Suppression of Quantum Fluctuations by Classical Backgrounds

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- $$S = \int d^4x \left\{ \frac{1}{2}(\partial\pi)^2 - \frac{\nu}{2}(\partial\pi)^2 \Box\pi \right\}.$$
- The cubic Galileon theory describes the dynamics of the scalar mode that survives in the decoupling limit of the DGP model (Dvali, Gabadadze, Porrati).
- The action contains a higher-derivative term, cubic in the field $\pi(x)$, with a dimensionful coupling that sets the scale Λ at which the theory becomes strongly coupled.
- $\nu = 1/\Lambda^3$
- $\Lambda \sim (m^2 M_{\rm Pl})^{1/3}$ with $m \sim H \sim M_5^3/M_{\rm Pl}^2$

- The action is invariant under the Galilean transformation $\pi(x) \to \pi(x) + b_\mu x^\mu + c$, up to surface terms.
- In the Galileon theory additional terms can also be present, but the theory is ghost-free: EOM is second order (Nicolis, Rattazzi, Trincherini).
- Nonlinearities become important below the Vainshtein radius $r_V \sim (M/\Lambda^3 M_{\rm Pl})^{1/3}$.
- Does this contruction survive quantum corrections?
- The DBI action $S = \int d^4x \mu \sqrt{1 + \partial_\mu \pi \partial^\mu \pi}$ corresponds to the simplest term of a theory of embedded surfaces.
- The effective theory of embedded surfaces can be used in order to reproduce the Galileon theory at low energies $(\partial\pi)^2 << 1$ (de Rham, Tolley).

Outline

- Classical solutions and Vainshtein mechanism.
- Renormalization of the cubic Galileon theory, perturbative background.
- Heat-kernel method for nontrivial backgrounds.
- Suppression of quantum corrections by the Vainshtein mechanism.
- Classicalon.

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Classical solution, Vainshtein mechanism

- The classical EOM for cubic Galileon is

$$\Box\pi - \frac{1}{\Lambda^3}(\Box\pi) + \frac{1}{\Lambda^3}\partial_\mu\partial_\nu\pi\partial^\mu\partial^\nu\pi = T\delta^3(\vec{x})$$

- Spherically symmetric solution ($w = r^2$)

$$\pi'_{cl}(w) = \frac{1}{8\nu}\left(1 - \sqrt{1 + \frac{16\nu c}{w^{3/2}}}\right).$$

- $r_V \sim (c\nu)^{\frac{1}{3}}$
- For $r \ll r_v$ we have $\pi \sim \sqrt{c/\nu}\sqrt{r}$.
- For $r \gg r_v$ we have $\pi \sim c/r$.

Renormalization of the Galileon theory

- Perturbative background.
- $$S=\int d^4x\left\{\frac{1}{2}(\partial\pi)^2-\frac{\nu}{2}(\partial\pi)^2\Box\pi+\frac{\bar{\kappa}}{4}(\partial\pi)^2\left((\Box\pi)^2-(\partial_\mu\partial_\nu\pi)^2\right)+\ldots\right\}.$$
- If a momentum cutoff is used, of the order of the fundamental scale Λ of the theory, and the couplings are taken of order Λ, the one-loop effective action of the Galileon theory is, schematically, (Luty, Porrati, Nicolis, Rattazzi)

$$\Gamma_1\sim\int d^4x\sum_m\left[\Lambda^4+\Lambda^2\partial^2+\partial^4\log\left(\frac{\partial^2}{\Lambda^2}\right)\right]\left(\frac{\partial^2\pi}{\Lambda^3}\right)^m.$$

- Non-renormalization of the Galileon couplings (de Rham, Gabadadze, Heisenberg, Pirtskhalava, Hinterbichler, Trodden, Wesley).
- Explicit one-loop calculation using dimensional regularization (Paula Netto, Shapiro).

