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Vertex Functions in Quantum Gravity

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N.C., Knorr, Pawlowski, Rodigast (1403.1232)

N.C., Knorr, Pawlowski, Rodigast (on arxiv next week...or so)

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Outline

- Introduction
- Vertex expansions for flows in quantum gravity: General setup
- Global phase structure: ultraviolet and infrared fixed points
- The three point function
- Outlook

Quantum gravity and asymptotic safety in a nutshell

- classical gravity is described by general relativity on all scales observed so far

$$10^{-4}\text{m} \longrightarrow 10^{26}\text{m}$$

- in $d = 4$: $[G_\text{N}] = \text{energy}^{-2}$ → perturbative quantization fails

Newtons Constant

- Non-perturbative quantization: asymptotic safety: UV fixed point

$$\lim_{k\to\infty} g_i(k) = g_{i,*}$$

dimensionless coupling constant

→ predictive theory, finite on all scales

finite observables!

Einstein-Hilbert Truncation

- basic approximation scheme: Einstein Hilbert (EH) truncation

$$\Gamma_k = \frac{Z_k}{16\pi G_\mathrm{N}} \int \omega_d(g)(-R(g) + 2\Lambda_k)$$

scale dependent, running couplings:

$$G_\mathrm{N} \longrightarrow G_k = \frac{G_\mathrm{N}}{Z_k} \qquad \Lambda \longrightarrow \Lambda_k$$

