

# do interacting UV fixed points exist fundamentally, and if so: what can we do with them?

Daniel F Litim

US

University of Sussex

**ERG2014**

7th International Conference on the Exact Renormalization Group

# standard model

local QFT for fundamental interactions

**strong** nuclear force

**weak** force

**electromagnetic** force

degrees of freedom

spin 0 (the **Higgs** has finally arrived)

spin 1/2 (quite a few)

spin 1

perturbatively renormalisable & **predictive**

# standard model

local QFT for fundamental interactions

**strong** nuclear force

**weak** force

**electromagnetic** force

challenges

**Higgs, QED**: maximum UV extension?

how does **quantum gravity** fit in?

...

**interacting UV fixed points**

# UV fixed points



**ERG2014**

**7th International Conference on the Exact Renormalization Group**

# perturbation theory


theory with coupling  $\alpha$ :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = -B \alpha^2$$

$$\alpha_* \ll 1$$

free fixed point


$$\alpha_* = 0$$

# perturbation theory


theory with coupling  $\alpha$ :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = -B \alpha^2$$

$$\alpha_* \ll 1$$

free fixed point


$$\alpha_* = 0$$

**QED, Higgs**

$$B < 0$$

**IR fixed point**

predictive up to maximal UV extension

# asymptotic freedom


theory with coupling  $\alpha$ :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = -B \alpha^2$$

$$\alpha_* \ll 1$$

free fixed point


$$\alpha_* = 0$$

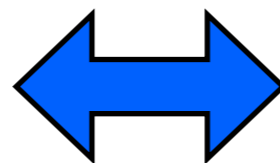
QCD

$$B > 0$$

UV fixed point

perturbative renormalisability & asymptotic freedom  
predictive up to highest energies

fundamental  
definition of QFT



UV fixed point

Wilson '71

# interacting fixed point

theory with coupling  $\alpha$ :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* \ll 1$$

perturbative non-renormalisability:  $A > 0$



# interacting fixed point

theory with coupling  $\alpha$ :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* \ll 1$$

fixed points

$$\alpha_* = 0$$

$$\alpha_* = A/B$$

# interacting fixed point

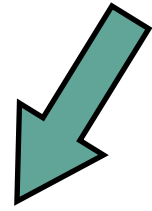
theory with coupling  $\alpha$ :

$$t = \ln \mu / \Lambda$$


$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* \ll 1$$

fixed points  
if  $A > 0, B > 0$ :


$$\alpha_* = 0$$

**IR**


$$\alpha_* = A/B$$

**UV**

# interacting fixed point

theory with coupling  $\alpha$ :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* \ll 1$$



fixed points

$$\alpha_* = 0$$

$$\alpha_* = A/B$$

**epsilon** expansion:

$$\epsilon = D - D_c$$

**large-N** expansion:

many fields

# perturbation theory

theory with coupling  $\alpha$ :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* \ll 1$$

**gravitons**

$$D = 2 + \epsilon : \quad \alpha = G_N(\mu) \mu^{D-2}$$

Gastmans et al '78  
Weinberg '79  
Kawai et al '90

**fermions**

$$D = 2 + \epsilon : \quad \alpha = g_{\text{GN}}(\mu) \mu^{2-D}$$

Gawedzki, Kupiainen '85  
de Calan et al '91

**gluons**

$$D = 4 + \epsilon : \quad \alpha = g_{\text{YM}}^2(\mu) \mu^{4-D}$$

Peskin '80  
Morris '04

non-perturbative  
renormalisability

$$A = \epsilon \ll 1, \quad B = \mathcal{O}(1) > 0$$

# exact interacting UV fixed points in 4D quantum gauge theories

with F Sannino  
1406.2337

**ERG2014**

**7th International Conference on the Exact Renormalization Group**

# gauge theory with fermions

SU(**NC**) YM with **NF** fermions:

$$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$$t = \ln \mu/\Lambda$$

$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3$$

$$\alpha_* \ll 1$$



$$\alpha_* = 0$$

$$\alpha_g^* = B/C$$

# gauge theory with fermions


SU(**NC**) YM with **NF** fermions:

$$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$$t = \ln \mu/\Lambda$$

$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3$$

$$\alpha_* \ll 1$$


$$\alpha_* = 0 \quad \alpha_g^* = B/C$$

**large-NF,NC (Veneziano) limit:**

$\epsilon$  continuous


$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

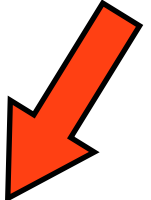
# gauge theory with fermions

SU(**NC**) YM with **NF** fermions:  $\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$   $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3$$

$$\alpha_* \ll 1$$


$$\alpha_* = 0$$


$$\alpha_g^* = B/C$$

interacting  
fixed points:

$B < 0$  &  $C < 0$  : **UV fixed point**  
no asymptotic freedom

$B > 0$  &  $C > 0$  : Caswell-Banks-Zaks  
**IR fixed point**




# gauge theory with fermions

SU(**NC**) YM with **NF** fermions:  $\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$   $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3$$

$$\alpha_* \ll 1$$


$$\alpha_* = 0 \quad \alpha_g^* = B/C$$

**here:**  $B = -\frac{4\epsilon}{3} < 0$  &  $C > 0$


**no physical  
fixed point**

# gauge theory with fermions

SU(**NC**) YM with **NF** fermions:  $\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$   $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3$$

$$\alpha_* \ll 1$$


$$\alpha_* = 0 \quad \alpha_g^* = B/C$$

 **scalar** fields & **Yukawa** couplings required

# gauge-Yukawa theory

## Lagrangian

$$L_{\text{YM}} = -\frac{1}{2} \text{Tr} F^{\mu\nu} F_{\mu\nu}$$

$$L_F = \text{Tr} (\bar{Q} i \not{D} Q)$$

$$L_Y = y \text{Tr} (\bar{Q} H Q)$$

$$L_H = \text{Tr} (\partial_\mu H^\dagger \partial^\mu H)$$

$$L_U = -u \text{Tr} (H^\dagger H)^2$$

$$L_V = -v (\text{Tr} H^\dagger H)^2.$$

small parameter

## couplings

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}$$

$$\alpha_h = \frac{u N_F}{(4\pi)^2}, \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}.$$

**no asymptotic freedom**

$$0 < \epsilon \ll 1 \quad \epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

# gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left( 25 + \frac{26}{3}\epsilon \right) \alpha_g - 2 \left( \frac{11}{2} + \epsilon \right)^2 \alpha_y \right\}$$

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon) \alpha_y - 6 \alpha_g \right\} .$$

$$\beta_h = -(11 + 2\epsilon) \alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v(\alpha_v + 4\alpha_h + \alpha_y) .$$

# gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left( 25 + \frac{26}{3}\epsilon \right) \alpha_g - 2 \left( \frac{11}{2} + \epsilon \right)^2 \alpha_y \right\}$$

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon) \alpha_y - 6 \alpha_g \right\} .$$

$$\beta_h = -(11 + 2\epsilon) \alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v(\alpha_v + 4\alpha_h + \alpha_y) .$$

# results

UV fixed point

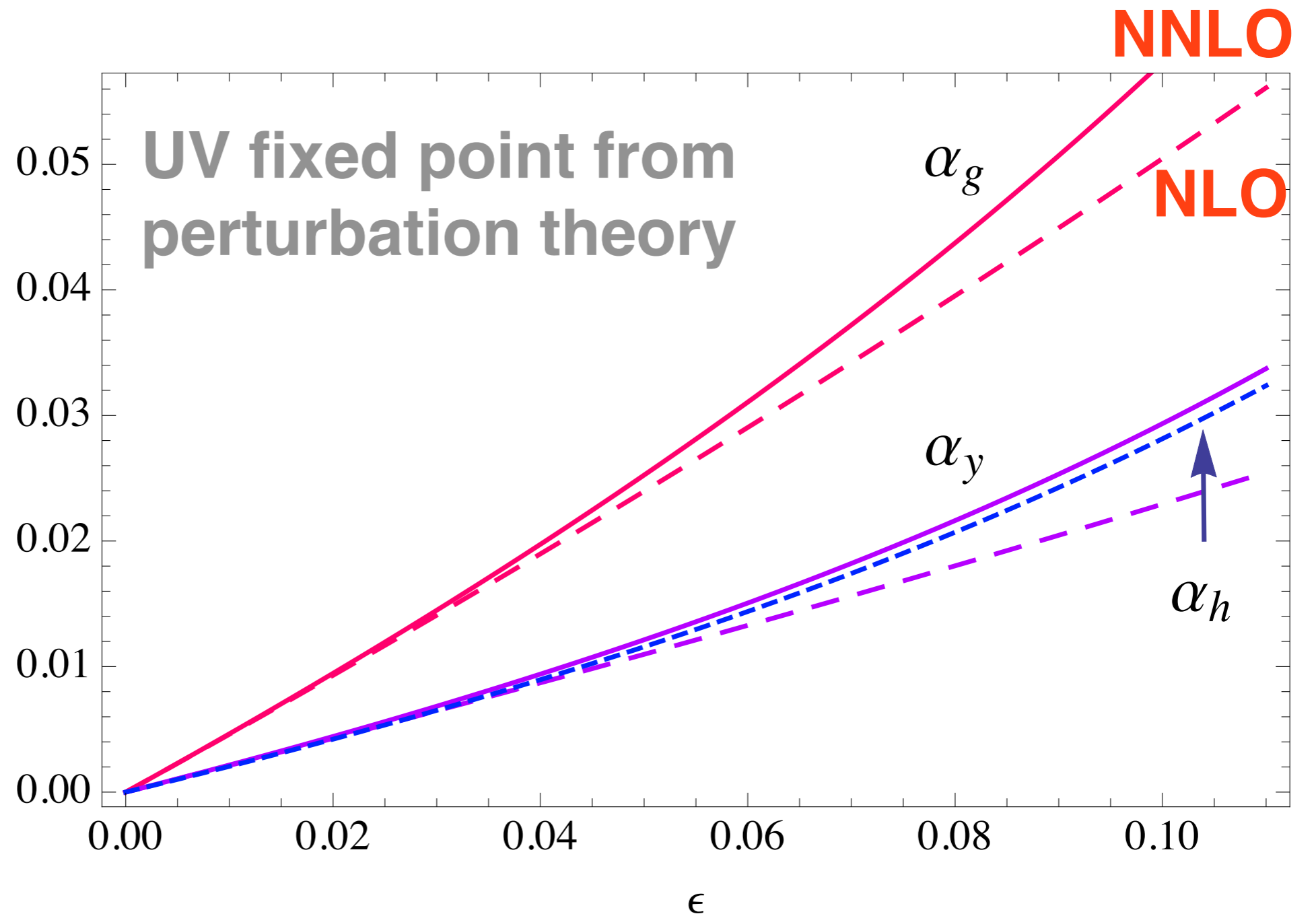
$$\begin{aligned} \alpha_g^* &= \text{NLO} \quad \text{NNLO} \quad \text{NNNLO} \\ &= 0.4561 \epsilon + 0.7808 \epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_y^* &= 0.2105 \epsilon + 0.5082 \epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_h^* &= 0.1998 \epsilon + 0.5042 \epsilon^2 + \mathcal{O}(\epsilon^3). \end{aligned}$$

