Dimensional Reduction in Asymptotically Safe Gravity

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Motivation I

- Asymptotically Safe Quantum Gravity might be tested at the LHC.
- Functional Renormalisation Group:
  Employed to investigate the Asymptotic Safety Scenario.
- ``Large'' extra dimensions (ADD model):
  Introduced to solve hierarchy problem of Particle Physics.

Motivation II

- ``Large'' extra dimensions (ADD model):
  - At short distances: Gravity not tested below 10^{-4} m, ample space for 'new' physics, e.g. extra dimensions as in the ADD model [1] to unify 'true' Planck with e.w. scale.
  - Low-scale Quantum Gravity: No hierarchy problem as $m_{EW} \approx$ gravity scale in $D=4+n$.
  - ``Large'' extra dimensions: $L^n \approx M_{\rm Pl}^2 / m_{\rm EW}^{2+n}$
  - Appearance of Kaluza-Klein modes: Experimentally testable at LHC [2].
  - Is the ADD model UV complete?

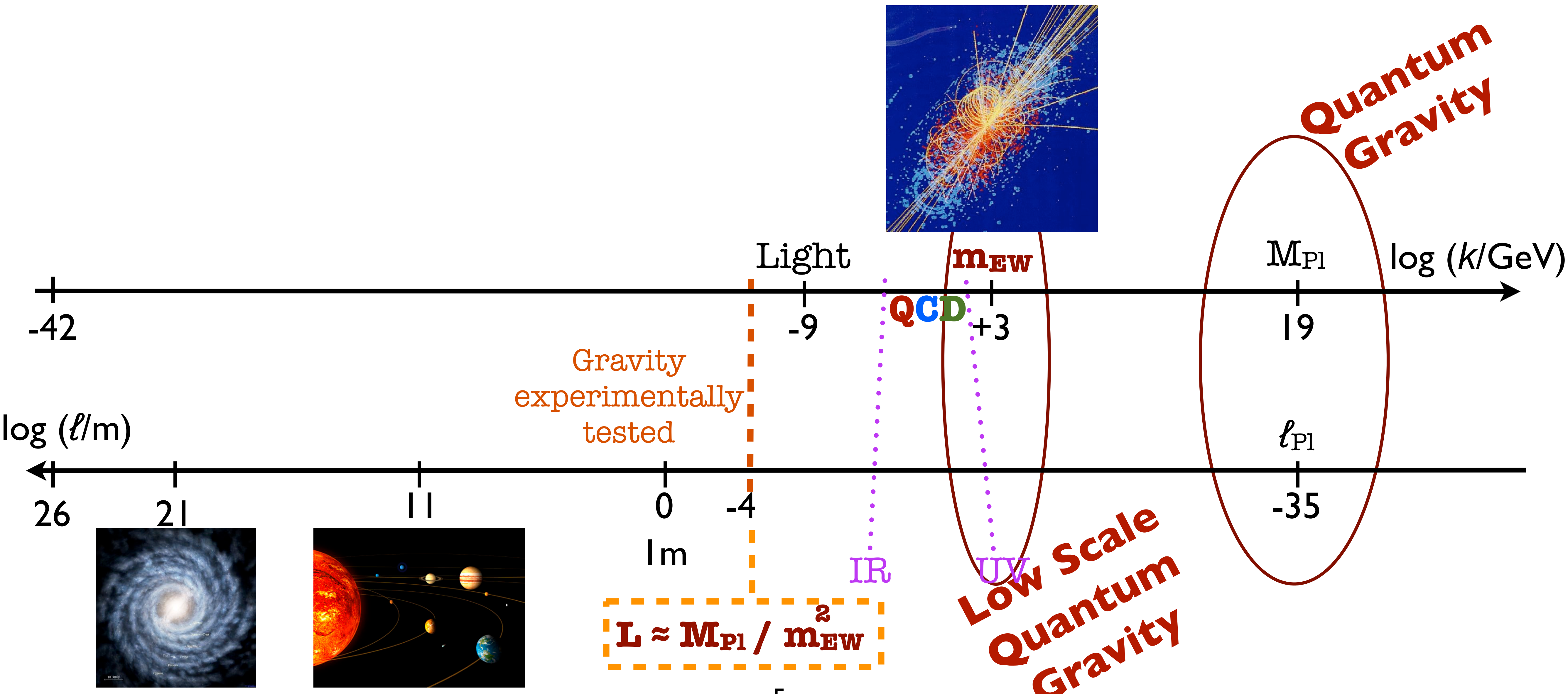
[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429 (1998) 263, [hep-ph/9803315].
[2] see e.g., E. Gerwick, D. Litim and T. Plehn, Phys. Rev. D 83 (2011) 084048, [hep-ph/1101.5548] and references therein.