→ parametrization of the effective action

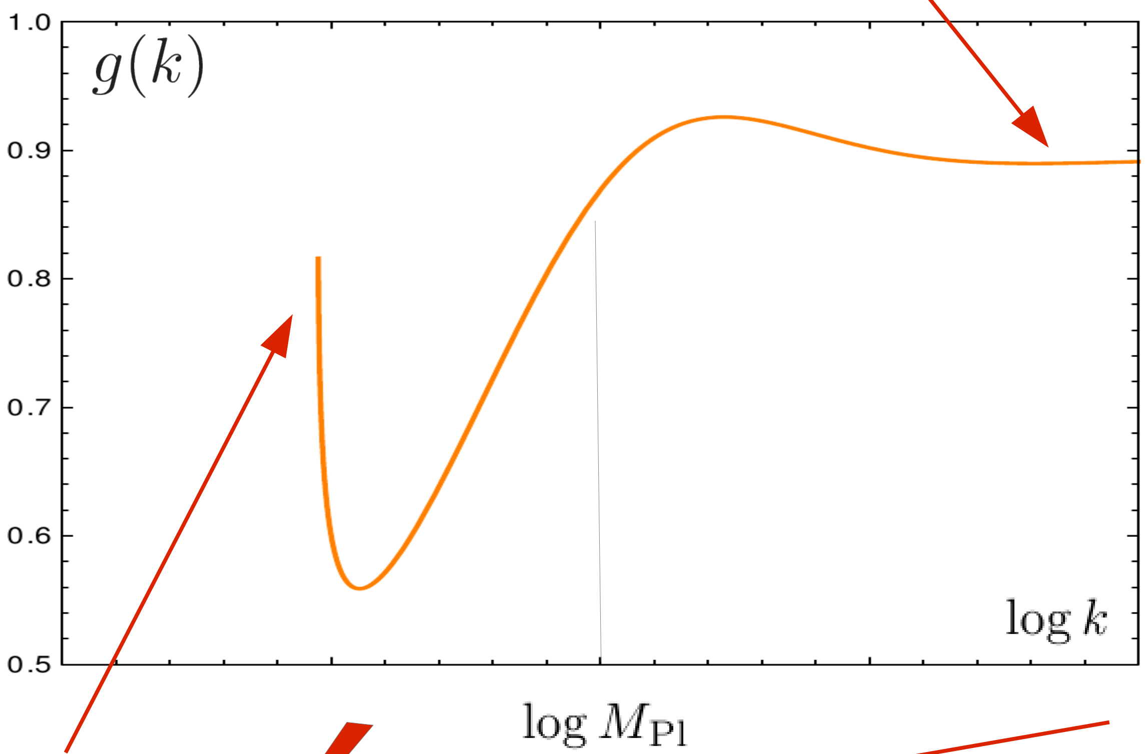
→ calculate $\partial_t \Gamma$

→ non-perturbative beta-functions for the dimensionless couplings

$$g_k = G_k k^2 \quad \text{and} \quad \lambda_k = \frac{\Lambda_k}{k^2}$$

Asymptotic Safety in basic Einstein-Hilbert

- Einstein Hilbert approximation leads to UV fixed point ✓



confirm this!

change this!

- singularity in the IR

The background field and all that

- usual background field approach:

$$k\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k[\bar{g},h] = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2}{\delta h^2}\Gamma_k + \mathcal{R}\right)^{-1}\frac{\mathrm{d}}{\mathrm{d}k}\mathcal{R}$$

evaluated at $h = 0$ with $g = \bar{g} + h$ ← note: linear split is not the most general form!!!

→ flow is not closed since $\frac{\delta^2}{\delta h^2}\Gamma_k \neq \frac{\delta^2}{\delta \bar{g}^2}\Gamma_k$

→ background approximation: using equality

- this can change the sign of the 1-loop YM-beta function!

- couplings of fluctuation fields $\neq$ couplings of the background field

related to fluctuation correlators (→ couplings of fluctuation fields)

related to background correlators (→ couplings of the background field)

Hierarchy of flow equations

→ calculate the flow of the propagator itself!

$$\partial_t \left. \frac{\delta^2 \Gamma}{\delta h^2}[\bar{g}; h] \right|_{h=0} = \text{Flow}^{(2)}[\Gamma^{(2)}, \Gamma^{(3)}, \Gamma^{(4)}] \quad , \quad \Gamma^h = \frac{\delta^n \Gamma}{\delta h^n}$$

→ no reason to stop at the two point function!

→ flow of higher order vertex functions:

- $\partial_t \Gamma^{(3)}$
- $\partial_t \Gamma^{(4)}$
- ....

$\partial_t \Gamma^{(n)}$ depends on $\Gamma^{(n)}, \Gamma^{(n+1)}, \Gamma^{(n+2)}$

→ generates the full hierarchy of flow equations for the vertex functions $\Gamma^{(n)}$

Structure of the vertex functions and approximations

- Structure of the vertices

$$\Gamma^{(n)} = \prod_{i=1}^{n} Z_k^{\frac{1}{2}}(p_i) G_k^{\frac{n}{2}-1} \mathcal{T}_k^{(n)} \left(p_i; \Lambda_k^{(n)} \right)$$

Fully momentum dependent !

momentum independent parts

- two point function:

$$M_k^2 = -2\Lambda_k^{(2)}$$

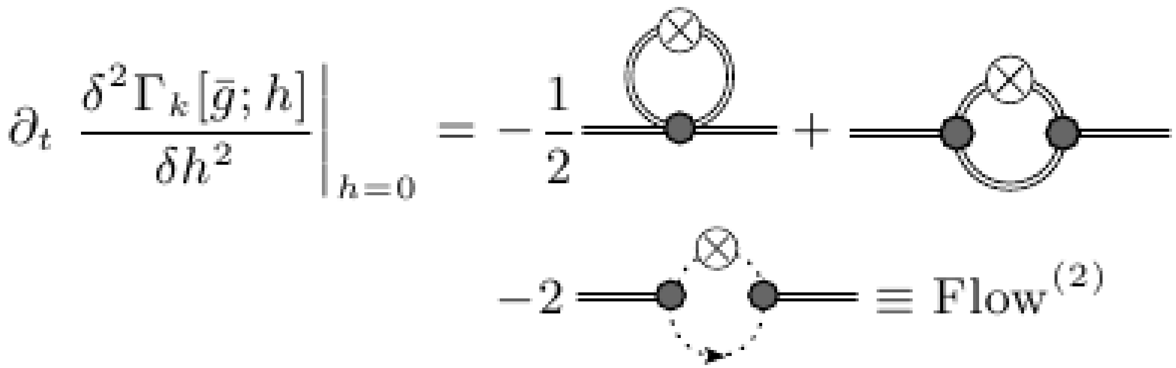
$$\Gamma^{(2)} = Z_k(p^2) \left(p^2 + M_k^2 \right) \Pi_{\mathrm{TT}}$$

