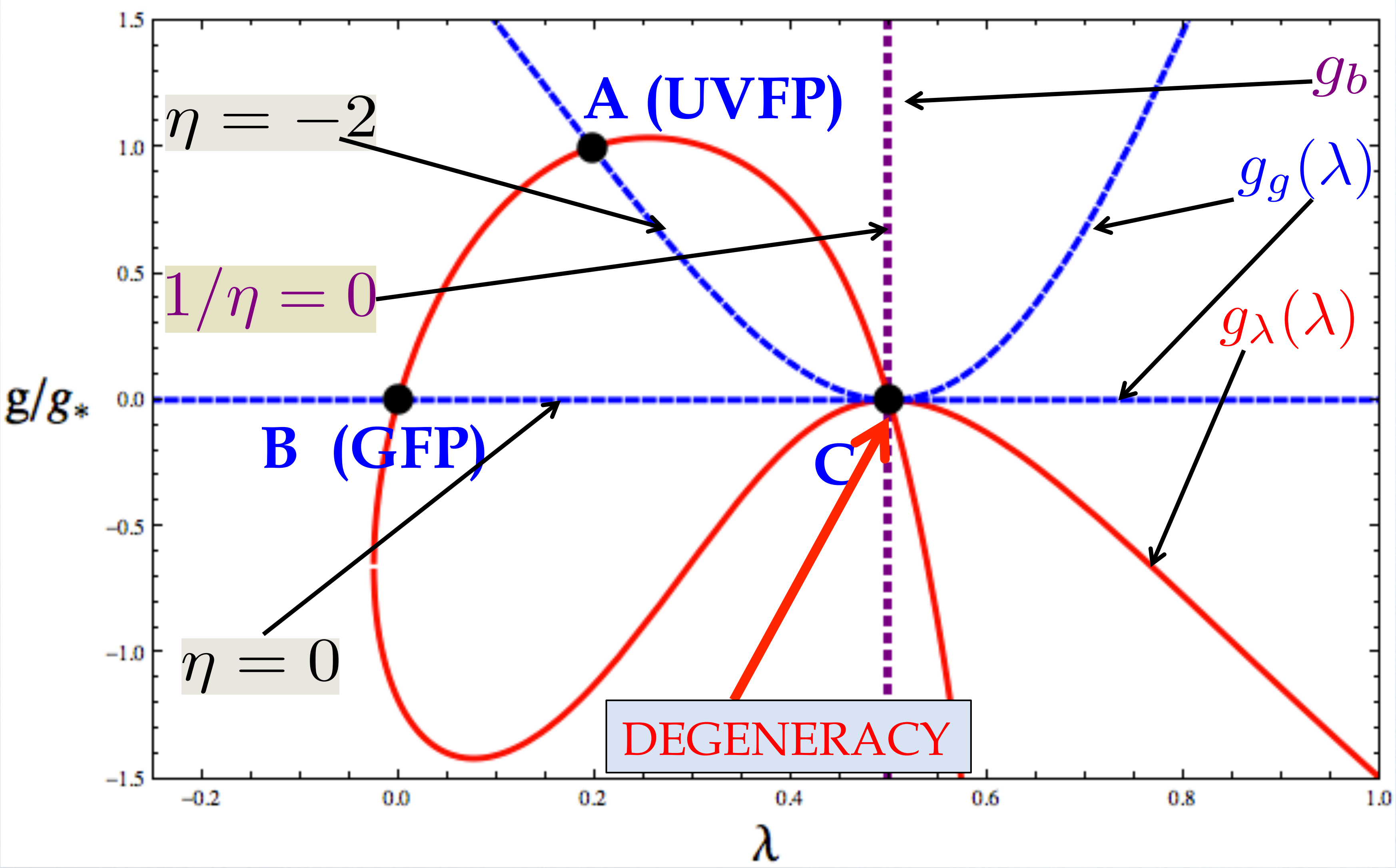
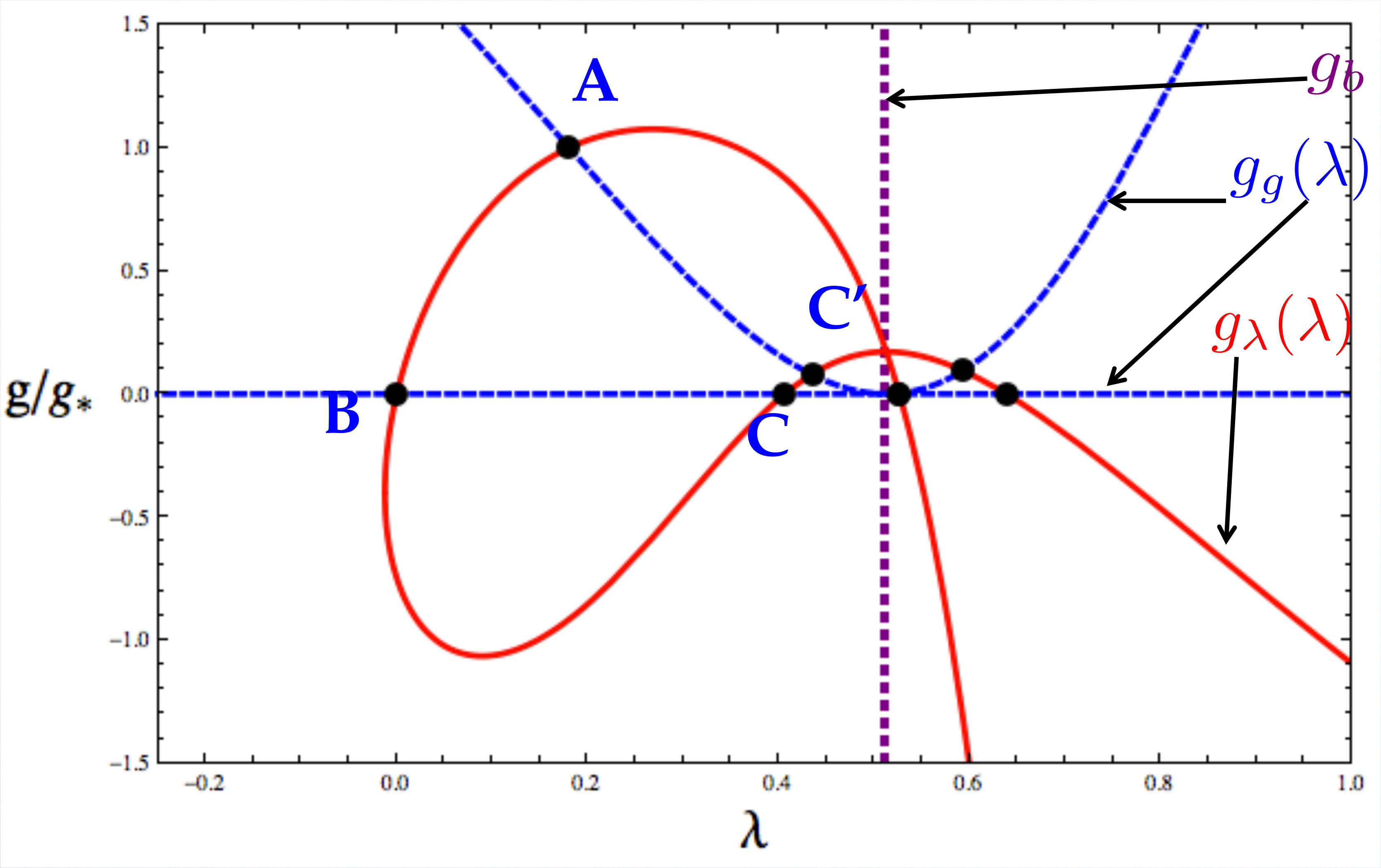