One-loop corrections to the cubic Galileon

- Tree-level action in Euclidean *d*-dimensional space

$$S = \int d^d x \left\{ \frac{1}{2}(\partial\pi)^2 - \frac{\nu}{2}(\partial\pi)^2 \Box\pi \right\}.$$

- Field fluctuation $\delta\pi$ around the background π. The quadratic part is

$$S^{(2)} = \int d^d x \left\{ -\frac{1}{2}\delta\pi\Box\delta\pi + \frac{\nu}{2}\delta\pi\left[2(\Box\pi)\Box\delta\pi - 2(\partial^\mu\partial^\nu\pi)\partial_\mu\partial_\nu\delta\pi\right] \right\}.$$

- Define

$$K = -\Box \qquad \Sigma_1 = 2\nu(\Box\pi)\,\Box \qquad \Sigma_2 = -2\nu(\partial_\mu\partial_\nu\pi)\,\partial^\mu\partial^\nu$$

- One-loop contribution to the effective action

$$\Gamma_1 = \frac{1}{2}\mathrm{tr}\,\log\left(K + \Sigma_1 + \Sigma_2\right) = \frac{1}{2}\mathrm{tr}\,\log\left(1 + \Sigma_1 K^{-1} + \Sigma_2 K^{-1}\right) + \mathcal{N}.$$

- Expanding the logarithm up to $\mathcal{O}\left(\nu^2\right)$ we obtain

$$\mathrm{tr}\left(\Sigma_1 K^{-1}\Sigma_1 K^{-1}\right) = 4\nu^2(2\pi)^d \int d^d k\, k^4 \tilde{\pi}(k)\tilde{\pi}(-k) \int \frac{d^d p}{(2\pi)^d}$$

$$\mathrm{tr}\left(\Sigma_1 K^{-1}\Sigma_2 K^{-1}\right) = -4\nu^2(2\pi)^d \int d^d k\, k^4 \tilde{\pi}(k)\tilde{\pi}(-k)\frac{1}{d} \int \frac{d^d p}{(2\pi)^d}$$

$$\mathrm{tr}\left(\Sigma_2 K^{-1}\Sigma_2 K^{-1}\right) = 4\nu^2(2\pi)^d \int d^d k\, \tilde{\pi}(k)\tilde{\pi}(-k)\left\{\frac{3}{d(d+2)}k^4 \int \frac{d^d p}{(2\pi)^d} + \frac{(d-8)(d-1)}{d(d+2)(d+4)}k^6 \int \frac{d^d p}{(2\pi)^d}\frac{1}{p^2} - \frac{(d-24)(d-2)(d-1)}{d(d+2)(d+4)(d+6)}k^8 \int \frac{d^d p}{(2\pi)^d}\frac{1}{p^4}\right\}.$$

- Putting everything together, we obtain in position space, the one-loop correction to the effective action

$$\Gamma_1^{(2)} = \nu^2 \int d^d x\, \pi(x) \Bigg\{ -\frac{d^2-1}{d(d+2)} \left(\int \frac{d^d p}{(2\pi)^d} \right) \Box^2 + \frac{(d-8)(d-1)}{d(d+2)(d+4)} \left(\int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2} \right) \Box^3 + \frac{(d-24)(d-2)(d-1)}{d(d+2)(d+4)(d+6)} \left(\int \frac{d^d p}{(2\pi)^d} \frac{1}{p^4} \right) \Box^4 \Bigg\} \pi(x).$$

- The momentum integrals are defined with UV and IR cutoffs.
- If dimensional regularization near $d = 4$ is used, the first two terms are absent. The third one corresponds to a counterterm $\sim 1/\epsilon$ (Paula Netto, Shapiro).
- No corrections to the Galileon couplings.
- Terms outside the Galileon theory are generated.

