Understanding hysteresis - same universality class of a system with disorder in and out of equilibrium

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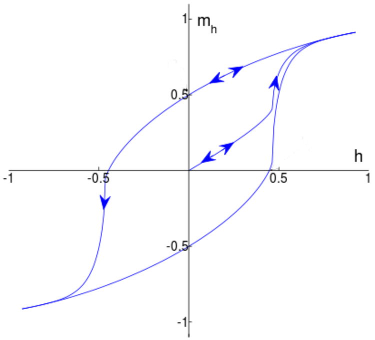
Collaboration

- "Laboratoire de Physique Théorique de la Matière Condensée de l'Université Pierre et Marie Curie",
- Gilles Tarjus
- Matthieu Tissier
- Bertrand Delamotte

I.Balog,G.Tarjus and M.Tissier: Phys. Rev. B 89, 104201 (2014)

What is hysteresis?

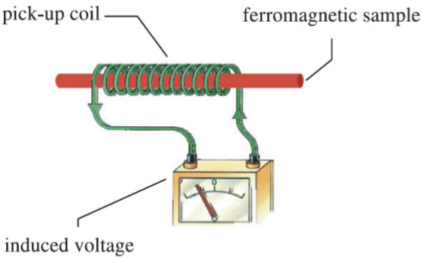
Where does it appear?

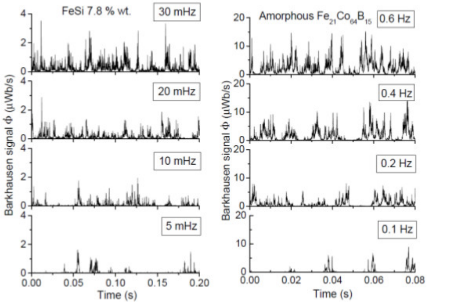


- Ferromagnetic amorphous alloys O'Brien, Weissman: PRE, 50, 3446
- capillary condensation of helium in pores of an aerogel Lilly, Finley, Hallock: PRL, 71, 4186
- structural martensic transition (Cu-Zn-Al alloys) Vives etal.: PRL, 72, 1694
- charge density wave systems Middleton: PRL, 68, 670
- "SOC" self organized criticality Bak, Tang: PRL 59 381

Barkhausen noise

Heinrich Barkhausen Z.Physik 20, 401 (1919)



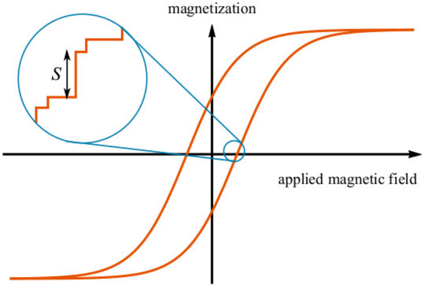


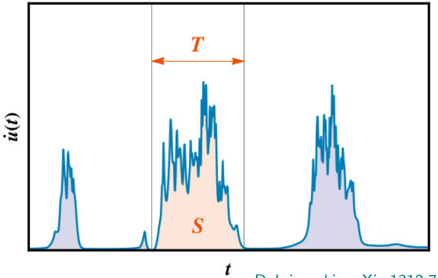
Dobrinevski: arXiv:1312.7156

- noninvasive testing and characterization of samples

→ residual stress Curr. Appl. Phys. 4, 308

→ grain sizes J. Appl. Phys. 61, 3199, Acta Mat. 49, 3019





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What is in common?

- interplay of two phases
- discontinuous phase transition in the background
- thermal activation is a secondary effect
- non-equilibrium
- disorder

Is there a theoretical model?
Can it be solved?

random field Ising model (RFIM):

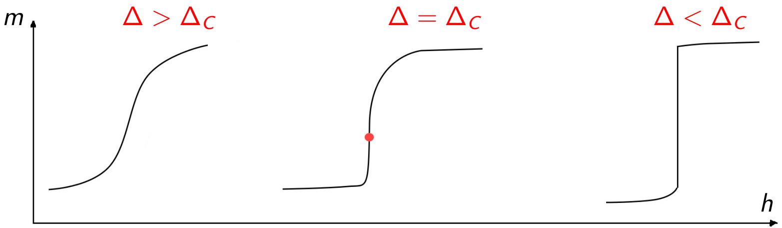
- Hamiltonian $H = \sum_{<i,j>} -J\sigma_i\sigma_j - \sum_i h_i\sigma_i - h\sum_i \sigma_i$
- Gaussian disorder distribution: $P[h_i] = \frac{1}{\sqrt{2\pi\Delta^2}} e^{-\frac{h_i^2}{2\Delta}}$

Can RFIM explain all the concepts? - YES

Sethna & Dahmen (1990-2000): “plain old criticality...”