Introduce a small parameter δ to lift the degeneracy.

Lifting the degeneracy.

Lifting the Degeneracy-Approximation.



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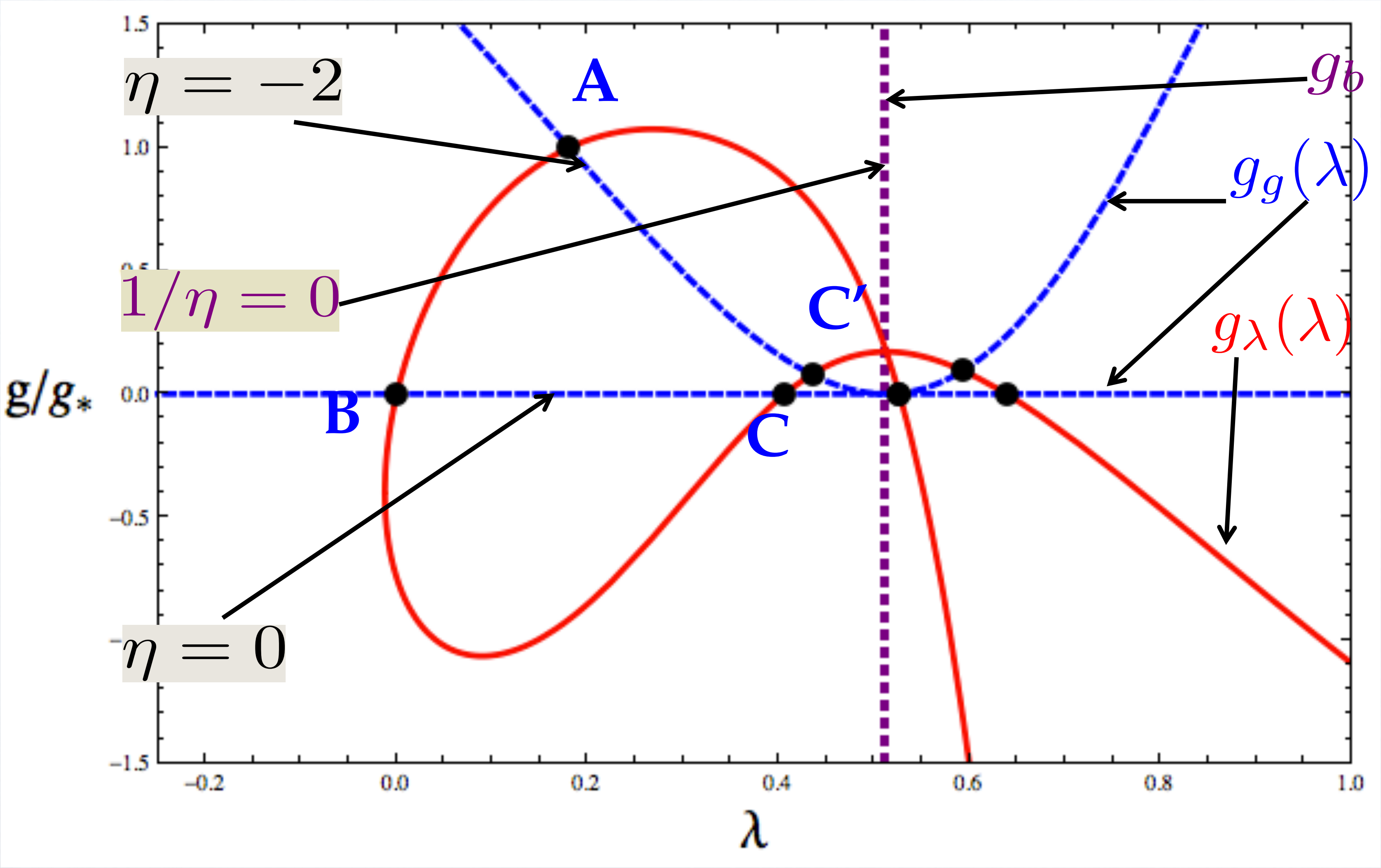