- three point function:

$$\Gamma^{(3)} = \sqrt{G_k} \sqrt{Z_k(p_1)} \sqrt{Z_k(p_2)} \sqrt{Z_k(p_3)} \left(\mathcal{T}_1(p) + \mathcal{T}_2 \Lambda_k^{(3)} \right)$$

Flow of the propagator

- flow equation for the inverse propagator



→ LHS contains: $\left(\partial_t Z, \partial_t M^2\right)$

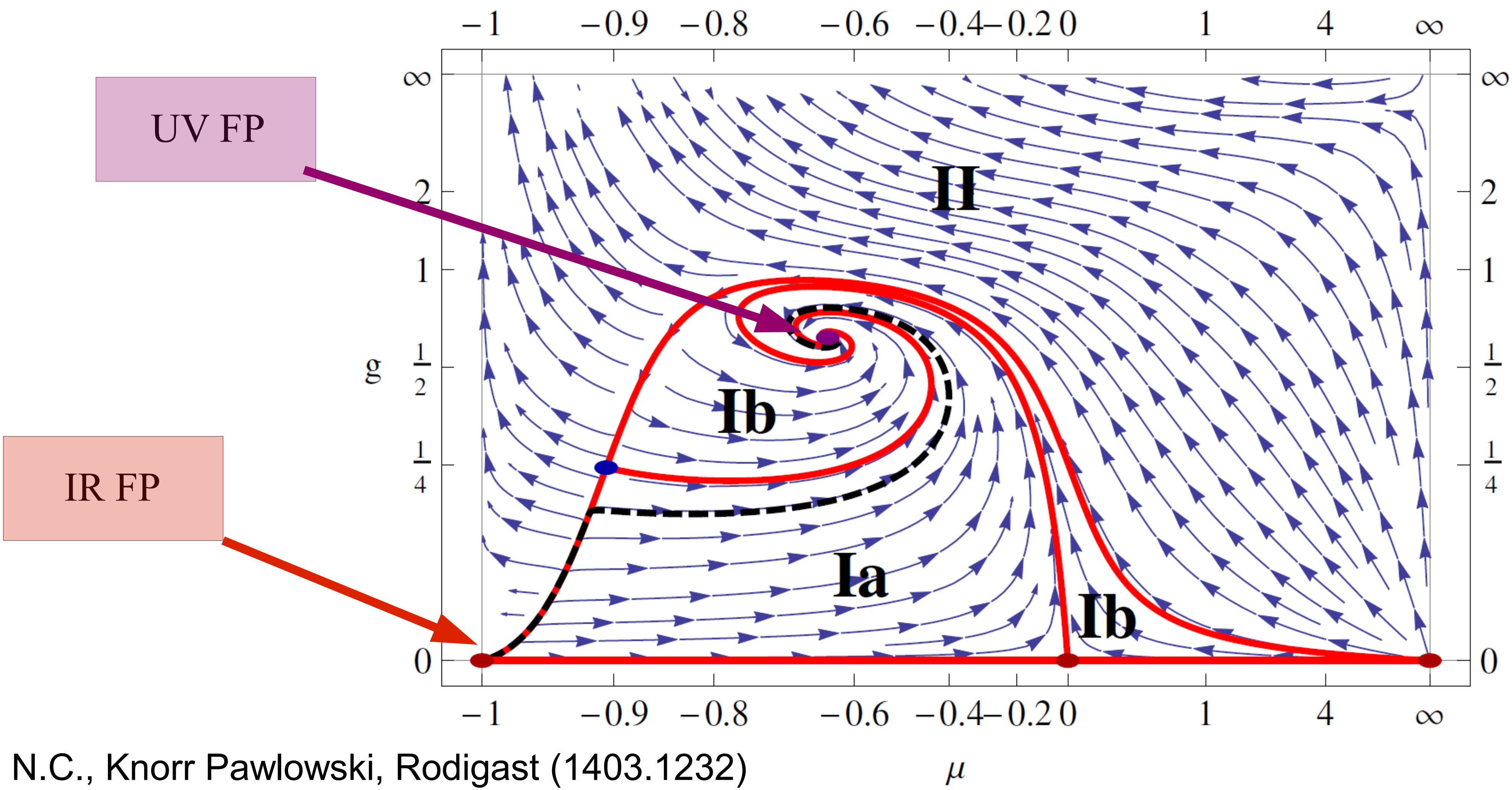
→ RHS contains $\left(Z, M^2, G, \Lambda^{(3)}, \Lambda^{(4)}\right)$