vacuum stability

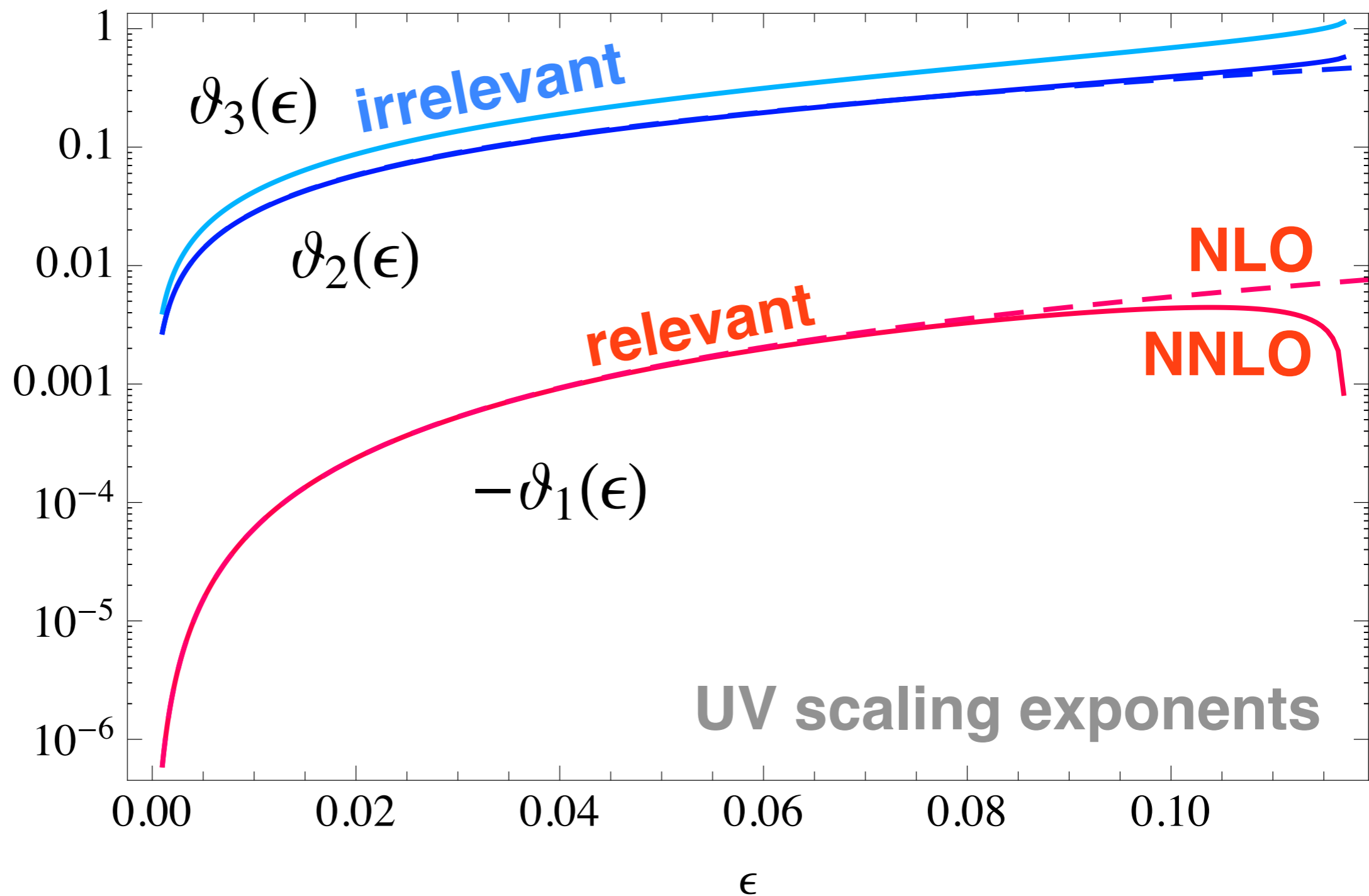
$$\alpha_h^* + \alpha_{v2}^* < 0 < \alpha_h^* + \alpha_{v1}^*$$

coupling	order in perturbation theory		
	1	2	3
$\alpha_g$	1	2	3
$\alpha_y$	0	1	2
$\alpha_h$	0	0	1
$\alpha_v$	0	0	1
approximation level	LO	NLO	NNLO