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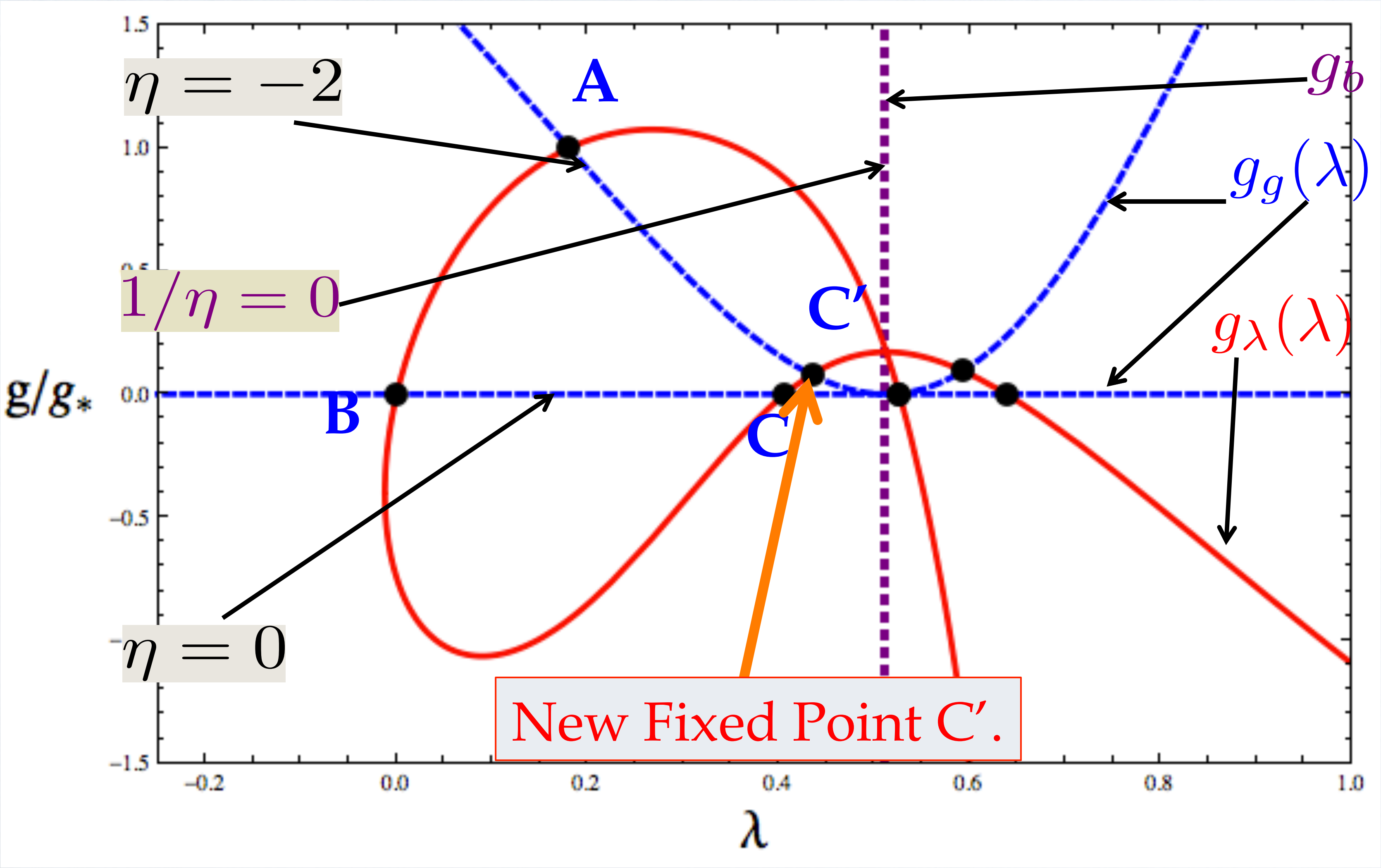
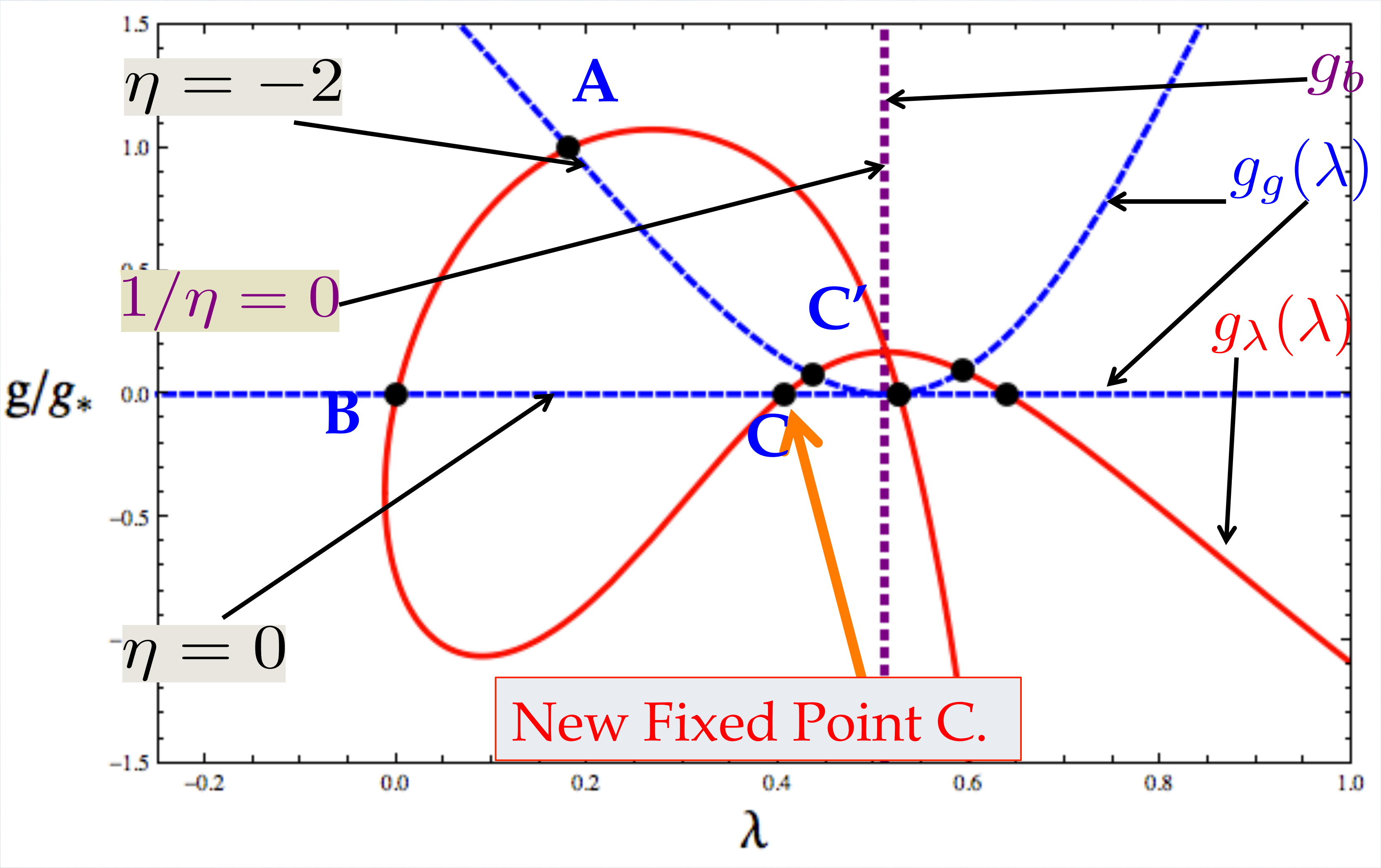