Renormalisation Group

- Different resolution scales: k (momentum) or ℓ (distance).

  Start from $M_{Pl} = (\hbar c/G_N)^{1/2} = 1.22 \times 10^{19}$ GeV, resp.,

  $\ell_{Pl} = 1.62 \times 10^{-35}$ m



GRAVITY I

Einstein-Hilbert Theory

- Einstein Gravity: metric field $g_{\mu\nu}(x)$

  curvature (Riemann) tensor $R_{\mu\nu\rho\sigma}$

  Ricci tensor $R_{\mu\nu} = R^{\rho}{}_{\mu\rho\nu}$

  curvature scalar $R = R^{\mu}{}_{\mu}$

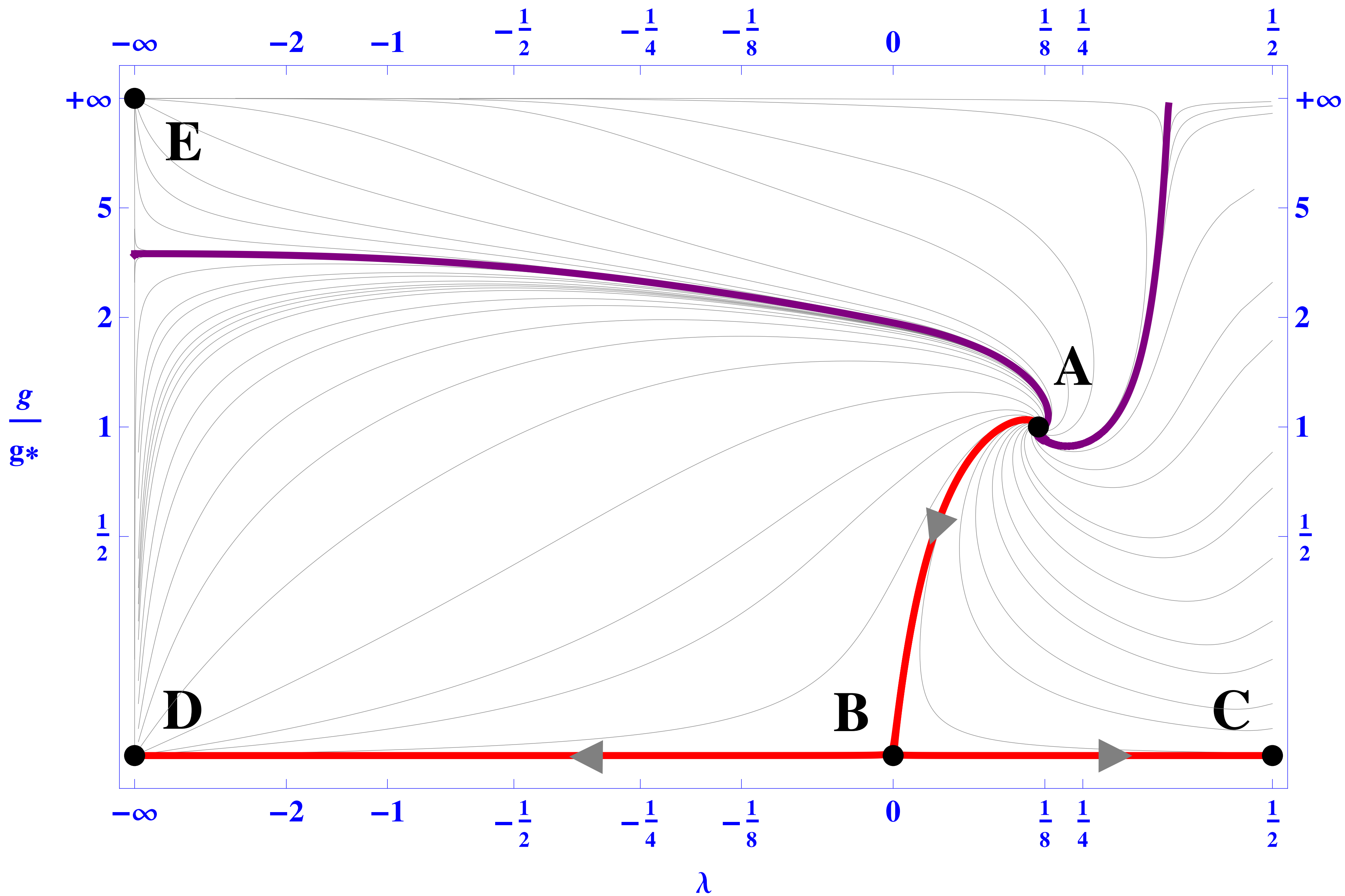
- Einstein-Hilbert action (Euclidean signature):