# results

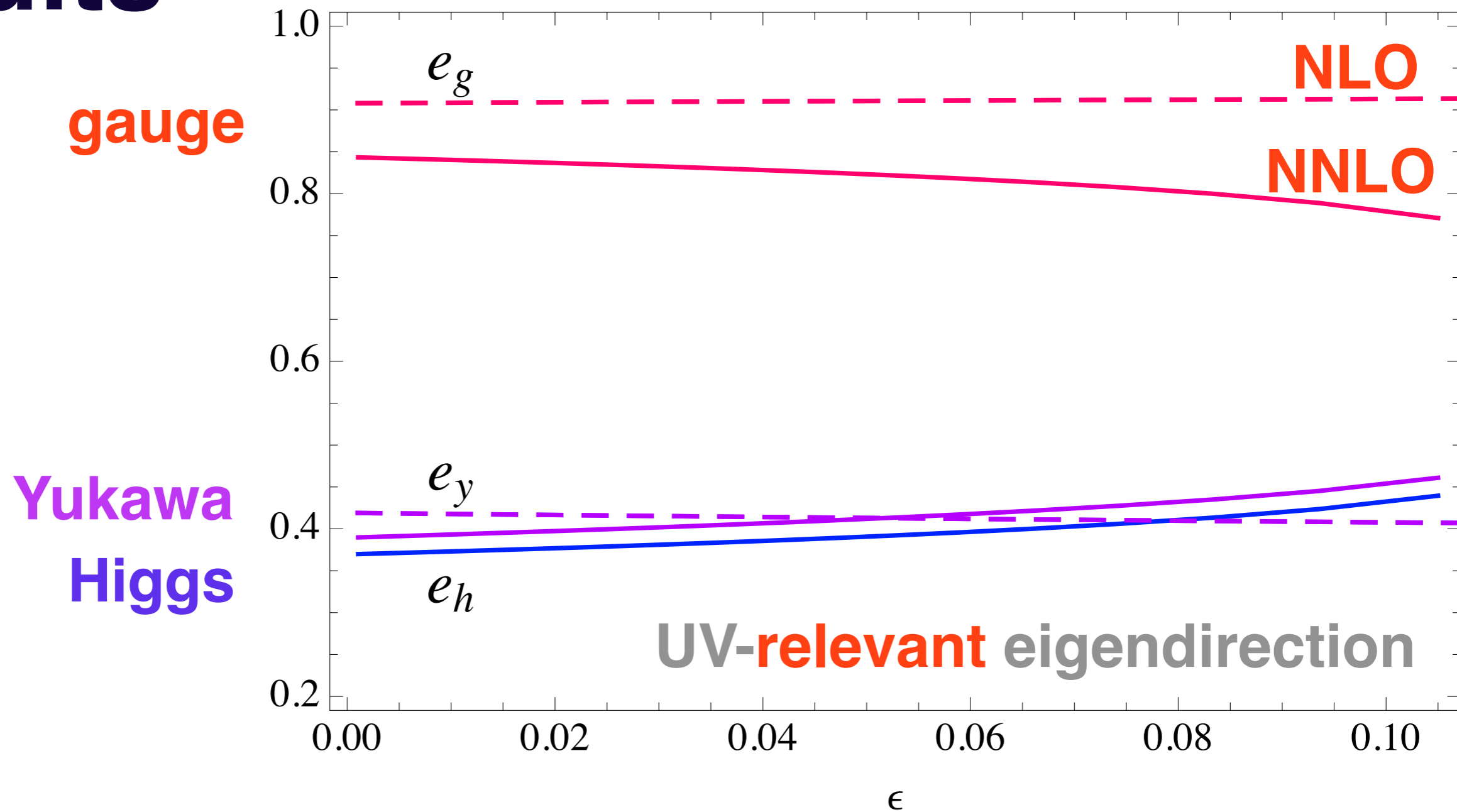


# results



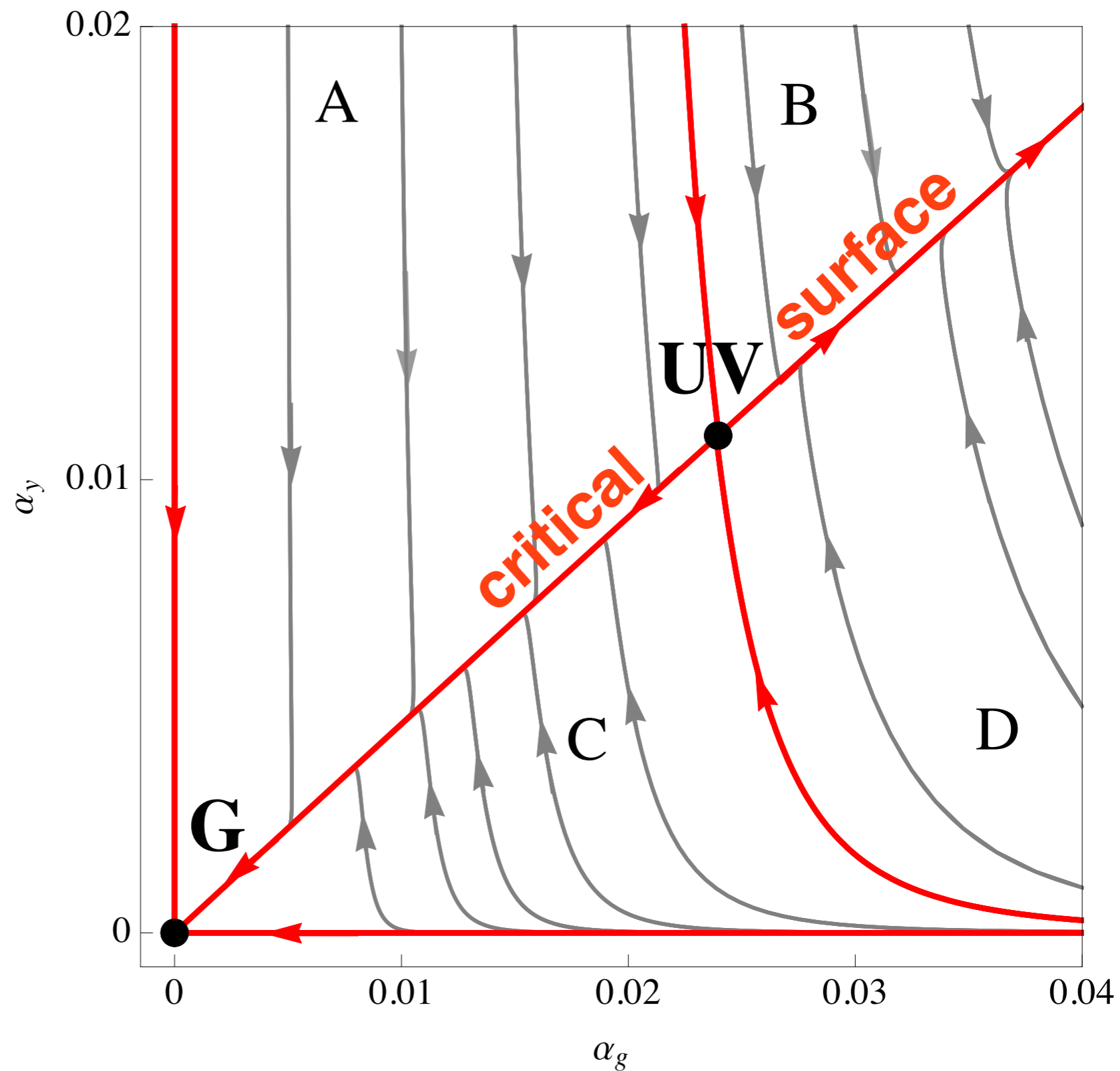


# results

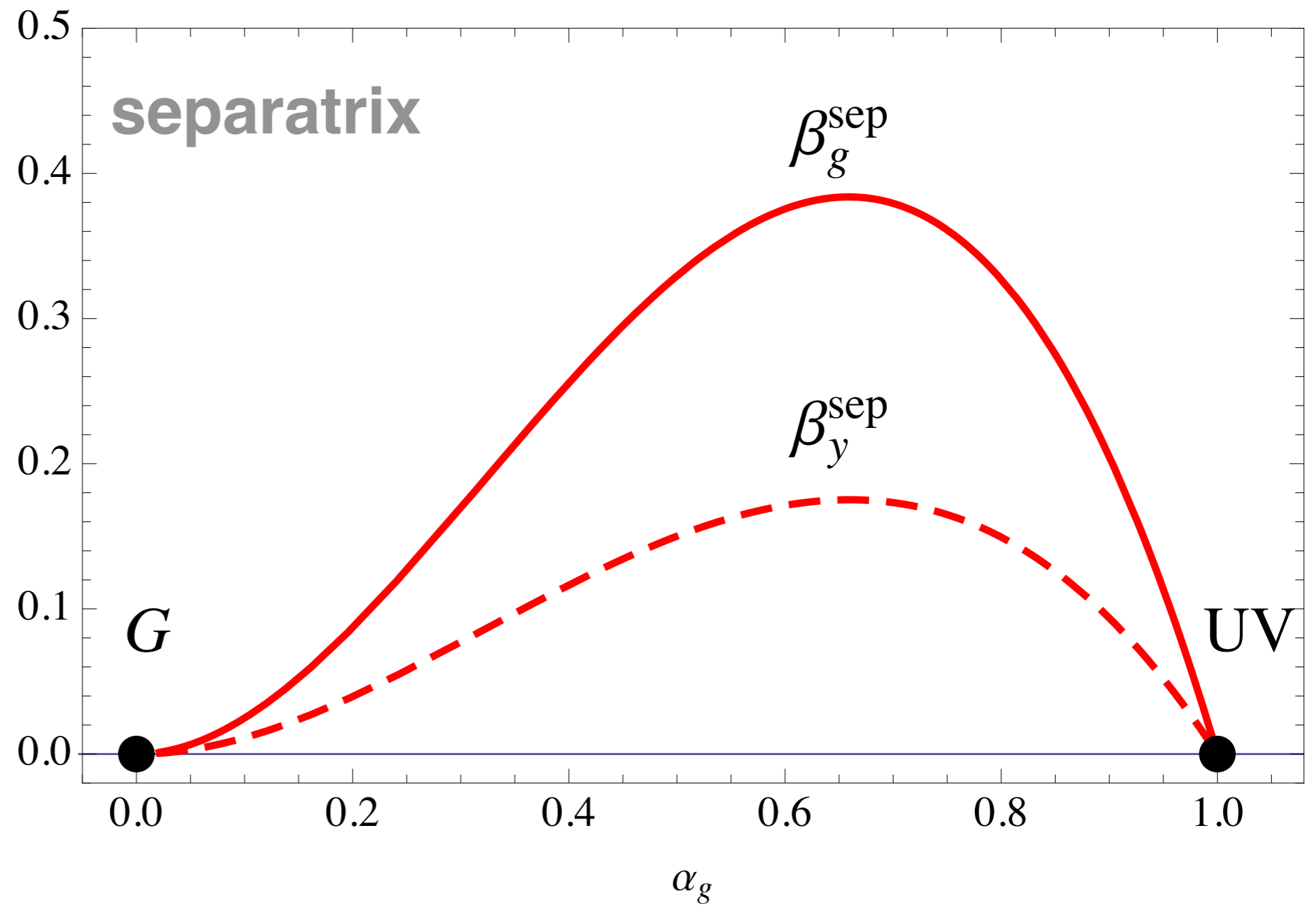


# results

## phase diagram



# results



# results

asymptotic safety at small epsilon

unitarity

non-triviality and elementary scalars

asymptotic safety at large epsilon

# what can we do with it?

interacting UV fixed points in 4D

**cooperation** decisive

generic, largely independent of gauge group, representation & gauge charges

exploit ideas from AdS/CFT & holography

Seiberg duality - without supersymmetry?

# what can we do with it?

interacting UV fixed points in 4D

UV-complete extensions of the Standard Model

applicable for finite N    e.g.  $(NC, NF) = (5, 28)$

new handle on

(i) **hierarchy problem**

(ii) (non-)triviality of scalar sector in 4D

**scalars elementary**

# what can we do with it?

interacting UV fixed points in 4D

UV-complete extensions of the Standard Model

helps with canonical quantisation of gravity

conformal matter + conformal mode (‘t Hooft)

asymptotically safe gravity with matter

# **UV fixed points in 4D quantum gravity**

with K Falls, DL, K Nikolakopoulos & C Rahmede  
1301.4191 and forthcoming



# evidence for UV fixed point

overviews: DL 0810.3675 and 1102.4624

## gravitation

**Einstein-Hilbert** (Souma '99, Reuter, Lauscher '01, DL '03)

**higher dimensions, dimensional reduction** (DL '03, Fischer, DL '05)

**f(R), polynomials in R** (Lauscher, Reuter, '02, Codello, Percacci, Rahmede '08, Machado, Saueressig '09, Benedetti, Caravelli '12, Dietz, Morris '12, Falls, DL, Nikolakopoulos, Rahmede '13)

**local potential approximation** (Benedetti, Caravelli '12, Dietz, Morris, '12, Demmel, Saueressig, Zanusso '12, Falls, DL, Nikolakopoulos, Rahmede '13, Benedetti '13, Benedetti, Guarnieri '13)

**higher-derivative gravity** (Codello, Percacci '05)  
(Benedetti, Saueressig, Machado '09, Niedermaier '09)

**conformally reduced gravity** (DL, Rahmede, in prep.)  
(Reuter, Weyer '09, Machado, Percacci '10, DL, Satz '12)

**Holst action + Immirzi parameter** (Daum, Reuter '10, Benedetti, Speciale '11)

**signature effects** (Manrique, Rechenberger, Saueressig '11)

## gravitation + matter

**matter** (Percacci '05, Perini, Percacci '05, Narain, Percacci '09, Narain, Rahmede '09, Codello '11, Eichhorn et al '13)

### Yang-Mills gravity

**1-loop:** (Robinson, Wilzcek '05, Pietrokowski, '06, Toms '07, Ebett, Plefka, Rodigast '08)


**beyond:** (Manrique, Reuter, Saueressig '09, Folkerts, DL, Pawlowski, '11, Harst, Reuter '11)

# quantum gravity


running coupling  $g(k) = G_N(k)k^{D-2}$

$$\partial_t g = (D - 2 + \eta_N) g$$

$$t = \ln k / \Lambda_c$$


$$g_* \neq 0$$

UV


$$g_* = 0$$

IR

fixed points

**large** anomalous dimension

$$\eta_N = \eta_N(g, \text{all other couplings})$$

**large** UV scaling exponents

$$\vartheta \approx \mathcal{O}(1)$$

**strong** coupling effects

$$g_* \approx \mathcal{O}(1)$$

**relevant** vs **irrelevant**

invariants not known a priori

# asymptotic freedom

vs

# asymptotic safety

$$g_* = 0$$

anomalous dimensions

$$\eta_A = 0$$

**canonical** power counting

$\{\mathcal{V}_{G,n}\}$  are known

$F^{256}$  irrelevant !

$$g_* \neq 0$$

anomalous dimensions

$$\eta_N \neq 0$$

**non-canonical** power counting

$\{\mathcal{V}_n\}$  are **not** known

$R^{256}$

relevant  
marginal  
irrelevant



# bootstrap

**hypothesis** ordering follows  
canonical dimension

strategy

**Step 1** retain invariants up to mass dimension  $D$

**Step 2** compute  $\{\mathcal{V}_n\}$  (eg. RG, lattice, holography)

**Step 3** enhance  $D$ , and iterate

convergence (no convergence) of the iteration:

**hypothesis** supported (refuted)

# $f(R)$

$$\Gamma_k \propto f(R)$$

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n k^{d_n} \int d^4x \sqrt{g} R^n$$

invariants up to  $D = 2(N - 1)$

functional renormalisation:

$$k \frac{d\Gamma_k}{dk} = \frac{1}{2} \text{Tr} \left[ \left( \frac{\delta^2 \Gamma_k[\phi]}{\delta\phi \delta\phi} + R_k \right)^{-1} k \frac{dR_k}{dk} \right] = \frac{1}{2} \text{Diagram}$$

here:

M Reuter hep-th/9605030

Falls, DL, Nikolakopoulos, Rahmede

[1301.4191.pdf](#)

DL [hep-th/0103195](#)  
[hep-th/0312114](#)

A Codello, R Percacci, C Rahmede 0705.1769, 0805.2909  
P Machado, F Saueressig 0712.0445

# f(R)

recursive solution

$$\lambda_n(\lambda_0, \lambda_1) = \frac{P_n(\lambda_0, \lambda_1)}{Q_n(\lambda_0, \lambda_1)}$$

boundary condition

$$\lambda_N = 0 \quad \& \quad \lambda_{N+1} = 0$$

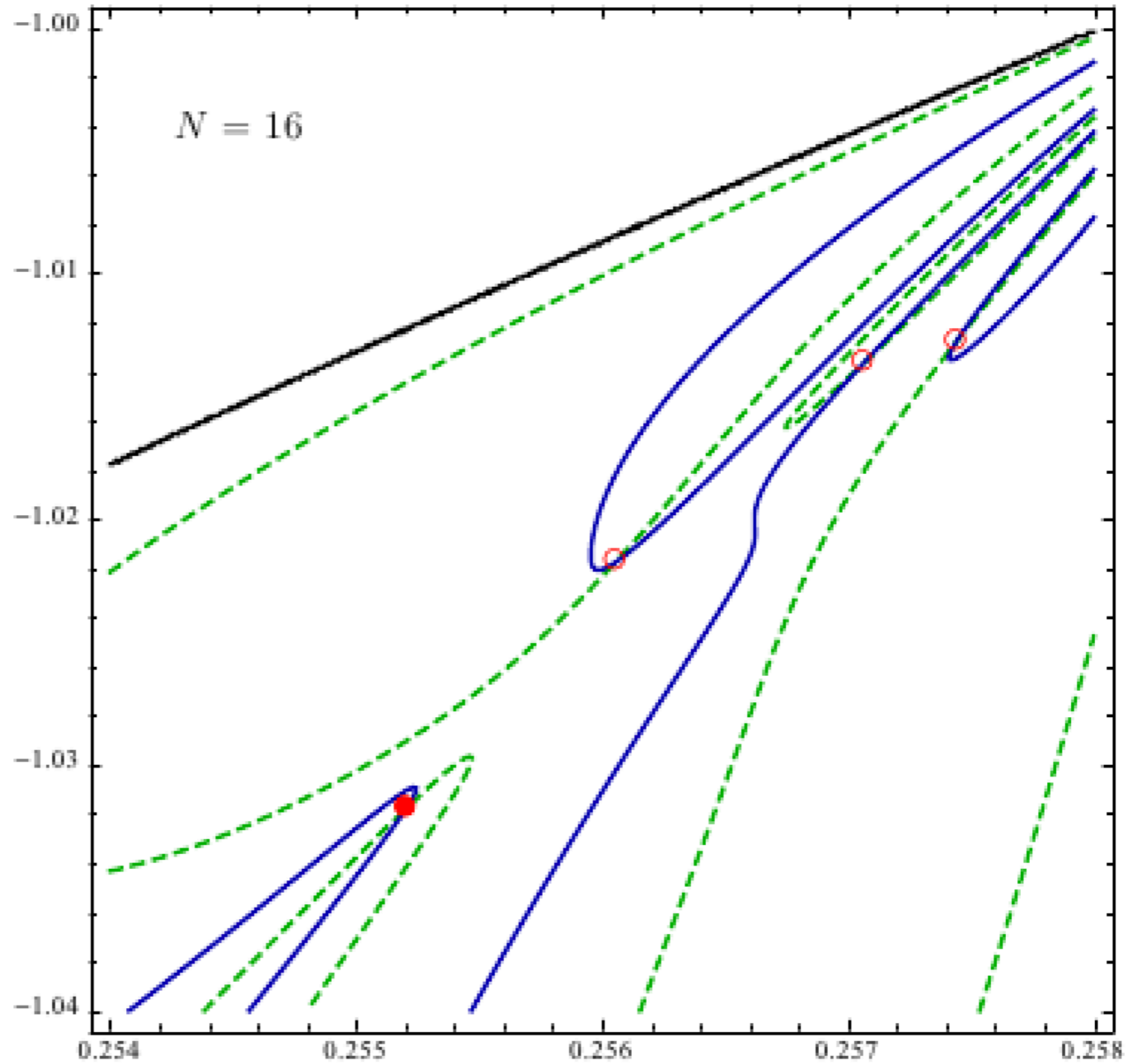
polynomials grow large, eg.

