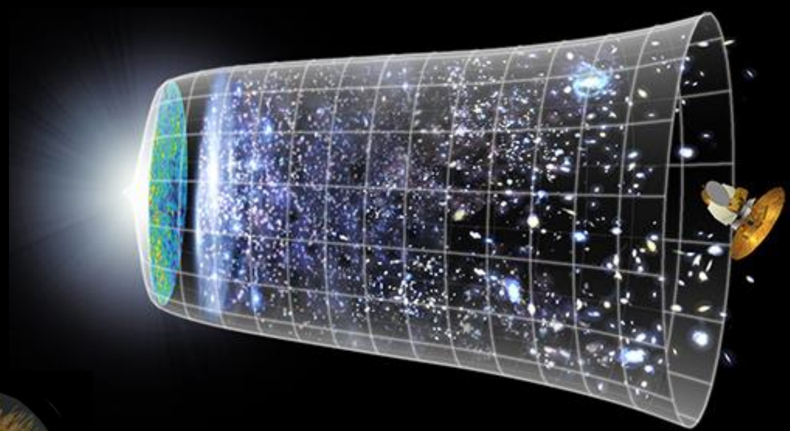
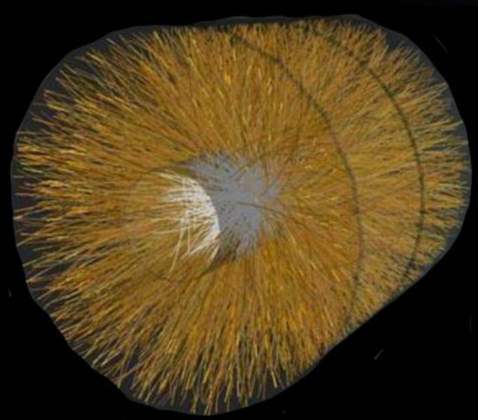
Universality far from equilibrium: From superfluid Bose gases to heavy-ion collisions



WMAP Science Team



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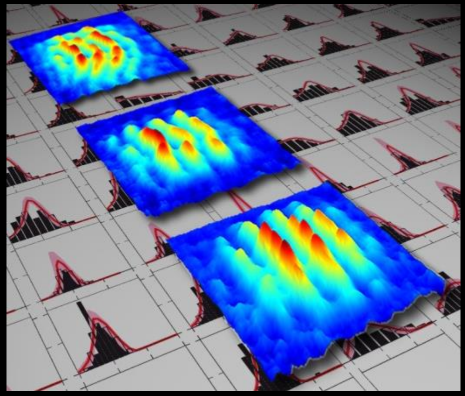

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Lefkada, September 2014

Renormalization group fixed points

RG: 'microscope' with varying resolution of length scale

$\sim 1/k$

Fixed point: physics looks the same for 'all' resolutions (in rescaled units)

→ scaling form, e.g. anti-commutator expectation value:

$$F_k = \frac{1}{2}\langle\{\Phi,\Phi\}\rangle_k \sim \frac{1}{k^{2+\kappa}}$$

'occupation number' exponent

→ similarly, spectral function (commutator, $\rho = G_R - G_A$):

$$\rho_k \sim \langle[\Phi,\Phi]\rangle_k \sim \frac{1}{k^{2-\eta}}$$

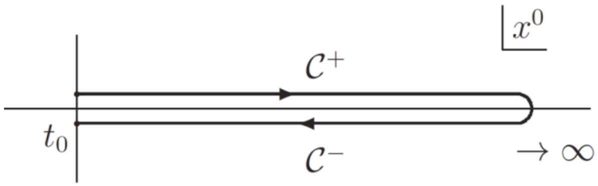
← anomalous dimension

Typically not for *all* resolutions:

- IR fixed point for k → 0
- UV fixed point for k → '∞'

Thermal & *nonthermal* fixed points: Wetterich equation on the closed time path

Scale derivative $\dot{\Gamma}_k[\phi,\tilde{\phi}] \equiv k\dfrac{\partial \Gamma_k[\phi,\tilde{\phi}]}{\partial k}$ gives



$\Rightarrow$ $$\boxed{\dot{\Gamma}_k[\phi,\tilde{\phi}] = -\frac{i}{2}\,\mathrm{Tr}\left\{G_k^{\mathrm{R}}\dot{R}_k^{\mathrm{R}} + G_k^{\mathrm{A}}\dot{R}_k^{\mathrm{A}}\right\}} = -\frac{i}{2}\Big\{$$  $+$  $\Big\}$

closed equation: $$G_k^{\mathrm{R}}[\phi,\tilde{\phi}] = -\left\{\Gamma_k^{\tilde{\phi}\phi} + R_k - \Gamma_k^{\tilde{\phi}\tilde{\phi}}\left(\Gamma_k^{\phi\tilde{\phi}} + R_k\right)^{-1}\Gamma_k^{\phi\phi}\right\}^{-1}[\phi,\tilde{\phi}]$$

Flow interpolates between effective action (k = 0) and classical action ($k \to \Lambda$)

for: $$R_{k,ab}^{\mathrm{R,A}}(x,y) = R_k(-\Box_x)\,\delta(x-y)\delta_{ab}$$ with $\lim\limits_{k\to\Lambda} R_k \to \infty$

IR fixed point hierarchy

Berges, Hoffmeister, Nucl. Phys. B 813 (2009) 383

Hierarchy of infrared fixed point solutions ($\lambda\phi^4$):

- vacuum: $\kappa = -\eta$
- thermal: $\kappa = -\eta + z$
- *nonequilibrium:* $\kappa = -\eta + z + d$, …

increasing complexity

Fluctuation-dissipation relation for vacuum/thermal equilibrium:

$$\frac{F_k(\omega,\mathbf{p})}{\rho_k(\omega,\mathbf{p})} \sim n_{\mathrm{BE}}(\omega) + \frac{1}{2} \qquad \text{for} \quad n_{BE}(\omega) \overset{\omega \ll T}{\simeq} \frac{T}{\omega}$$