$$S_{EH} = \int d^D x \sqrt{\det g_{\mu\nu}} \left(\frac{-R + 2\Lambda}{16\pi G_N} \right) + S_{\mathrm{matter}}$$

Asymptotically safe!

Gravity II

Asymptotic Safety Scenario



- Non-trivial UV fixed point (A).
- Gaussian IR fixed point (B).

Gravity III

Exact Functional Identity

- **Wetterich equation [3] for QEG [4]:**

Litim regulator [5]

$$k\frac{d}{dk}\,\Gamma_k[g_{\mu\nu};\bar{g}_{\mu\nu}] = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k{}^{(2)}[g_{\mu\nu};\bar{g}_{\mu\nu}] + R_k\right)^{-1} k\frac{d}{dk}\,R_k\right]$$

- **Effective action:**

$$\Gamma_k = \int d^Dx\,\sqrt{\det g_{\mu\nu}}\left(\frac{-R+2\Lambda_k}{16\pi G_k}+\ldots\right) + \cancel{S_{\mathrm{matter},k}} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},\cancel{k}}$$

[3] C. Wetterich, Phys. Lett. B 301 (1993) 90.
[4] M. Reuter, Phys. Rev. D 57 (1998) 971, [hep-th/9605030].
[5] D. F. Litim, Phys. Lett. B 486 (2000) 92, [hep-th/0005245]; Phys. Rev. D 64 (2001) 105007, [hep-th/0103195].

Gravity V

β-functions

$$\partial_t = k \frac{\partial}{\partial k}$$
$$g_k \equiv k^{D-2} \; G_k$$
$$\lambda_k \equiv k^{-2} \; \Lambda_k$$