$$P_{35} \approx 45.000 \quad \text{terms}$$

# f(R)

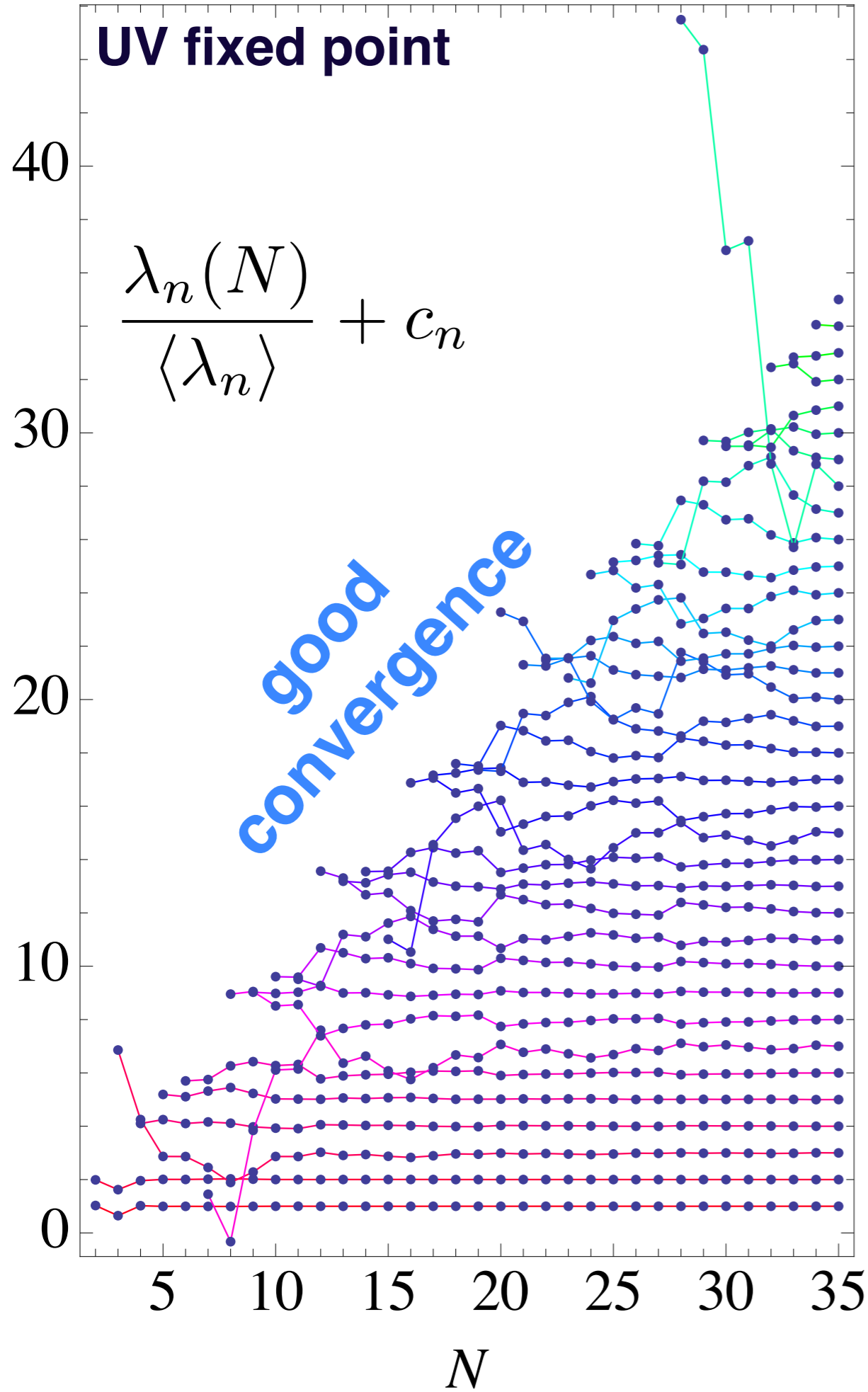
boundary condition

$$\lambda_N = 0$$
$$\lambda_{N+1} = 0$$



# UV fixed point

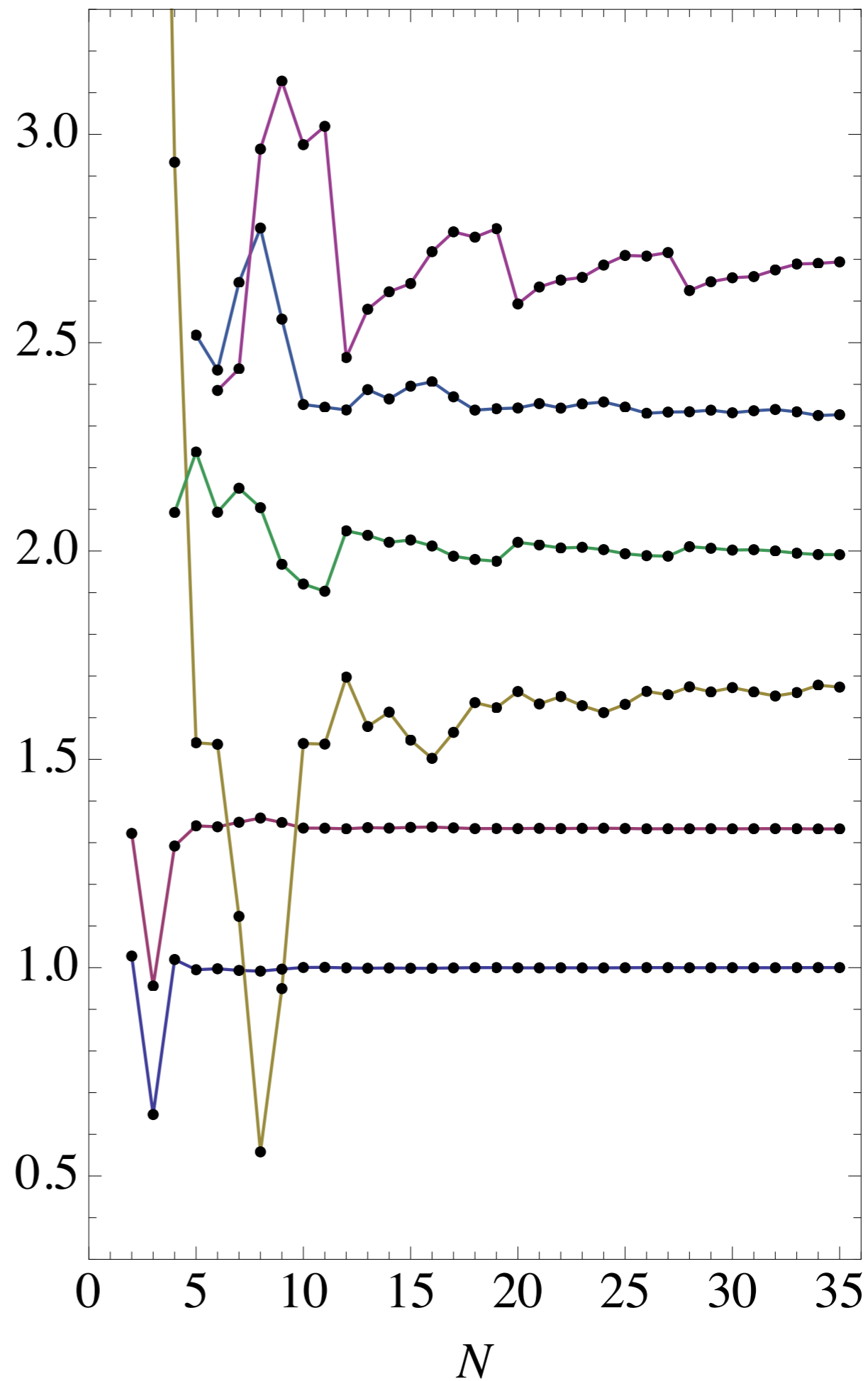
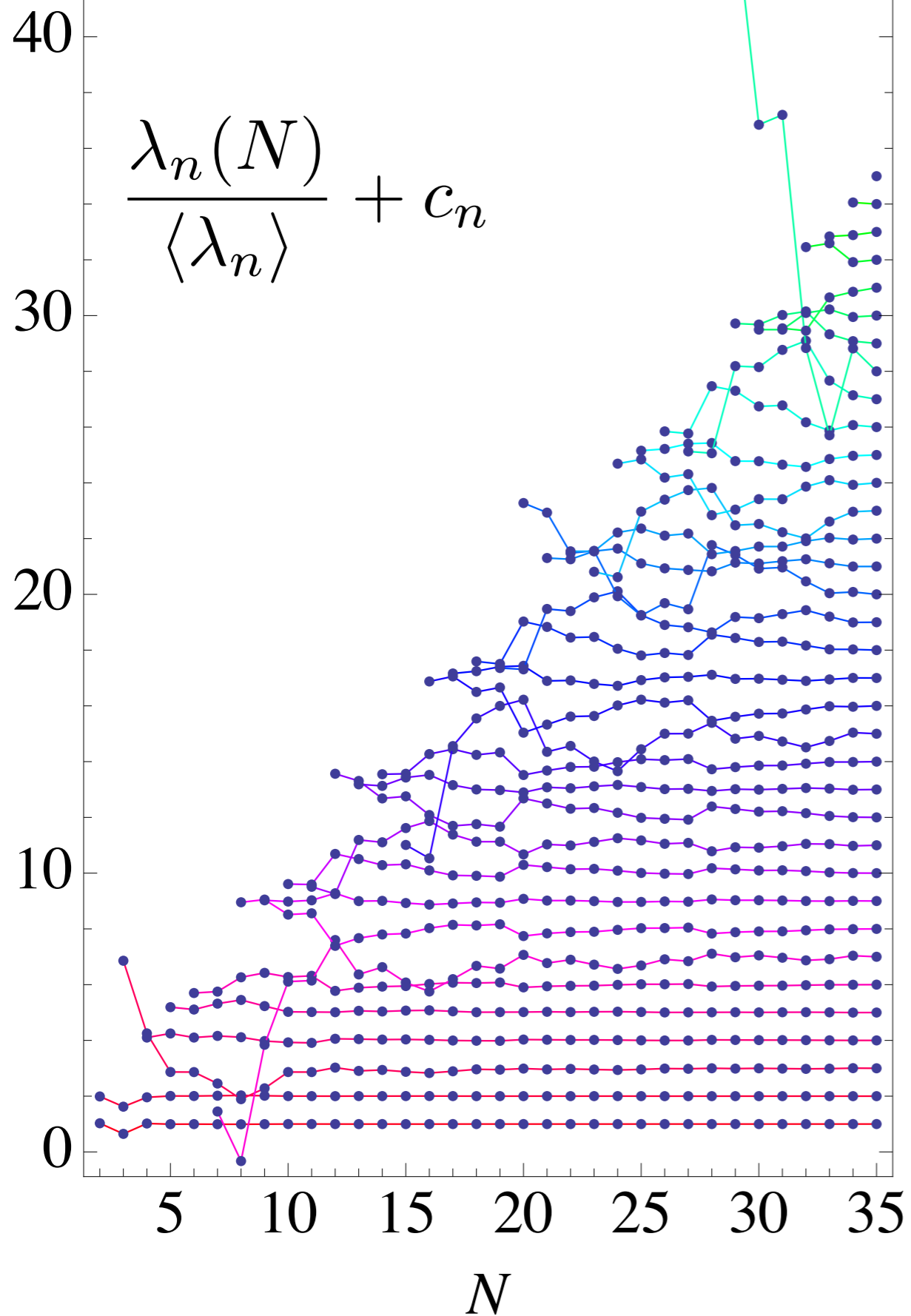
$$\frac{\lambda_n(N)}{\langle \lambda_n \rangle} + c_n$$