- power law behavior
- critical exponents
- ...

Phase transition by changing disorder strength Δ:



GOAL:rigorously understand the phase transition in hysteresis

What next?

random field Ising model

$\Downarrow$

Field theory

$\Downarrow$

NonPerturbative Renormalization Group

$\Downarrow$

Result

field theoretic approach

A) formulate the field theory for RFIM (Hubbard-Stratonowich):

$H \to \int_x \left\{ \frac{1}{2}(\nabla_x \phi_x)^2 + \frac{r}{2}\phi_x^2 + \frac{u}{4!}\phi_x^4 - (h_x + J_B)\phi_x \right\}$

B) sum over solutions of the stationary stochastic equation (+ source $\hat{J}$):

$Z = \int D\phi \delta[\frac{\delta S}{\delta \phi_x} - h_x - J_B] | \frac{\delta^2 S}{\delta \phi_x \delta \phi_y} | e^{\int_x \hat{J}_x \phi_x}$

C) write Z in exponential form (superfield f. at $T = 0$ Parisi, Sourlas: PRL 43 744)

$Z = \int \underbrace{D(\phi, \hat{\phi}, \psi, \bar{\psi})}_{D\Phi} e^{\int_x \left\{ -\hat{\phi}_x (\frac{\delta S}{\delta \phi_x} - h_x - J_B) + \hat{J}_x \phi_x + \psi_x \bar{K}_x + \bar{\psi}_x K_x \right\} + \int_x \int_y \bar{\psi}_x \frac{\delta^2 S}{\delta \phi_x \delta \phi_y} \psi_y}$

D) introduce a sum over infinite replicas Tarjus, Mouhanna: PRE 81, 051101 (2010)

$Z^n = \int \Pi_a D\Phi_a e^{\sum_a \left\{ \int_x \left\{ -\hat{\phi}_x^a (\frac{\delta S}{\delta \phi_x^a} - h_x - J_B) + \hat{J}_x^a \phi_x^a + \psi_x^a \bar{K}_x^a + \bar{\psi}_x^a K_x^a \right\} + \int_x \int_y \bar{\psi}_x^a \frac{\delta^2 S}{\delta \phi_x^a \delta \phi_y^a} \psi_y^a \right\}}$

E) average over disorder:

$\overline{Z^n} = \int \Pi_a D\Phi_a e^{\sum_a \left\{ \int_x \left\{ -\hat{\phi}_x^a (\frac{\delta S}{\delta \phi_x^a} - J_B) + \hat{J}_x^a \phi_x^a + \psi_x^a \bar{K}_x^a + \bar{\psi}_x^a K_x^a \right\} + \int_x \int_y \cdots \right\} + \frac{\Delta}{4} \sum_{a,b} \int_x \hat{\phi}_a \hat{\phi}_b}$

Steps A)-E) define free energy

$$\overline{Z^n} = e^{W[\{\mathcal{J}_a,\mathcal{J}_b,\cdots\}]} = Exp\Big[\sum_a W_a[\{\mathcal{J}_a\}] + \frac{1}{2}\sum_{a,b} W_{a,b}[\{\mathcal{J}_a,\mathcal{J}_b\}] + \cdots\Big]$$

F) Legendre transform$[W[\{\mathcal{J}_a, \mathcal{J}_b, \cdots\}]] \Rightarrow \Gamma[\{\mathcal{M}_a, \mathcal{M}_b, \cdots\}]$

$\Rightarrow$ functional of the effective average action

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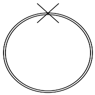
$\Rightarrow$ functional of the effective average action

$\Rightarrow$ use NPRG

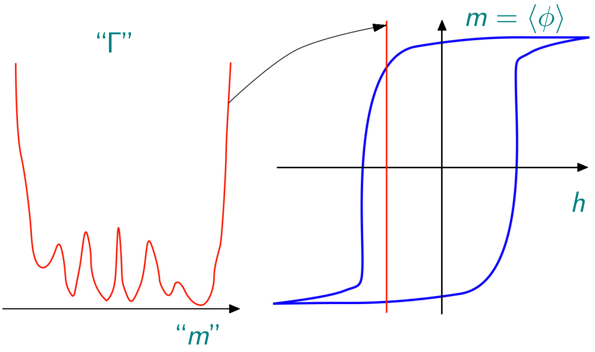
- introduce a proper infrared regulator
- set the Wetterich equation:

$$\partial_t \Gamma = \frac{1}{2} Tr \int_q$$ 

Why is all this about hysteresis?