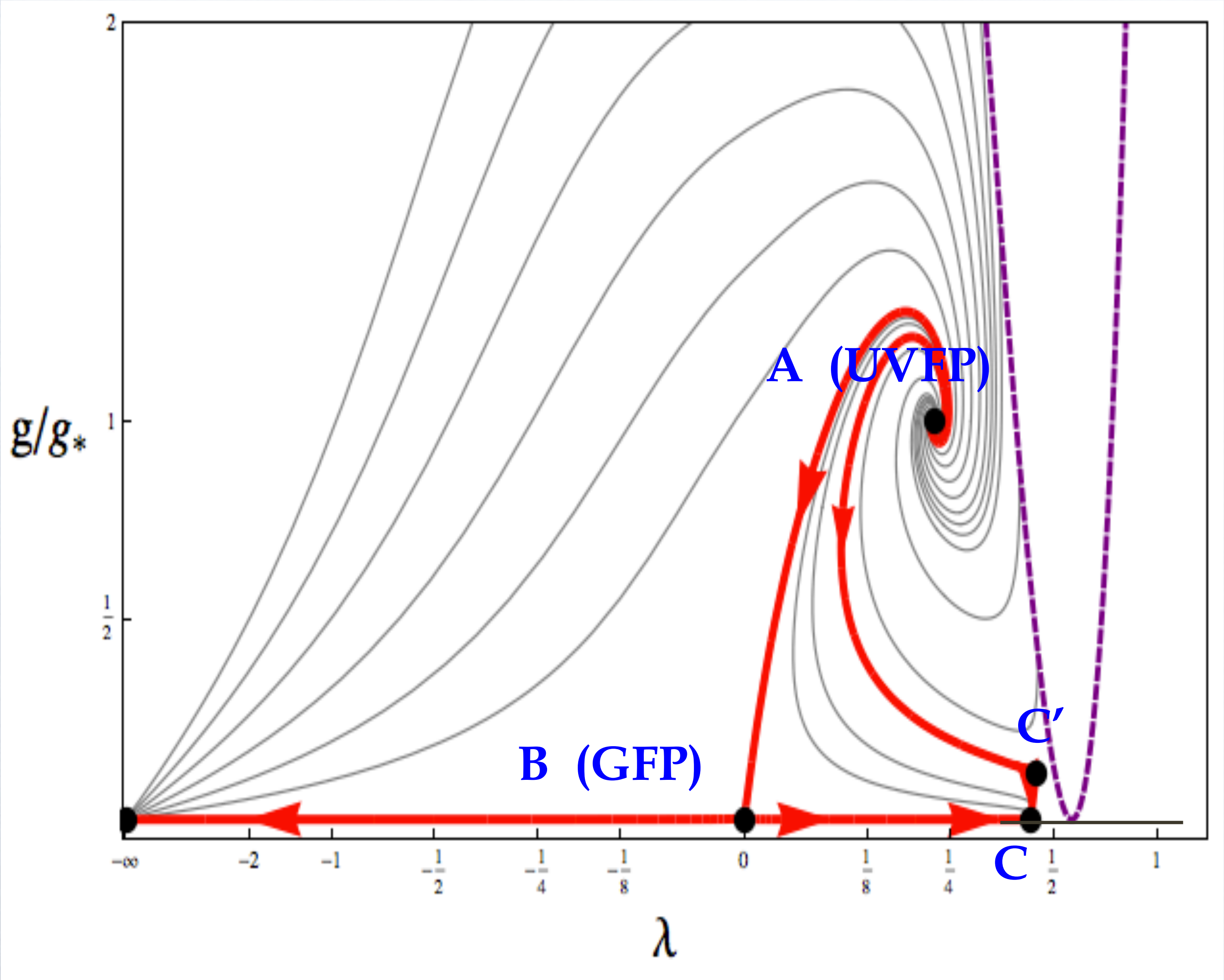
$C' : \quad g_* \neq 0, \quad \lambda_* \neq 0, \quad \eta = -2 \longrightarrow$ Candidate for IRFP.