↳ dynamic exponent *z*

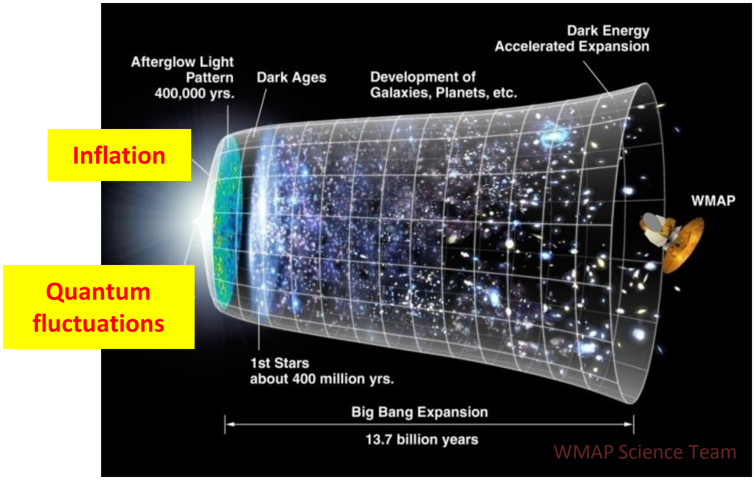
No fluctuation-dissipation relation *out of equilibrium*:

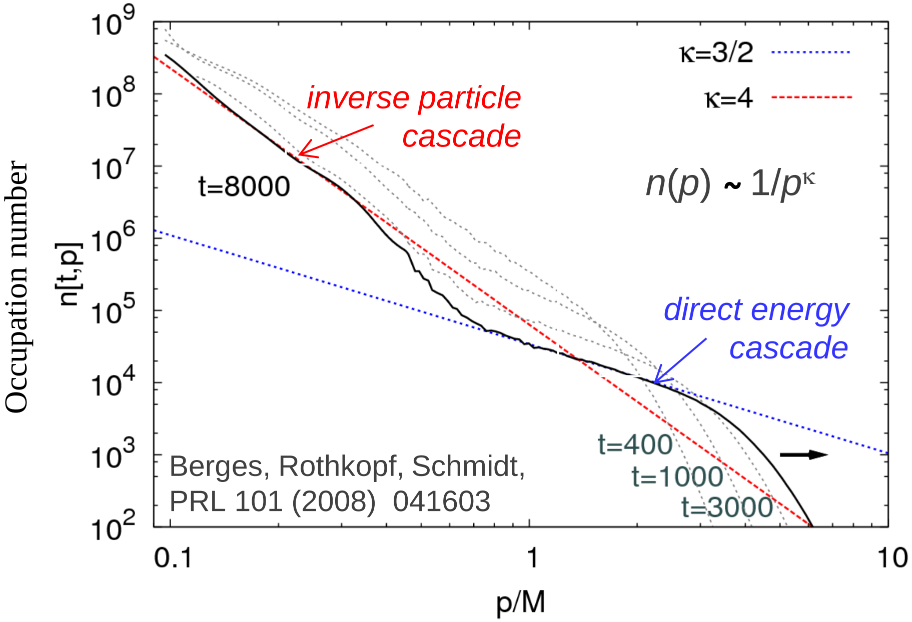
$$\text{"}n(\omega,\mathbf{p})\text{"}$$

↳ spatial dimension *d* , …

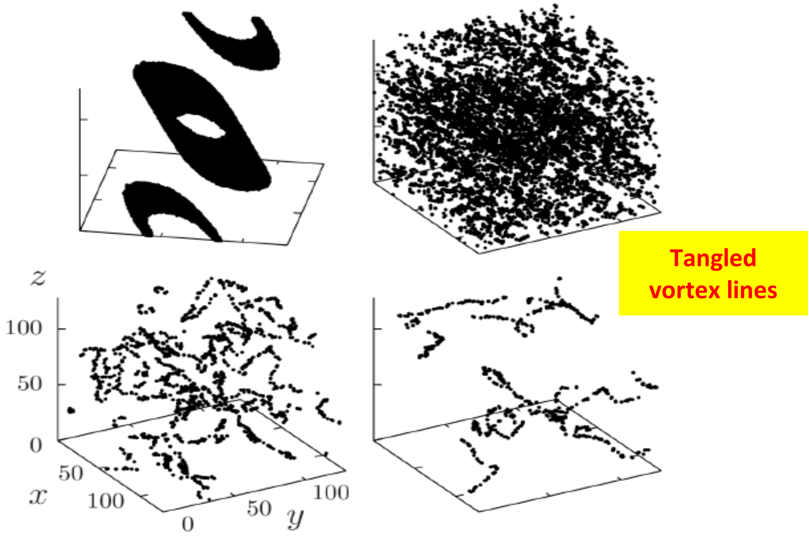
Applications: Nonthermal fixed points and turbulence

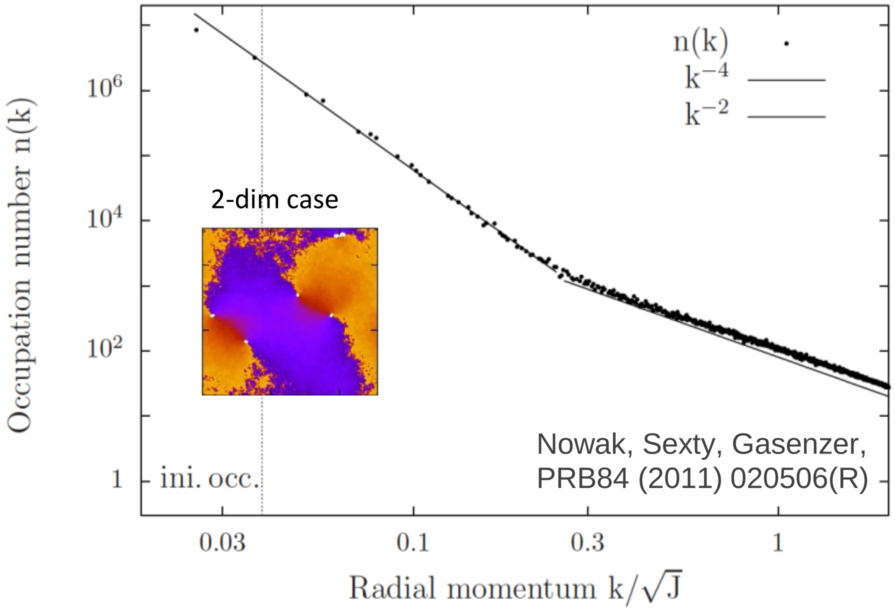
- Reheating dynamics after chaotic inflation