→ $\partial_t G$ from geometrical flow equations

→ $\Lambda^{(3)}, \Lambda^{(4)}$ constrained via scaling analysis

The Phase Diagram

- The phase diagram in the (g,μ) plane : $g = Gk^2 \quad \mu = M^2 k^{-2}$



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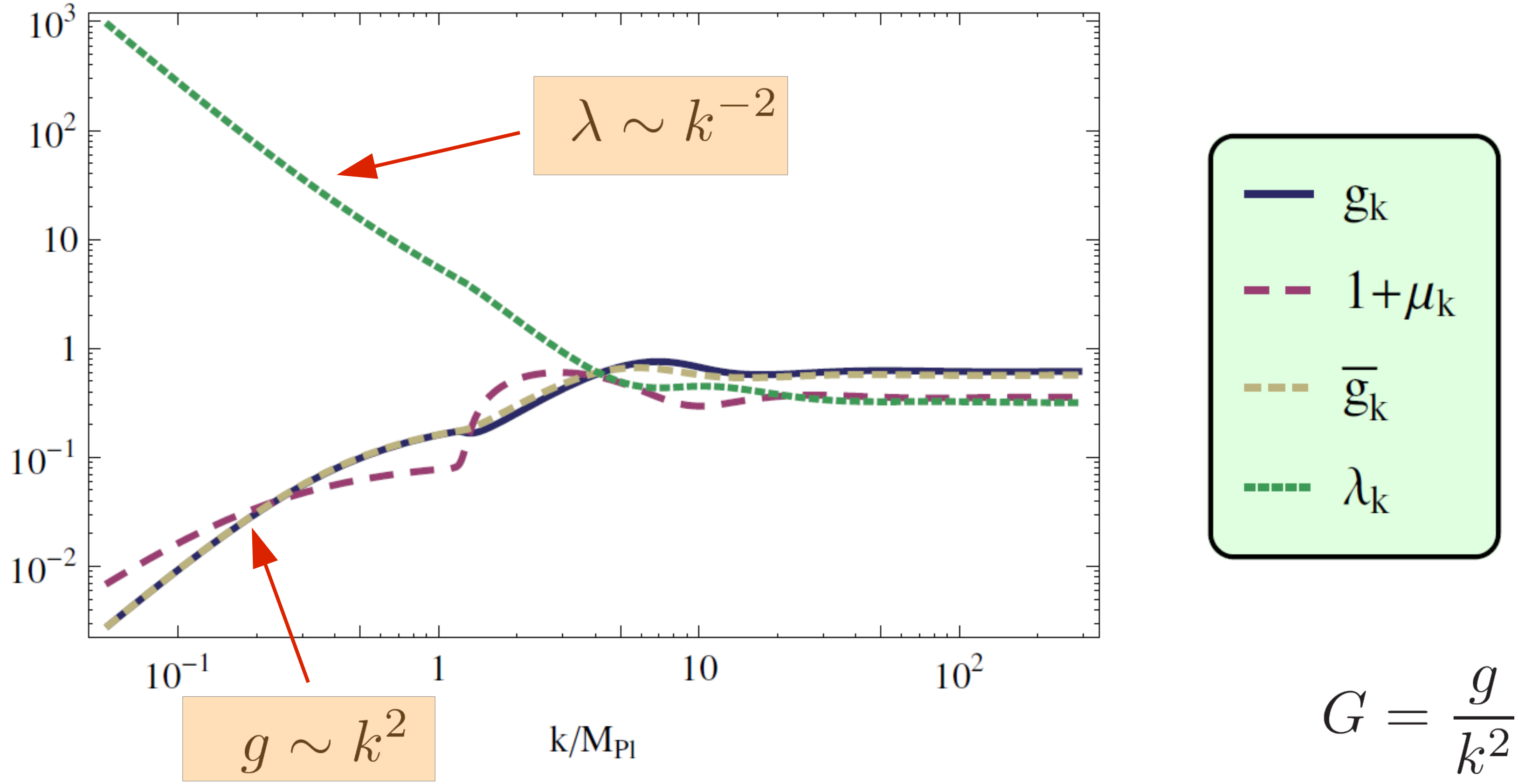
Running of couplings