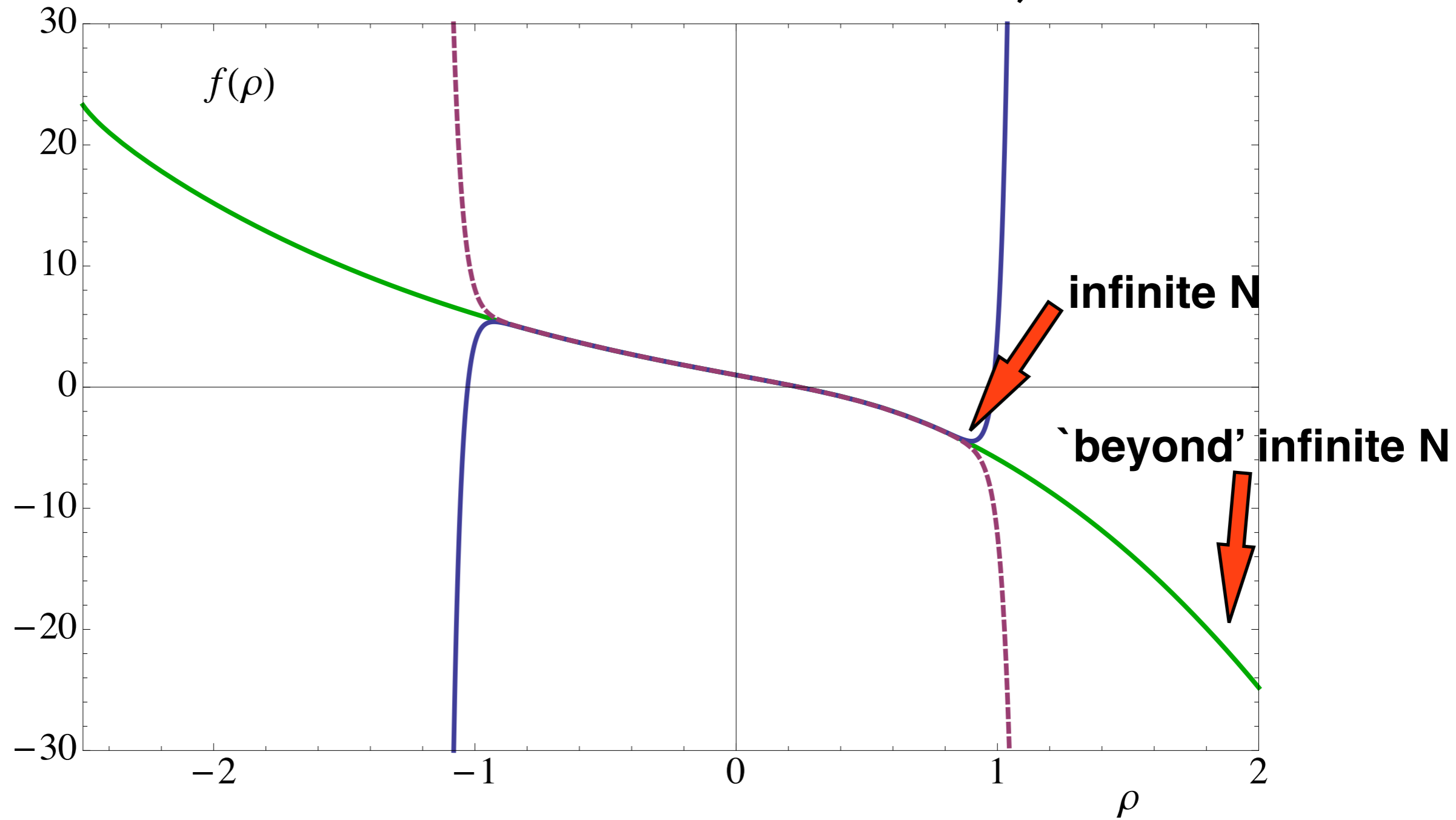
# UV fixed point

$$\frac{\lambda_n(N)}{\langle \lambda_n \rangle} + c_n$$



# f(R) quantum gravity

UV scaling solution

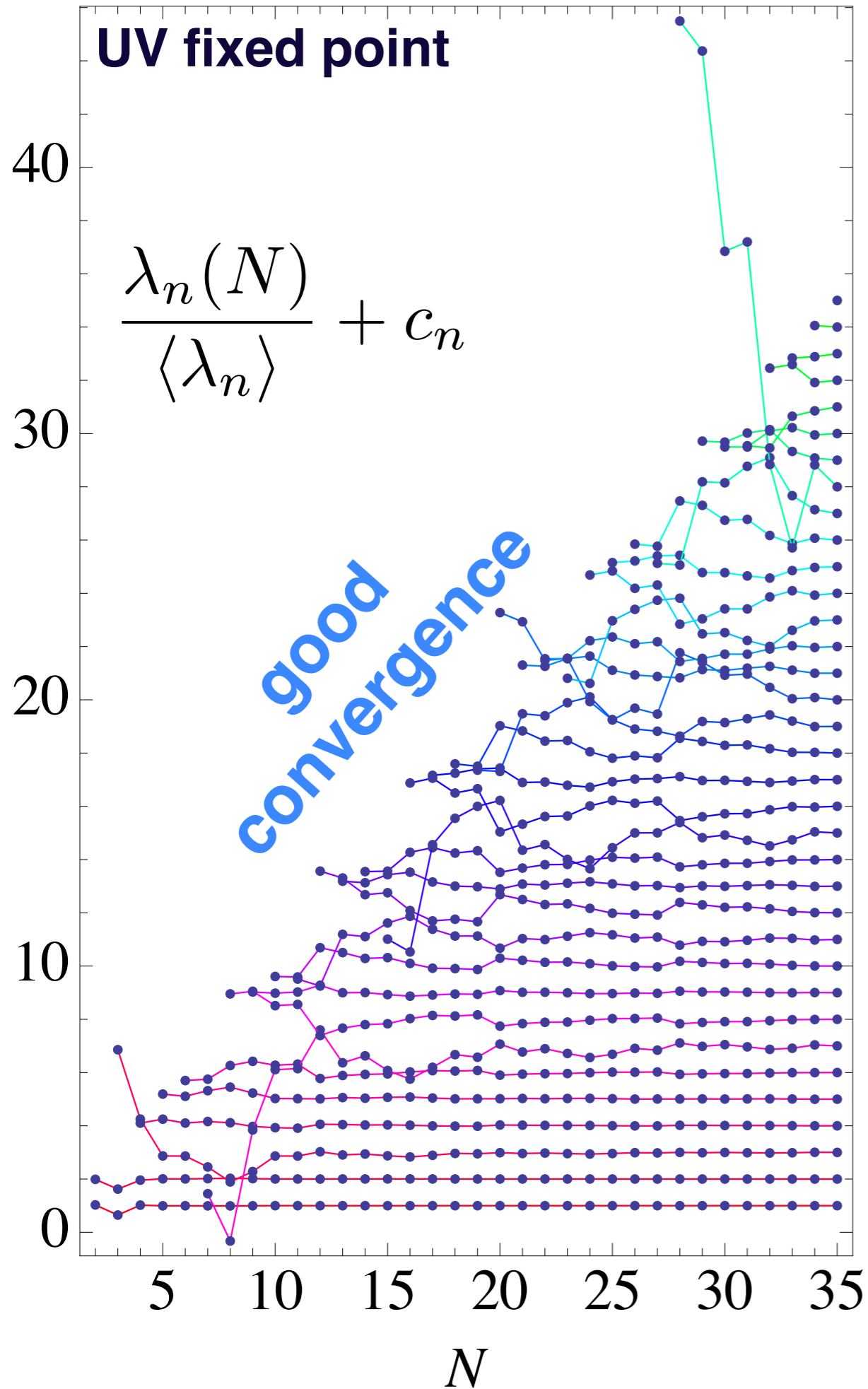


**radius of convergence**

$$\rho_c \approx 0.82 \pm 5\%$$

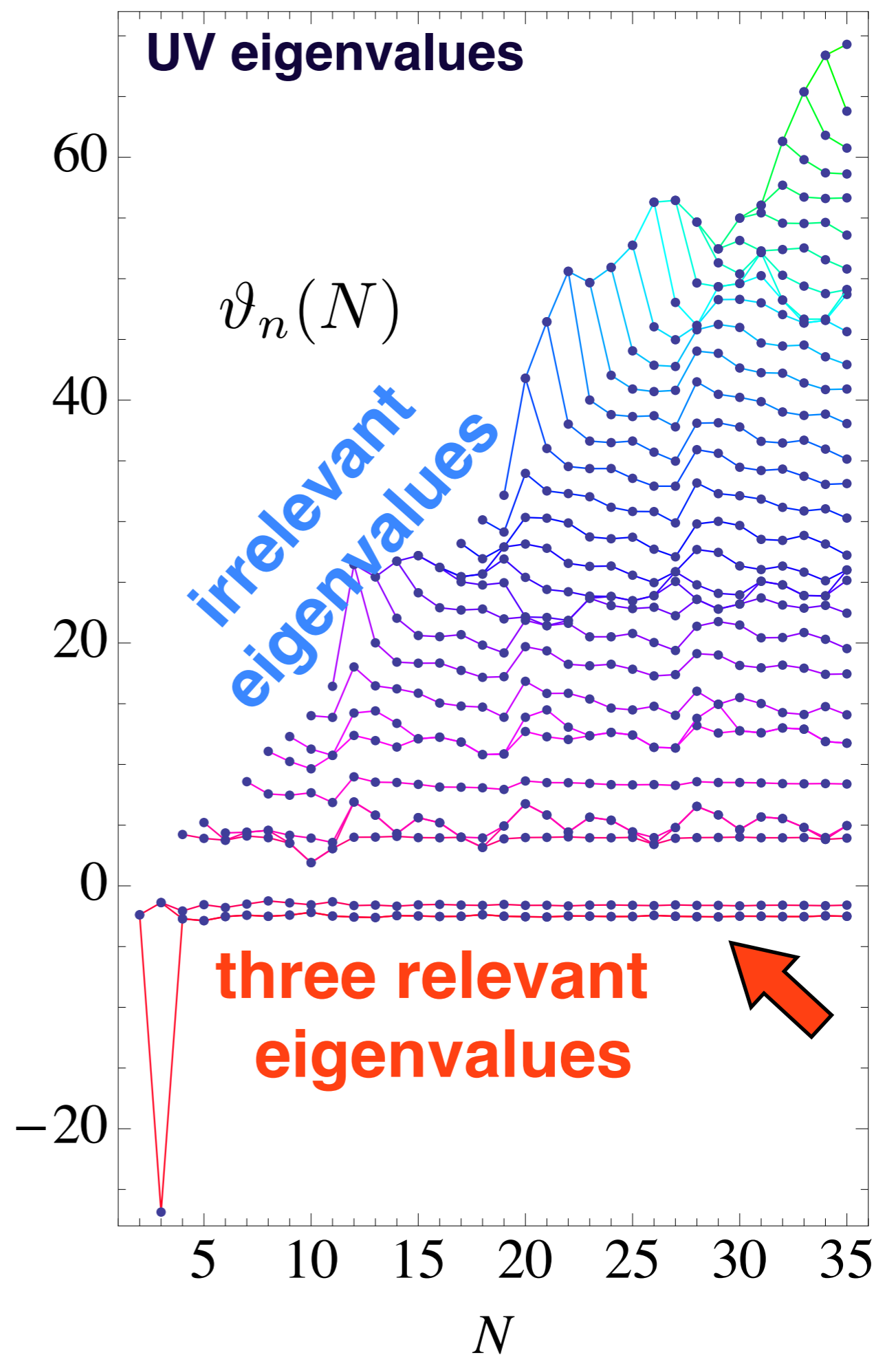
### UV fixed point

$$\frac{\lambda_n(N)}{\langle \lambda_n \rangle} + c_n$$

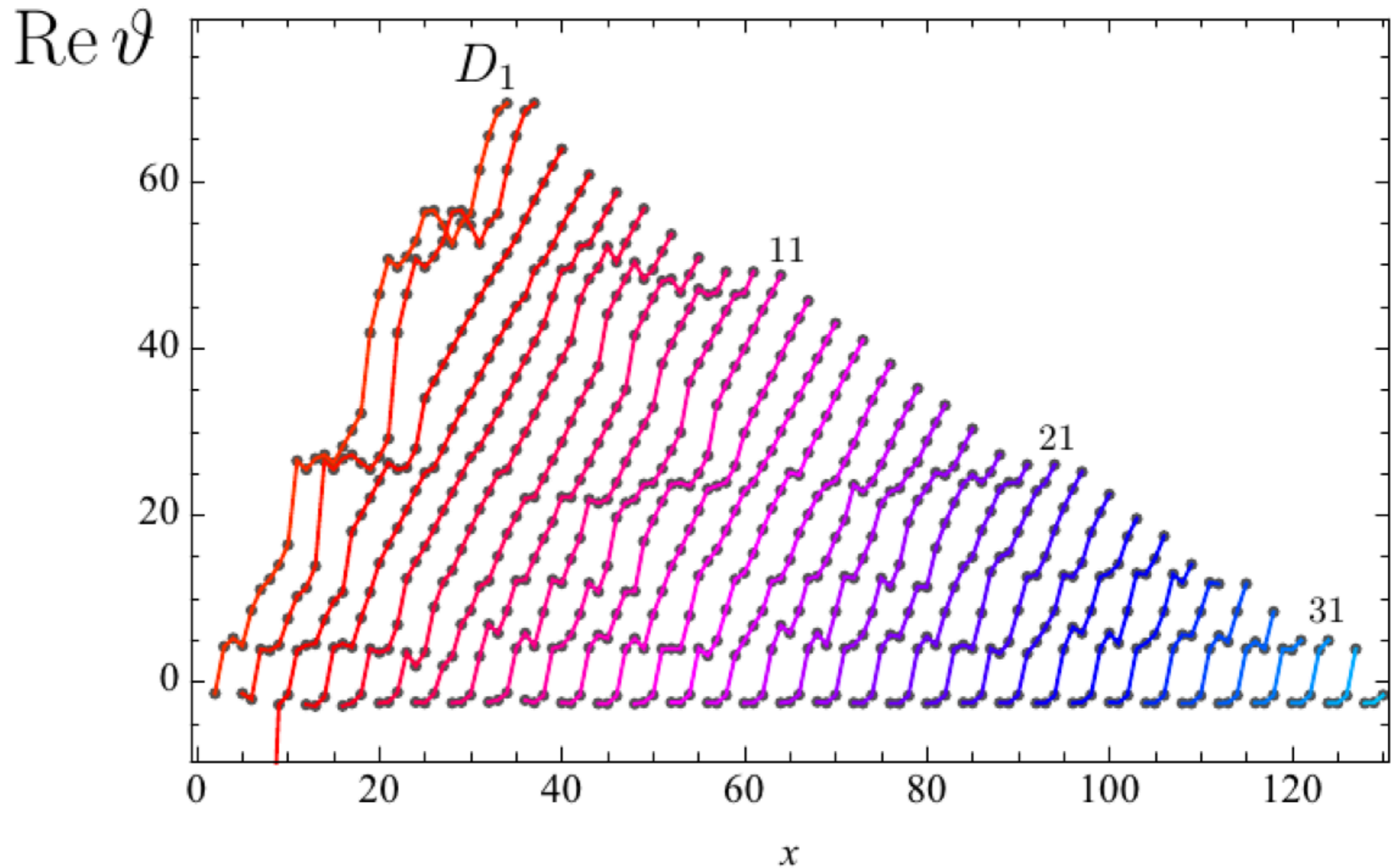


### UV eigenvalues

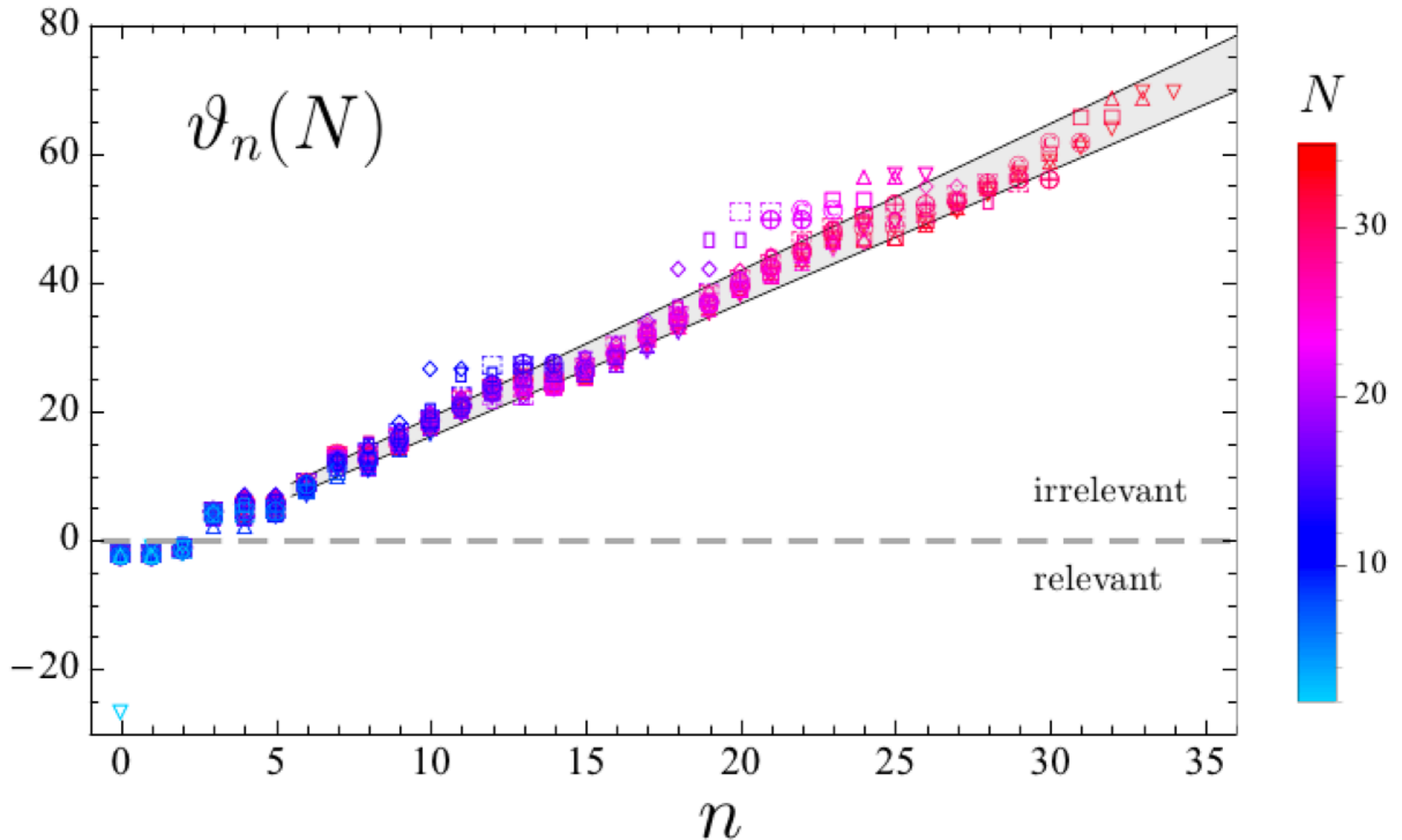
$$\vartheta_n(N)$$



# bootstrap test



# near-Gaussian



# f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g} [f_k(R_{\mu\nu} R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu} R^{\mu\nu})]$$

$$\begin{aligned} \partial_t \Gamma[\bar{g}, \bar{g}] = & \frac{1}{2} \text{Tr}_{(2T)} \left[ \frac{\partial_t \mathcal{R}_k^{h^T h^T}}{\Gamma_{h^T h^T}^{(2)}} \right] + \frac{1}{2} \text{Tr}'_{(1T)} \left[ \frac{\partial_t \mathcal{R}_k^{\xi\xi}}{\Gamma_{\xi\xi}^{(2)}} \right] + \frac{1}{2} \text{Tr}''_{(0)} \left[ \frac{\partial_t \mathcal{R}_k^{\sigma\sigma}}{\Gamma_{\sigma\sigma}^{(2)}} \right] + \frac{1}{2} \text{Tr}_{(0)} \left[ \frac{\partial_t \mathcal{R}_k^{hh}}{\Gamma_{hh}^{(2)}} \right] \\ & + \text{Tr}''_{(0)} \left[ \frac{\partial_t \mathcal{R}_k^{\sigma h}}{\Gamma_{\sigma h}^{(2)}} \right] - \text{Tr}_{(1T)} \left[ \frac{\partial_t \mathcal{R}_k^{\bar{c}^T c^T}}{\Gamma_{\bar{c}^T c^T}^{(2)}} \right] - \text{Tr}_{(0)'} \left[ \frac{\partial_t \mathcal{R}_k^{\bar{\eta}\eta}}{\Gamma_{\bar{\eta}\eta}^{(2)}} \right] - \text{Tr}''_{(0)} \left[ \frac{\partial_t \mathcal{R}_k^{\bar{\lambda}\lambda}}{\Gamma_{\bar{\lambda}\lambda}^{(2)}} \right] \\ & + \frac{1}{2} \text{Tr}''_{(0)} \left[ \frac{\partial_t \mathcal{R}_k^{\omega\omega}}{\Gamma_{\omega\omega}^{(2)}} \right] - \text{Tr}'_{(1T)} \left[ \frac{\partial_t \mathcal{R}_k^{\bar{c}^T c^T}}{\Gamma_{\bar{c}^T c^T}^{(2)}} \right] + \frac{1}{2} \text{Tr}'_{(1T)} \left[ \frac{\partial_t \mathcal{R}_k^{\zeta^T \zeta^T}}{\Gamma_{\zeta^T \zeta^T}^{(2)}} \right] + \text{Tr}'_{(0)} \left[ \frac{\partial_t \mathcal{R}_k^{\bar{s}s}}{\Gamma_{\bar{s}s}^{(2)}} \right] \end{aligned}$$

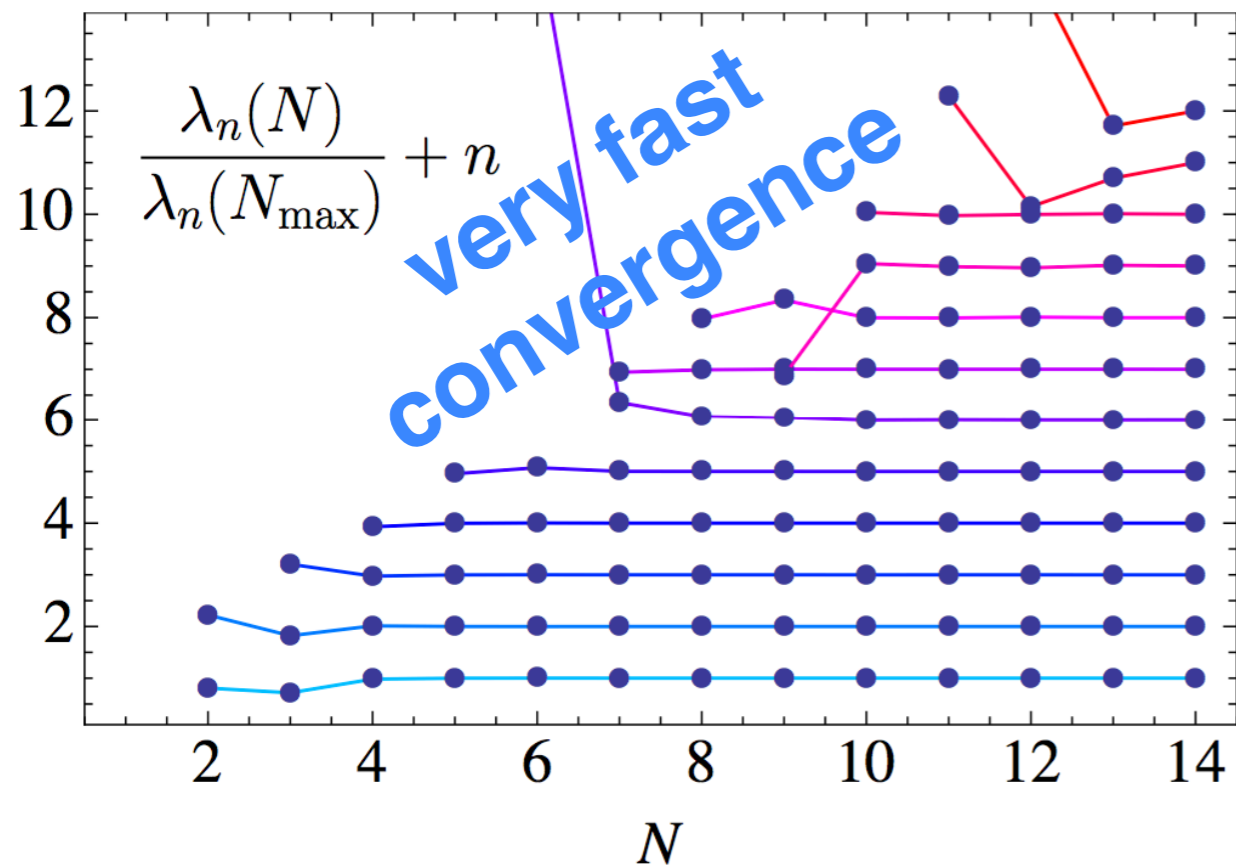
K Falls, DL, K Nikolakopoulos & C Rahmede, (to appear)