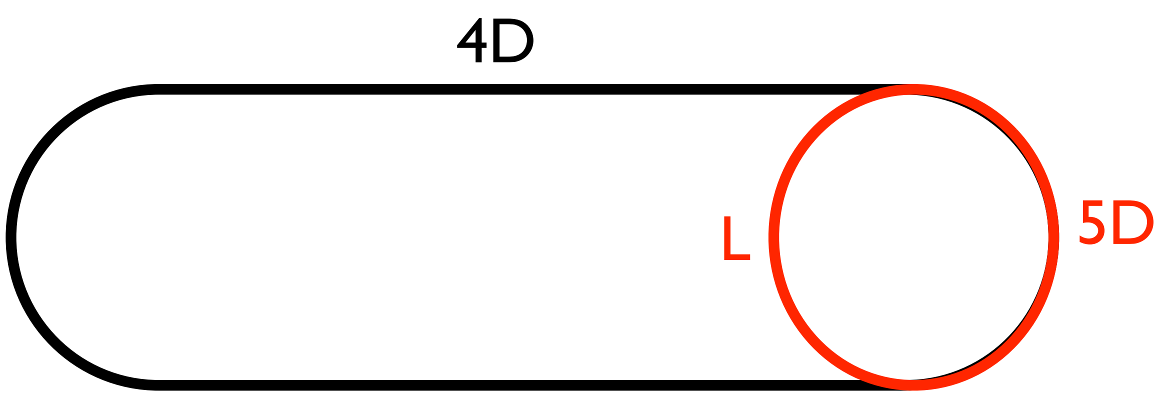
$$\partial_t g_k = \beta_g \left(g_k , \lambda_k \right) = \left(D - 2 + \eta_{Nk} \right) g_k$$

$$\partial_t \lambda_k = \beta_\lambda \left(g_k , \lambda_k \right) = \eta_{Nk} \, \lambda_k - 2 \, \lambda_k + g_k \; \Big(A_0 \left(\lambda_k \right) - \eta_{Nk} \; A_1 \left(\lambda_k \right) \Big)$$

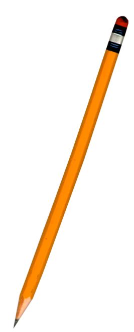
$$\eta_{Nk} = \frac{g_k \; B_0(\lambda_k)}{1 + g_k \; B_1(\lambda_k)}$$

Dimensional Reduction I

- 4+1 ADD model: Choose one extra dimension to be compact (periodic boundary conditions), sum over Kaluza-Klein modes.



$$\int \frac{d^D q}{(2\pi)^D} \to \frac{1}{L} \sum_n \int \frac{d^{D-1} q}{(2\pi)^{D-1}} \,, \qquad q_D \to \frac{2\pi\, n}{L}$$

- After many  and 20  : Expressions for the β-functions,

  e.g., in an approximate Background Field Flow:

$$A_0 \left(\lambda_k \,;\, kL \right) = \frac{1}{8\,\pi} \left[\frac{15}{\sqrt{1 - 2\,\lambda_k}} \,\coth \left(\frac{kL\sqrt{1-2\,\lambda_k}}{2} \right) - 10 \coth \left(\frac{kL}{2} \right) \right]$$