$$G = \frac{g}{k^2}$$

$$\Lambda = k^2 \lambda$$

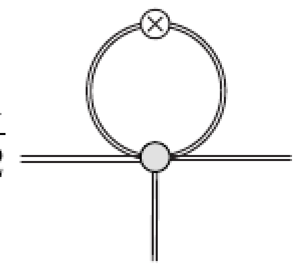
→ $G = \text{const.}$ $\Lambda = \text{const.}$

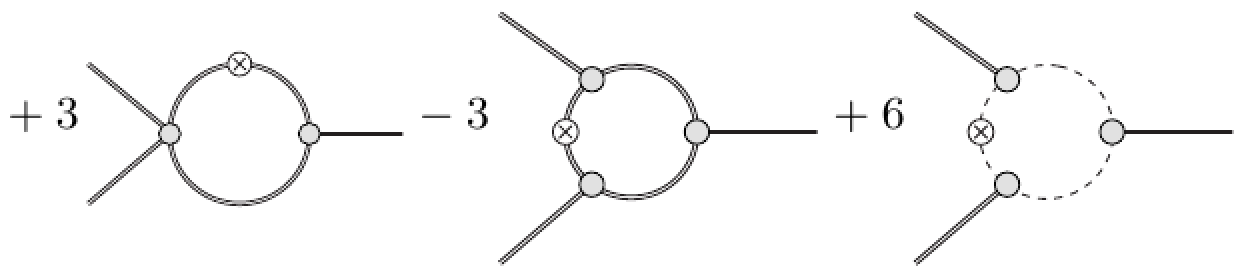
classical gravity in the IR!

The three point function

- coupling constants of the three point function: $\left(G, \Lambda^{(3)}\right)$

$$\partial_t \left. \frac{\delta^3 \Gamma[\bar{g}, \phi]}{\delta h^3} \right|_{\phi=0} (p_1^2, p_2^2, p_3^2) = -\frac{1}{2}$$