- Pertubation theory:

$$\Gamma_1 \sim \int d^4x \sum_m \left[\Lambda^4 + \Lambda^2\partial^2 + \partial^4 \log\left(\frac{\partial^2}{\Lambda^2}\right)\right] \left(\nu\partial^2\pi\right)^m .$$

- Split the field as $\pi = \pi_{cl} + \delta\pi$.
- The action includes terms $\sim \nu^2\Lambda^4(\nu\Box\pi_{cl})^n(\Box\delta\pi)^2$
- But $\nu\Box\pi_{cl} \sim (r_V/r)^{3/2} \gg 1$ below the Vainshtein radius.

Heat-kernel approach around a nontrivial background

- Our task is to evaluate the one-loop effective action

$$\Gamma_1 = \frac{1}{2}\mathrm{tr}\log \Delta$$

with $\Delta = -\Box + 2\nu\left(\Box\pi\right)\Box - 2\nu\left(\partial_\mu\partial_\nu\pi\right)\partial^\mu\partial^\nu$
around the background ($w = r^2$)

$$\pi'_{cl}(w) = \frac{1}{8\nu}\left(1 - \sqrt{1 + \frac{16\nu c}{w^{3/2}}}\right).$$

- The propagation of classical fluctuations in suppressed below the Vainshtein radius $r_V \sim (\nu c)^{1/3}$, where $\nu\Box\pi_{cl} \sim (r_V/r)^{3/2} \gg 1$.
- What about the quantum fluctuations?

- Calculate the heat kernel

$$h(x, x', \epsilon) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx'} e^{-\epsilon\Delta} e^{ikx}$$

- The one-loop effective action can be obtained as

$$\Gamma_1 = -\frac{1}{2} \int_{1/\Lambda^2}^{\infty} \frac{d\epsilon}{\epsilon} \int d^4x \, h(x, x, \epsilon).$$

- $$h(x, x, \epsilon) = \int \frac{d^4k}{(2\pi)^4 \epsilon^2} e^{-k^2} e^{\sqrt{\epsilon}X(k,\partial)+\epsilon Y(k,\partial)}. \tag{1}$$

- Expand in powers of $\sqrt{\epsilon}$. The result is the derivative expansion of the effective action.

- The diagonal part of the heat kernel becomes

$$
\begin{aligned}
h(x,x,\epsilon) \quad = \quad & \int \frac{d^4k}{(2\pi)^4}\frac{1}{\epsilon^2}\exp\Big\{-k^2 + 2i\sqrt{\epsilon}k^\mu\partial_\mu + \epsilon\Box \\
& +2\nu\Box\pi\left(k^2 - 2i\sqrt{\epsilon}k^\mu\partial_\mu - \epsilon\Box\right) \\
& -2\nu\partial_\mu\partial_\nu\pi\left(k^\mu k^\nu - 2i\sqrt{\epsilon}k^\mu\partial^\nu - \epsilon\partial^\mu\partial^\nu\right)\Big\}
\end{aligned}
$$

- Expand in ϵ and ν.
- The leading perturbative result is reproduced:

$$
h(x,x,\epsilon) = \frac{15}{32\pi^2\epsilon^2}\nu^2(\Box\pi)^2
$$

$$
\Gamma_1^{(2)} = -\frac{1}{2}\int_{1/\Lambda^2}^{\infty}\frac{d\epsilon}{\epsilon}\int d^4x\, h(x,x,\epsilon) = -\frac{15}{128\pi^2}\nu^2\Lambda^4\int d^4x\,(\Box\pi)^2.
$$