- Superfluid turbulence in a cold Bose gas



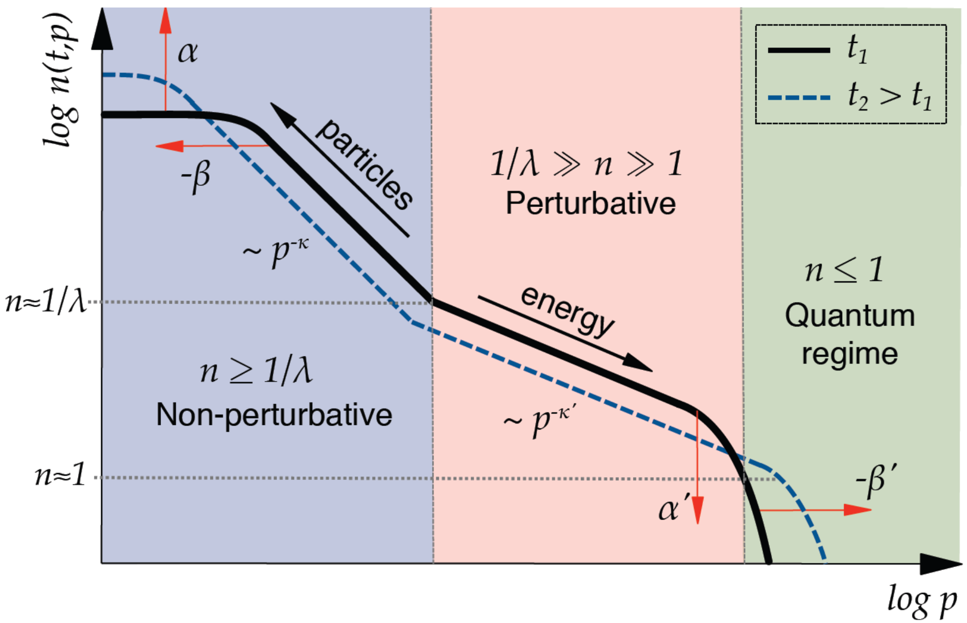


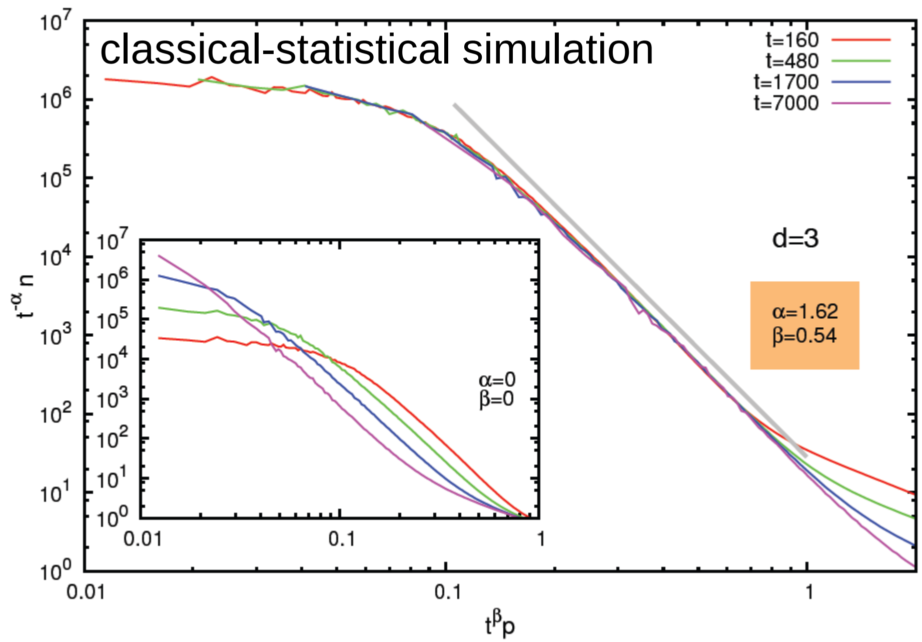
Self-similar dynamics of isolated systems

Self-similar time evolution: $n(t,p) = t^{\alpha}\, n_s\,(t^{\beta} p)$

stationary fixed-point distribution

E.g. *O(N)* symmetric scalar field theory with $\lambda\phi^4$ interaction:



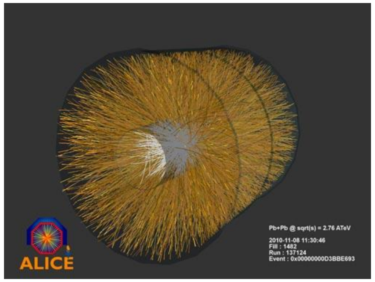


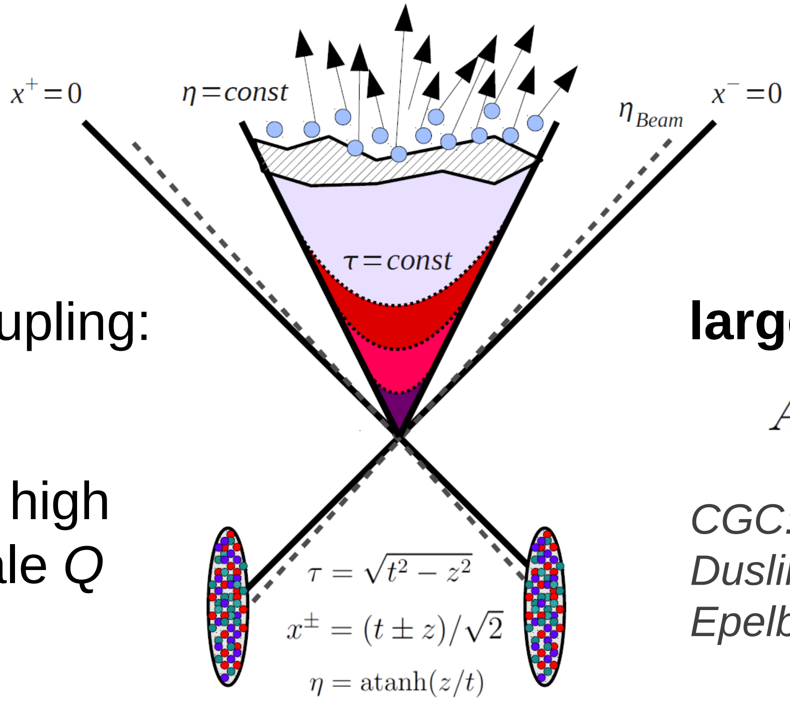
2PI/RG at NLO large-*N*: $\beta = \frac{1}{2-\eta}$ $\alpha = d\beta$

with A. Pineiro, K. Boguslavski

Application to heavy-ion collisions

Particle production in the presence of large fields:





small gauge coupling:

$$g \ll 1$$

at characteristic high momentum scale *Q*

large initial gauge fields:

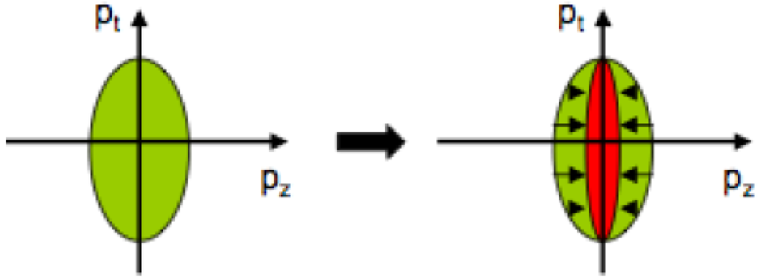
$$A^a_\mu(x) \sim \mathcal{O}(1/g)$$

CGC: Lappi, McLerran, Dusling, Gelis, Venugopalan, Epelbaum…

small initial (vacuum) fluctuations $\rightarrow$ ***plasma instabilities!***

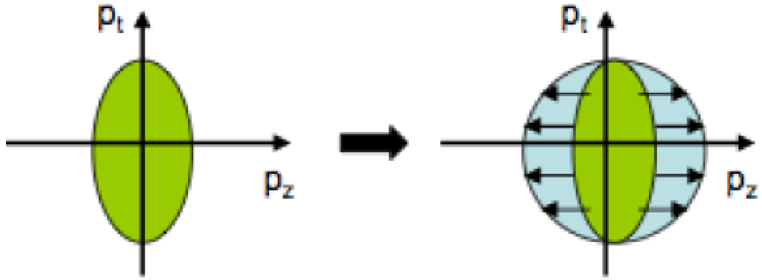
Mrowczynski; Rebhan, Romatschke, Strickland; Arnold, Moore, Yaffe …
Romatschke, Venugopalan; Berges, Scheffler, Schlichting, Sexty; Fukushima, Gelis; Wetterich, Flörchinger …

Competition between interactions and expansion



Longitudinal Expansion:

- Red-shift of longitudinal momenta p_z
  $\rightarrow$ increase of anisotropy
- Dilution of the system



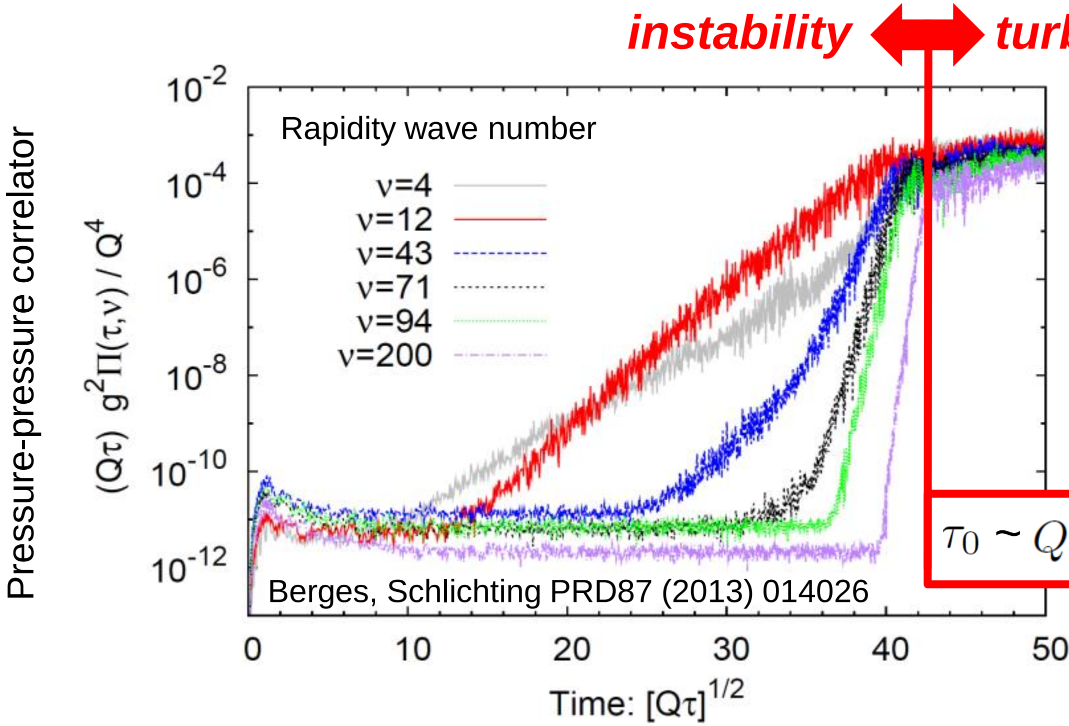
Interactions:

- Isotropize the system

$\rightarrow$ New universality class for expanding system?