# f(Ricci)

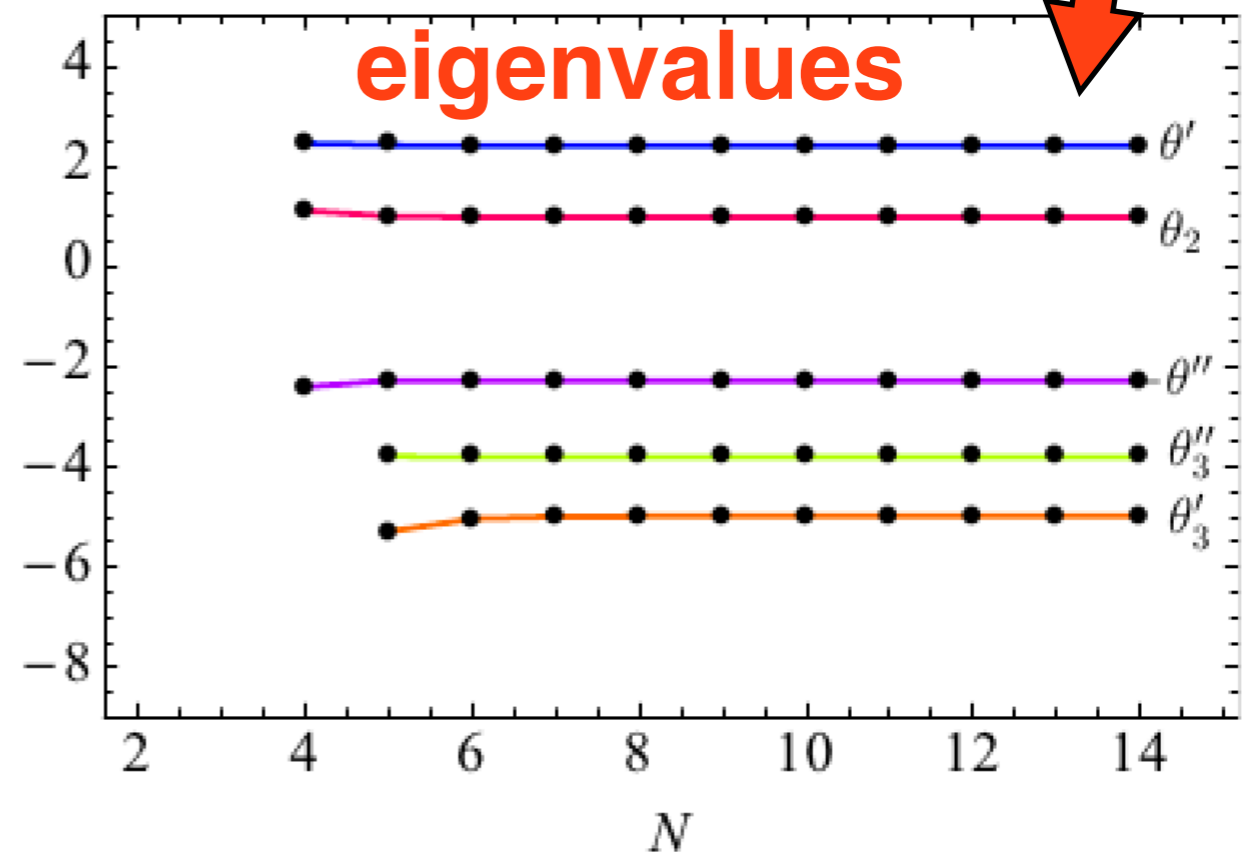
K Falls, DL, K Nikolakopoulos & C Rahmede, (to appear)

$$\Gamma_k \propto \int d^d x \sqrt{g} [f_k(R_{\mu\nu} R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu} R^{\mu\nu})]$$

fixed point



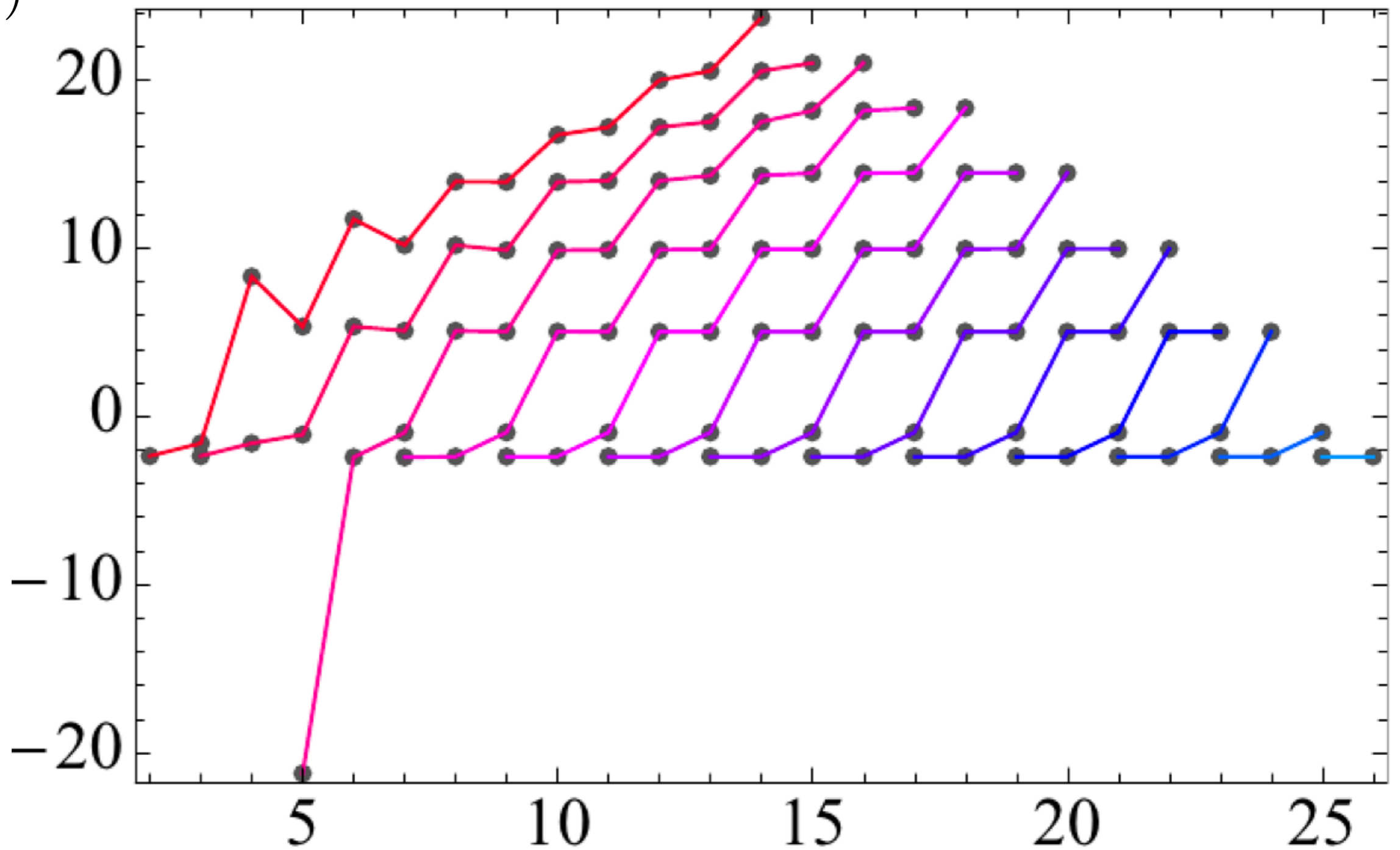
three relevant  
eigenvalues



# bootstrap test

K Falls, DL, K Nikolakopoulos, C Rahmede (to appear, 2014)

$$\vartheta_n(N)$$





# UV fixed points and black hole thermodynamics

with K Falls, K Nikolakopoulos & A Raghuraman

1002.0260 (IJMPA)

1212.1821 (PRD)

1308.5630 (JHEP)

# black hole thermodynamics

**entropy = horizon area**  
**temperature = surface gravity**

Bekenstein '73  
Bardeen, Carter, Hawking '73  
Gibbons, Hawking '77  
...  
Jacobson '95  
...

# black hole thermodynamics

**input:**  
**saddle point of effective action**

Gibbons, Hawking '77

$$S \approx \int d^4x \sqrt{-\det g_{\mu\nu}} \left[ \frac{1}{8\pi G_0} R + \frac{1}{4\alpha_0} F^{\mu\nu} F_{\mu\nu} \right] + S_m$$

central building block

IR limit

$$G_0 \approx 6.674 \times 10^{-11} \text{ N (m/kg)}^2$$

$$\alpha_0 \approx 1/137$$

# renormalisation group

**new input:**  
**scale-dependent effective action**

Falls & DL, 1212.8121 (PRD)

$$\Gamma_k \approx \int d^4x \sqrt{-\det g_{\mu\nu}} \left[ \frac{1}{8\pi G_k} R + \frac{1}{4\alpha_k} F^{\mu\nu} F_{\mu\nu} \right] + S_m$$

running couplings



**family of Kerr-Newman BH solutions**

$$A = A(M, J, q; k)$$

$$S = \frac{A}{4G_k}$$

**choice of RG scale**

determined by physical parameters of the BH:  $k = k_{\text{opt}}(M, J, q)$

# renormalisation group

Bekenstein's thought experiment

infinitesimal amount of matter crossing the horizon, with heat flow

$$\frac{\delta Q}{T} = \frac{\delta A}{4G\kappa}$$

BH settles in a new state

$$M \rightarrow M + \delta M \quad J \rightarrow J + \delta J$$

$$q \rightarrow q + \delta q \quad k_{\text{opt}} \rightarrow k_{\text{opt}} + \delta k_{\text{opt}}$$

total change of horizon area

$$\left(1 - \frac{2\pi}{\kappa} T\right) \delta A = \frac{\partial A(M, J, q; k)}{\partial \ln k} \Bigg|_{k=k_{\text{opt}}} \frac{\delta k_{\text{opt}}}{k_{\text{opt}}}$$