Lifting the Degeneracy-Approximation.



$C: \quad g_* = 0, \quad \lambda_* \neq 0, \quad \eta = 0 \longrightarrow$ Classical Gravity.

Approximation 2: Hartree-Fock Resummation.



Stability Analysis of C and C′.

- **Critical Exponents:** - Eigenvalues of the Stability Matrix.

1. IRFP C′:

$$\theta^1_{C'} = 4 - \frac{16\sqrt{2}}{3}\delta^{1/2} + \frac{80}{3}\delta - \frac{160\sqrt{2}}{3}\delta^{3/2}$$

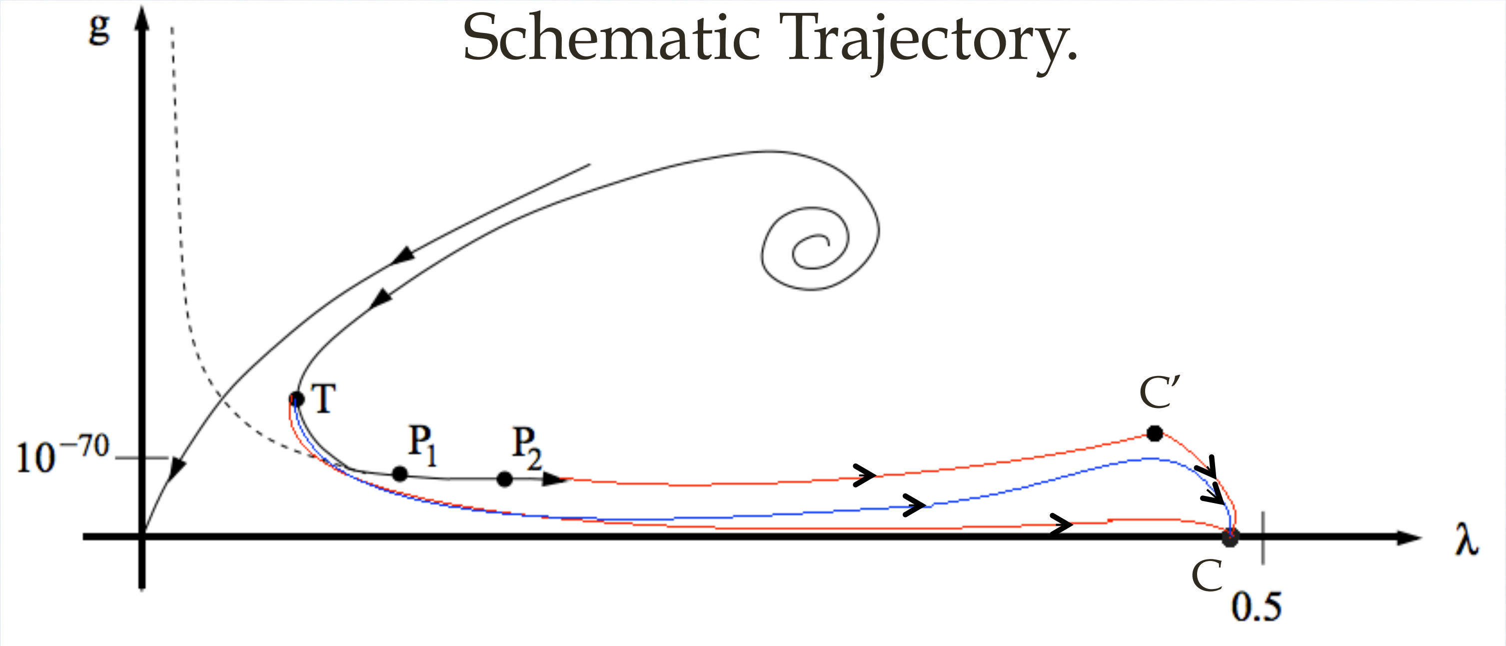
$$\theta^2_{C'} = -2\sqrt{2}\delta^{-1/2} - \frac{8}{3} + \frac{40\sqrt{2}}{9}\delta^{1/2} - \frac{256}{9}\delta + \frac{6056\sqrt{2}}{81}\delta^{3/2}$$