Auxiliary source $\hat{J}$ (introduced in steps A)-F)):

- enters the partition function as $Z = \int D\phi e^{\dots + \int_x \hat{J}_x \phi_x}$
-



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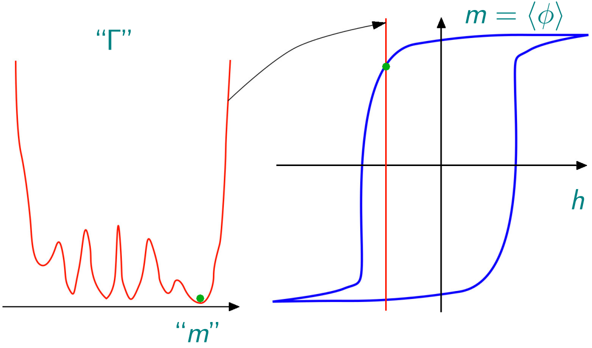

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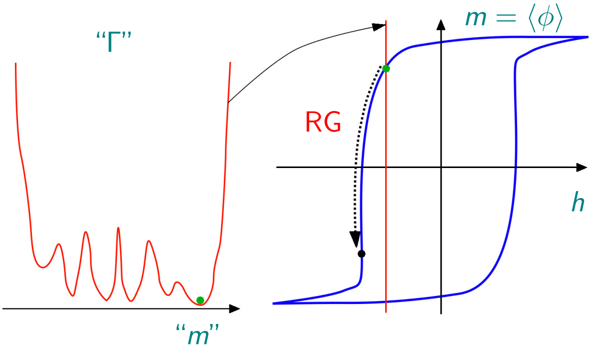
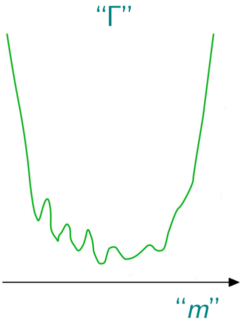
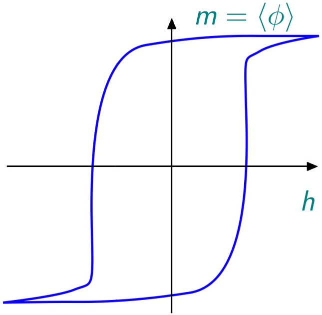


B) equilibrium RFIM Tarjus & Tissier 2004-2012

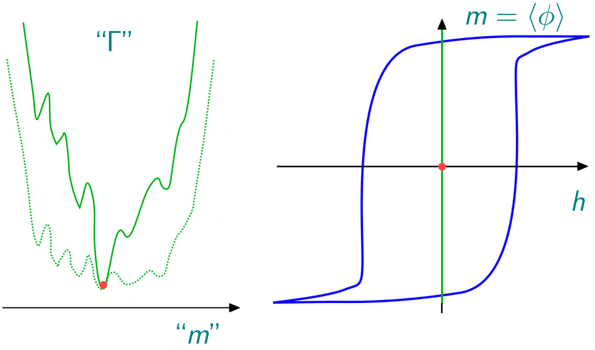
- a problem of choosing the ground state!





B) equilibrium RFIM Tarjus & Tissier 2004-2012

- introduce auxiliary temperature T_a: $Z = \int D\phi e^{\int_x \frac{1}{T_a} S[\phi] + \cdots}$
- lifts the ground state degeneracy!



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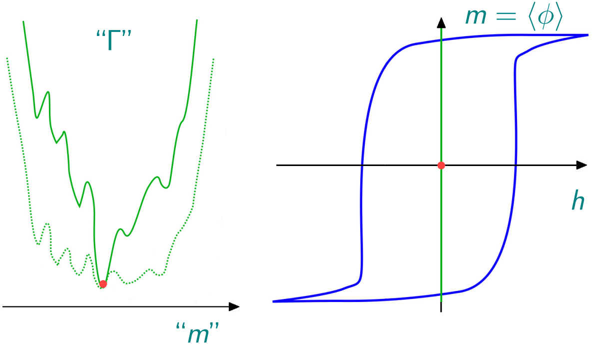

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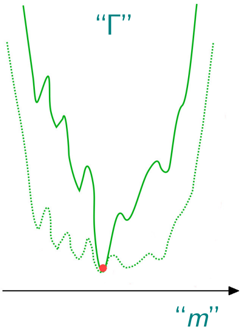

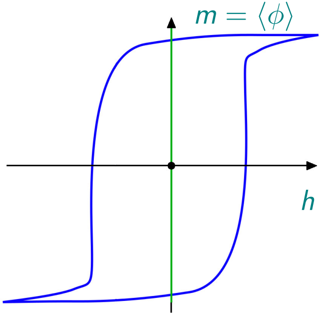

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B) equilibrium RFIM ($\hat{J} \to 0$ + auxiliary temperature T_a)

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$$\Downarrow$$

formally identical flow equation

$$\boxed{\partial_t \Gamma = \beta_\Gamma}$$

Are we done???