Plasma instabilities at early times

CGC expanding (classical-statistical lattice simulations):



$\tau_0 \sim Q^{-1} \ln^2(\alpha_s^{-1})$

quasi-exponential growth of initial fluctuations

'overpopulated' occupancies $\sim 1/\alpha_s$ after instability regime

$\alpha_s = g^2/4\pi$

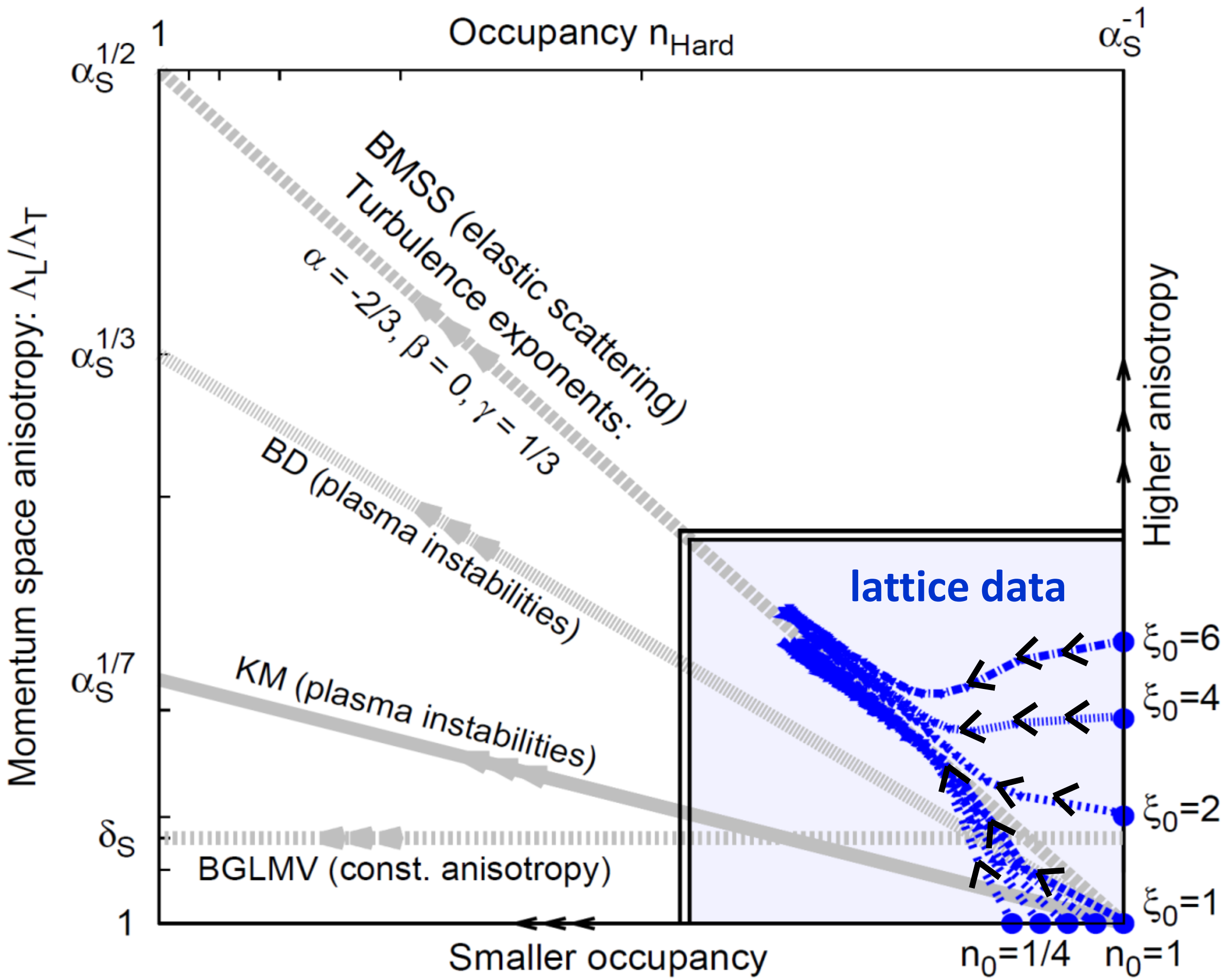
gluon distribution: $$f(\mathrm{p_T}, \mathrm{p_z}, \tau_0) = \frac{n_0}{2g^2}\,\Theta\left(Q - \sqrt{\mathrm{p_T^2} + (\xi_0 \mathrm{p_z})^2}\right)$$

occupancy parameter (n_0)

anisotropy parameter (ξ_0)