Dimensional Reduction II

Effective Coupling

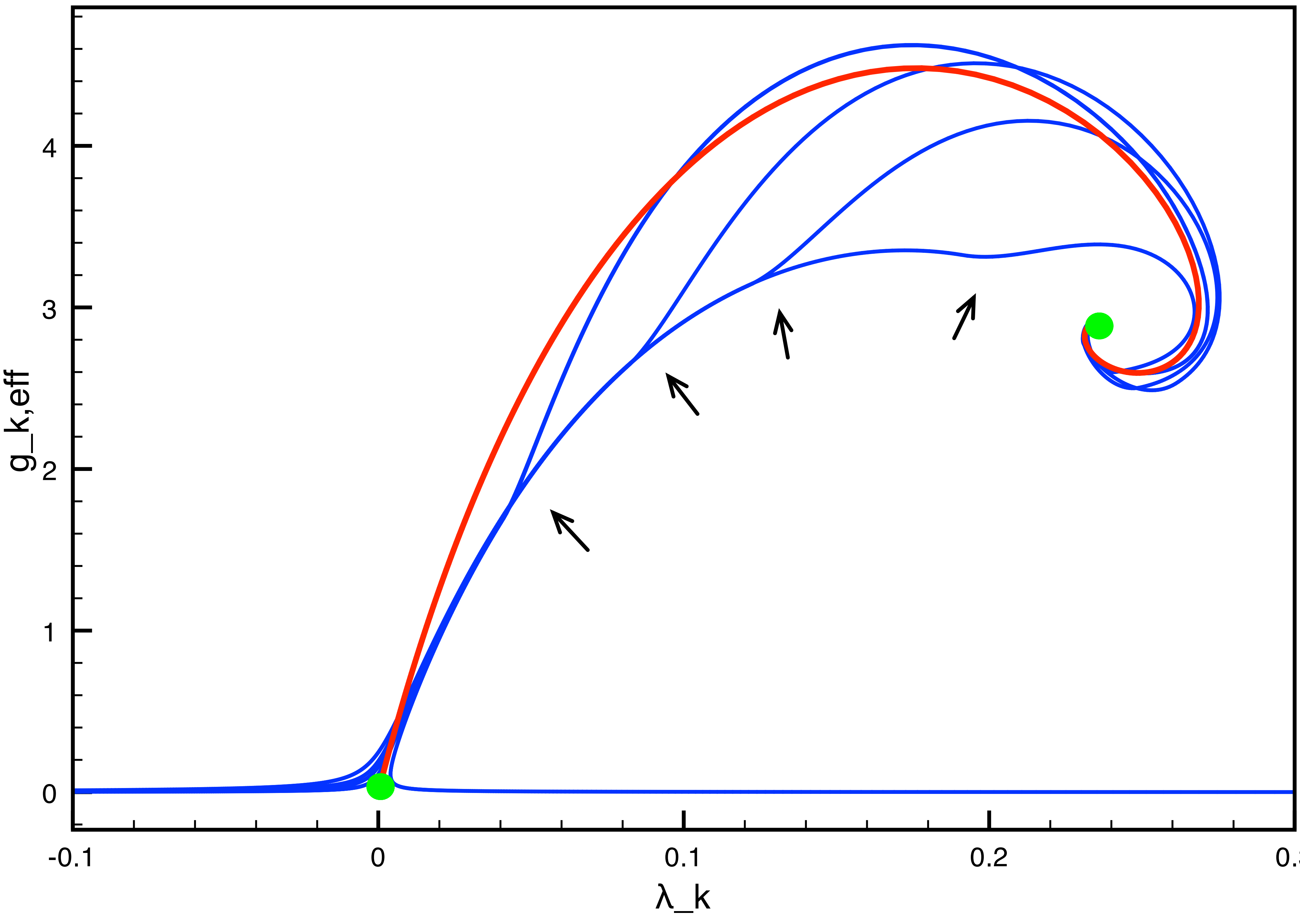
Consistency of limits requires: $g_k^{4D} = \frac{g_k^{5D}}{kL} \quad \text{for} \quad kL \ll 1.$