2. IRFP C:

$$\theta^1_{C} = -\frac{4}{\sqrt{3}}\delta^{-1/2} - \frac{8}{3} - \frac{14}{3\sqrt{3}}\delta^{1/2} + 8\delta$$

$$\theta^2_{C} = -2$$

New Schematic Flow.



- One special trajectory (separatrix (red)) will hit the FP C′ without feeling the effects of C. And another similar but connecting C.
- Trajectories between the separatrices (blue) will be dragged abruptly towards C′, spending some time in its vicinity (strong gravity). After that, they will be pushed smoothly to C where it will finish.

Conclusions.

- ⋆ Deep Infrared regime of the flow contain a degenerated FP.
- ⋆ We have lifted the degeneracy and found new FP.
  - ⋆ C′: $g_* \neq 0$ $G_k = \frac{g_*}{k^2}$
  - ⋆ C: $g_* = 0$ $G_k \to constant$

1. Use the result in Cosmology (Transition to FP epoch, Accelerated Expansion without Dark Matter?).
2. Find a dynamical way to lift the degeneracy.

Thank you!

Plaudite, cives.

Poles in the flow.

* The graviton propagator displays a pole around:

$$\sim \frac{1}{1-2\lambda} \qquad \textit{or} \qquad \sim \frac{1}{1-2\alpha\lambda}$$

* Then, fixed point solutions must obey

$$\lambda_* \leq \lambda_{bound} = min\left\{\frac{1}{2}, \frac{1}{2\alpha}\right\}$$

Gauge dependence.

* For $0 \leq \alpha \leq 1$ we computed the mean value and the standard deviation for FP and Critical Exponents:

	λ_A	g_A	θ_A^{Re}	θ_A^{Im}	λ_C	g_C	θ_C^1	θ_C^2	$\lambda_{C'}$	$g_{C'}$	$\theta_{C'}^1$	$\theta_{C'}^2$
$\langle X_{LO} \rangle$	0.1972	0.9124	1.3943	2.5287	0.4316	0	-27.0797	-2	0.4513	0.0424	3.5088	-38.1576
$\langle \Delta X_{LO} \rangle$	0.0040	0.0053	0.1264	0.0355	0.0010	0	0.3064	0	0.001	0.0003	0.0338	0.2687
$\langle X_{HF} \rangle$	0.1651	0.8362	1.909	2.5061	0.4635	0	-31.7670	-2	0.4692	0.0126	3.5554	-38.1802
$\langle \Delta X_{HF} \rangle$	0.0018	0.0547	0.0926	0.0752	0.0003	0	0.2688	0	0.0003	0.0001	0.0319	0.3591

* The relative standard deviation ranges:

  * **LO (d = 1/50)**: 0.22 % for λ'_C to 9.06% for $Re(\theta_A)$
  * **HF (d = 1/300)**: 0.06% for λ'_C to 6.54% for g_A