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B) equilibrium RFIM ($\hat{J} \to 0$ + auxiliary temperature T_a)

$$\Downarrow$$

formally identical flow equation

$$\boxed{\partial_t \Gamma = \beta_\Gamma}$$

Are we done???

NO: what are the initial conditions?

What is the meaning of the eff. average action? ($\mathcal{M} = (M, \hat{M})$):

$$\Gamma[\{\mathcal{M}_a, \mathcal{M}_b, \cdots\}] = \sum_a \Gamma_1[\mathcal{M}_a] - \frac{1}{2} \sum_{a,b} \Gamma_2[\mathcal{M}_a, \mathcal{M}_b] + \cdots$$

Derivatives by $\hat{M}$ have a physical meaning:

- $\frac{\delta \Gamma_1[\mathcal{M}_a]}{\delta \hat{M}_\alpha} = \frac{\delta}{\delta M_\alpha}(\frac{1}{2}(\nabla M_\alpha)^2 Z_k[M_\alpha] + U_k[M_\alpha])$
- $\frac{\delta^2 \Gamma_2[\mathcal{M}_a, \mathcal{M}_b]}{\delta \hat{M}_\beta \delta \hat{M}_\alpha} = \Delta_k[M_\alpha, M_\beta]$
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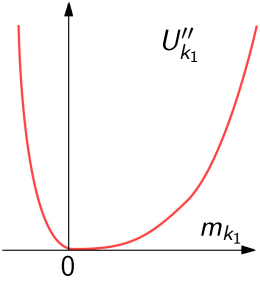
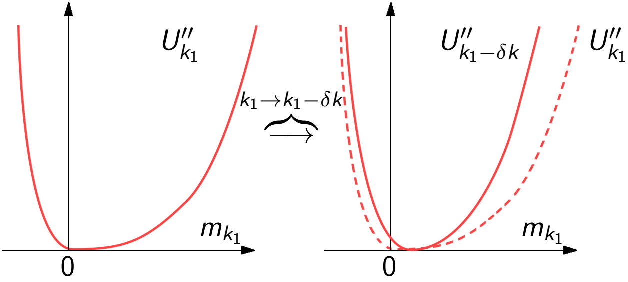
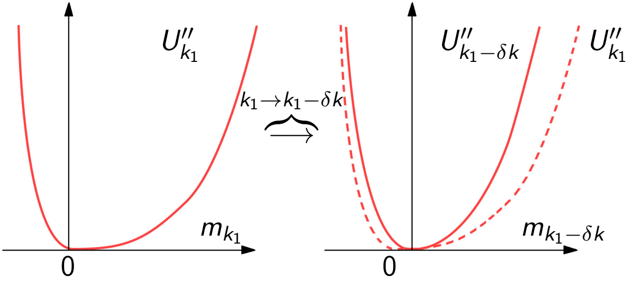
- Z_k - field renormalization (critical exponent η)
- Δ_k - renormalized disorder strength (critical exponent $\bar{\eta}$)
- U_k - effective potential

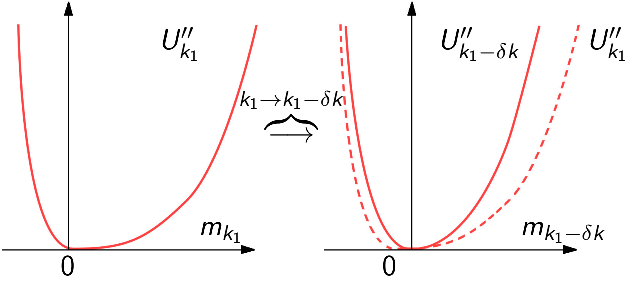

U''_{k_1}
m_{k_1}
0



U''_{k_1}
m_{k_1}
0
$k_1 \to k_1 - \delta k$
$U''_{k_1-\delta k}$
U''_{k_1}
m_{k_1}
0