Heat kernel

- The exponent of the heat-kernel is ($\pi = \pi_{cl} + \delta\pi$)

$$\begin{aligned} F \;=\; & -G_{\mu\nu}k^\mu k^\nu - (1 - 2\nu\Box\pi_{cl})D_\epsilon(k) + 2\nu\partial_\mu\partial_\nu\pi_{cl}\, L_\epsilon^{\mu\nu}(k) \\ & +2\nu\Box\delta\pi\left(k^2 + D_\epsilon(k)\right) + 2\nu\partial_\mu\partial_\nu\delta\pi\left(-k^\mu k^\nu + L_\epsilon^{\mu\nu}(k)\right) \end{aligned}$$

with the "metric" $G_{\mu\nu} = g_{\mu\nu} - 2\nu\Box\pi_{cl}\, g_{\mu\nu} + 2\nu\partial_\mu\partial_\nu\pi_{cl}$ and

$$\begin{aligned} D_\epsilon(k) &= -2i\sqrt{\epsilon}k^\mu\partial_\mu - \epsilon\Box \\ L_\epsilon^{\mu\nu}(k) &= 2i\sqrt{\epsilon}k^\mu\partial^\nu + \epsilon\partial^\mu\partial^\nu . \end{aligned}$$

- Make the "metric" $G_{\mu\nu}$ trivial by rescaling $k^\mu = S^\mu{}_\nu k'^\nu$, with

$$S^\mu{}_\rho G_{\mu\nu} S^\nu{}_\sigma = g_{\rho\sigma} .$$

- The most divergent term quadratic in $\delta\pi$ in the heat kernel is

$$\begin{aligned} h(x,x,\epsilon) \;=\; & \int \frac{d^4k}{(2\pi)^4}(\det S)\frac{1}{2\epsilon^2}e^{-k^2}\Big(2\nu\Box\delta\pi(Sk)^2 \\ & +2\nu\partial_\mu\partial_\nu\delta\pi\left(-Sk^\mu Sk^\nu\right)\Big)^2 . \end{aligned}$$

- On the background that realizes the Vainshtein mechanism

$$\Gamma_1^{(2)} = -\frac{1}{128\pi^2}\nu^2\Lambda^4 \int d^4x \left((\Box\delta\pi)^2 P(r^2) - 2(\Box\delta\pi)(\partial_\mu\partial_\nu\delta\pi)\, V^{\mu\nu}(r^2) + (\partial_\mu\partial_\nu\delta\pi)(\partial_\rho\partial_\sigma\delta\pi)\, W^{\mu\nu\rho\sigma}(r^2) \right).$$

with $P(r^2), V^{\mu\nu}(r^2), W^{\mu\nu\rho\sigma}(r^2) \sim (r/r_V)^6$ and $r_v \sim (\nu c)^{1/3}$.

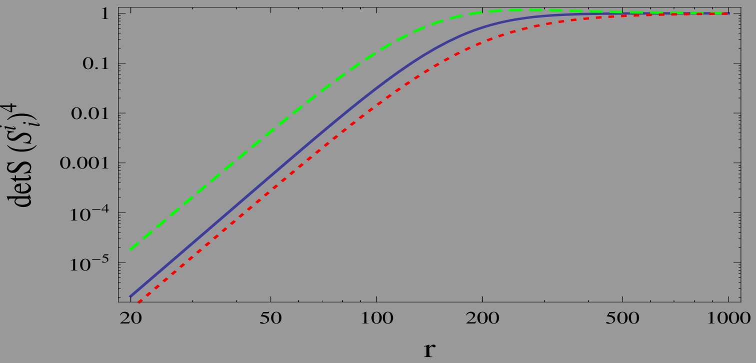

Figure: $(\det S)\left(S^i_i\right)^4$ *as a function of* r *with* $\nu = 1$, $c = 10^6$. *The solid, blue line corresponds to* $i = 0$, *the dotted, red line to* $i = 1$ *and the dashed, green line to* $i = 2$ *or* 3.