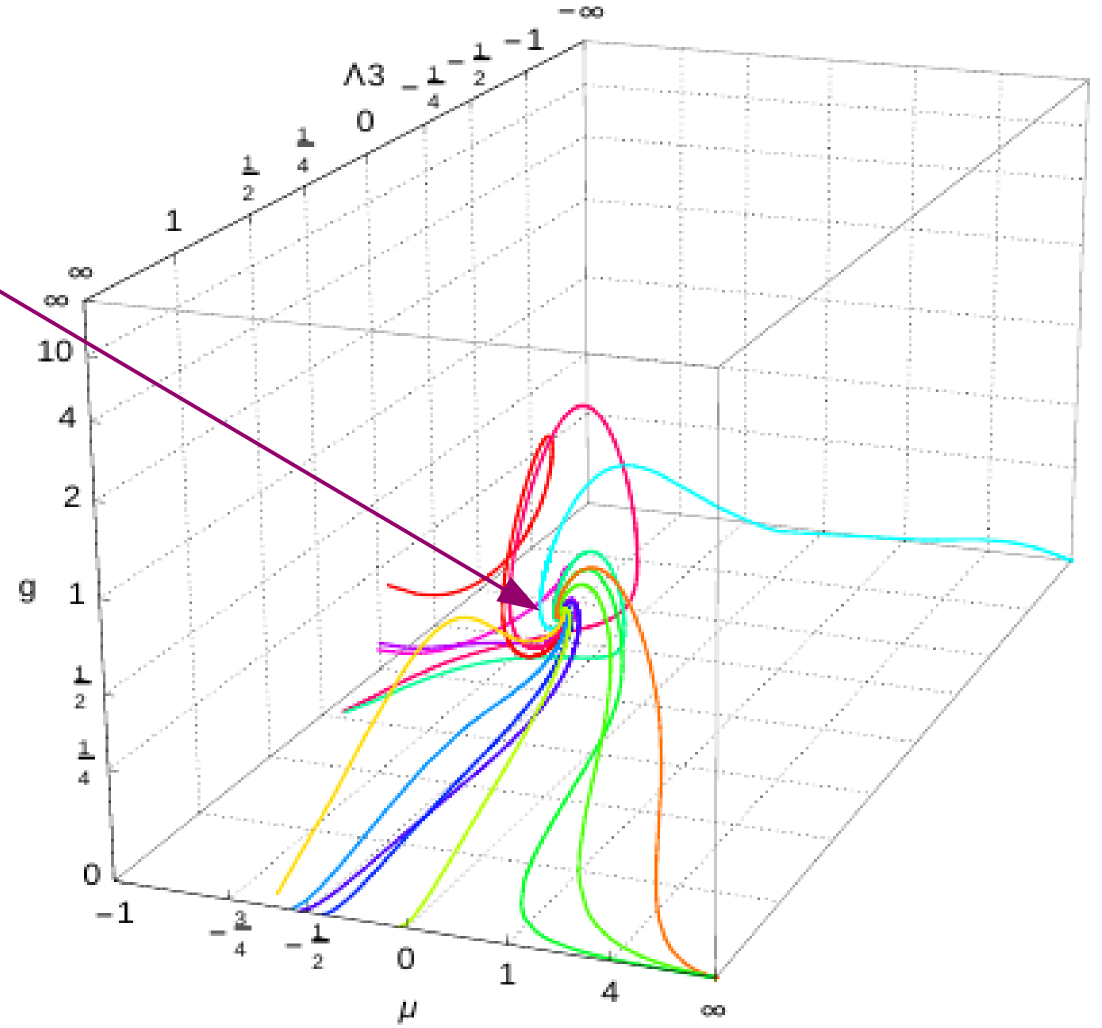




The phase diagram II

- phase diagram with couplings $(g, \lambda^{(3)}, \mu)$

UV FP



Summary

- dynamical vs background couplings
- full momentum dependence of the propagator
- UV - IR stability with classical IR fixed point
- flow of the three point function: UV fixed point

Outlook:

- higher derivative tensor structures
- fermions ← see poster: J.Meibohm, M.Reichert
- gluons
- curved backgrounds
- four point function

Thank you!