# renormalisation group

## results

RG scale

$$k_{\text{opt}}^2(M, J, q) \equiv k_{\text{opt}}^2(A) = \frac{4\pi\xi^2}{A}$$

state function

$$M^2 \equiv \frac{4\pi}{A} \left[ \left( \frac{A + 4\pi G(A)e^2(A)q^2}{8\pi G(A)} \right)^2 + J^2 \right]$$

temperature

$$T = 4G(A) \frac{\partial M}{\partial A}$$

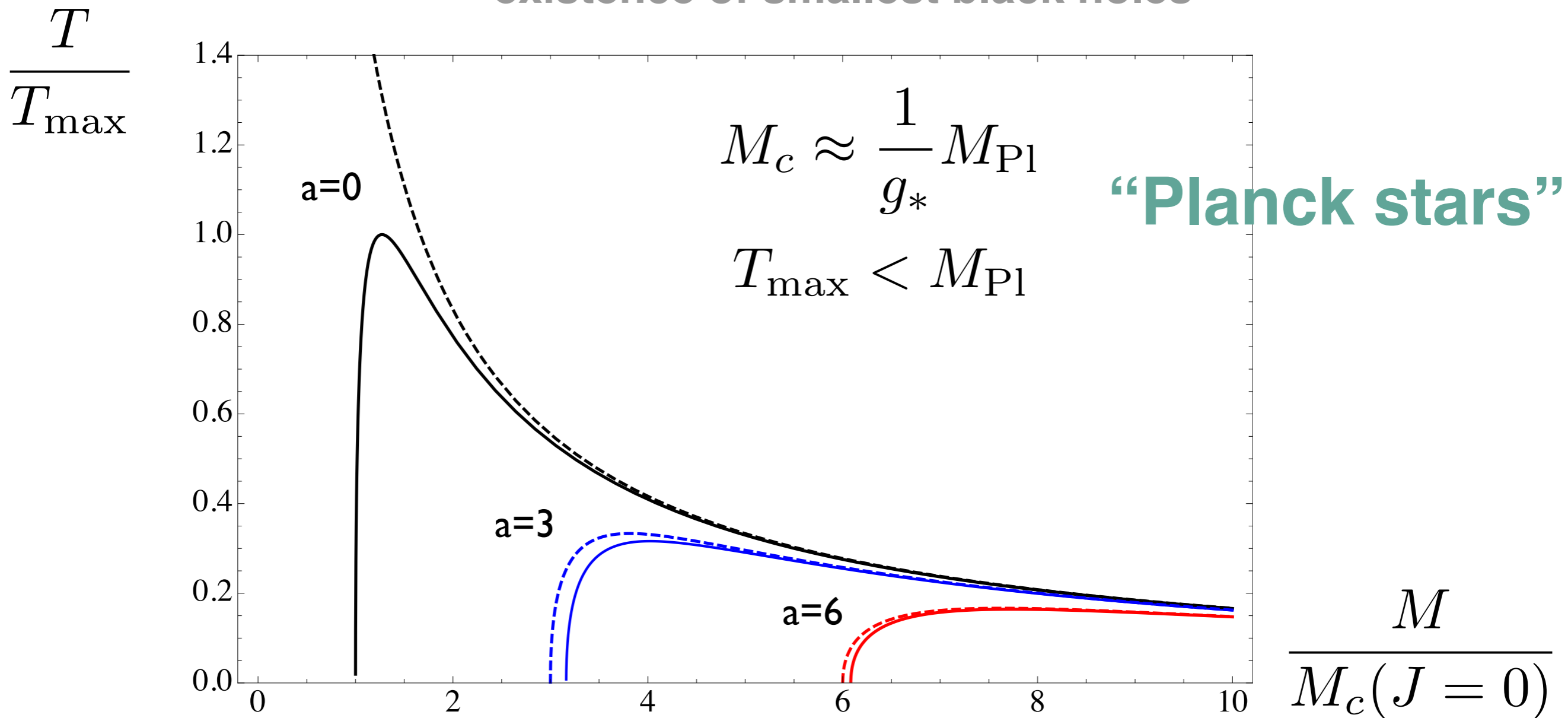
entropy

$$S = \frac{A}{4G_k} \quad \text{with} \quad k = k_{\text{opt}}$$

# asymptotic safety

## prediction I: temperature & mass

finite maximum temperature  
existence of smallest black holes



Falls & DL, 1212.8121 (PRD)

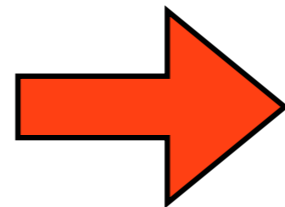
# conformal scaling

Aharony, Banks '98, Shomer '07

conformal scaling in QFT

$$S \sim (RT)^{d-1}, \quad E \sim R^{d-1} T^d$$

$$S \sim E^\nu$$

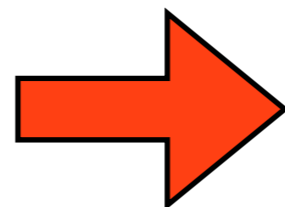


$$\nu_{\text{CFT}} = \frac{d-1}{d}$$

Schwarzschild BH scaling

$$S \sim R^{d-2}/G_N \quad E \sim R^{d-3}/G_N$$

$$S \sim E^\nu$$



$$\nu_{\text{BH}} = \frac{d-2}{d-3}$$



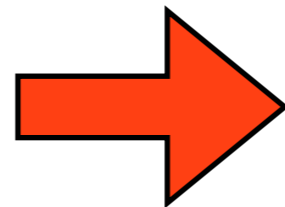
# conformal scaling

Aharony, Banks '98, Shomer '07

conformal scaling in QFT

$$S \sim (RT)^{d-1}, \quad E \sim R^{d-1} T^d$$

$$S \sim E^\nu$$

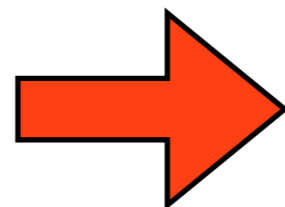


$$\nu_{\text{CFT}} = \frac{d-1}{d}$$

Schwarzschild BH scaling

$$S \sim R^{d-2}/G_N \quad E \sim R^{d-3}/G_N$$

$$S \sim E^\nu$$



$$\nu_{\text{BH}} = \frac{d-2}{d-3}$$

$$\nu_{\text{BH}} \neq \nu_{\text{CFT}}$$

except for  $d = \frac{3}{2}$

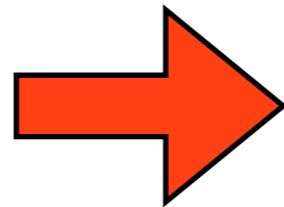
# conformal scaling

Falls, Litim '12

conformal scaling in QFT

$$S \sim (RT)^{d-1}, \quad E \sim R^{d-1}T^d$$

$$\frac{S}{R^{d-1}} \sim \left( \frac{E}{R^{d-1}} \right)^\nu$$

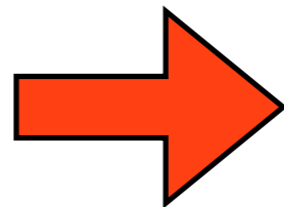


$$\nu_{\text{CFT}} = \frac{d-1}{d}$$

Schwarzschild BH scaling

$$S \sim R^{d-2}/G_N \quad E \sim R^{d-3}/G_N$$

$$\frac{S}{R^{d-1}} \sim \left( \frac{E}{R^{d-1}} \right)^\nu$$



$$\nu_{\text{BH}} = \frac{1}{2}$$

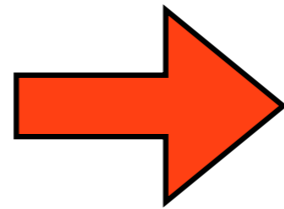
# conformal scaling

Falls, Litim '12

conformal scaling in QFT

$$S \sim (RT)^{d-1}, \quad E \sim R^{d-1}T^d$$

$$\frac{S}{R^{d-1}} \sim \left( \frac{E}{R^{d-1}} \right)^\nu$$

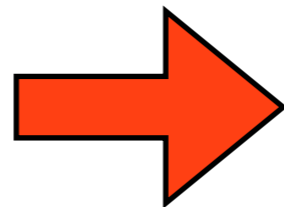


$$\nu_{\text{CFT}} = \frac{d-1}{d}$$

Schwarzschild BH scaling

$$S \sim R^{d-2}/G_N \quad E \sim R^{d-3}/G_N$$

$$\frac{S}{R^{d-1}} \sim \left( \frac{E}{R^{d-1}} \right)^\nu$$



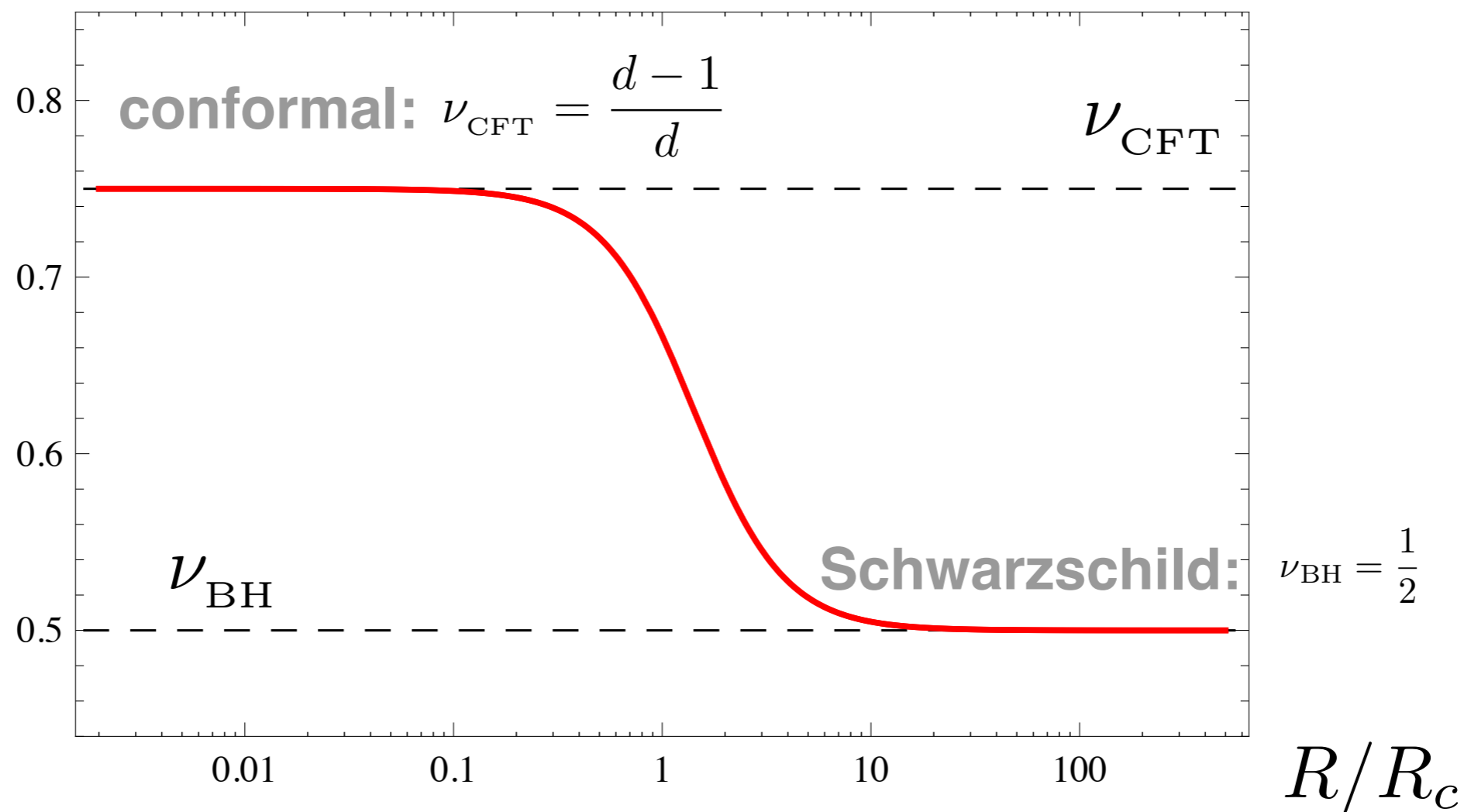
$$\nu_{\text{BH}} = \frac{1}{2}$$

$$\nu_{\text{BH}} \neq \nu_{\text{CFT}}$$

except for  $d = 2$

# asymptotic safety

**prediction II: conformal scaling from AS black holes**



# asymptotic safety

## prediction III: entropy

thermodynamical  
entropy

$$S = \frac{A}{4G(A)} = \frac{A}{4G_N} + \frac{\pi}{g_*}$$

Clausius' entropy

$$S = \int \frac{dA}{4G(A)} = \frac{A}{4G_N} + \frac{\pi}{g_*} \left( 1 + \ln \frac{A}{A_c} \right)$$

statistical entropy

$$F = M - S T$$

**valid for all RG scales**

[using “off-shell” conical singularity method] (Soludkhin '96)

Falls & DL, 1212.8121 (PRD)

# conclusions

- **UV fixed points offer maximal predictivity for QFTs including gravity, no artificial UV cutoff**

# conclusions

- **UV fixed points offer maximal predictivity for QFTs including gravity, no artificial UV cutoff**
- **4D gauge-Yukawa: exact interacting UV fixed point, full perturbative control**

# conclusions

- **UV fixed points offer maximal predictivity for QFTs including gravity, no artificial UV cutoff**
- **4D gauge-Yukawa: exact interacting UV fixed point, full perturbative control**
- **4D gravity: self-consistent fixed point bootstrap test available**



# conclusions

- **UV fixed points offer maximal predictivity for QFTs including gravity, no artificial UV cutoff**
  - **4D gauge-Yukawa: exact interacting UV fixed point, full perturbative control**
    - **4D gravity: self-consistent fixed point bootstrap test available**
      - **black hole thermodynamics: conformal scaling, entropy**