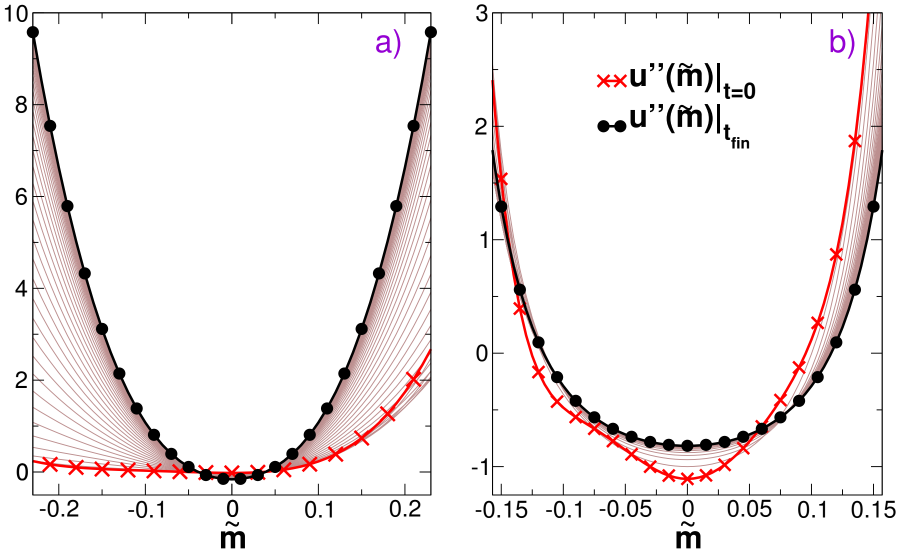
move $m_{k_1-\delta k} = m_{k_1} + \delta m_{k_1,0}$ so that $U'''_k[0] = 0\ \forall k$!



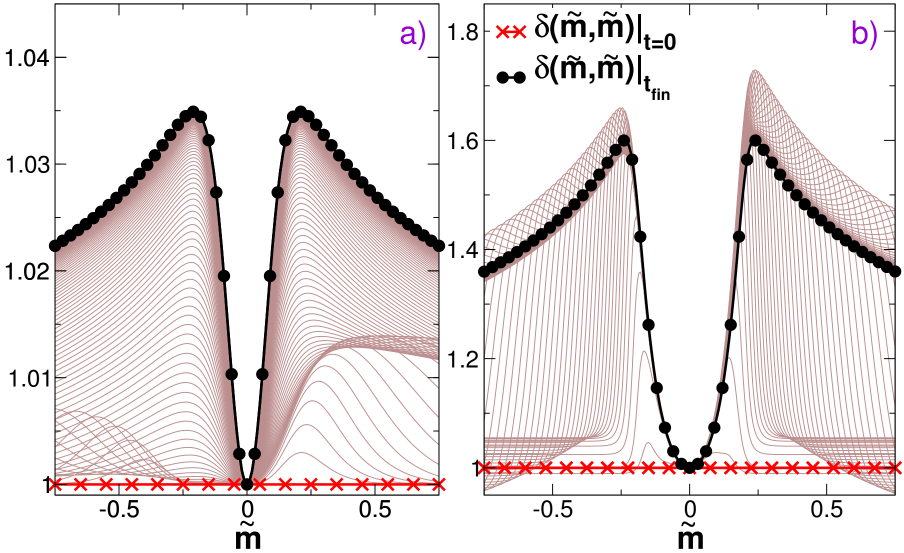
move $m_{k_1-\delta k} = m_{k_1} + \delta m_{k_1,0}$ so that $U'''_k[0] = 0 \; \forall k$!

Solve in dimensionless quantities (gives me critical exponents):

$\Rightarrow$ $M = k^{\frac{d-4+\bar{\eta}}{2}} m$; $Z[M] = k^{-\eta} z[m]$; $U''[M] = k^{2-\eta} u''[m]$;
$\Delta[M_1, M_2] = k^{-2\eta+\bar{\eta}} \delta[m_1, m_2]$



a) $d = 5.5$, b) $d = 4$



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Conclusions

- Z2 symmetry is asymptotically restored in $k \to 0$
- equations + renormalization functions at fixed point + critical exponents equal as in equilibrium RFIM = SAME UNIVERSALITY CLASS
- "COROLLARY": fluid gas transition in the pores of aerogel is in the same universality class (binary system with an asymmetric initial condition!)

Perspectives

- study relaxation in the vicinity of the critical point
- implications on the physics of structural and spin glasses
- RFI model under external driving field

Thanks for your attention!