Nonthermal fixed point

Evolution in the `anisotropy-occupancy plane´



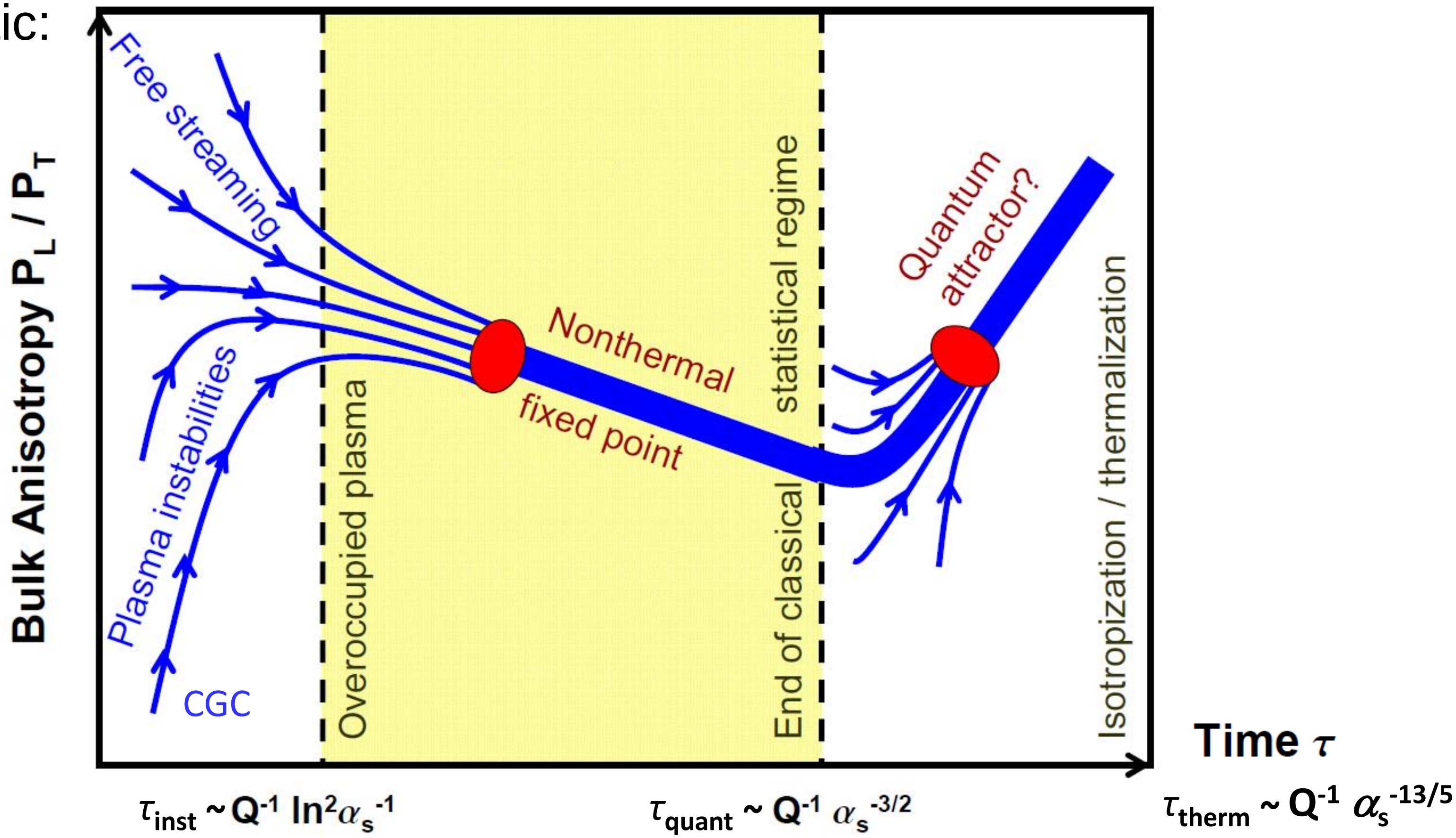
Berges, Boguslavski, Schlichting, Venugopalan,
PRD 89 (2014) 074011; ibid. 114007

Bottom-up* scenario emerges as a consequence of the fixed point!

**Baier et al, PLB 502 (2001) 51*

Turbulent thermalization process

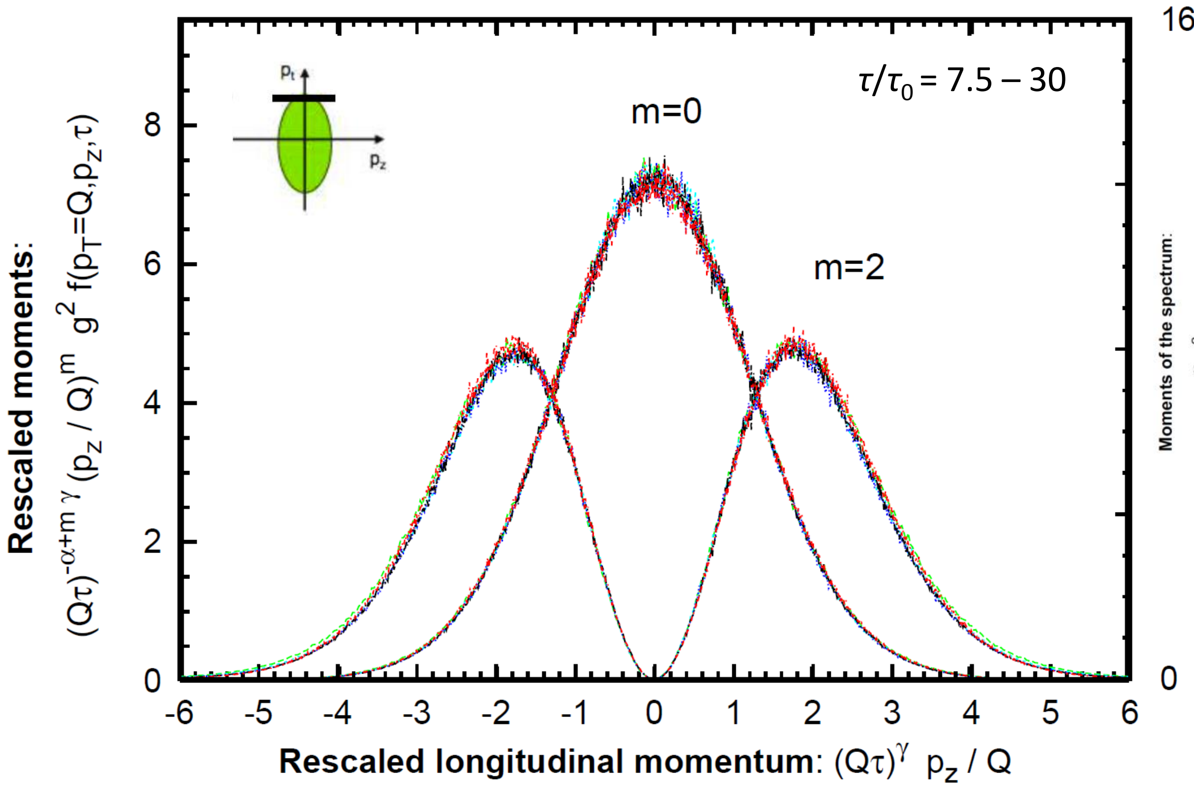
Schematic:

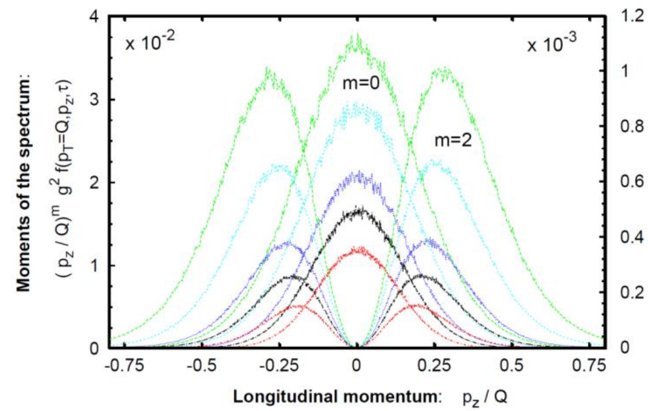


Extrapolation to realistic coupling $\alpha_s \sim 0.3$ for $Q \sim 2$ GeV:

$\tau_{inst} \sim$ **0.1 fm/c**	$\tau_{quant} \sim$ **0.6 fm/c**	$\tau_{therm} \sim$ **2 fm/c**
$P_L/P_T \sim$ **20-30%**	$P_L/P_T \sim$ **10-20%**	$P_L \sim P_T$

Self-similar evolution





Scaling exponents: $\alpha = -2/3\ , \qquad \beta = 0\ , \qquad \gamma = 1/3$

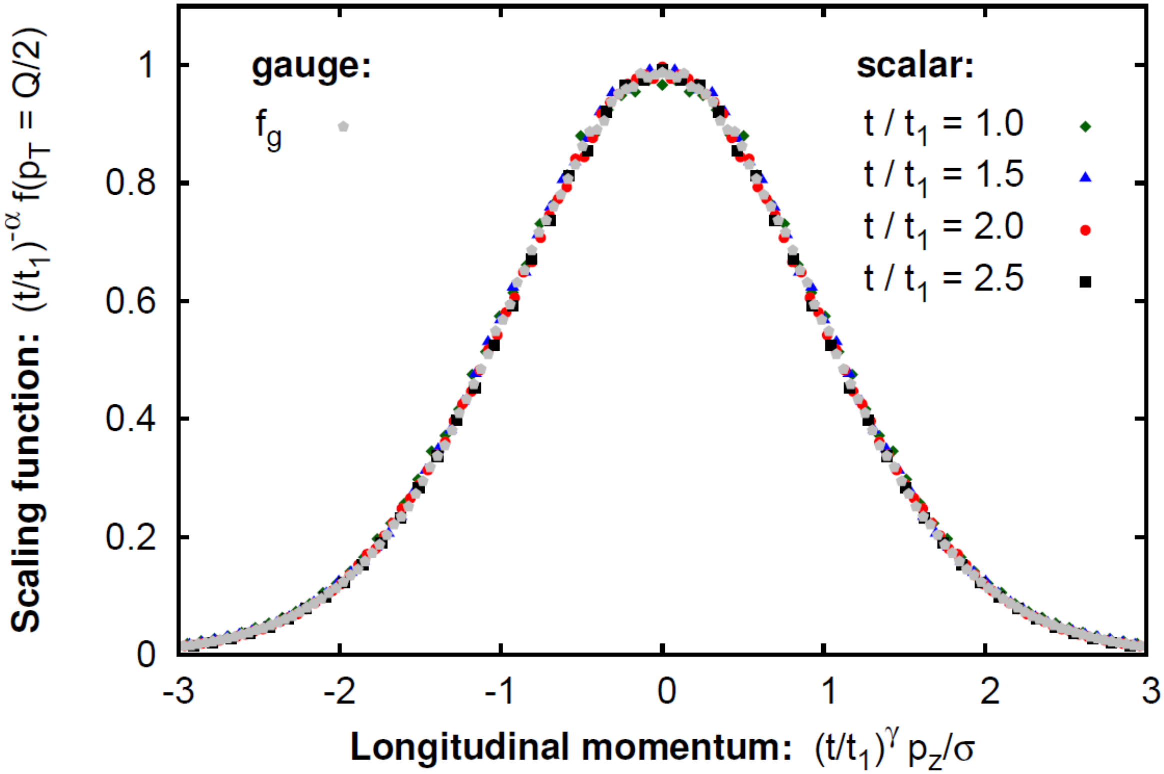
and scaling distribution function f_S:

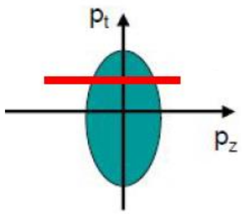
$$f(\mathrm{p_T}, \mathrm{p_z}, \tau) = (Q\tau)^{\alpha} f_S\Big((Q\tau)^{\beta}\mathrm{p_T}, (Q\tau)^{\gamma}\mathrm{p_z}\Big)$$

stationary fixed-point distribution

Comparing gauge and scalar field theories

Accurate agreement of scaling exponents and scaling function in characteristic momentum range : ($\lambda\phi^4$ with longitudinal expansion)