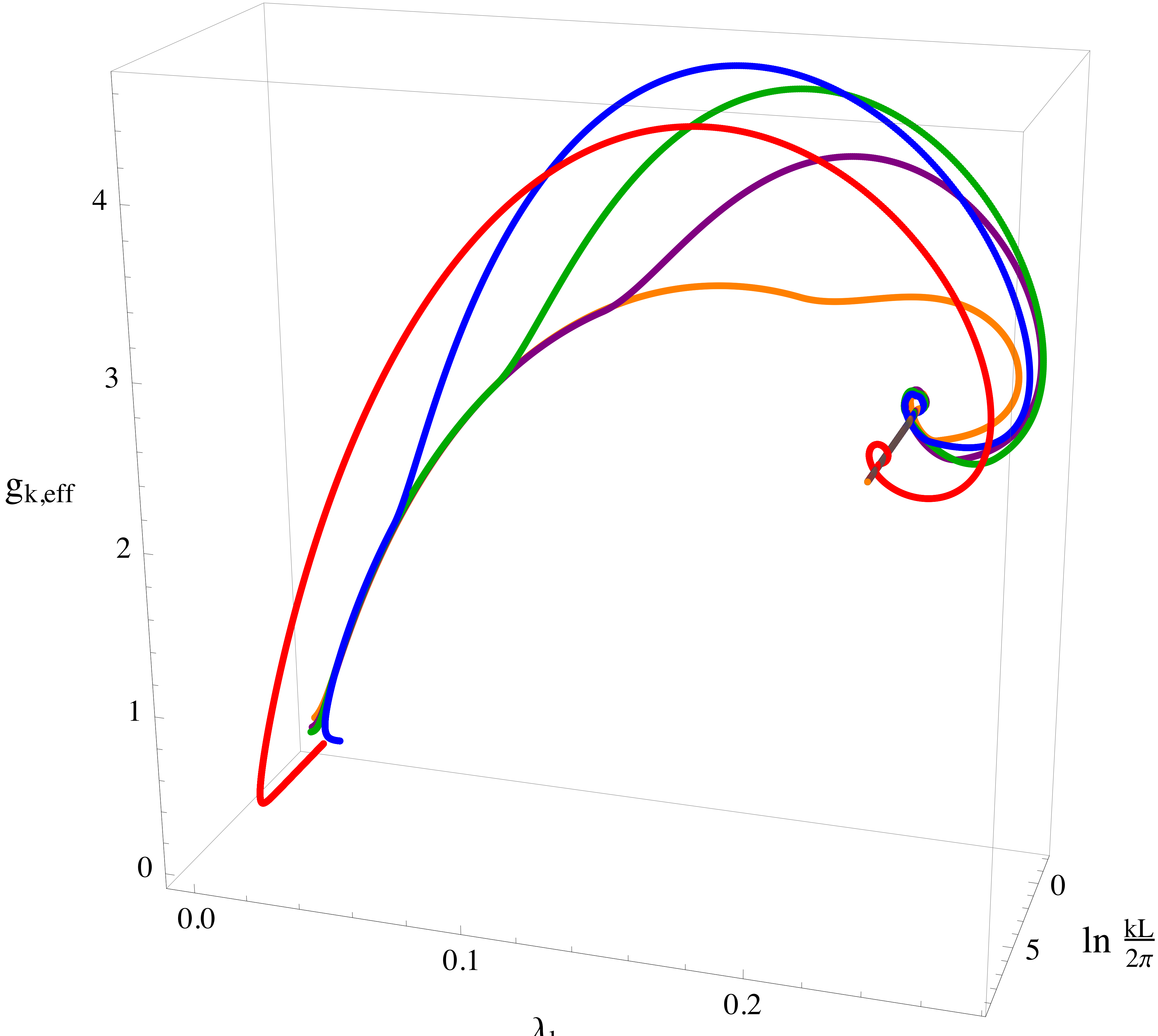
Follows also from the identification $G_N^{4D} = G_N^{5D}/L$

Define effective coupling such that
- it is well-defined and finite in both limits $L \to \infty$ and $L \to 0$.
- it connects smoothly both limits
- it behaves like k^2 for $k \ll 1/L$ and k^3 for $k \gg 1/L$, semi-class. regime.
- it displays the 4D to 5D crossover at $k \approx 1/L$.

$$\boxed{g_{\mathrm{k,eff}} = g_k B_0(\lambda_k; kL)/B_0^\infty} \quad \text{with} \quad B_0^\infty = \lim_{L\to\infty} B_0(\lambda_k; kL)$$