Higher order in ϵ

- The heat-kernel for the cubic Galileon takes the form

$$h(x,x,\epsilon) = \int \frac{d^4k}{(2\pi)^4}\frac{1}{\epsilon^2}\exp\left\{-G_{\mu\nu}k^\mu k^\nu + 2i\sqrt{\epsilon}G_{\mu\nu}k^\mu\partial^\nu + \epsilon G_{\mu\nu}\partial^\mu\partial^\nu\right\},$$

- $X = -G_{\mu\nu}k^\mu k^\nu, \quad Y = 2i\sqrt{\epsilon}G_{\mu\nu}k^\mu\partial^\nu + \epsilon G_{\mu\nu}\partial^\mu\partial^\nu.$
- $e^{X+Y} = e^X\left(1 - \frac{1}{2}Y[X,Y] - \frac{1}{2}[X,Y] + \ldots\right.$

- The general structure of the effective action is

$$\Gamma_1^{(2)} = \nu^2 \int d^4x$$

$$\left[\Lambda^4 \left(c_0 \frac{r^6}{R_V^6} \left(\delta\pi\partial^4\delta\pi\right)\right)\right.$$

$$+\Lambda^2 \left(c_{1a} \frac{r^{5/2}}{R_V^{9/2}} \left(\delta\pi\partial^4\delta\pi\right) + c_{1b} \frac{r^{7/2}}{R_V^{9/2}} \left(\delta\pi\partial^5\delta\pi\right) + c_{1c} \frac{r^{9/2}}{R_V^{9/2}} \left(\delta\pi\partial^6\delta\pi\right)\right)$$

$$+\log(\Lambda/\mu)\left(c_{2a} \frac{1}{rR_V^3} \left(\delta\pi\partial^4\delta\pi\right) + c_{2b} \frac{1}{R_V^3} \left(\delta\pi\partial^5\delta\pi\right)\right.$$

$$\left.\left. + c_{2c} \frac{r}{R_V^3} \left(\delta\pi\partial^6\delta\pi\right) + c_{2d} \frac{r^2}{R_V^3} \left(\delta\pi\partial^7\delta\pi\right) + c_{2e} \frac{r^3}{R_V^3} \left(\delta\pi\partial^8\delta\pi\right)\right)\right].$$

Classicalon

- We repeat the same procedure for the Classicalon field.

$$S = \int d^4x \left(\frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{1}{\Lambda^4}\left(\partial_\mu\pi\partial^\mu\pi\right)^2 \right).$$

- $$G_{\mu\nu} = g_{\mu\nu}\left(1 + \frac{\nu}{2}\partial_\rho\pi\partial^\rho\pi\right) + \nu\partial_\mu\pi\partial_\nu\pi.$$

- $$r_c = \frac{1}{\Lambda}\left(\frac{M}{\Lambda}\right)^{\frac{1}{2}}$$

- $$h(x,x,\epsilon) = \frac{1}{16\pi^2\epsilon^2} detS$$

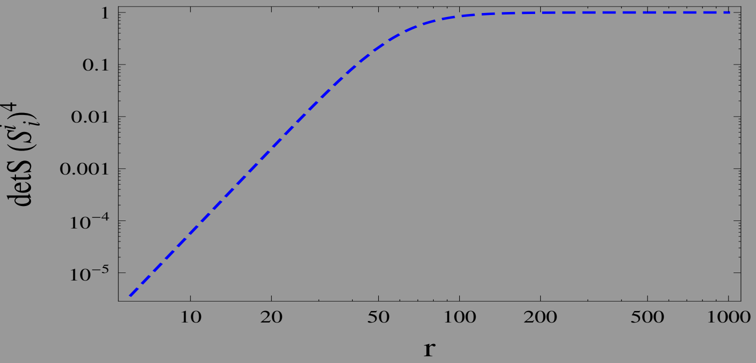

Figure: $(\det S)\left(S^i_i\right)^4$ *as a function of r with* $\Lambda = 1$, $r_c = 30$.

Conclusions

- The couplings of the Galileon theory do not get renormalized. However, the Galileon theory is not stable under quantum corrections. Additional terms are generated.
- Quantum corrections are suppressed below the Vainshtein radius.
- The Classicalon model possibly shares the same properties.