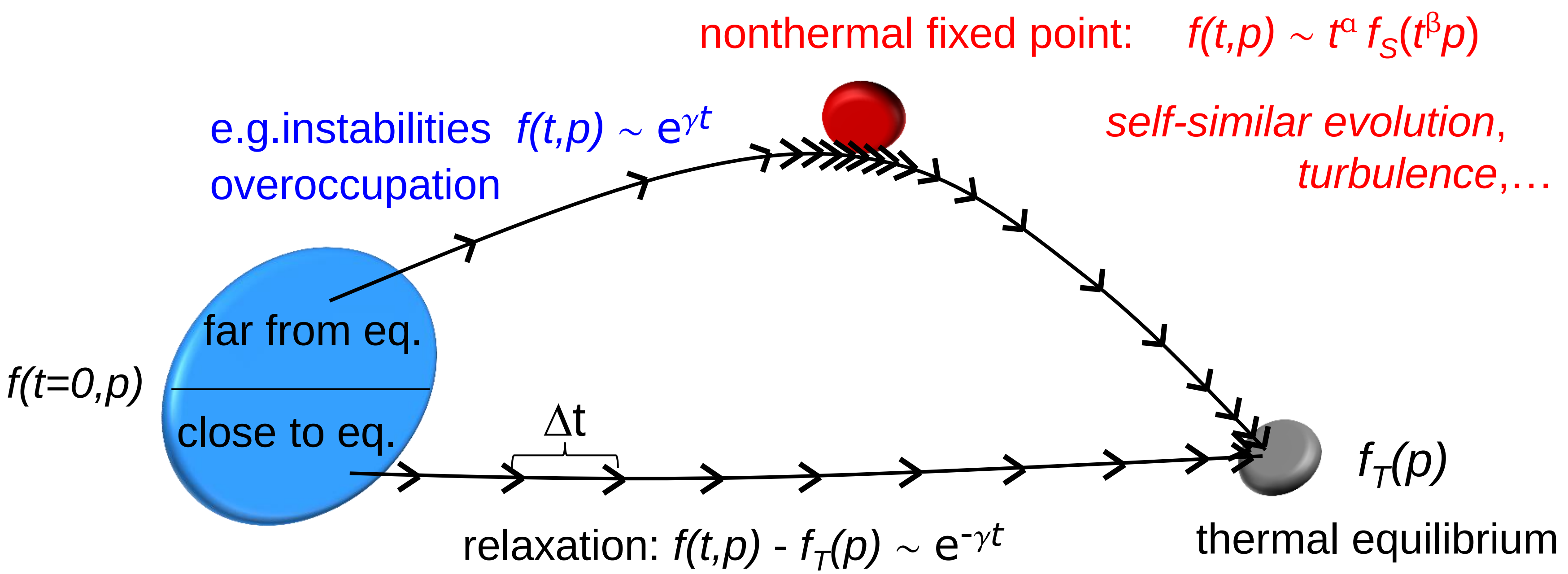
$$\alpha = -2/3\,, \qquad \beta = 0\,, \qquad \gamma = 1/3$$

Universality far from equilibrium!

Berges, Boguslavski, Schlichting, Venugopalan, arXiv:1408.1670

Conclusions

- ***Nonthermal fixed points:*** Self-similar scaling solutions far from equilibrium (universality classes, \`self-tuned´)
- ***Wide range of applications:*** Early Universe reheating, heavy-ion collisions, superfluid Bose gases, …

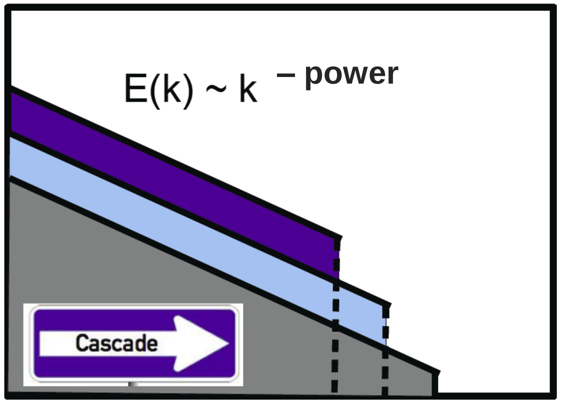


Nature of nonthermal fixed point: wave turbulence

Boltzmann equation with generic collision term for longitudinal expansion:

$$\left[\partial_t - \frac{p_z}{t}\partial_{p_z}\right] f(p_T, p_z, t) = C[p_T, p_z, t; f]$$

Turbulent Thermalization



Self-similar evolution:

$$f(p_T, p_z, t) = t^{\alpha} f_S(t^{\beta} p_T, t^{\gamma} p_z)$$

$$C[p_T, p_z, t; f] = t^{\mu}\, C[t^{\beta} p_T, t^{\gamma} p_z; f_S]$$

→ *a)* ***fixed point equation for stationary distribution****:*

$$\alpha f_S(p_T, p_z) + \beta p_T \partial_{p_T} f_S(p_T, p_z + (\gamma - 1)\, p_z \partial_{p_z} f_S(p_T, p_z) = C[p_T, p_z; f_S]$$

→ *b)* ***scaling condition****:*

$$\boxed{\alpha - 1 = \mu(\alpha, \beta, \gamma)}$$

Nonthermal fixed point

Interpret scaling condition with **energy/number conserving*** Fokker-Planck-type dynamics for the collisional broadening of longitudinal momentum distribution:

$$C^{(\text{elast})}[p_T, p_z; f] = \hat{q}\, \partial^2_{p_z} f(p_T, p_z, t)$$

with **momentum diffusion parameter**: $\hat{q} \sim \alpha_S^2 \int \frac{d^2 p_T}{(2\pi)^2} \int \frac{dp_z}{2\pi}\, f^2(p_T, p_z, t)$

→ 1) $\mu = 3\alpha - 2\beta + \gamma$ $\xrightarrow{\alpha - 1 = \mu(\alpha,\beta,\gamma)}$ $2\alpha - 2\beta + \gamma + 1 = 0$

2) number conservation $\longrightarrow$ $\alpha - 2\beta - \gamma + 1 = 0$

3) energy conservation $\longrightarrow$ $\alpha - 3\beta - \gamma + 1 = 0$

remarkable agreement with lattice data!

$$\alpha = -2/3\,, \qquad \beta = 0\,, \qquad \gamma = 1/3$$

*cf. early stages of Baier, Mueller, Schiff, Son, PLB 502 (2001) 51 (‘BMSS‘)