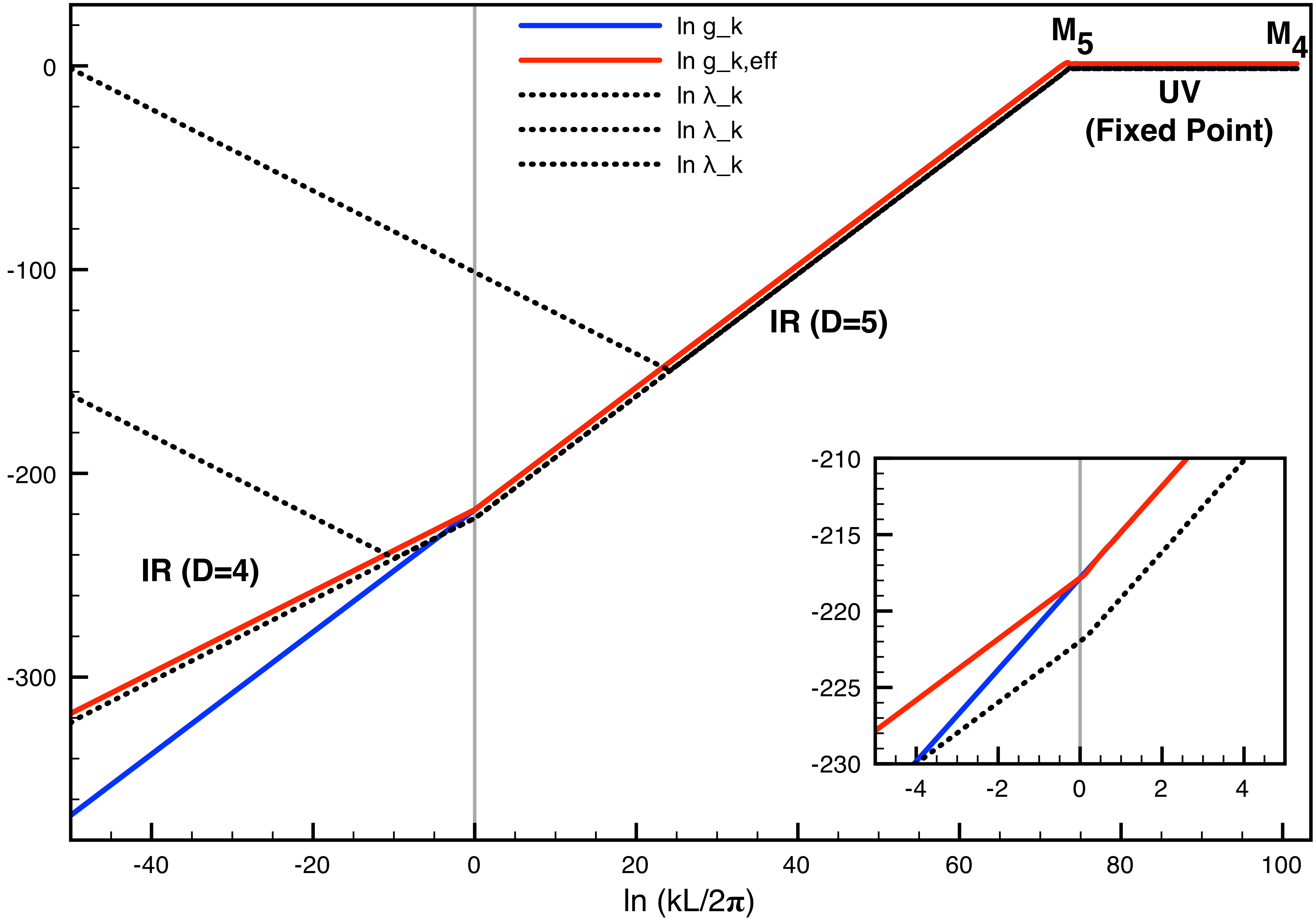
Results I





$g_{k,\mathrm{eff}}$
4
3
2
1
0
0.0
0.1
0.2
λ_k
0
5
$\ln \frac{kL}{2\pi}$

Results II

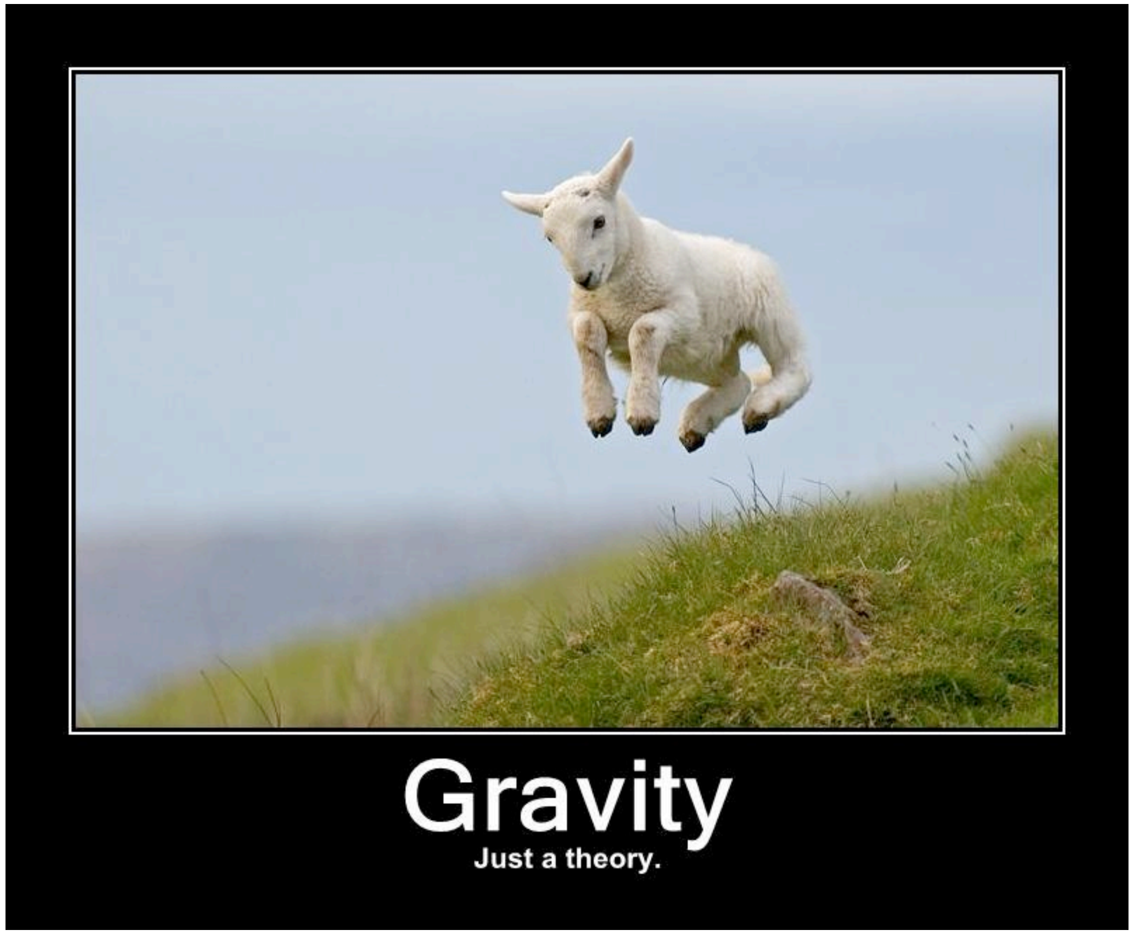


ln g_k
ln g_k,eff
ln λ_k
ln λ_k
ln λ_k
M5
M4
UV
(Fixed Point)
IR (D=5)
IR (D=4)
0
-100
-200
-300
-40
-20
0
20
40
60
80
100
ln (kL/2π)
-210
-215
-220
-225
-230
-4
-2
0
2
4

Conclusions & Outlook

- Asymptotic Safety Scenario to Einstein-Hilbert quantum gravity in four extended + one compact dimensions.
- Explicit example for an UV completion of the ADD model!!!
- 4D-5D crossover identified.
- [ ] Include several compact dimensions.
- [ ] Improve on truncation,

  e.g., f(R) gravity.
- [ ] Include matter.
- [ ] ...



Thank you !

Backup Slide

Result: Exact Functional Identity